



UNIT V

BACKTRACKING AND BRANCH-AND-BOUND

Backtracking – N-Queens Problem – Hamiltonian Circuit
Problem – Subset Sum Problem – Branch-and- Bound –
Travelling Salesman Problem



Finding Hamiltonian Circuits in Graphs

A Hamiltonian circuit or tour of a graph is a path that starts at a given vertex, visits each vertex in the graph exactly once, and ends at the starting vertex. Some graphs do not contain Hamiltonian circuits.

- Problem: Find a Hamiltonian circuit in a graph $G = (V, E)$
 - Sub-problem: Does G contain a Hamiltonian circuit?
 - No known easy algorithm for checking this...
- One solution: Search through *all paths* to find one that visits each vertex exactly once
 - Can use your favorite graph search algorithm (DFS!) to find various paths
- This is an *exhaustive search* (“brute force”) algorithm
- Worst case \rightarrow need to search all paths **How many paths??**



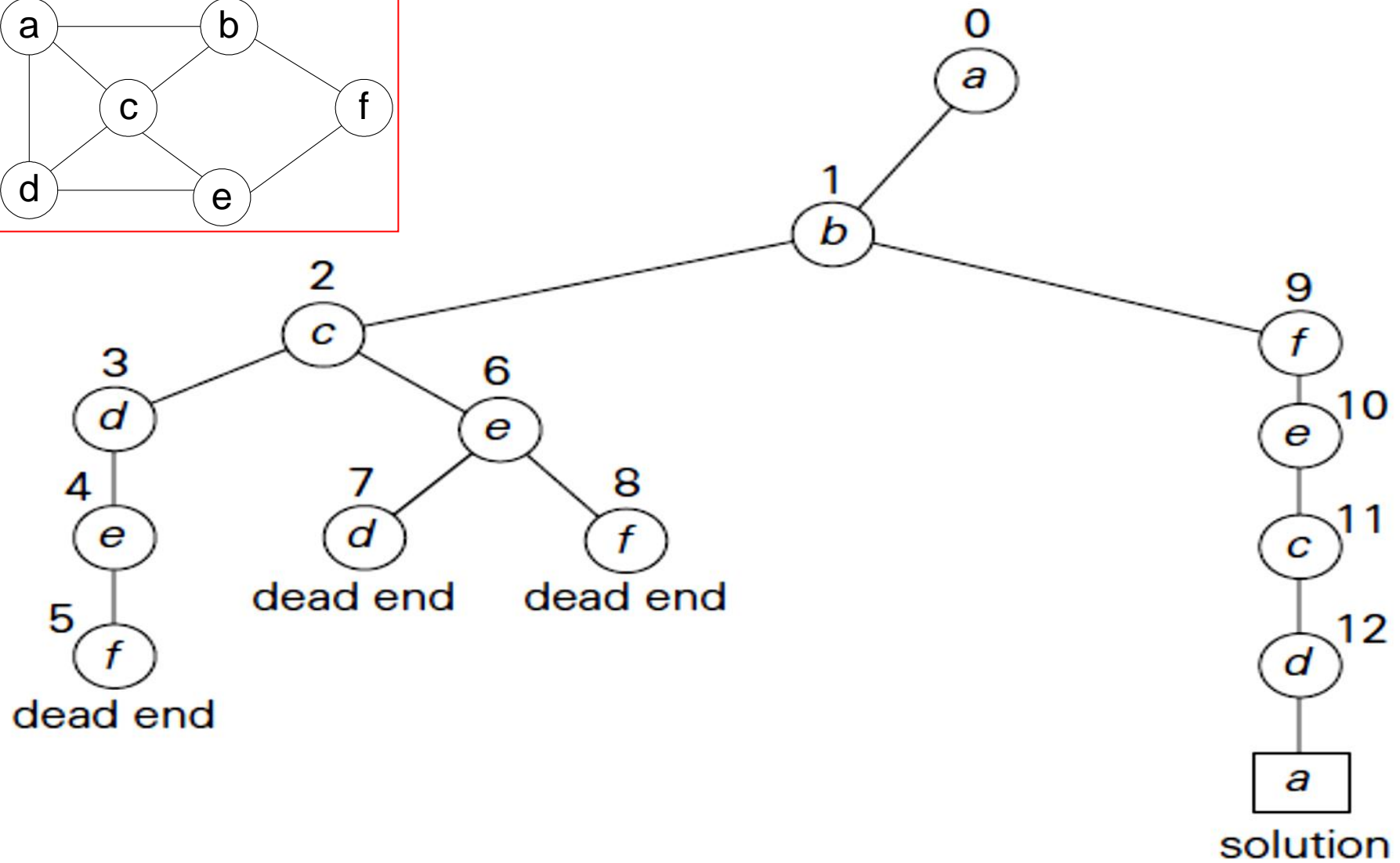
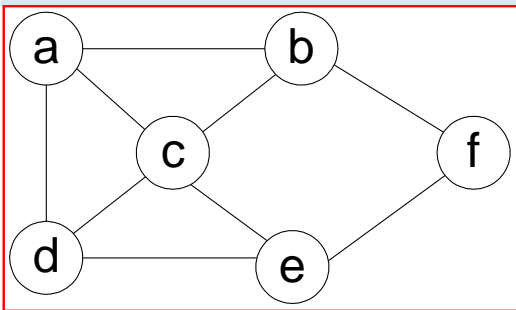
Hamilton Circuits and Paths

- A circuit that visits every vertex of a graph is called a Hamilton circuit.
- A path that visits every vertex of a graph is called a Hamilton path.

The three Rules

- **Rule 1.** If a vertex x has degree 2, both edges incident to x must be part of any Hamilton circuit.
- **Rule 2.** No proper subcircuit can be formed when building a Hamilton circuit.
- **Rule 3.** Once the Hamilton circuit is required to use two edges at a vertex x , all other edges incident to x must be removed from consideration.

Example: Hamiltonian Circuit Problem

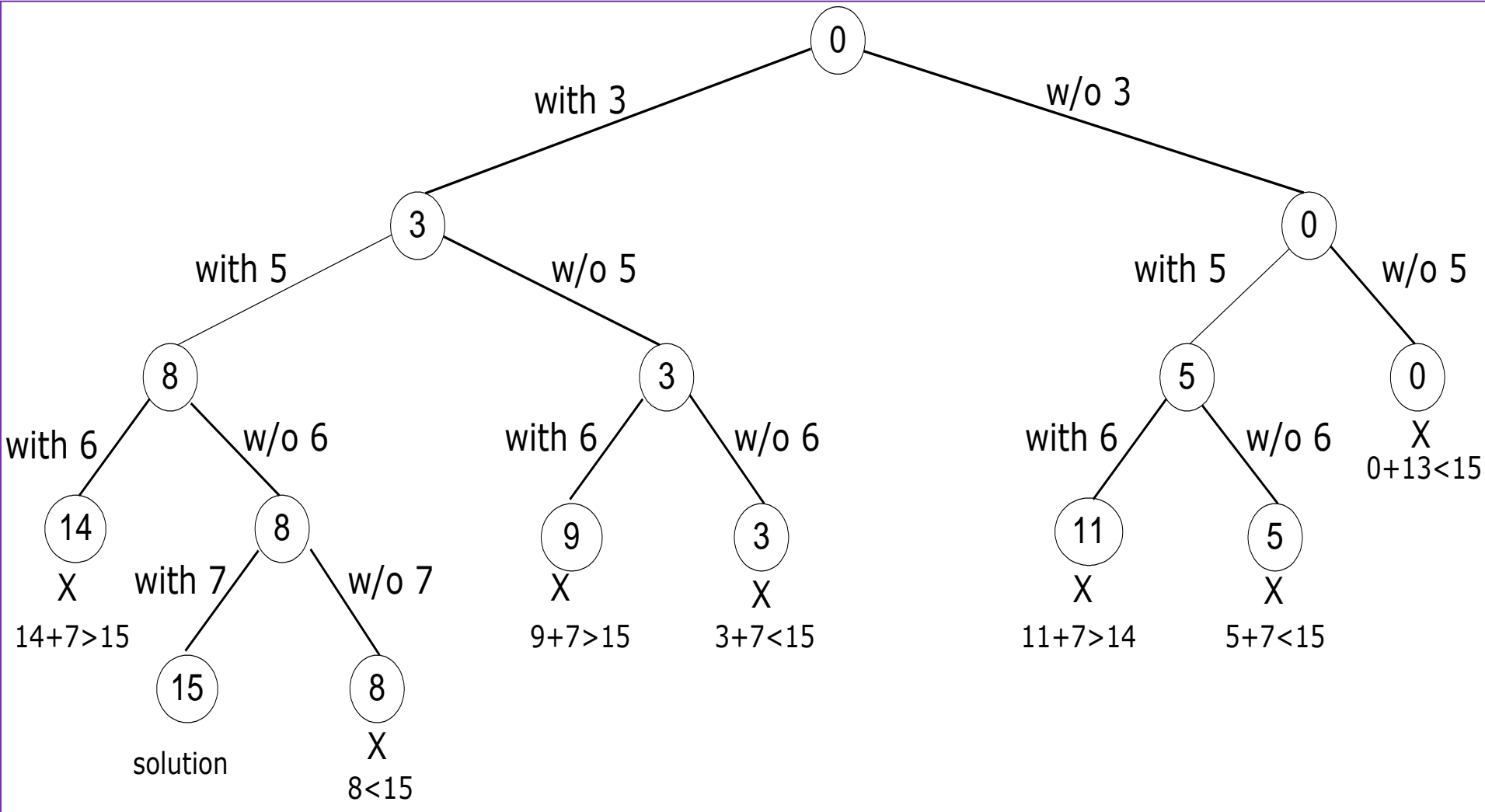




Subset Sum Problem

- ❖ As our last example, we consider the subset-sum problem: find a subset of a given set $A = \{a_1, \dots, a_n\}$ of n positive integers whose sum is equal to a given positive integer d .
- ❖ For example, for $A = \{1, 2, 5, 6, 8\}$ and $d = 9$ there are two solutions: $\{1, 2, 6\}$ and $\{1, 8\}$ Of course, some instances of this problem may have no solutions.
- ❖ It is convenient to sort the set's elements in increasing order. So, we will assume that $a_1 < a_2 < \dots < a_n$.
- ❖ The state-space tree can be constructed as a binary tree like that in For the instance $A = \{3, 5, 6, 7\}$ and $d = 15$.

Example: Subset Sum Problem {3,5,6,7} sum=15





Thank You