Canonical Forms

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Sum-Of-Minterm (SOM)

Sum-Of-Minterm (SOM) canonical form:
Sum of minterms of entries that evaluate to '**1**'

x	y	z	F	Minterm
0	0	0	0	
0	0	1	1	$m_1 = \bar{x}\,\bar{y}\,z$
0	1	0	0	
0	1	1	0	
1	0	0	0	
1	0	1	0	
1	1	0	1	$m_6 = x\,y\,\bar{z}$
1	1	1	1	$m_7 = x\,y\,z$

Focus on the '**1**' entries

$$F = m1 + m6 + m7 = \sum (1, 6, 7) = x\,y\,\bar{z} + x\,y\,z + \bar{x}\,y\,z$$

Sum-Of-Minterm Examples

$F(a, b, c, d) = \sum(2, 3, 6, 10, 11)$

$F(a, b, c, d) = m_2 + m_3 + m_6 + m_{10} + m_{11} =$

$$\bar{a}\,\bar{b}\,c\,\bar{d} + \bar{a}\,\bar{b}\,c\,d + \bar{a}\,b\,c\,\bar{d} + a\,\bar{b}\,c\,\bar{d} + a\,\bar{b}\,c\,d$$

$G(a, b, c, d) = \sum(0, 1, 12, 15)$

$G(a, b, c, d) = m_0 + m_1 + m_{12} + m_{15} =$

$$\bar{a}\,\bar{b}\,\bar{c}\,\bar{d} + \bar{a}\,\bar{b}\,\bar{c}\,d + a\,b\,\bar{c}\,\bar{d} + a\,b\,c\,d$$

Product-Of-Maxterm (POM)

Product-Of-Maxterm (POM) canonical form:

Product of maxterms of entries that evaluate to '**0**'

x	y	z	F	Maxterm
0	0	0	1	
0	0	1	1	
0	1	0	**0**	$M_2 = (x + \bar{y} + z)$
0	1	1	1	
1	0	0	**0**	$M_4 = (\bar{x} + y + z)$
1	0	1	1	
1	1	0	**0**	$M_6 = (\bar{x} + \bar{y} + z)$
1	1	1	1	

Focus on the '**0**' entries

$$F = M2 \cdot M4 \cdot M6 = \prod (2, 4, 6) = (x + y \mp z)(x + \bar{y} + z)(x + y \mp z)$$

Product-Of-Maxterm Examples

1. $F(a, b, c, d) = \prod(1, 3, 6, 11)$

$F(a, b, c, d) = M_1 \cdot M_3 \cdot M_6 \cdot M_{11}$

$$(a+b+c+\bar{d})\ (a+b+\bar{c}+\bar{d})\ (a+\bar{b}+\bar{c}+d)\ (\bar{a}+b+\bar{c}+\bar{d})$$

2. $G(a, b, c, d) = \prod(0, 4, 12, 15)$

$G(a, b, c, d) = M_0 \cdot M_4 \cdot M_{12} \cdot M_{15}$

$$(a+b+c+d)\ (a+\bar{b}+c+d)\ (\bar{a}+\bar{b}+c+d)\ (\bar{a}+\bar{b}+\bar{c}+\bar{d})$$

Conversions Between Canonical Forms

x	y	z	F	Minterm	Maxterm
0	0	0	**0**		$M_0 = (x + y + z)$
0	0	1	**1**	$m_1 = \bar{x}\,\bar{y}\,z$	
0	1	0	**1**	$m_2 = \bar{x}\,y\,\bar{z}$	
0	1	1	**1**	$m_3 = \bar{x}\,y\,z$	
1	0	0	**0**		$M_4 = (\bar{x} + y + z)$
1	0	1	**1**	$m_5 = x\,\bar{y}\,z$	
1	1	0	**0**		$M_6 = (\bar{x} + \bar{y} + z)$
1	1	1	**1**	$m_7 = x\,y\,z$	

$$F = m_1 + m_2 + m_3 + m_5 + m_7 = \sum(1, 2, 3, 5, 7) =$$

$$\bar{x}\,\bar{y}\,z + \bar{x}\,y\,\bar{z} + \bar{x}\,y\,z + x\,y\,\bar{z} + x\,y\,z$$

$$F = M_0 \cdot M_4 \cdot M_6 = \prod(0, 4, 6) = (x+y+z)(\bar{x}+y+z)(\bar{x}+\bar{y}+z)$$

Algebraic Conversion to Sum-of-Minterms

- Expand all terms first to explicitly list all minterms
- AND any term missing a variable v with $(v + \bar{v})$
- Example 1: $f = x + \bar{x}\,\bar{y}$ (2 variables)

$f = x\,(y + \bar{y}) + \bar{x}\,\bar{y}$

$f = x\,y + x\,\bar{y} + \bar{x}\,\bar{y}$

$f = m_3 + m_2 + m_0 = \sum(0, 2, 3)$

- Example 2: $g = a + \bar{b}\,c$ (3 variables)

$g = a\,(b + \bar{b})(c + \bar{c}) + (a + \bar{a})\,\bar{b}\,c$

$g = a\,b\,c + a\,b\,\bar{c} + a\,\bar{b}\,c + a\,\bar{b}\,\bar{c} + a\,\bar{b}\,c + \bar{a}\,\bar{b}\,c$

$g = \bar{a}\,\bar{b}\,c + a\,\bar{b}\,\bar{c} + a\,\bar{b}\,c + a\,b\,\bar{c} + a\,b\,c$

$g = m_1 + m_4 + m_5 + m_6 + m_7 = \sum(1, 4, 5, 6, 7)$

Conversions Between Canonical Forms

References:
- Digital Principles and Applications, A.P. Malvino, D. P. Leach and Saha, 7th Ed., 2011, Tata McGraw Hill
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- Modern Digital Electronics, R P Jain, 4th Ed., 2010, Tata McGraw Hill