CLASSICAL STATISTICS

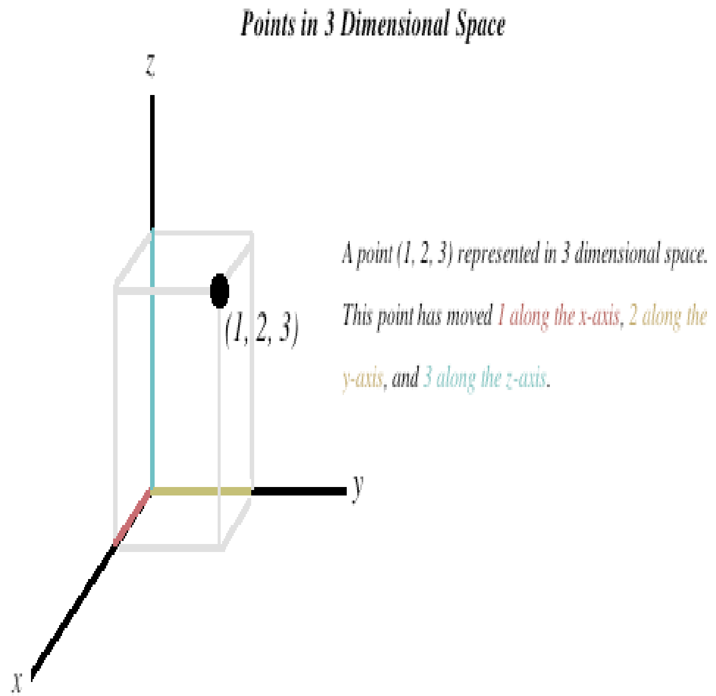
TOPICS COVERED:

- Phase Space

GENERAL OVERVIEW

The microstate of a classical system can be described in terms of the position and momentum of its constituent particles

The position of a particle is specified by three components of position i.e. x,y,z.



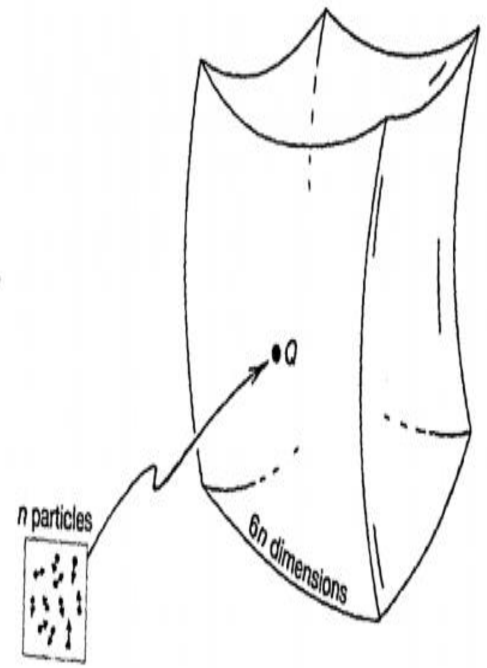
The three dimensional space in which the location of a particle is completely specified by the three position co-ordinates, is known as 'Position space'.

- **The momentum of a particle is specified by three components of momenta i.e. Px,Py,Pz**

The three dimensional space in which the momentum of a particle is completely specified by the three momentum co-ordinates P_x P_y and P_z is known as 'Momentum space'.

- ***The combination of the position space and momentum space is known as 'Phase space'.***

PHASE SPACE- DEFINITION



- A phase space is a mathematical tool that allows us to grasp important aspects of complicated systems.
- Each point of the phase space represents one specific configuration a given system can be in.
- The state of a system is recorded in a phase space point through all the location and all the momenta the objects in the system have at a given point in time.
- The time evolution of a system can then be represented as a path in phase space.

SINGLE PARTICLE IN ONE DIMENSION

- Consider a **Single Particle in 1 Dimension**. In classical mechanics, it can be ***completely described*** in terms of it's generalized position coordinate **q** & it's momentum **p**.

  - The usual case is to consider the ***Hamiltonian Formulation*** of classical mechanics, where we talk of generalized coordinates **q** & generalized momenta **p**, rather than the **Lagrangian Formulation**, where we talk of coordinates **q** & velocities **(dq/dt)**.

  - The particle obeys ***Hamilton's Equations of Motion***

$$\dot{q}_i = \frac{\partial H}{\partial p_i}, \quad \dot{p}_i = -\frac{\partial H}{\partial q_i}$$

Consider a two dimensional space defined by q,p

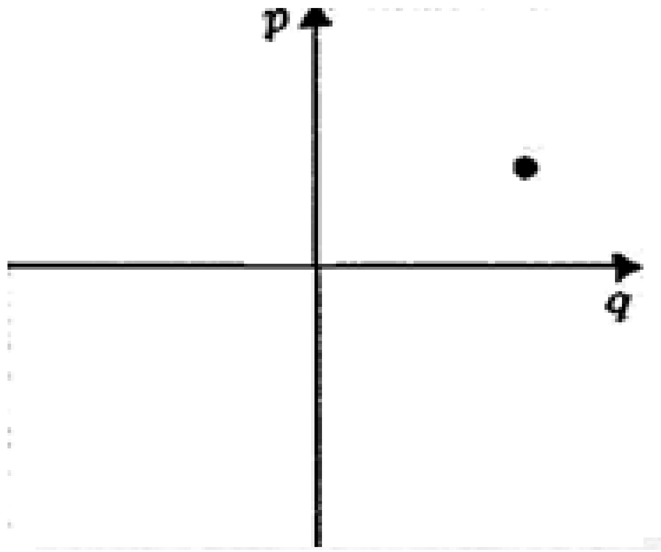
At any time t, stating the (q,p) of the particle describes it's "State".

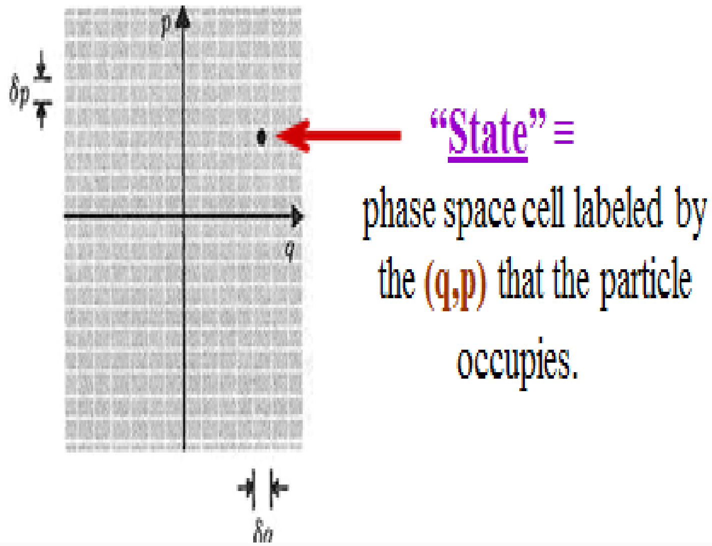
As q and p change in time, the point representing the particle state moves in the plane

q and p are continuous variables, so an infinite number of points are in this Classical phase space

It is convenient to subdivide the ranges of q and p into small rectangles of size **δq x δp** . We can think of this 2D space as divided as divided into cells of equal area **$\delta q \times \delta p = h_0$** , where h_0 is a small constant with units of angular momentum

We can call these cells as **phase cells.** The particle state is specified by stating which cell in the phase space the q,p of the particle is in





We can use Heisenberg Uncertainty Principle to determine the constant h_0

According to Heisenberg Uncertainty Principle " It is impossible to simultaneously specify a particle's position and momentum to a greater accuracy than **$\delta q \times \delta p >= h/4\pi$**

So the minimum value of h_0 is $h/4\pi$

Single particle moving in three dimensions

- Now let us consider a single particle moving in three dimensions.
- To specify state of a system, we should know three q-p pairs: qx-px,qy-py,qz-pz (Three degrees of freedom)
- The time evolution of the q and p can be visualized by plotting q-p in six dimensional phase space---- μ space
- Meaning of point in phase understood based on uncertainty principle
- For this divide a phase into small six dimensional cells with sides dqx, dqy, dqz, dp_x, dp_y, dp_z
- Volume of these cells given as

$$d\tau = dqxdqydqzdp_xdp_ydp_z$$

- By uncertainty principle
- $dqxdp_x \geq h/4\pi$, $dqydp_y \geq h/4\pi$, $dqzdp_z \geq h/4\pi$
- So $d\tau = h^3$
- A point in phase space is considered to be cell whose minimum volume is of the order h^3
- A particle can be understood as being located in such a cell centered at some location instead of being precisely at a point

N classical particles moving in three dimensions

- To specify state of a system, we should know a large number of q-p pairs
- For N particles system, degrees of freedom(f)= 3N
- Therefore in order to visualize the time evolution of the system we require:
- 2f= 2x3N=6N dimensional phase space !!!

REFERENCES

- Statistical Mechanics, Satya Prakash
- A Textbook of Statistical Mechanics, Prof Suresh Chandra, CBS Publishers
- Statistical Mechanics, R.K. Pathria, Butterworth Heinemann: 2nd Ed., 1996, Oxford Univers
- Slide Share
- Google images