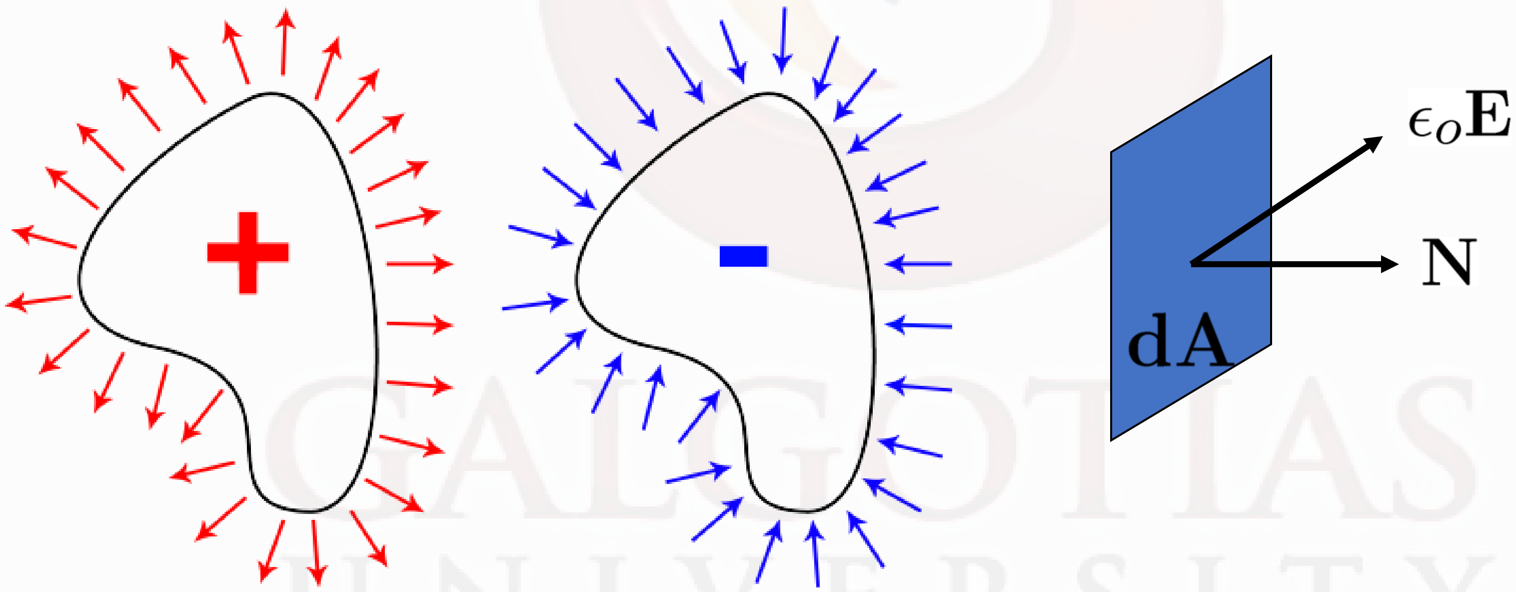
Electricity and Magnetism

Topic Covered: Maxwell's Equations (In Free Space)
Gauss' Law & Faraday's Law
Applications of Gauss' Law
Electrostatic Boundary Conditions

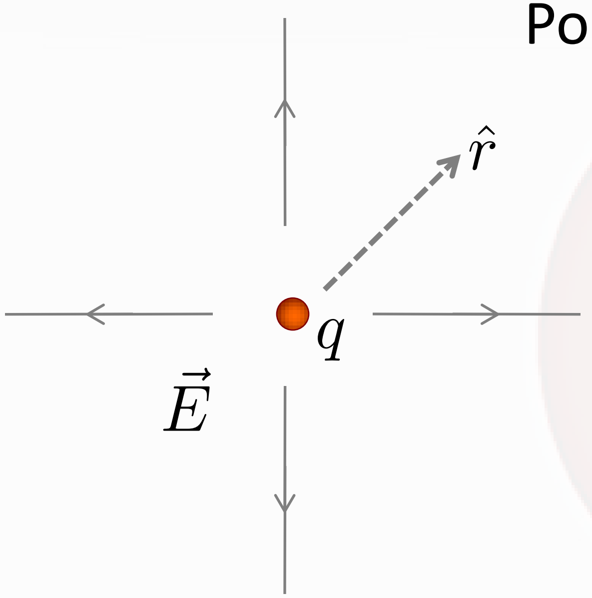
Gauss' Law

Flux of $\epsilon_o \mathbf{E}$ through closed surface ***S*** = net charge inside V

$$\int_S \epsilon_o \mathbf{E} \cdot \mathbf{dA} = \int_{\mathbf{V}} \rho \mathbf{dV} = Q_{enclosed}$$

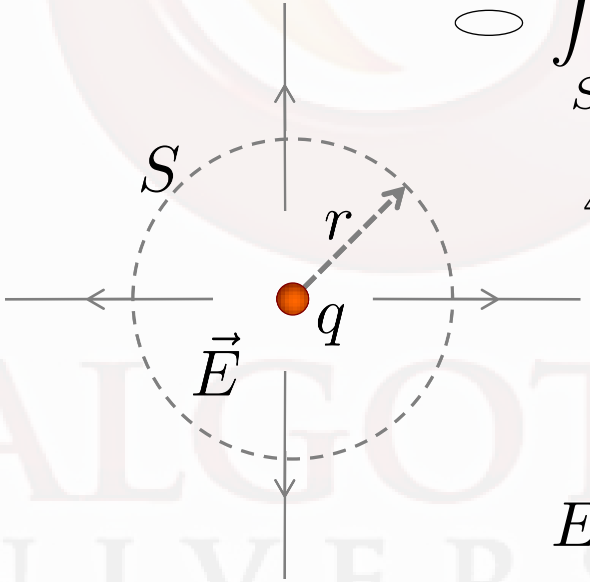


Point Charge Example



Apply Gauss' Law in integral form making use of symmetry to find $\vec{E}$

$$\oint\!\!\oint_S \epsilon_o \vec{E} \cdot dS = q$$



$$4\pi\epsilon_o E_r r^2 = q$$

- Assume that the image charge is uniformly distributed at $\mathrm{r} = \infty$. Why is this important ?
- Symmetry

$$\Rightarrow \vec{E} = E_r(r)\,\hat{r}$$

$$E_r = \frac{q}{4\pi\epsilon_o r^2}$$

Gauss' Law Tells Us ...

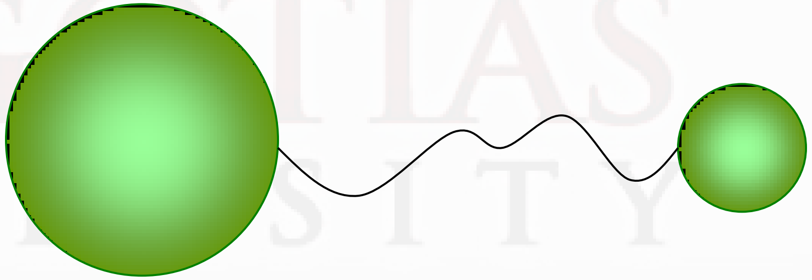
... the electric charge can reside only on the surface of the conductor.

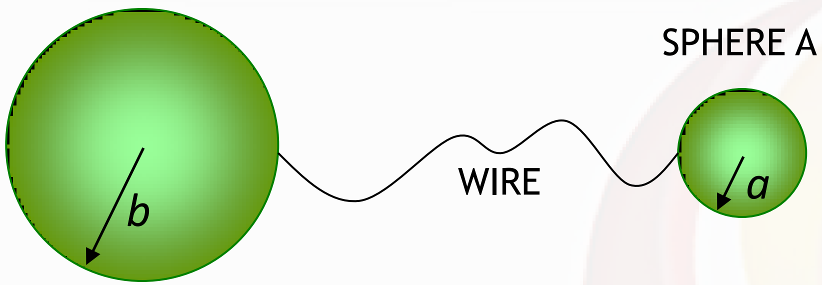
[If charge was present inside a conductor, we can draw a Gaussian surface around that charge and the electric field in vicinity of that charge would be non-zero ! A non-zero field implies current flow through the conductor, which will transport the charge to the surface.]

... there is no charge at all on the inner surface of a hollow conductor.

... that, if a charge carrying body has a sharp point, then the electric field at that point is much stronger than the electric field over the smoother part of the body.

Lets show this by considering two spheres of different size, connected by a long, thin wire ...





Because the two spheres are far apart, we can assume that charges are uniformly distributed across the surfaces of the two spheres, with charge q_a *on the surface of sphere A and* q_b *on the surface of sphere B*

$$q_a + q_b = q$$

$$V_b = \frac{q_b}{4\pi\epsilon b} \qquad V_a = \frac{q_a}{4\pi\epsilon a}$$

$$since \quad V_b = V_a \quad then$$

$$q_b = q\frac{b}{a+b} \qquad q_a = q\frac{a}{a+b}$$

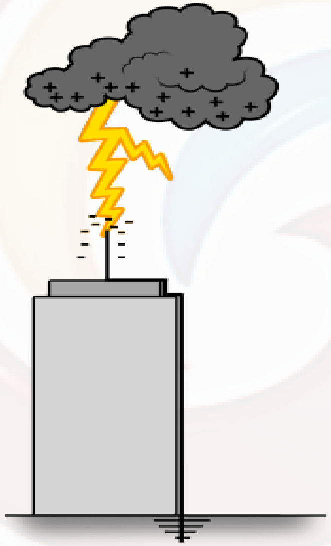
… and the E-field on the surface of the spheres is:

$$E_b = \frac{q_b}{4\pi\epsilon b^2} = \frac{q}{4\pi\epsilon(a+b)b} \qquad E_a = \frac{q_a}{4\pi\epsilon a^2} = \frac{q}{4\pi\epsilon(a+b)a}$$

Note that $E_a >> E_b$ if $b >> a$

Lightning Rod

When a conductive body contains sharp points, the electric field on these points is much stronger than that on the smooth part of the conducting body.



Lighting Rods are connected to the ground. When a cloud carrying electric charges approaches, the rod attracts opposite charges from the ground. The Electric field at the tip of the rod is much stronger than anywhere else. When the E-field exceeds the air breakdown strength (of 33 kV/cm), charges start to travel to ground.

Faraday's Law

Dynamic form:

$$\nabla \times \vec{E} = \frac{\partial}{\partial t}\mu_o \vec{H}$$

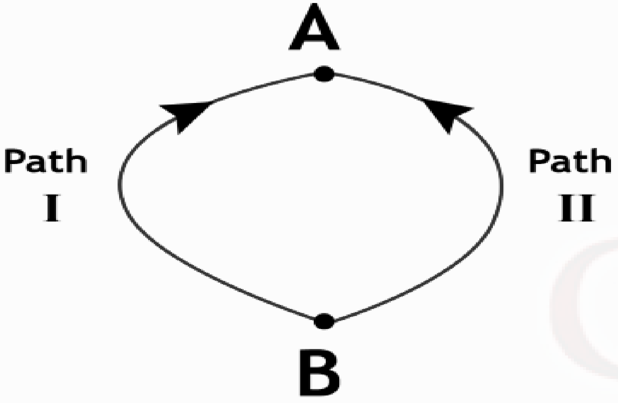
Static form:

$$\nabla \times \vec{E} = 0 \quad \Rightarrow \vec{E} = -\nabla\Phi$$

an
d

$$\Phi_a - \Phi_b = \int_a^b \vec{E} \cdot d\vec{C}$$

$$\oint \vec{E} \cdot d\vec{C} = 0 \quad (KVL)$$



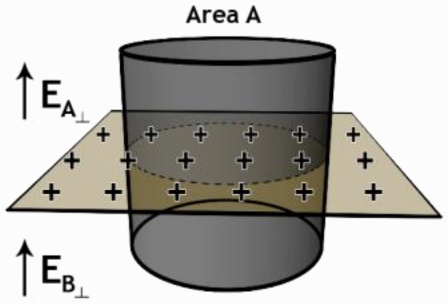
$$\Rightarrow \int_b^a \vec{E} \cdot d\vec{C} + \int_a^b \vec{E} \cdot d\vec{C} = 0$$

Path I Path II

$$\Rightarrow \int_b^a \vec{E} \cdot d\vec{C} = \int_a^b \vec{E} \cdot d\vec{C}$$

Path I Path II

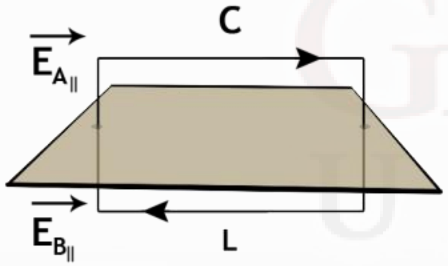
Boundary Conditions



$$\lim_{\delta \to 0} Gauss \Rightarrow (\epsilon_0 E_{A\perp} - \epsilon_0 E_{B\perp})A = \rho_s A$$

$$\hat{n} \cdot (\epsilon_0 E_A - \epsilon_0 E_B) = \rho_s$$

Normal $\vec{E}$ is discontinuous at a surface charge.

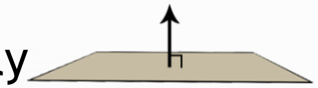


$$\lim_{\delta \to 0} Faraday \Rightarrow (E_{A\parallel} - E_{B\parallel})L = 0$$

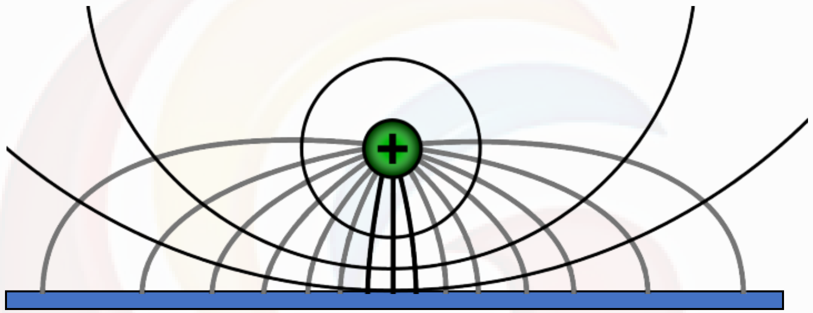
$$\hat{n} \times (E_A - E_B) = 0$$

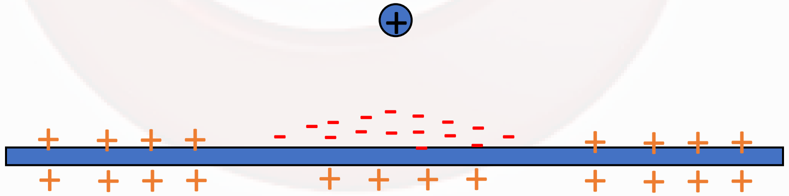
$\vec{E}$ Tangential is continuous at a surface.

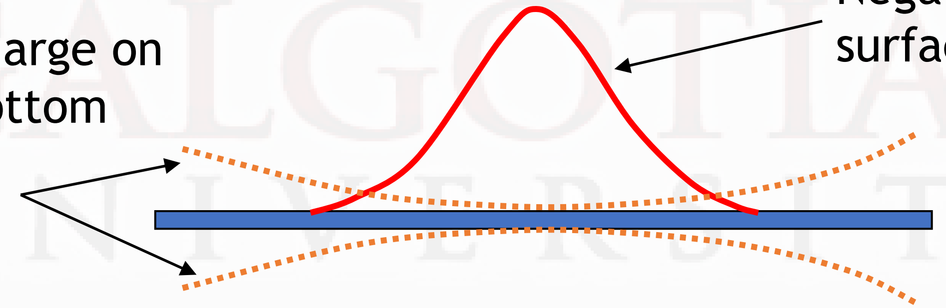
A static field terminates perpendicularly on a conductor



Point Charges Near Perfect Conductors



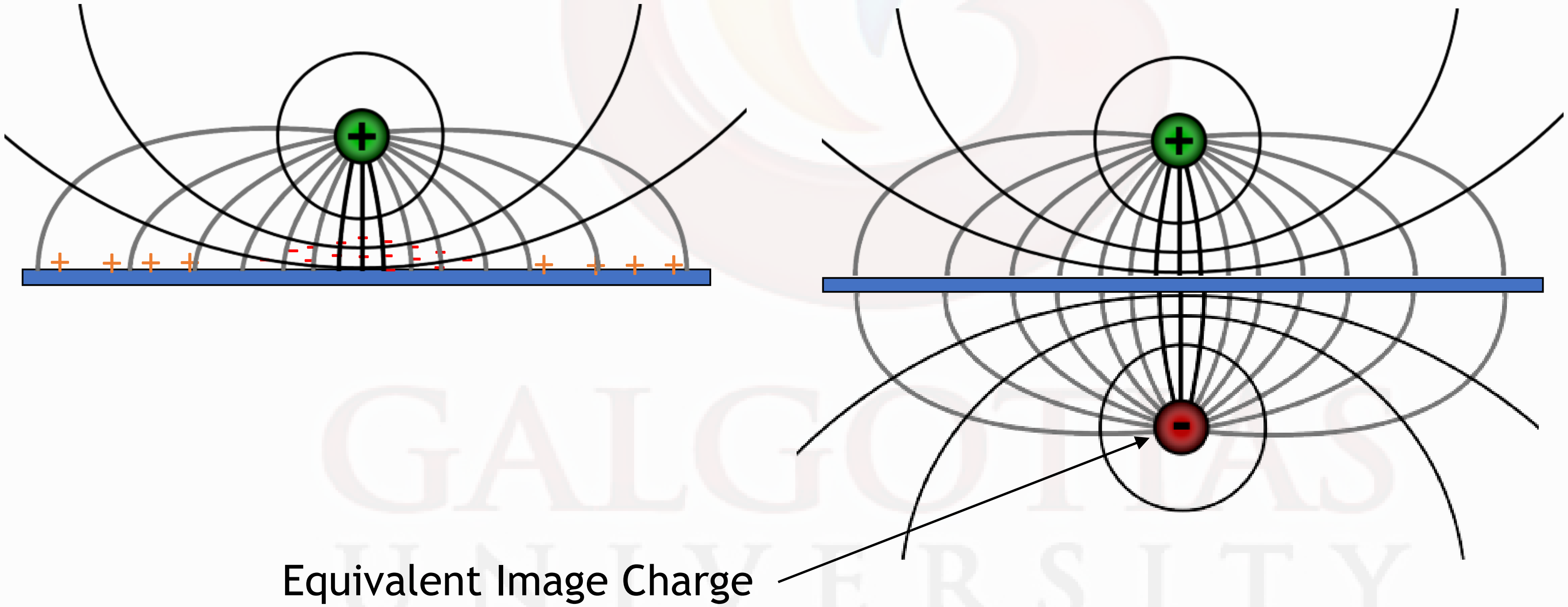




Positive charge on top and bottom surface of conductor

Negative charge on top surface of conductor

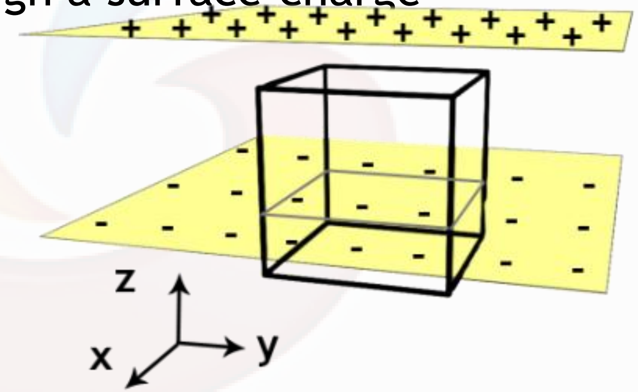
Uniqueness and Equivalent Image Charges



Electrostatic Boundary Conditions

$$\overline{n} \cdot \left(\epsilon_0 \overline{E}^a - \epsilon_0 \overline{E}^b\right) = \sigma_s$$

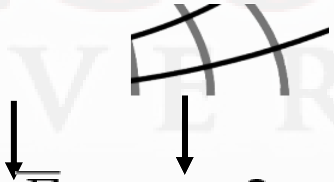
There is a jump in the normal electric field as one passes through a surface charge



$$\overline{n} \times \left(\overline{E}^a - \overline{E}^b\right) = 0$$

$$\epsilon_0 \overline{E}_{||} = 0$$

Tangential field is continuous



$$\overline{E}_{cond} = 0$$

References:

•Griffiths, D. J. (1999). *Introduction to electrodynamics*. Upper Saddle River, N.J: Prentice Hall.
•*Concepts of Physics Part-2*, Bharati Bhawan Publishers & Distributors, 1992, ISBN
•Electricity and Magnetism, Edward M. Purcell, 1986 McGraw-Hill Education