E-Content

Mathematical Economics

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COURSE CONTENT

Income and Leisure
&
Linear Expenditure Systems

Income and Leisure

- If the consumer's income is payment for work performed by hi, the optimum amount of work that he performs can be derived from the analysis of utility maximization. One can also derive the consumer's demand curve for income from this analysis. Assume that the consumer's satisfaction depends on income and leisure.

His utility function is

$$U = g(L, y) \ldots\ldots\ldots\ldots (1)$$

Where L denotes leisure

Both income and leisure are desirable. It is assumed that consumer buys the various commodities at constant prices and income is there by treated as generalized purchasing power.

For maximizing the utility, we will run partial derivative with equation 1.

$$dU = g1\ dL + g2\ dy \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Setting dU = 0

$$g1\ dL + g2\ dy = 0 \text{---------} (3)$$

$$g1dL = -g2dy$$

$$-dy/dL = g1/g2 \text{-----------} (4)$$

Change in income with respect leisure is equal to the ratio of partial derivative of the utility function.

Denote the amount of work performed by the consumer by W and wage rate by r

By definition

$$L = T - W \text{-------------} (5)$$

Where T is total amount of available time. (Normally we use T = 24 hours)

The budget constraint is

$$y = rW \text{-------------} (6)$$

Substituting equations 5 and 6 in to equation 1.

U = g (T-W, rW)------------(7)

To maximize utility set the derivative of equation 7 with respect to W.

dU/dW = -g1 +g2r

Setting dU/dW = 0 for maximum utility

-g1 + g2r = 0

r = g1/g2---------------(8)

-dy/dL = g1/g2 from equation (4)

-dy/dL = g1/g2 = r .... (9)

Equation no. 9 states that the rate of substitution of income for leisure equals to wage rate.

- Question.1 Assume that the utility function defined for a time period of one day is given by

$$U = 48 (T-W) + (T-W)Wr - (T-W)^2$$

$$T=24 \text{ hours}$$

find out the value of W for utility maximization.

$$U = 48 (T-W) + (T-W)Wr - (T-W)^2$$

$$= 48T - 48W + TWr - W^2r - T^2 - W^2 + 2TW$$

$$dU/dW = d/dW\ 48T - d/dW\ 48W + d/dW\ TWr - d/dW\ W^2r - d/dW\ T^2 - d/dW\ W^2 + d/dW\ 2TW$$

$$= 0 - 48 + Tr - 2Wr - 0 - 2W + 2T$$

$$0 = -48 + Tr - 2Wr - 2W + 2T$$

$$W = \frac{T(r+2) - 48}{2(r+1)}$$

$$y = rW$$

- $y = rW$

$$= r.\ \frac{T(r+2) - 48}{2\,(r+1)}$$

(i). Prove that at zero wage rate, the individual will not work at all. Since T, the total available time is 24 hours

$$W = \frac{T(r+2) - 48}{2\,(r+1)}$$

(ii). At wage rate 10 rupees per hour and 15 rupees per hour, calculate the working hours.

$$W = \frac{T(r+2) - 48}{2\,(r+1)}$$

Ans- At 10 rupees working hours will be 10 H and 54 min

At 15 rupees working hours will be 11 H and 15 min

(iii). Irrespective of how high the wage becomes the individual will never work more than 12 hours per day.

$$W = \frac{T(r+2) - 48}{2\,(r+1)}$$

$$W = \lim_{r \to \infty} \left| \frac{T(r+2) - 48}{2(r+1)} \right|$$

$$\frac{\left(T + \frac{2T}{r} - \frac{48}{r}\right)}{\left(2 + \frac{2}{r}\right)}$$

$$= T/2$$

$$W = 24/2 = 12$$

Linear Expenditure Systems

These are models which deal with groups of commodities rather than individual commodities. Such groups, when added, yield total consumer expenditure. Linear expenditure systems are thus of great interest in aggregate econometric models, where they provide desirable disaggregation of the consumption function. One of the earliest linear expenditure models was suggested by **R. Stone.**

$$U = U_{(A)} + U_{(B)} + U_{(C)} + U_{(D)} + U_{(E)} + \ldots\ldots\ldots$$

The consumers buy some minimum quantity from each group, irrespective of prices. The minimum quantities are called 'subsistence quantities' because they are the minimum requirements for keeping the consumer alive. The income left (after the expenditure on the minimum quantities is covered) is allocated among the various groups on the basis of prices.

Cobb-Douglas utility function

$U(x1,x2) = x1^{\alpha} x2^{\beta}$

where $\alpha + \beta = 1$

$\beta = 1 - \alpha$

- **Stone Geary utility maximization**

The Stone Geary function is often used to model problems involving subsistence levels of consumption. In these cases, a certain minimal level of some goods have to consumed, irrespective of its price or the consumer's income.

In the two goods case, consumers will first set aside a subsistence level of consumption of two goods x1 and x2 of $\bar{x}1$ for good x1 and $\bar{x}2$ for good x2. The Stone Geary utility function is based on the traditional Cobb- Douglas utility function.

$$U(x1,x2) = (x1-\bar{x}1)^{\alpha} (x2-\bar{x}2)^{\beta}$$

and α and β are the proportion of each of goods x1 and x2 consumed.

As a result the Stone-Geary utility function for two goods case can be presented as following

$U(x1,x2) = (x1-\bar{x}1)^{\alpha} (x2-\bar{x}2)^{1-\alpha}$ ....(1)

We can take Log in both side of equation (1)

$U(x1, x2) = \ln U(x1,x2) = \ln (x1-\bar{x}1)^{\alpha} (x2-\bar{x}2)^{1-\alpha}$

$U(x1,x2) = \alpha \ln (x1-\bar{x}1) + 1-\alpha \ln (x2-\bar{x}2)$ -------(2)

Income equation

$y = p1X1 + p2X2$.................. (3)

To apply Lagrange theorem

$L = \alpha \ln (x1-\bar{x}1) + 1-\alpha \ln (x2-\bar{x}2) + \lambda (y - p1X1 - p2X2)$

$L = \alpha \ln(x1-\bar{x}1) + 1-\alpha \ln(x2-\bar{x}2) + \lambda(y - p1X1 - p2X2)$

Differentiating above equation with respect of x1, x2 and λ with the first order condition.

$dL/dx1 = (\alpha/ x1-\bar{x}1) - \lambda p1 = 0$ ---------------(4)

$dL/dx2 = (1-\alpha)/(x2-\bar{x}2) - \lambda p2 = 0$ --------------- (5)

$dL/d\lambda = y - p1x1 - p2x2 = 0$ ------------------(6)

$x1 = \bar{x}1 + (\alpha/p1)(y - p1X1 - p2X2)$ --------(7)

$x2 = \bar{x}2 + ((1-\alpha)/p2)(y - p1X1 - p2X2)$ -------(8)

We can describe expenditure equation in given form also.

$y = p1X1 + p2X2$ (Income equation)

Expenditure functions will be

Here we are multiplying p1 in both side of equation (7)

$p1X1 = p1\,\bar{x}1 + p1(\alpha/p1)(y - p1X1 - p2X2)$

$p1X1 = p1\,\bar{x}1 + \alpha(y - p1X1 - p2X2)$……………..(9)

we are multiplying p2 in both side of equation (8)

$p2x2 = p2\bar{x}2 + (p2(1-\alpha)/p2)(y - p1X1 - p2X2)$

$p2X2 = p2\bar{x}2 + (1-\alpha)(y - p1X1 - p2X2)$

$p2X2 = p2\bar{x}2 + \beta(y - p1X1 - p2X2)$……………………(10)

Here equation 9 and 10 are liner expenditure equations. Which is liner in income and prices and thus suitable for linear regression analysis.

$y = p1\,\bar{x}1 + \alpha(y - p1X1 - p2X2) + p2\bar{x}2 + \beta(y - p1X1 - p2X2)$

TEXT BOOKS/REFERENCE BOOKS

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