

Lecture Notes

on

Counting & Radix Sort



July 2020 (Be safe and stay at home)





Assumption: Keys (attached to items) are Ints in range $1, \ldots, m$.



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- 2. The counting information stored in C can be used to determine the position of each element in the sorted array. Suppose we modify the values of the C[j] so that now
 - C[j] = the number of keys less than or equal to j.

Then we know that the elements with key "j" must be stored at the indices $C[j-1]+1,\ldots,C[j]$ of the final sorted array.



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 - C[j] = the number of keys less than or equal to j.
 - Then we know that the elements with key "j" must be stored at the indices $C[j-1]+1,\ldots,C[j]$ of the final sorted array.
- 3. We use a "trick" to move the elements to the right position of an auxiliary array. Then we copy the sorted auxiliary array back to the original one.



Implementation of Counting Sort

```
Algorithm Counting Sort(A, m)
 1. n \leftarrow A.length
 2. Initialise array C[1 \dots m]
 3 for i \leftarrow 1 to n do
 4. j \leftarrow A[i].key
 5. C[i] \leftarrow C[i] + 1
 6. for j \leftarrow 2 to m do
          C[j] \leftarrow C[j] + C[j-1] \quad \triangleright C[j] \text{ stores } \sharp \text{ of keys} < j
     Initialise array B[1 \dots n]
 9 for i \leftarrow n downto 1 do
10
     i \leftarrow A[i].key \triangleright A[i] highest w. key i
11. B[C[i]] \leftarrow A[i] \triangleright Insert A[i] into highest free index for j
12. C[i] \leftarrow C[i] - 1
13. for i \leftarrow 1 to n do
              A[i] \leftarrow B[i]
```



Analysis of Counting Sort

- ▶ The loops in lines 3–5, 9–12, and 13–14 all require time $\Theta(n)$.
- ▶ The loop in lines 6–7 requires time $\Theta(m)$.
- ▶ Thus the overall running time is

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Note: Counting-Sort is STABLE.

► (After sorting, 2 items with the same key have their *initial relative* order).



Radix Sort

Basic Assumption

Keys are sequences of digits in a fixed range 0, ..., R-1, all of equal length d.

Examples of such keys

- 4 digit hexadecimal numbers (corresponding to 16 bit integers) R=16, d=4
- ▶ 5 digit decimal numbers (for example, US post codes) R = 10, d = 5
- Fixed length ASCII character sequences
 R = 128
- ► Fixed length byte sequences R = 256



Stable Sorting Algorithms

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Examples

- COUNTING-SORT, MERGE-SORT, and INSERTION SORT are all stable.
- ▶ QUICKSORT is not stable.
- ▶ If keys and elements are exactly the same thing (in our setting, an element is a structure containing the key as a sub-element) then we have a much easier (non-stable) version of COUNTING-SORT. (How? ... CLASS?).

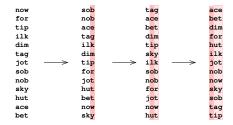


Radix Sort (cont'd)

Idea

Sort the keys digit by digit, *starting with the least significant digit*.

Example



Each of the three sorts is carried out with respect to the digits in that column. "Stability" (and having previously sorted digits/suffixes to the right), means this achieves a sorting of the suffixes starting at the current column.

ADS: lect 9 - slide 9 - 8th February, 2016



Radix Sort (cont'd)

Algorithm RADIX-SORT(A, d)

- 1. for $i \leftarrow 0$ to d do
- 2. use stable sort to sort array A using digit i as key

Most commonly, Counting Sort is used in line 2 - this means that once a set of digits is already in sorted order, then (by stability) performing Counting Sort on the *next-most significant* digits preserves that order, within the "blocks" constructed by the new iteration.

Then each execution of line 2 requires time $\Theta(n+R)$. Thus the overall time required by RADIX-SORT is

$$\Theta(d(n+R))$$



Sorting Integers with Radix-Sort

Theorem 2

An array of length n whose keys are b-bit numbers can be sorted in time

$$\Theta(n\lceil b/\lg n\rceil)$$

using a suitable version of RADIX-SORT.

Proof: Let the digits be blocks of $\lceil \lg n \rceil$ bits. Then $R = 2^{\lceil \lg n \rceil} = \Theta(n)$ and $d = \lceil b / \lceil \lg n \rceil \rceil$. Using the implementation of RADIX-SORT based on COUNTING SORT the integers can be sorted in time

$$\Theta(d(n+R)) = \Theta(n\lceil b/\lg n\rceil).$$

Note: If all numbers are at most n^k , then $b = k \lg n \ldots \Rightarrow \text{Radix Sort}$ is $\Theta(n)$ (assuming k is some constant, eg 3, 10).