BTME 3060
Computer Aided Design
Lecture 18

2nd Year

III Semester

Galgotias University

2020-21

Unit II: Transformation

- **Syllabus**
- Geometric Transformation - Basic transformation,
  - translation,
  - rotation,
  - scaling,
  - reflection,
- homogeneous coordinates;
- Composite Transformation- Introduction, translation, rotation, scaling,
  - 3-D transformation- translation, rotation, scaling, reflection;
  - **3-D composite transformation- generalized rotation,**
  - **generalized reflection.**

Objective of the lecture

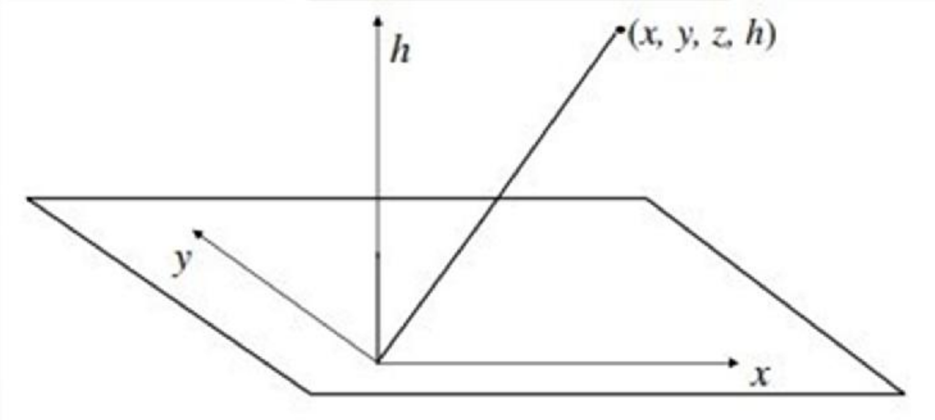
- 3-D composite transformation- generalized rotation,
- generalized reflection.

3-D composite transformation

- Geometric transformations are mappings from one coordinate system onto itself.
- The geometric model undergoes change relative to its MCS (Model CoordinateSystem)
- The Transformations are applied to an object represented by point sets.
- Rigid Body Motion: The relative distances between object particles remain constant
- Affine and Non-Affine maps
- Transformed point set X* = f(P, transformation parameters)

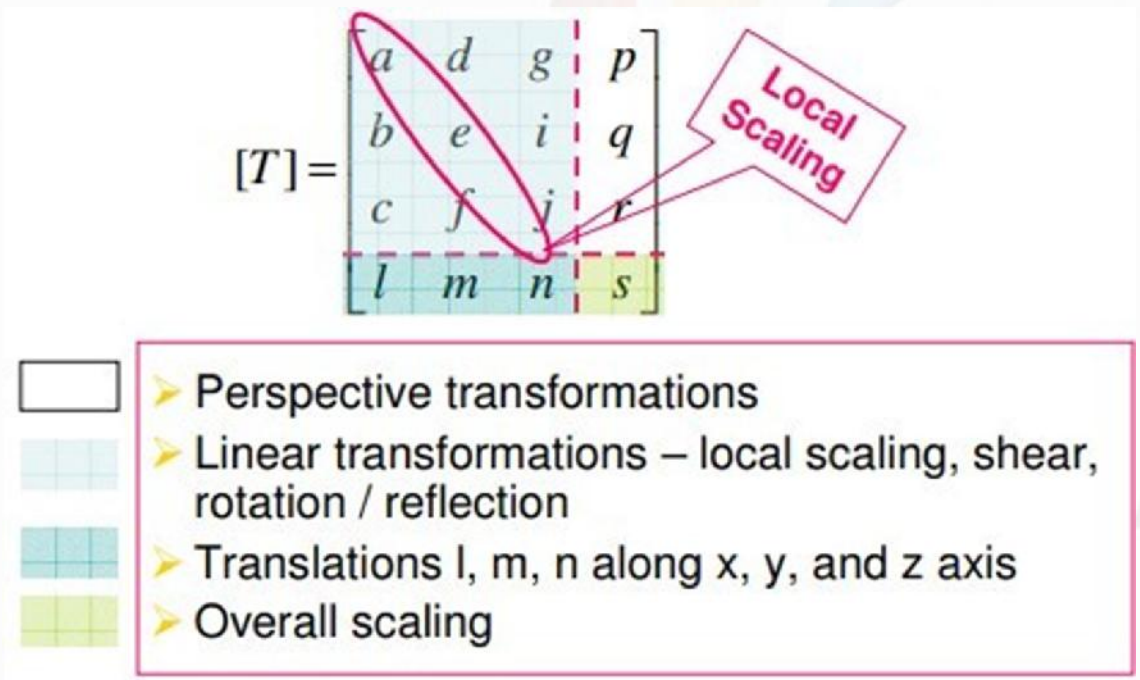
Contd..

- Homogeneous coordinates in 3 dimensions
- A point in homogeneous coordinates (x, y, z,h), h 0, corresponds to the 3-D vertex (x/h, y/h, z/h) in Cartesian coordinates.
- Homogeneous coordinates in 3D give rise to 4 dimensional position vector.



Contd..

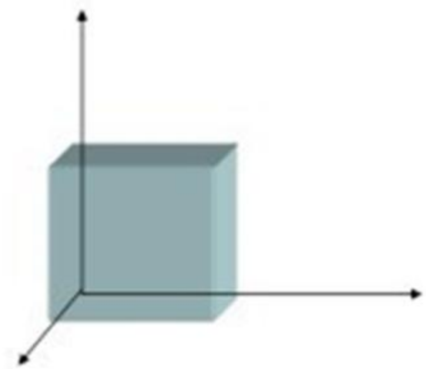
- Generalized 4 x 4 transformation matrix in homogeneous coordinates



3D Scaling

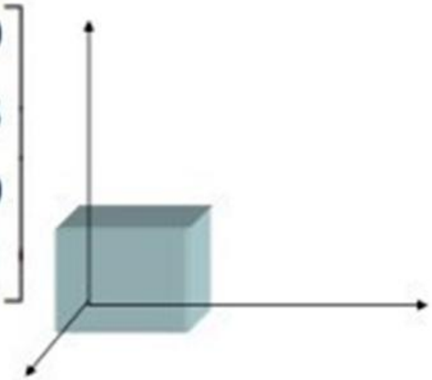
- 3D Scaling

$$[T_s] = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & e & 0 & 0 \\ 0 & 0 & j & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Ex: Required scaling to scale the RPP to a unit cube is ½, 1/3, 1

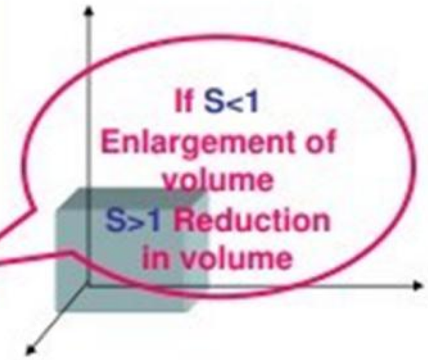
$$[X'] = [T_s] \begin{bmatrix} 0 & 2 & 2 & 0 & 0 & 2 & 2 & 0 \\ 0 & 0 & 3 & 3 & 0 & 0 & 3 & 3 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$



Overall scaling

- Overall Scaling

$$[T_s] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & s \end{bmatrix}$$

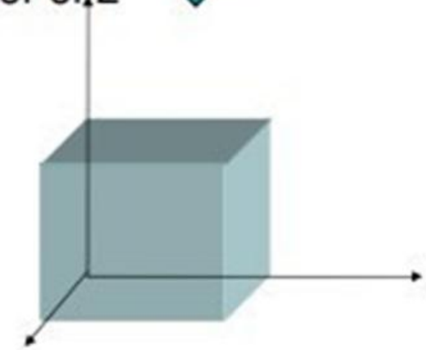


$$[T_s][X] = [x' \quad y' \quad z' \quad s]^T$$

$$= [x'/s \quad y'/s \quad z'/s \quad 1]^T$$

Ex: Uniformly scale the unit cube by a factor of 2 requires $s = \frac{1}{2}$

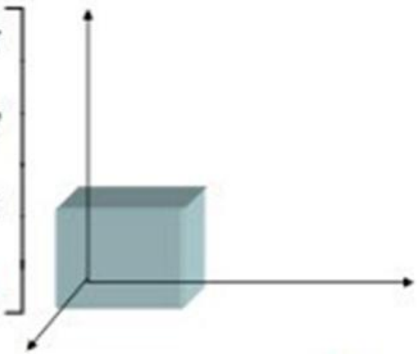
$$[X'] = \begin{bmatrix} 0 & 2 & 2 & 0 & 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 2 & 0 & 0 & 2 & 2 \\ 2 & 2 & 2 & 2 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$



3D-Shearing

- 3D SHEARING

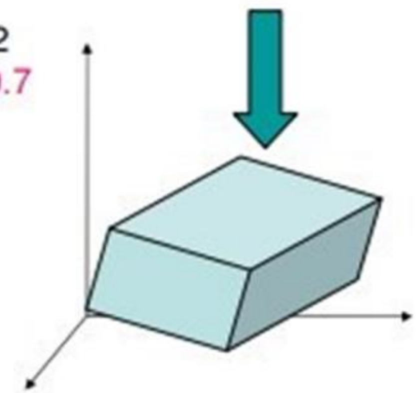
$$[T_{SH}] = \begin{bmatrix} 1 & d & g & 0 \\ b & 1 & i & 0 \\ c & f & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



$$[T_s][X] = [x + yd + gz \quad bx + y + iz \quad cx + fy + z \quad 1]^T$$

Ex: Uniformly scale the unit cube by a factor of 2 requires d= -0.75, g=0.5,i=1, b=-0.85,c=0.25,f=0.7

$$[X'] = \begin{bmatrix} 0.5 & 1.5 & 0.75 & -0.25 & 0 & 1 & 0.25 & -0.75 \\ 1 & 0.15 & 1.15 & 2 & 0 & -0.85 & 0.15 & 1 \\ 1 & 1.25 & 1.95 & 1.7 & 0 & 0.25 & 0.95 & 0.7 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

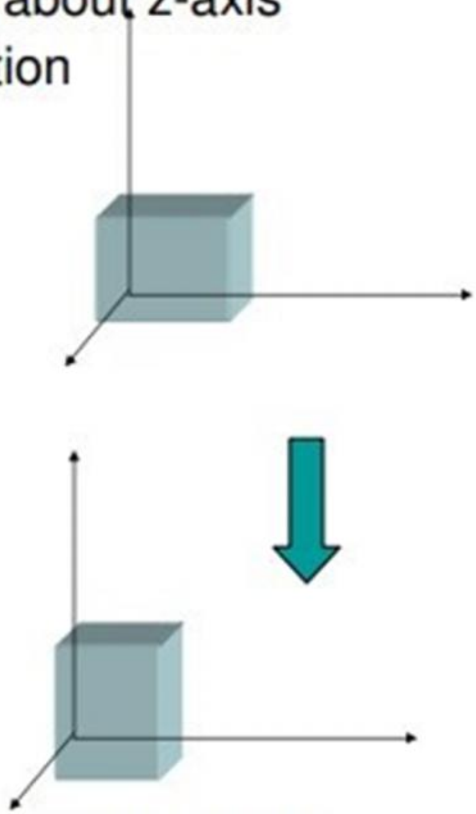


3-D Rotation about z-axis

➢ 3D ROTATIONS – (i) Rotation about z-axis

We are already familiar with rotation about z-axis (in 2D rotations)

$$[T_{Rz}] = \begin{bmatrix} \cos\psi & \sin\psi & 0 & 0 \\ -\sin\psi & \cos\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Det[T]=+1

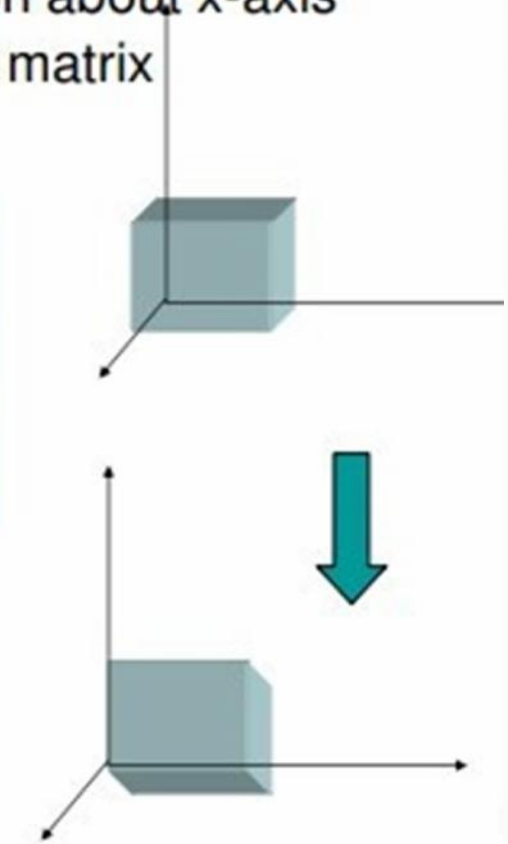
The position vector is assumed to be a row vector in right handed system

3DRotation about x-axis

➤ 3D ROTATIONS – (ii) Rotation about x-axis

Similarly we can obtain rotation matrix about x-axis

$$[T_{Rx}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\vartheta & \sin\vartheta & 0 \\ 0 & -\sin\vartheta & \cos\vartheta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

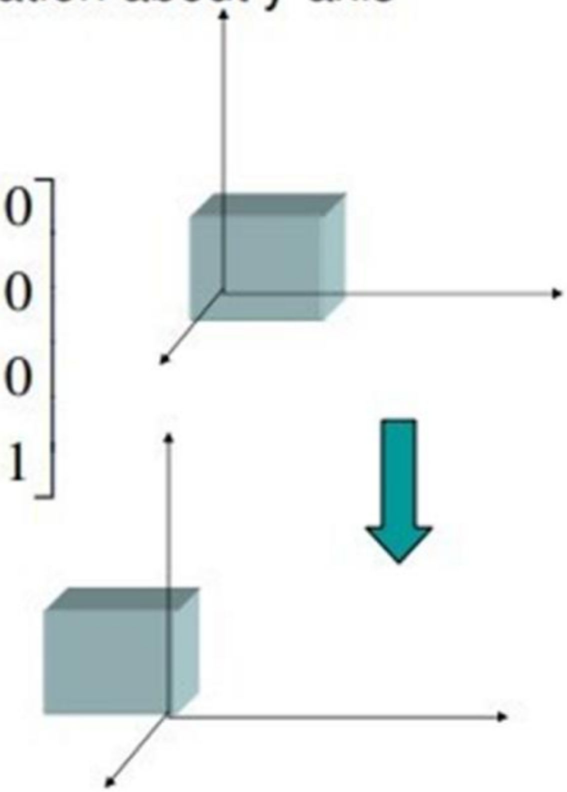


3D-Rotation about y-axis

3D ROTATIONS – (iii) Rotation about y-axis

We can obtain rotation about y-axis as

$$[T_{Ry}] = \begin{bmatrix} \cos\phi & 0 & -\sin\phi & 0 \\ 0 & 1 & 0 & 0 \\ \sin\phi & 0 & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



3D Reflection

- 3D REFLECTIONS – As in 2D, we can perform 3D transformations about a plane now.
- Rotation of 180° about an axis passing through origin out into 4-D space and projection back onto 3D space.

Through x-y plane

$$[T_{xy}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Det[T]=-1

Similarly through y-z and x-z planes are

$$[T_{yz}] = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad [T_{xz}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

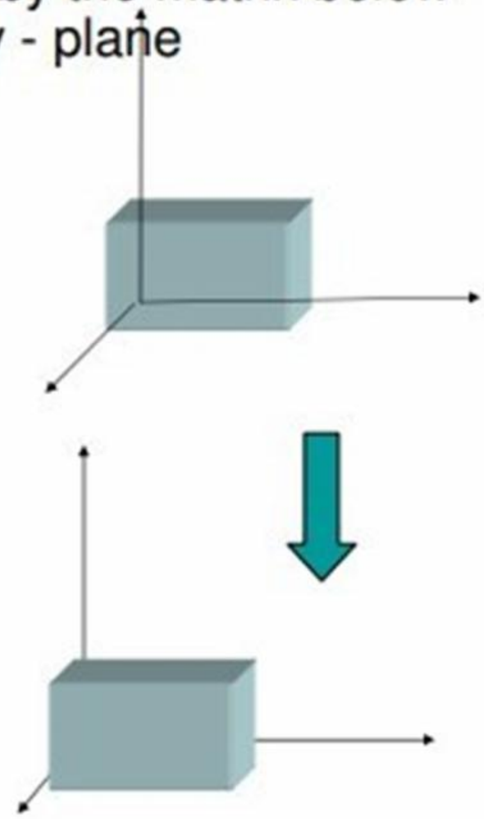
Example:

➤ Example – for the RPP given by the matrix below obtain 3D reflection through xy - plane

$$[X] = \begin{bmatrix} 1 & 2 & 2 & 1 & 1 & 2 & 2 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ -1 & -1 & -1 & -1 & -2 & -2 & -2 & -2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$[T_{xy}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[X] = \begin{bmatrix} 1 & 2 & 2 & 1 & 1 & 2 & 2 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$



COMBINATION OF TRANSFORMATIONS

- COMBINATION OF TRANSFORMATIONS – As in 2D, we can perform a sequence of 3D linear transformations.
- This is achieved by concatenation of transformation matrices to obtain a combined transformation matrix

A combined matrix $[T][X] = [X][T_1][T_2][T_3][T_4]\ldots[T_n]$

Where $[T_i]$ are any combination of

- Translation
- Scaling
- Shearing
- Rotation
- Reflection

(Scaling, Shearing, Rotation, Reflection) linear trans. but not perspective transformation (Results in loss of info)

ROTATION ABOUT AN ARBITRARY AXIS IN SPACE

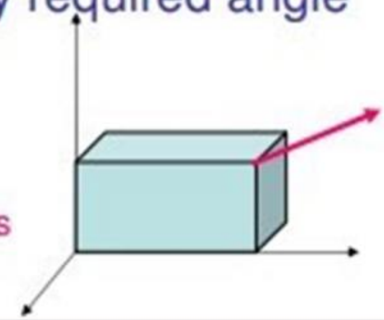
ROTATION ABOUT AN ARBITRARY AXIS IN SPACE

Make the arbitrary axis coincide with one of the coordinate axes.

- Consider an arbitrary axis passing through a point (x_0, y_0, z_0)

Procedure

- Translate (x_0, y_0, z_0) so that the point is at origin
- Make appropriate rotations to make the line coincide with one of the axes, say z-axis
- Rotate the object about z-axis by required angle
- Apply the inverse of step 2
- Apply the inverse of step 1
- Coinciding the arbitrary axis with any axis the rotations are needed about other two axes



Summery

- General concept is same as the 2D transformation
- The difference in the transformation matrix is mentioned

Questions

Books

- Text book
  - 1. Newman & Sprawl (1978), Principles of interactive Computer Graphics, Mcgraw hill college, ISBN- 978-0-074-63293-2
  - 2. Michel E. Mortenson (2006), Geometric modeling, Industrial press, ISBN-978-0-201-84840-3
  - 3. Van Dam, Hughes John, James Foley (2002), Computer graphics, principles and practices Pearson, ISBN- 978-0-201-84840-3
- Reference book
  - 1. Foley & van dam (1982), Fundamental of Interactive computer graphics, Addison Wesley longman publishing co, ISBN- 978-1-852-33818-3
  - 2. David Rogers (2001), Procedural elements of Computer graphics, TMH, ISBN- 978-0-070-53529-9
  - 3. Rogers and Adams (2002), Mathematical elements of Computer Graphics, TMH,ISBN- 978-0-070-53529-9
  - 4. Hearn & baker (2011), Computer Graphics, Pearson,ISBN- 978-8-177-58765-4.

Thank You !