STIFFNESS MATRIX FOR BAR

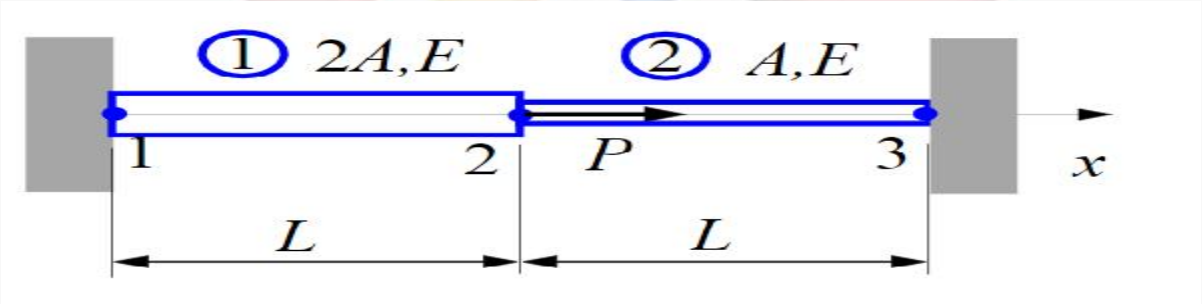
FINITE ELEMENT ANALYSIS

LECTURE OBJECTIVE:

- Global Stiffness Matrix for 1 D element BAR .
- FEA application in BAR Problem.

.

Find the stresses in the two bar assembly which is loaded with force P and constrained at the two ends, as shown in the figure.



Solution: Use two 1-D bar elements.

Element 1,

$$\mathbf{k}_1 = \frac{2EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad \begin{matrix} u_1 & u_2 \end{matrix}$$

Element 2,

$$\mathbf{k}_2 = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad \begin{matrix} u_2 & u_3 \end{matrix}$$

$$\frac{EA}{L}\begin{bmatrix} 2 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

Load and boundary conditions (BC) are,

$$u_1 = u_3 = 0, \qquad F_2 = P$$

FE equation becomes,

$$\frac{EA}{L}\begin{bmatrix} 2 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{Bmatrix} 0 \\ u_2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ P \\ F_3 \end{Bmatrix}$$

$$\frac{EA}{L}[3]\{u_2\} = \{P\}$$

Thus,

$$u_2 = \frac{PL}{3EA}$$

and

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \frac{PL}{3EA}\begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$

$$\sigma_1 = E\varepsilon_1 = E\mathbf{B}_1\mathbf{u}_1 = E\begin{bmatrix}-1/L & 1/L\end{bmatrix}\begin{Bmatrix}u_1\\u_2\end{Bmatrix}$$

$$= E\frac{u_2 - u_1}{L} = \frac{E}{L}\left(\frac{PL}{3EA} - 0\right) = \frac{P}{3A}$$

Similarly, stress in element 2 is

$$\sigma_2 = E\varepsilon_2 = E\mathbf{B}_2\mathbf{u}_2 = E\begin{bmatrix}-1/L & 1/L\end{bmatrix}\begin{Bmatrix}u_2\\u_3\end{Bmatrix}$$

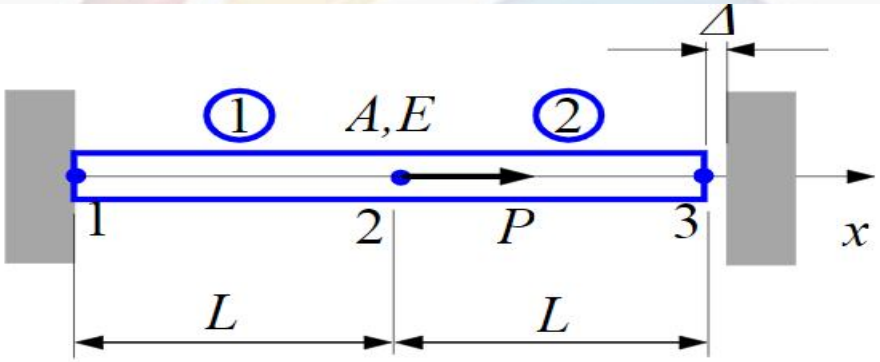
$$= E\frac{u_3 - u_2}{L} = \frac{E}{L}\left(0 - \frac{PL}{3EA}\right) = -\frac{P}{3A}$$

which indicates that bar 2 is in compression.

In this case, the calculated stresses in elements 1 and 2 are exact within the linear theory for 1-D bar structures. It will not help if we further divide element 1 or 2 into smaller finite elements.

For tapered bars, averaged values of the cross-sectional areas should be used for the elements.

- **Question-**



Problem: Determine the support reaction forces at the two ends of the bar shown above, given the following,

$$P = 6.0 \times 10^4 \text{ N}, \quad E = 2.0 \times 10^4 \text{ N/mm}^2,$$

$$A = 250 \text{ mm}^2, \quad L = 150 \text{ mm}, \quad \Delta = 1.2 \text{ mm}$$

Text Book-

1. Finite Element Analysis by S.S bhavikatti six multicolour edition,2018.New age International publisher. ISBN: 678-26-74589-23-4.

2. A Textbook of Finite Element Analysis Formulation and Programming by D.K.mahraj,Edition 2019. Publisher Willey India ISBN : 978-93-88425-93-3.

Reference Book-

1. Finite element analysis ,Theory and application with Ansys by Moaveni ,2nd edition 2015 ,publisher Pearson, ISBN- 528-43-88435-9.

2. Finite element Analysis By David V. Hutton ,Publisher Elizabeth A. Jomes ,4th edition 2017. ISBN: 0-07-23-9536-2

THANK YOU