AMORTIZED ANALYSIS

- Dynamic tables
- Aggregate method
- Accounting method
- Potential method

How large should a hash table be?

Goal: Make the table as small as possible, but large enough so that it won't overflow (or otherwise become inefficient).

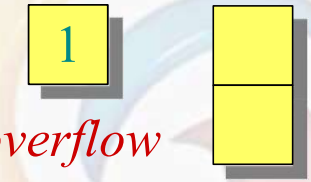
Problem: What if we don't know the proper size in advance?

Solution: ***Dynamic tables.***

IDEA: Whenever the table overflows, "grow" it by allocating (via **`malloc`** or **`new`**) a new, larger table. Move all items from the old table into the new one, and free the storage for the old table.

Example of a dynamic table

1. INSERT
2. INSERT



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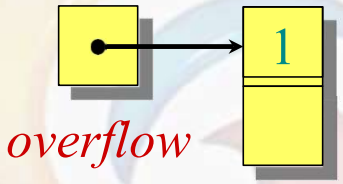

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Example of a dynamic table

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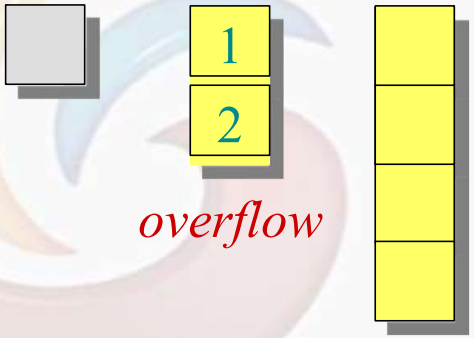

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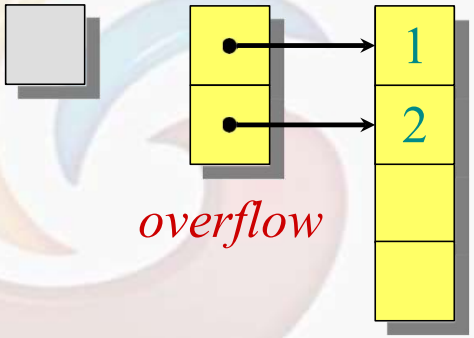

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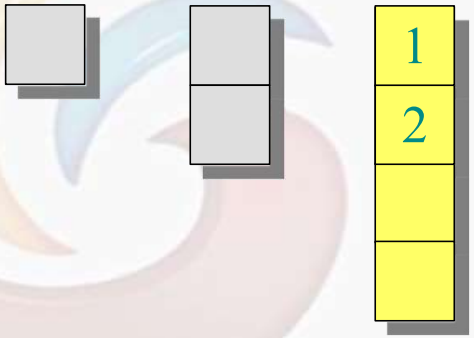

Example of a dynamic table

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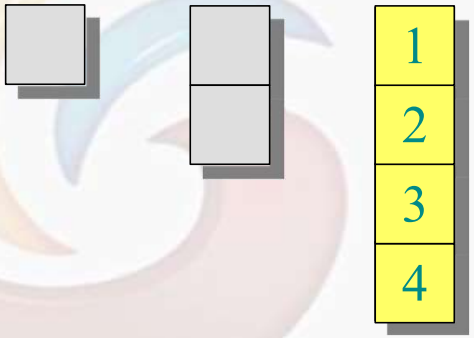
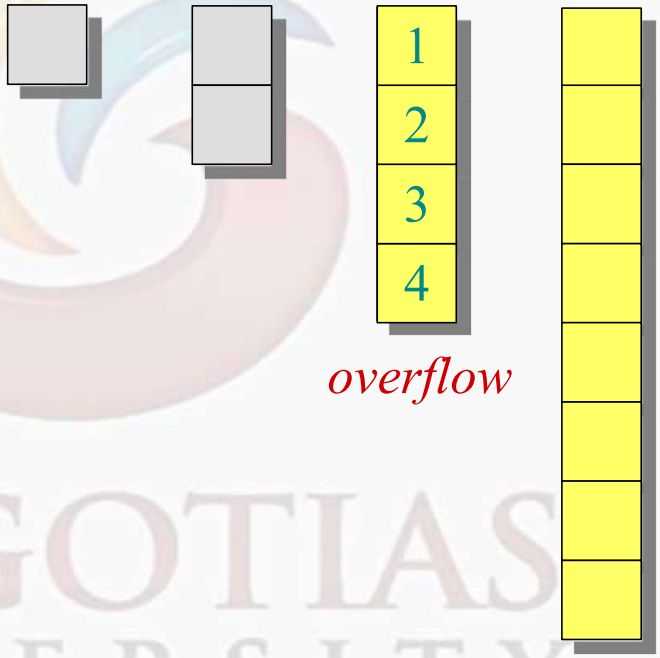
4. INSERT



Example of a dynamic table

1. INSERT
2. INSERT
3. INSERT
4. INSERT
5. INSERT



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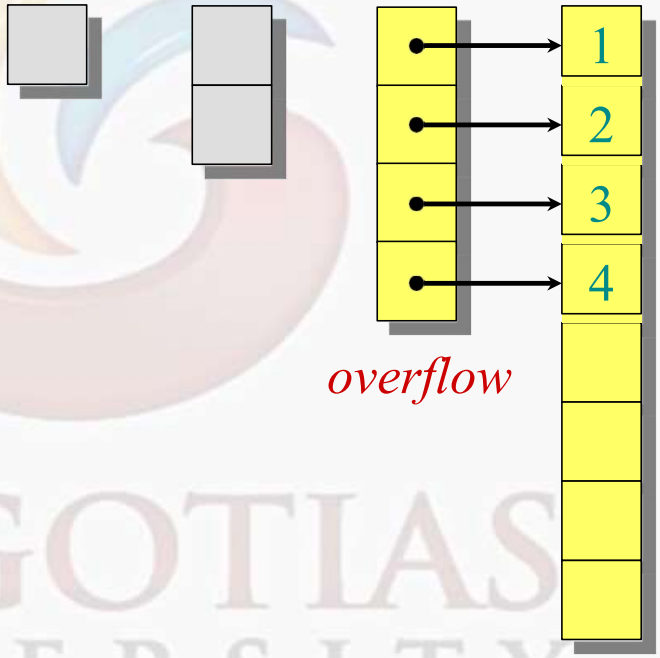

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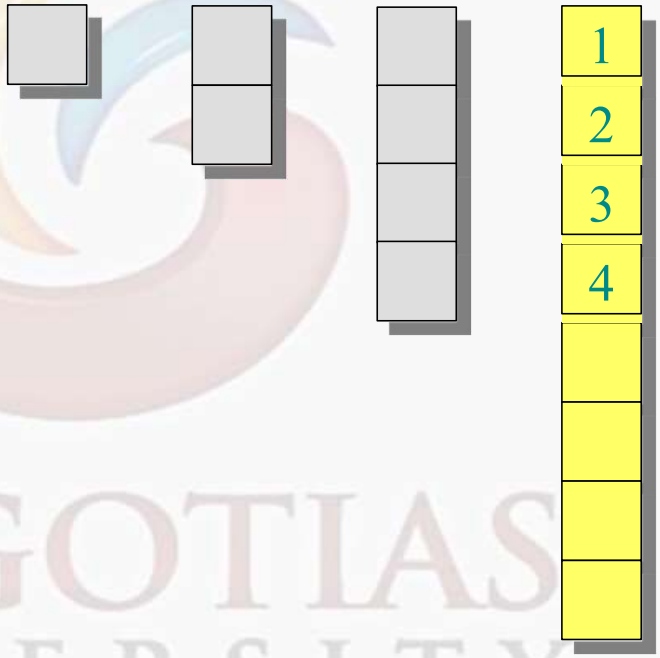
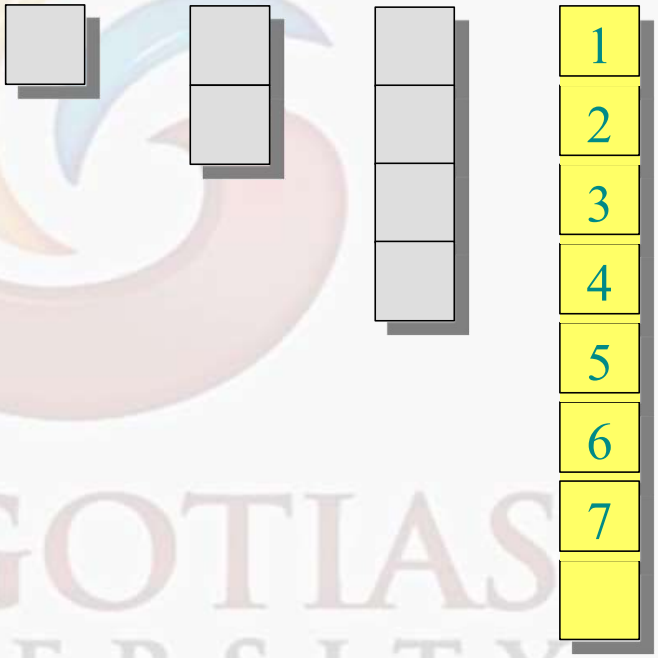


Example of a dynamic table

1. INSERT
2. INSERT
3. INSERT
4. INSERT
5. INSERT
6. INSERT
7. INSERT



Worst-case analysis

Consider a sequence of n insertions. The worst-case time to execute one insertion is $\Theta(n)$. Therefore, the worst-case time for n insertions is $n \cdot \Theta(n) = \Theta(n^2)$.

WRONG! In fact, the worst-case cost for n insertions is only $\Theta(n) \ll \Theta(n^2)$.

Let's see why.

Tighter analysis

Let $c_i =$ the cost of the i th insertion

$$= \begin{cases} i & \text{if } i-1 \text{ is an exact power of } 2, \\ 1 & \text{otherwise.} \end{cases}$$

i	1	2	3	4	5	6	7	8	9	10
$size_i$	1	2	4	4	8	8	8	8	16	16
c_i	1	2	3	1	5	1	1	1	9	1

Tighter analysis

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$size_i$	1	2	4	4	8	8	8	8	16	16
	1	1	1	1	1	1	1	1	1	1
c_i		1	2		4				8	

Tighter analysis (continued)

$$\text{Cost of } n \text{ insertions} = \sum_{i=1}^{n} c_i$$

$$\leq n + \sum_{j=0}^{\lfloor \lg(n-1) \rfloor} 2^j$$

$$\leq 3n$$

$$= \Theta(n).$$

Thus, the average cost of each dynamic-table operation is $\Theta(n)/n = \Theta(1)$.


Thank You