Lecture Notes

on

Information Theory and Coding

GALGOTIAS UNIVERSITY
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(Be safe and stay at home)

1. Course Information
2. What is Information Theory?
3. Review of Probability Theory
4. Information Measures

What is Information Theory?

- IT is a branch of math (a strictly deductive system). (C. Shannon, The bandwagon)
- General statistical concept of communication. (N. Wiener, What is IT?)
- It was build upon the work of Shannon (1948)
- It answers to two fundamental questions in Communications Theory:
  - What is the fundamental limit for information compression?
  - What is the fundamental limit on information transmission rate over a communications channel?

What is Information Theory?

- Mathematics: Inequalities
- Computer Science: Kolmogorov Complexity
- Statistics: Hypothesis Testings
- Probability Theory: Limit Theorems
- Engineering: Communications
- Physics: Thermodynamics
- Economics: Portfolio Theory

Communications Systems

- The fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at another point. (Claude Shannon: A Mathematical Theory of Communications, 1948)

Digital Communications Systems

- Source
- Source Coder: Convert an analog or digital source into bits.
- Channel Coder: Protection against errors/erasures in the channel.
- Modulator: Each binary sequence is assigned to a waveform
- Channel: Physical Medium to send information from transmitter to receiver. Source of randomness.
- Demodulator, Channel Decoder, Source Decoder, Sink.

Digital Communications Systems

- Modulator + Channel = Discrete Channel.
- Binary Symmetric Channel.
- Binary Erasure Channel.

Review of Probability Theory

- Axiomatic Approach
- Relative Frequency Approach

Axiomatic Approach

- Application of a mathematical theory called *M*easure Theory.
- It is based on a triplet

$$(\Omega, \mathcal{F}, P)$$

  where

  - Ω is the sample space, which is the set of all possible outcomes.
  - $\mathcal{F}$ is the $\sigma-$algebra, which is the set of all possible events (or combinations of outcomes).
  - P is the probability function, which can be any set function, whose domain is Ω and the range is the closed unit interval [0,1]. It must obey the following rules:
    - $P(\Omega) = 1$
    - Let A be any event in $\mathcal{F}$, then $P(A) \geq 0$.
    - Let A and B be two events in $\mathcal{F}$ such that $A \cap B = \emptyset$, then $P(A \cup B) = P(A) + P(B)$.

Axiomatic Approach: Other properties

- Probability of complement: $P(\overline{A}) = 1 - P(A)$.
- $P(A) \leq 1$.
- $P(\emptyset) = 0$.
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Conditional Probability

- Let A and B be two events, with $P(A) > 0$. The conditional probability of B given A is defined as:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

- Hence, $P(A \cap B) = P(B|A)P(A) = P(A|B)P(B)$
- If $A \cap B = \emptyset$ then $P(B|A) = 0$.
- If $A \subset B$, then $P(B|A) = 1$.