Simplex Method

In practice, most problems contain more than two variables and are consequently too large to be tackled by conventional means. Therefore, an algebraic technique is used to solve large problems using Simplex Method. This method is carried out through iterative process systematically step by step, and finally the maximum or minimum values of the objective function are attained.

The simplex method solves the linear programming problem in iterations to improve the value of the objective function. The simplex approach not only yields the optimal solution but also other valuable information to perform economic and 'what if' analysis.

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Three types of additional variables are used in simplex method such as,

- Slack variables (S_1, S_2, S_3.....S_n): Slack variables refer to the amount of unused resources like raw materials, labour and money.
- Surplus variables ($-S_1$, $-S_2$, $-S_3$.....$-S_n$): Surplus variable is the amount of resources by which the left hand side of the equation exceeds the minimum limit.
- Artificial Variables (a_1, a_2, a_3.. ...a_n): Artificial variables are temporary slack variables which are used for purposes of calculation, and are removed later.

The above variables are used to convert the inequalities into equality equations, as given in the Given Table below.

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Table 5.1: Types of Additional Variables

	Constraint Type		Variable added	Format
a)	Less than or equal to	$\leq$	Add Slack Variable	+S
b)	Greater than or equal to	$\geq$	Subtract surplus variable and add artificial variable	-S+a
c)	Equal to	=	Add artificial variable	+a

Procedure of simplex Method

Step 1: *Formulate the LP problem.*

Step 2: *Introduce slack /auxiliary variables.*

if constraint type is $\leq$ *introduce* $+S$

if constraint type is $\geq$*introduce* $-S+a$ *and*

if constraint type is $=$ *introduce* a

Step 3: *Find the initial basic solution.*

Explanation of procedure by an Example

Previous the packaging product mix problem is solved using simplex method.

Maximize $Z = 6x_1 + 4x_2$

Subject to constraints,

$2x_1+3x_2\leq 120$ (Cutting machine) ....(i)

$2x_1+ x_2\leq 60$ (Pinning machine) ....(ii)

where $x_1, x_2 \geq 0$

Considering the constraint for cutting machine,

$2x_1+ 3x_2 \leq 120$

To convert this inequality constraint into an equation, introduce a slack variable, S3 which represents the unused resources. Introducing the slack variable, we have the equation $2x_1+ 3x_2 + S_3 =120$

Similarly for pinning machine, the equation is $2x_1+ x_2 + S_4 = 60$

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i.e., x1 = 0 and x2 = 0, then

S3 = 120

S4 = 60

This is the basic solution of the system, and variables S_3 and S_4 are known as Basic Variables, S_B while x_1 and x_2 known as Non-Basic Variables. If all the variables are non

Rewriting the constraints with slack variables gives us,

$$Z_{max} = 6x_1 + 4x_2 + 0S_3 + 0S_4$$

Subject to constraints,

$2x_1 + 3x_2 + S_3 = 120$ ………………….(i)

$2x_1 + x_2 + S_4 = 60$ …………………(ii)

where $x_1, x_2 \geq 0$

Table 5.2: Co-efficients of Variables

Iteration Number	Basic Variables	Solution Value	X_1 K_C	X_2	S_3	S_4	Minimum Ratio	Equation
0	S_3	120	2	3	1	0	60	
	S_4	60	2	1	0	1	30	
	$-Z_j$	0	– 6	– 4	0	0		

Iteration Number	Basic Variables	Solution Value	X_1 K_C	X_2	S_3	S_4	Minimum Ratio	Equation
0	S_3	120	2	3	1	0	60	
K_r	S_4	60	2	1	0	1	30	
	$-Z_j$	0	-6	-4	0	0		

In the next iteration, enter the basic variables by eliminating the leaving variable (i.e., key row) and introducing the entering variable (i.e., key column).

Make the pivotal element as 1 and enter the values of other elements in that row accordingly. In this case,

convert the pivotal element value 2 as 1 in the next iteration table.

For this, divide the pivotal element by 2. Similarly divide the other elements in that row by 2. The equation is $S_4/2$.The other co-efficients of the key column in iteration Table 5.4 must be made as zero in the iteration Table 5.5.

Table 5.5: Iteration Table

Iteration Number	Basic Variables	Solution Value	X_1	X_2	S_3	S_4	Minimum Ratio	Equation
0	S_3	120	2	3	1	0	60	
	S_4	60	2	1	0	1	30	
K_r	$-Z_j$	0	-6	-4	0	0		
1	S_3	60	0	2	1	-1	30	$S_3 - 2p_e$
K_r	x_1	30	1	½	0	½	60	$S_{4/2}$
P_e	$-Z_j$	100	0	-1	0	3		$-Z + 6P_e$
2 P_e	X_2	30	0	1	½	–		$S_{3/2}$
	x_1	15	1	0	–	1/2		$S_3 - P_{e/2}$
	$-Z_j$	210	0	0	1/4	¾		$-Z + P_e$
					½	5/2		

The solution is,

x_1 = 15 corrugated boxes are to be produced and

x_2 = 30 carton boxes are to be produced to yield a

Profit, Z_{max} = Rs. 210.00

Thanks