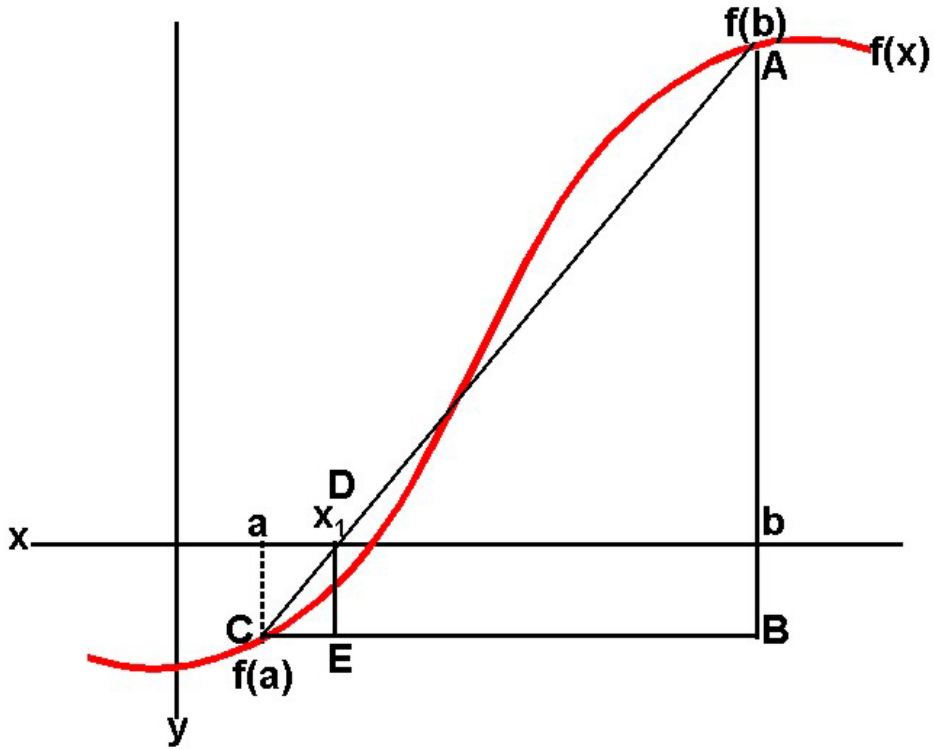
Lecture-03: Regula–Falsi Method

The **Regula–Falsi Method** is a numerical method for estimating the roots of a polynomial **f(x)**. A value **x** replaces the midpoint in the Bisection Method and serves as the new approximation of a root of **f(x)**. The objective is to make convergence faster. Assume that **f(x)** is continuous.

Algorithm for the Regula–Falsi Method:

Given a continuous function **f(x)**

- Find points **a** and **b** such that **a < b** and **f(a) * f(b) < 0**.
- Take the interval **[a, b]** and determine the next value of x_1.
- If $f(x_1) = 0$ then x_1 is an exact root, else if $f(x_1) * f(b) < 0$ then let $a = x_1$, else if $f(a) * f(x_1) < 0$ then let $b = x_1$.
- Repeat steps 2 & 3 until $f(x_i) = 0$ or $|f(x_i)| \leq DOA$,
- where **DOA** stands for **degree of accuracy**.



Continued…

Observe that

$$EC/BC=E/AB$$

$$[x–a]/[b–a]=[f(x)–f(a)]/[f(b)–f(a)]$$

$$x–a=[b–a][0–f(a)]/[f(b)–f(a)]$$

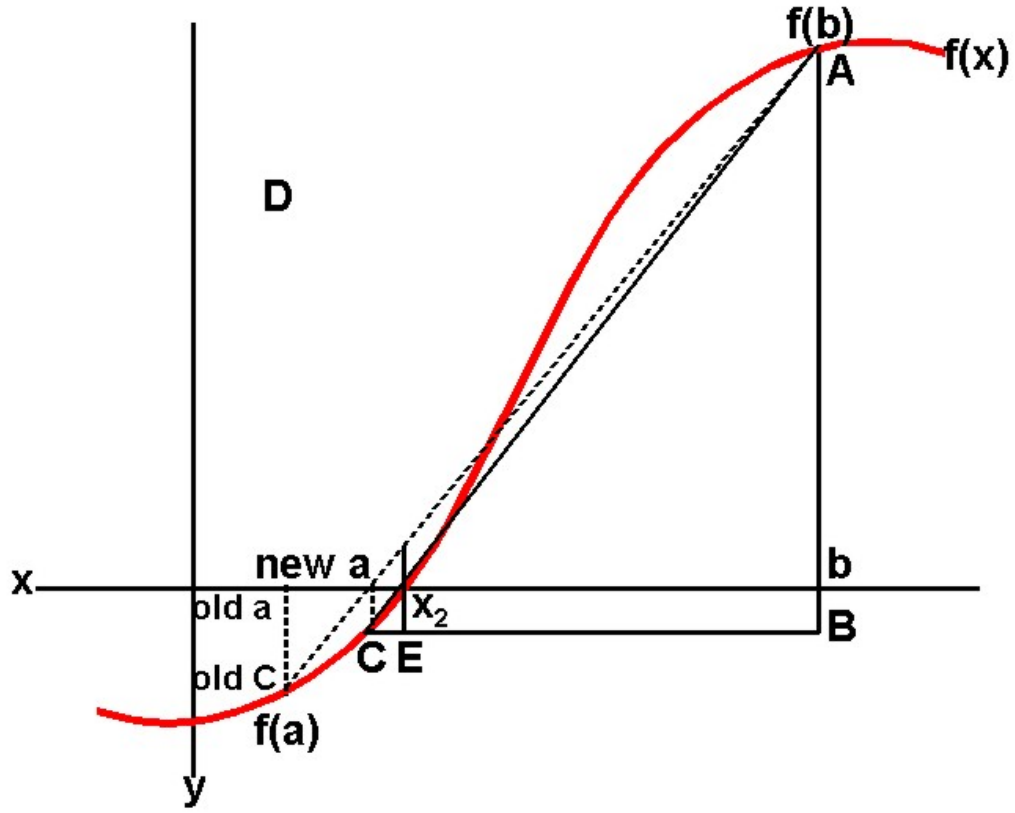
$$x=a+[b–a]\,[–f(a)\,]/[f(b)–f(a)]$$

$$x=a–[b–a]\,f(a)/[f(b)–f(a)]$$

Note that the line segment drawn from **f(a)** to **f(b)** is called the **interpolation line**.

Continued…

Graphically, if the root is in **[a, x_i]**, then the next interpolation line is drawn between **(a, f(a))** and **(x_i, f(x_i))**; otherwise, if the root is in **[x_i, b]**, then the next interpolation line is drawn between **(x_i, f(x_i))** and **(b, f(b))**.



EXAMPLE 1: Consider $f(x) = x^3 + 3x - 5$, where [a = 1, b = 2] and DOA = 0.001.

i	a	x	b	f(a)	f(x)	f(b)
1	1	1.1	2	– 1	– 0.369	9
2	1.1	1.13544668587896	2	– 0.369	– 0.129797592130931	9
3	1.13544668587896	1.14773797024856	2	– 0.129797592130931	– 0.044868050981328 6	9
4	1.14773797024856	1.15196570867269	2	– 0.044868050981328 6	– 0.015415586390991 7	9
5	1.15196570867269	1.15341577448	2	– 0.015415586390991 7	– 0.005285298529248 2	9
6	1.15341577448	1.15391264384212	2	– 0.005285298529248 2	– 0.001810778834876 46	9
7	1.15391264384212	1.15408284038531	2	– 0.001810778834876 46	– 0.000620231485743084	9

Hence root will be x=1.5408284038531

Example 2:Find a root of an equation $f(x)=2x^3-2x-5$ using False Position method (regula falsi method)

n	a	$f(a)$	b	$f(b)$	x	$f(x)$
1	1	-5	2	7	1.41667	-2.14699
2	1.41667	-2.14699	2	7	1.55359	-0.60759
3	1.55359	-0.60759	2	7	1.58924	-0.15063
4	1.58924	-0.15063	2	7	1.59789	-0.0361
5	1.59789	-0.0361	2	7	1.59996	-0.00858
6	1.59996	-0.00858	2	7	1.60045	-0.00203
7	1.60045	-0.00203	2	7	1.60056	-0.00048

Hence root x=1.60056

References

- http://www2.lv.psu.edu/ojj/courses/cmpsc-201/numerical/regula.html
- Chapra, Steven C. Applied Numerical Methods with MATLAB for Engineers and Scientists. McGraw-Hill, 2017.
- Class Notes from ENGRD 3200: Engineering Computation taught by Professor Peter Diamessis at Cornell University

GALGOTIAS UNIVERSITY