Classical Theory of Radiation

TOPICS COVERED:

- Stefan-Boltzmann law: Thermodynamic proof

STATEMENT- STEFAN BOLTZMANN LAW/STEFAN'S LAW

It states that the rate of emission of radiant energy by unit area of perfectly black body is directly proportional to the fourth power of its absolute temperature.

$$E=\sigma T^4 \quad .... (1)$$

Where σ is a constant and is called Stefan's constant. Its value is 5.672 X 10^{-8} $Jm^{-2}s^{-1}K^{-4}$

- A body which is not a black body absorbs and hence emit less radiation:

$$E = e\,\sigma\,T^4 \;........ (2)$$

where e = emissivity* which lies between 0 to 1.

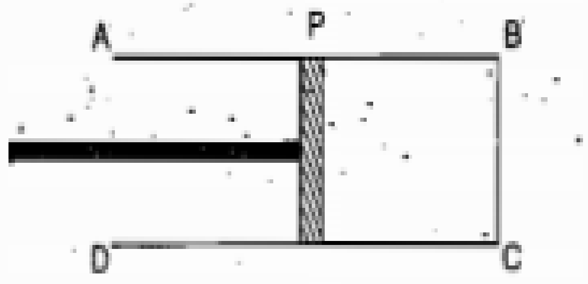
- With the surroundings of temperature T_0, net energy radiated by unit area per unit time:

$$\Delta E = E - E_o = e\sigma\,[T^4 - T_0^4] \;...... (3)$$

[*Emissivity is defined as the ratio of the energy radiated from a material's surface to that radiated from a perfect blackbody, at the same temperature and wavelength]

Thermodynamic proof

Consider a cylindrical enclosure ABCD provided with a Piston P.
Let it be filled with diffuse radiations of energy density ' u ' at uniform temperature T.



Then, total internal energy of radiations inside the enclosure:

U=uV , where V is the volume of the enclosure

If now dQ is the small amount of heat energy flowing in the enclosure from outside
➡ dU will be the change in internal energy and dW will be the external work done by radiation
In expansion of volume by dV.
Then by first law of thermodynamics:

dQ=dU+dW …………..(1)

U=uV and dw=pdV and p=(1/3)u ➡ p is the radiation pressure

[*Electromagnetic radiation exerts a minute pressure on everything it encounters. This is known as radiation pressure, and can be thought of as the transfer of momentum from photons as they strike the surface of the object]

Therefore, $dQ=d(uV)+(1/3)u\ dV$

$=udV+Vdu+(1/3)u\ dV$

$=Vdu+(4/3)u\ dV$……………..(2)

If dS is the change of entropy of the radiation, then from second law of thermodynamics:

$dS=dQ/T$…………………(3)

Substituting (2) in (3): ➡

$dS=(V/T)\ du + (4u/3T)\ dV$…….(4)

From (4), S is a function of u and V $S=f(u,V)$

$dS=(\partial S/\partial u)\ du + (\partial S/\partial V)\ dV$…….(5)

Comparing (4) and (5):

$\partial S/\partial u = V/T$ and $\partial S/\partial V= 4u/3T$

For a perfect differential, $\partial^2 S/ \partial u\, \partial V = \partial^2 S/ \partial V\, \partial u$

or, $\partial/ \partial u\, (\partial S/ \partial V) = \partial/ \partial V(\partial S/ \partial u)$

Therefore, $\partial/ \partial u\, (4u/ 3T) = \partial/ \partial V(V/ T)$

As T depends on u only and not on V, so finally

$$\partial u/u=4(\partial T/T)$$

Integrating we have:

$\log u= 4 \log T + \log A$ where, A is a constant

→ $u=AT^4$

As, $E=(1/4)uc$ → $E=(1/4)\, AcT^4$ → $E=\sigma T^4$ proved!!!!!