



Theorem: S is closed iff $R - S$ is open.

Proof: (Necessary part)

Assume: S is closed.

Aim: our aim is to prove that $R - S$ is open or we have to prove that every point of $R - S$ is an interior point of $R - S$.

Let us assume that $p \in R - S = S^c$ be any arbitrary point of $R - S$.

Implies $p \notin S$ implies p is not a limit point of S .

This implies $\exists \varepsilon > 0$ such that $(N_\varepsilon(p) - \{p\}) \cap S = \emptyset$ implies $N_\varepsilon(p) \cap S = \emptyset$ implies $N_\varepsilon(p) \subseteq S^c$.

Galgotias
UNIVERSITY

School of basic and applied sciences Galgotias University



Course Code : BSCM301

Course Name: Real Analysis-I

This implies p is an interior point of S^c .

S^c is open.

(Sufficient part) S^c is open.

Aim: S is closed.

Suppose S is not closed this implies that there is a point p in S which is not a limit point of S implies that p is an interior point of S . This implies that p is not an interior point of S^c . This implies that S^c is not open. A contradiction that S^c is open.

Our assumption is wrong implies S is closed.

GALGOTIAS
UNIVERSITY



Th: If A^o defined interior set of A .
Then,
(i) $A^o = \text{Int}(A) = \bigcup_{I_i} I_i$, I_i 's are open sets.
(ii) A^o is an open set.
(iii) A^o is the largest open set contained in A .



(10) A is open iff $A^o = A$.

Proof: Let $\mathcal{U} = \{G_\alpha / G_\alpha \text{ is an open set}\}$ be a class of open subsets of A .

Let $p \in \text{Int}(A) \Rightarrow p$ is a member of some open subset G_{α_0} for some α_0 .

School of basic and applied sciences Galgotias University



Course Code : BSCM301

Course Name: Real Analysis-I

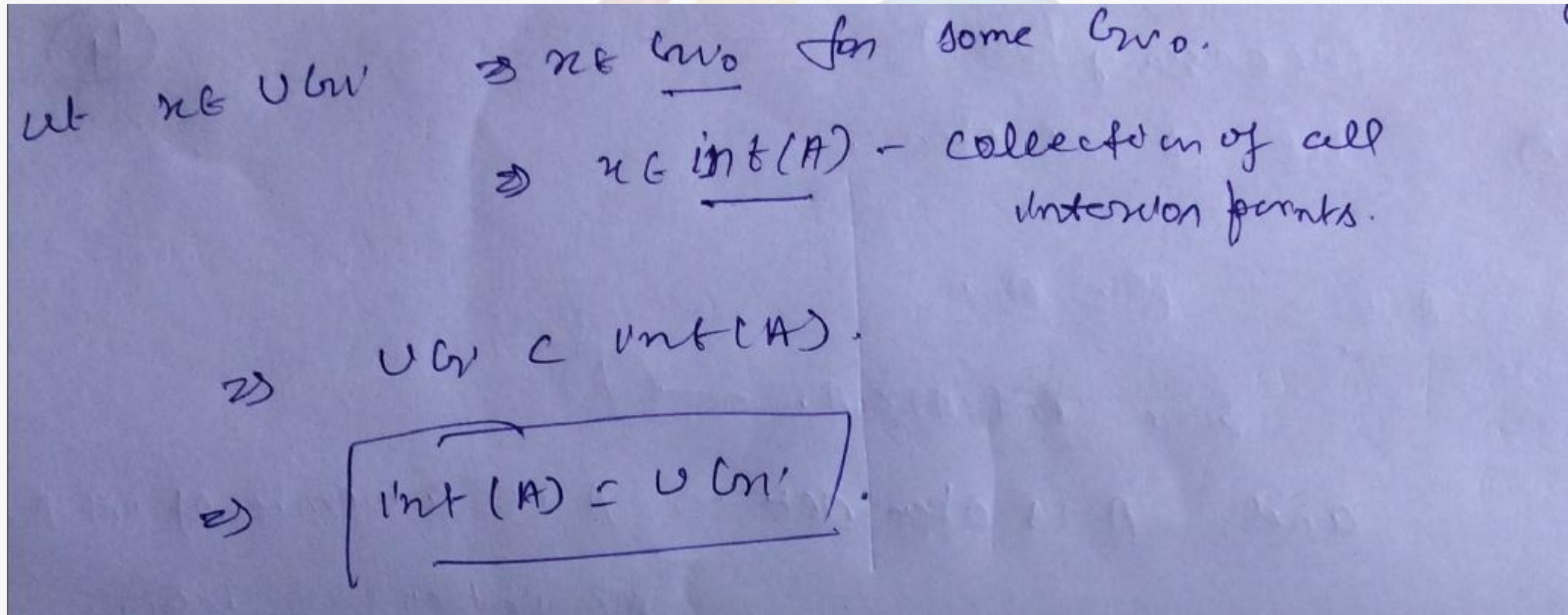
$\Rightarrow \exists \text{ am } \epsilon > 0$ such that

$$p \in N_\epsilon(p) \subset G_{2^0}$$

$$\Rightarrow p \in N_\epsilon(p) \subset \cup G_{2^i}$$

$$\Rightarrow p \in \cup G_{2^i}$$

$$\Rightarrow \text{int}(A) \subset \cup G_{2^i} .$$



School of basic and applied sciences Galgotias University



Course Code : BSCM301

Course Name: Real Analysis-I

Proof:
(ii) simple ~~observation~~ observation.
that arbitrary union of open sets is
open.

(iii). let G is an open set
 ~~$G = \bigcup_{\alpha} G_{\alpha}$~~ $G \subseteq \bigcup_{\alpha} G_{\alpha} = \text{int}(A)$
 $\Rightarrow G \subseteq \text{int}(A)$ — ①
largest open set.

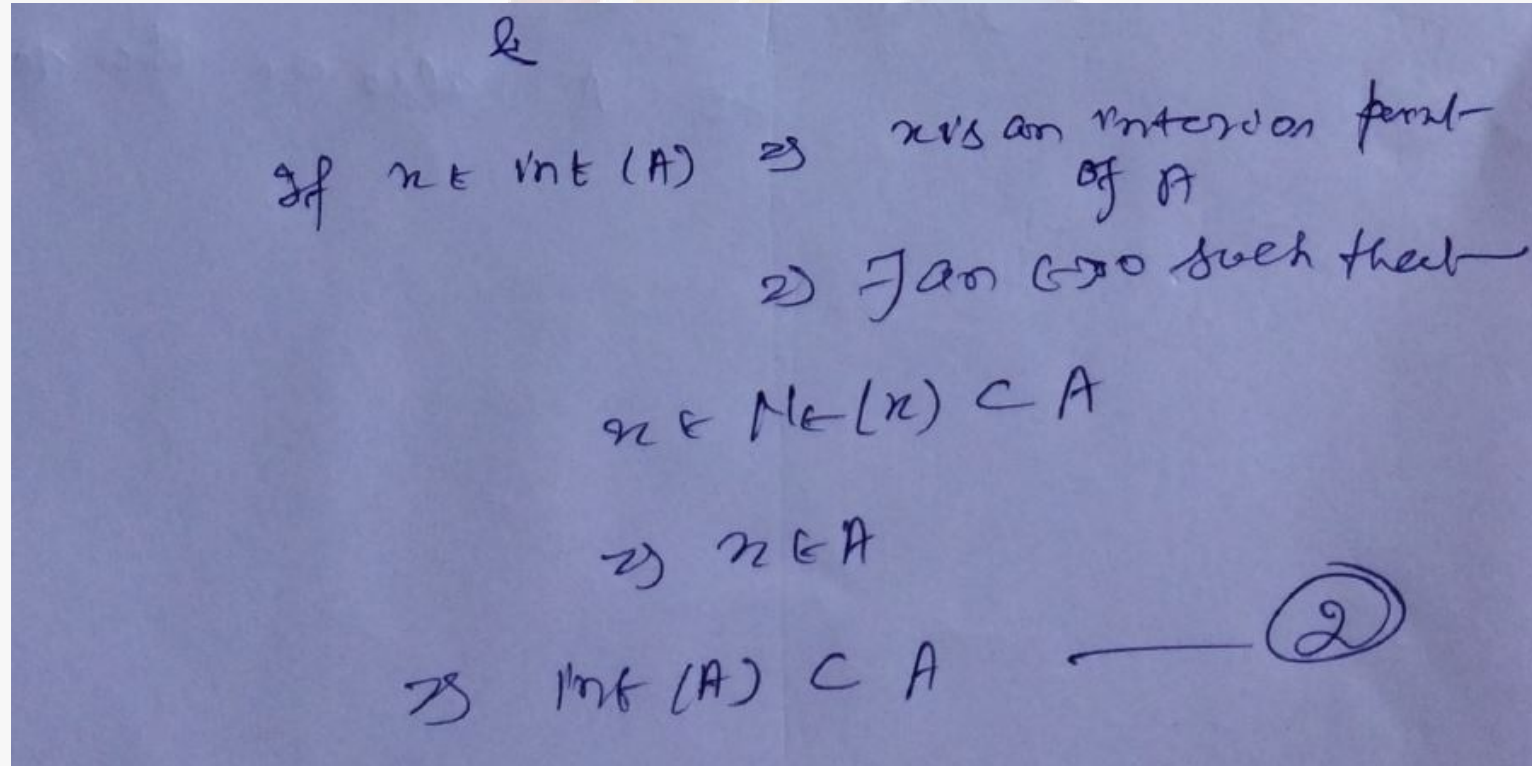
Reference book: Bansi Lal and Sanjay Arora; Introduction to Real Analysis, Satya Prakashan, 1st Vol (1991)

School of basic and applied sciences Galgotias University



Course Code : BSCM301

Course Name: Real Analysis-I



UNIVERSITY

School of basic and applied sciences Galgotias University



Course Code : BSCM301

Course Name: Real Analysis-I

②
 $\Rightarrow \text{Int}(A) \subset A$

②
 $G \subset \text{Int}(A) \subset A$
largest open set connected $\cup \cap A$

Reference book: Bansi Lal and Sanjay Arora; Introduction to Real Analysis, Satya Prakashan, 1st Vol (1991)

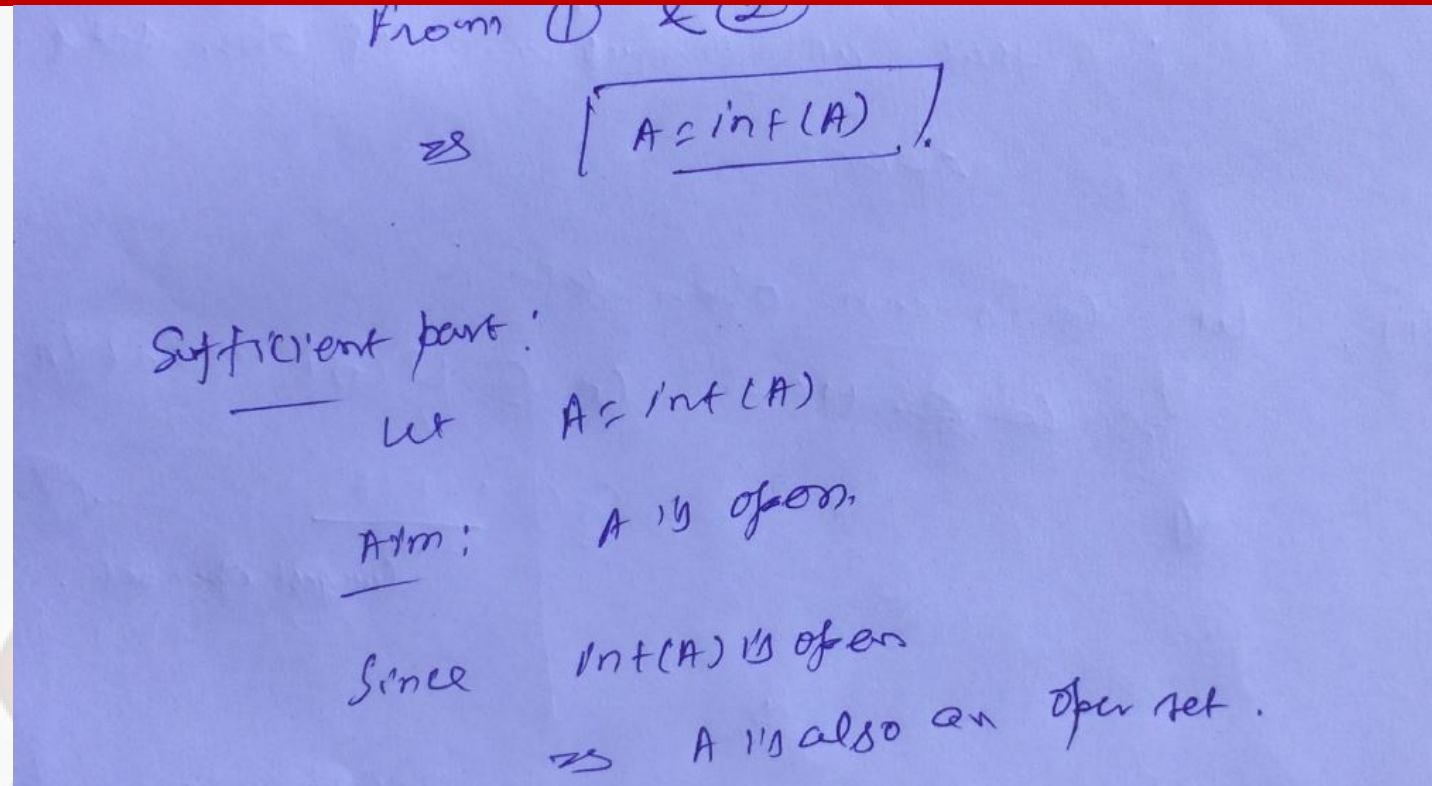
GALGOTIAS
UNIVERSITY

School of basic and applied sciences Galgotias University



Course Code : BSCM301

Course Name: Real Analysis-I



Reference book: Bansi Lal and Sanjay Arora; Introduction to Real Analysis, Satya Prakashan, 1st Vol (1991)