MMF Force and Torque

Acknowledgement: *The materials presented in this lecture has been taken from open source, reference books etc. This can be used only for student welfare and academic purpose.*

Recap

- Electromechanical energy conversion
- Energy balance
- Types of magnetic systems
- Magnetic Field Energy Stored
- Concept of Co-energy
- Magnetic force

Lecture-11 Objectives

- Torque in Rotational System
- Multiply Excited Magnetic System
- MMF of Distributed AC Windings

Torque in Rotational System

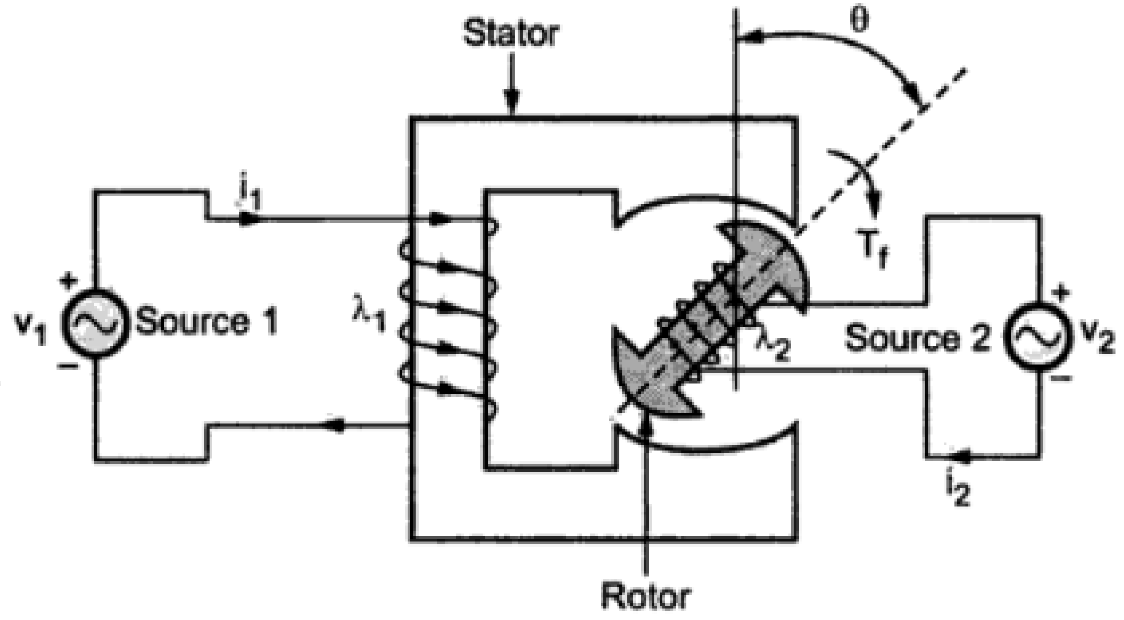
- In rotational system, force is replaced by torque and linear displacement dx is replaced by angular displacement dθ.

$$T_f = \frac{\partial W_f'(i,\theta)}{\partial \theta}$$

$$T_f = -\frac{\partial W_f(\lambda,\theta)}{\partial \theta}$$

Multiply Excited Magnetic System

- These systems are used where continuous energy conversion occurs.
- Example: motors, alternators etc…
- The doubly excited system has two independent sources of excitations.
- Due to the 2 sources, there are two sets of 3 independent variables.
- They are (λ_1, λ_2, θ) and (i_1, i_2, θ)



Case 1: Independent variables are ($\lambda_1, \lambda_2, \theta$)

- We know that,

$$T_f = -\frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \theta} -----1$$

- The field energy is,

$$W_f(\lambda_1, \lambda_2, \theta) = \int_0^{\lambda_1} i_1 d\lambda_1 + \int_0^{\lambda_2} i_2 d\lambda_2 -----2$$

$$\lambda_1 = L_{11} i_1 + L_{12} i_2 ----3$$

$$\lambda_2 = L_{12} i_1 + L_{22} i_2 ----4$$

MMF Force and Torque

- Solve eqn 3 and eqn 4 to express i_1 and i_2 in terms of λ_1 and λ_2.
- Multiply eqn 3 by L_{12} and eqn4 by L_{11},

$$L_{12}\lambda_1 = L_{12}L_{11}i_1 + L_{12}^2 i_2 ---- -5$$

$$L_{11}\lambda_2 = L_{11}L_{12}i_1 + L_{11}L_{22}i_2 --- -6$$

- Subtracting eqn 6 from eqn 5,

$$L_{12}\lambda_1 - L_{11}\lambda_2 = L_{12}L_{11}i_1 + L_{12}^2 i_2 - L_{11}L_{12}i_1 - L_{11}L_{22}i_2$$

$$L_{12}\lambda_1 - L_{11}\lambda_2 = (L_{12}^2 - L_{11}L_{22})i_2$$

$$i_2 = \left(\frac{L_{12}}{L_{12}^2 - L_{11}L_{22}}\right)\lambda_1 - \left(\frac{L_{11}}{L_{12}^2 - L_{11}L_{22}}\right)\lambda_2$$

MMF Force and Torque

$$i_2 = \beta_{21}\lambda_1 + \beta_{22}\lambda_2$$

- Similarly i_1 can be expressed in terms of λ_1 and λ_2 as,

$$i_1 = \beta_{11}\lambda_1 + \beta_{12}\lambda_2$$

$$\beta_{11} = \frac{L_{22}}{L_{11}L_{22} - L_{12}^2}$$

$$\beta_{22} = \frac{L_{11}}{L_{11}L_{22} - L_{12}^2}$$

$$\beta_{12} = \beta_{21} = -\frac{L_{12}}{L_{11}L_{22} - L_{12}^2}$$

MMF Force and Torque

- From eqn 2,

$$W_f(\lambda_1, \lambda_2, \theta) = \int_0^{\lambda_1} (\beta_{11}\lambda_1 + \beta_{12}\lambda_2)d\lambda_1 + \int_0^{\lambda_2} (\beta_{21}\lambda_1 + \beta_{22}\lambda_2)d\lambda_2$$

$$W_f(\lambda_1, \lambda_2, \theta) = \frac{1}{2}\beta_{11}\lambda_1^2 + \beta_{12}\lambda_1\lambda_2 + \frac{1}{2}\beta_{22}\lambda_2^2$$

- The self and mutual inductances of the coils depend on the angular position θ of the rotor.

Case 2: Independent variables are (i_1, i_2, θ)

- We know that,

$$T_f = \frac{\partial W_f'(i_1, i_2, \theta)}{\partial \theta} ----- -7$$

- The co – energy is given by,

$$W_f'(i_1, i_2, \theta) = \int_0^{i_1} \lambda_1 di_1 + \int_0^{i_2} \lambda_2 di_2 ----- -8$$

$$\lambda_1 = L_{11} i_1 + L_{12} i_2 --- -3$$

$$\lambda_2 = L_{12} i_1 + L_{22} i_2 --- -4$$

MMF Force and Torque

$$W'_f(i_1, i_2, \theta) = \frac{1}{2}L_{11}i_1^2 + L_{12}i_1i_2 + \frac{1}{2}L_{22}i_2^2$$

- Force in a doubly excited system,

$$F = \frac{\partial W'_f(i_1, i_2, \theta)}{\partial \theta}$$

$$F = \frac{\partial}{\partial \theta}\left[\frac{1}{2}L_{11}i_1^2 + L_{12}i_1i_2 + \frac{1}{2}L_{22}i_2^2\right]$$

MMF Force and Torque

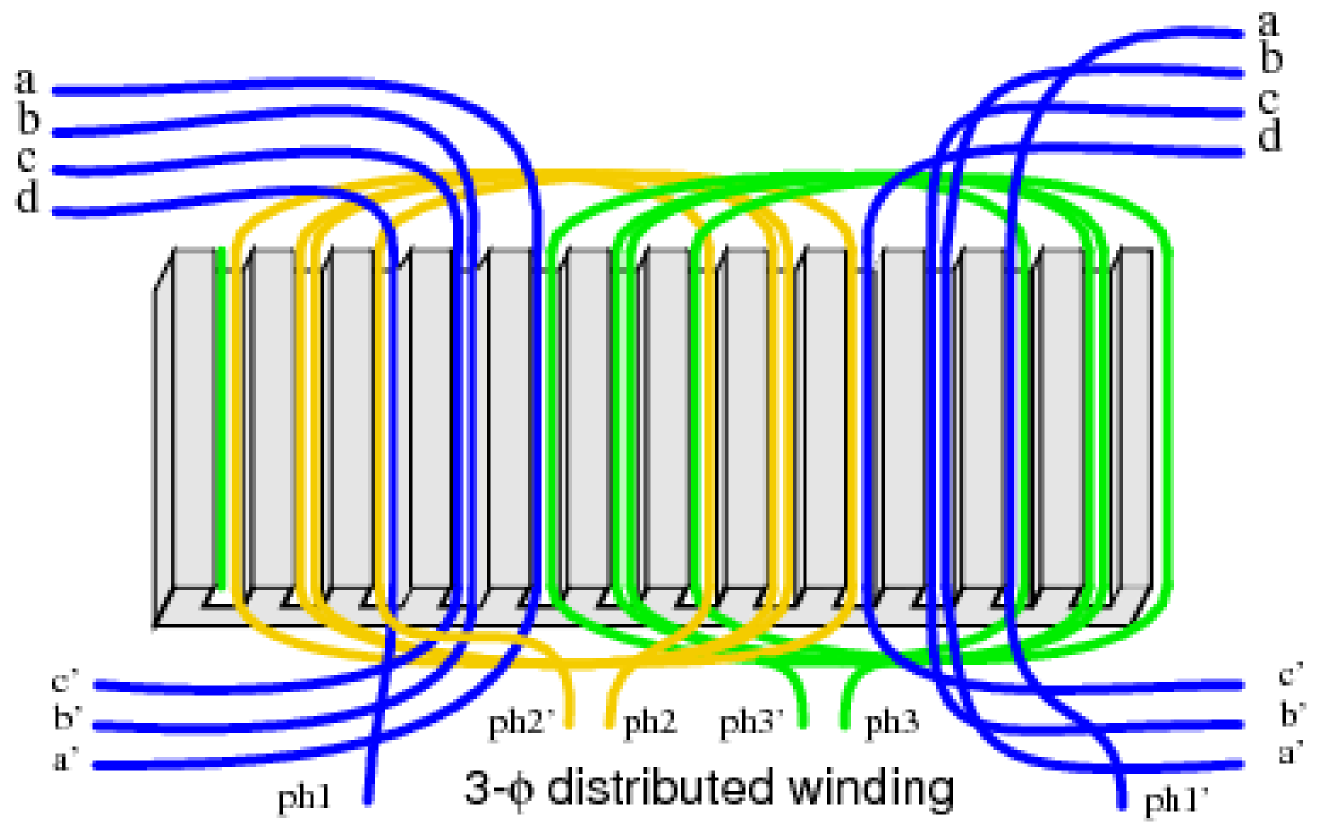
Two coupled coils have self and mutual inductance of

$$L_{11} = 2 + \frac{1}{2x}; L_{22} = 1 + \frac{1}{2x}; L_{12} = L_{21} = \frac{1}{2x}$$

over a certain range of linear displacement of x. The first coil is excited by a constant current of 20 A and the second by a constant current of –10 A. Find mechanical work done if x changes from 0.5 to 1 m and also the energy supplied by each electrical source.

MMF of Distributed AC Windings

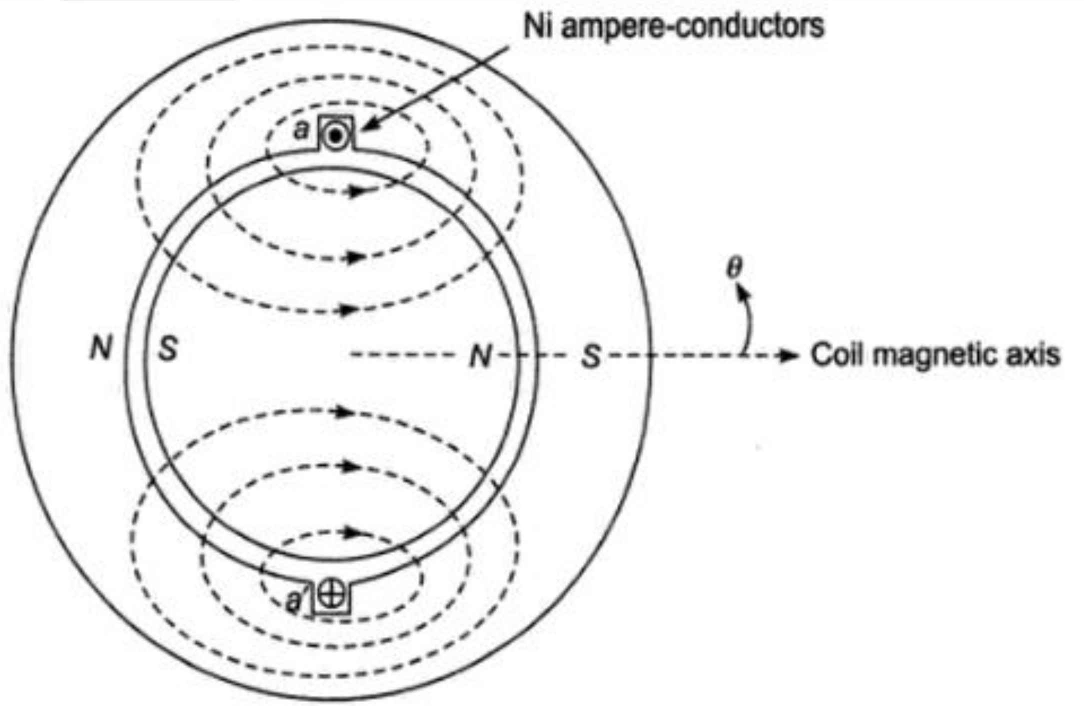
- The armature of any machine has distributed winding wound for the same number of poles as the field winding.



MMF of a Single Coil

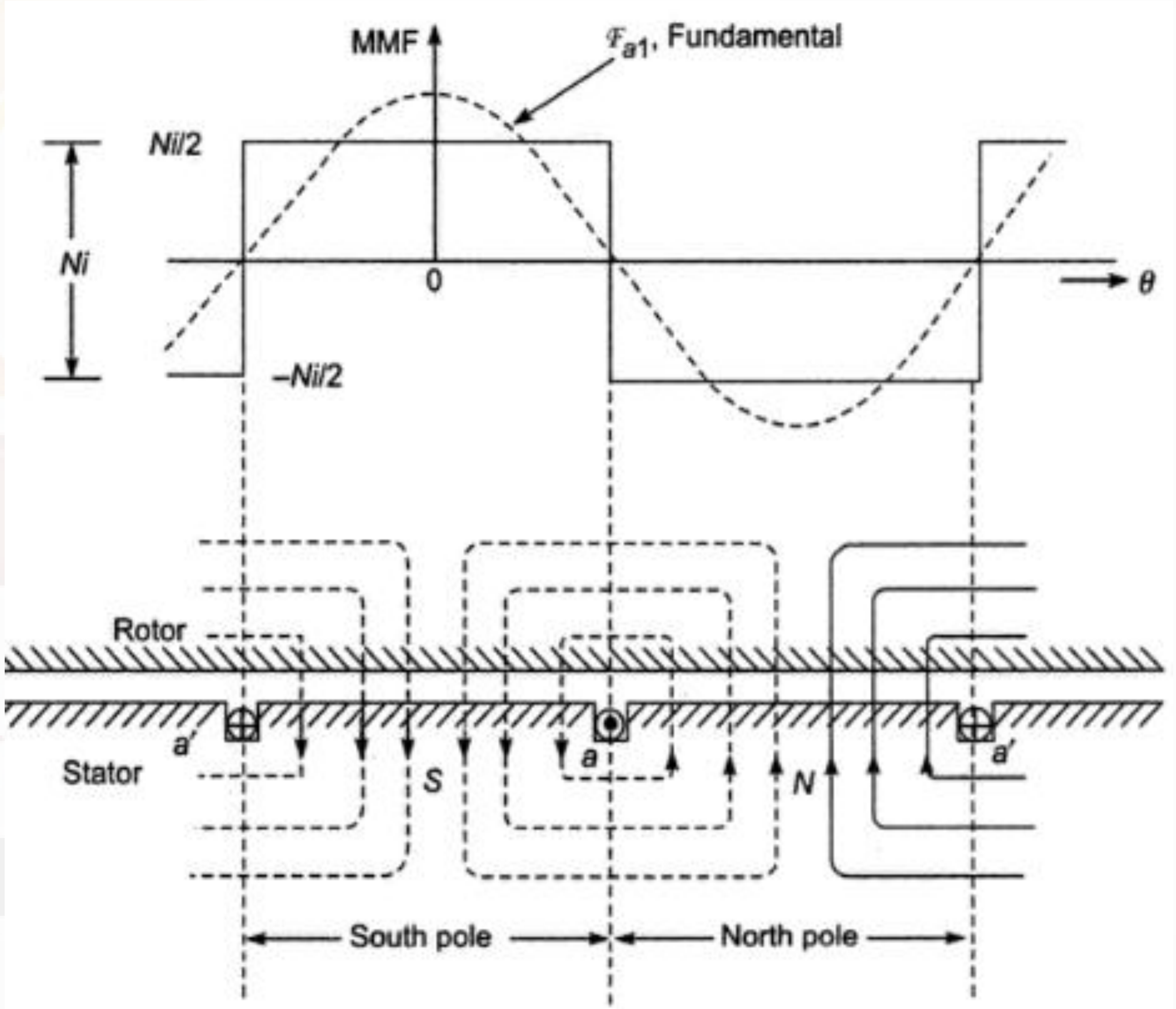
- Assume a cylindrical rotor machine with a small air-gap.
- The stator is wound for 2 poles with a single N turn coil carrying a current of ***i*** amps.

- The MMF produced by the single coil is Ni.
- This MMF creates a flux and each flux line crosses the air-gap radially twice.
- Half of the MMF is used to create flux from stator to rotor and other half is used to create flux from rotor to stator.



MMF of a Single Coil

- In the developed diagram shown, the stator is laid down with the rotor on the top of it.
- The shape of the MMF is seen to be rectangular.
- +Ni/2 is consumed in setting up flux from rotor to stator and –Ni/2 is consumed in setting up flux stator to rotor.



MMF Force and Torque

- MMF produced by the coil changes between +Ni/2 and –Ni/2 abruptly.
- Using fourier analysis, the fundamental component of MMF can be found as,

$$\mathcal{F}_{a1} = \frac{4}{\pi}\frac{Ni}{2}\cos\theta = F_{1p}\cos\theta$$

$$F_{1p} = \frac{4}{\pi}\frac{Ni}{2}$$

MMF of a Distributed Winding

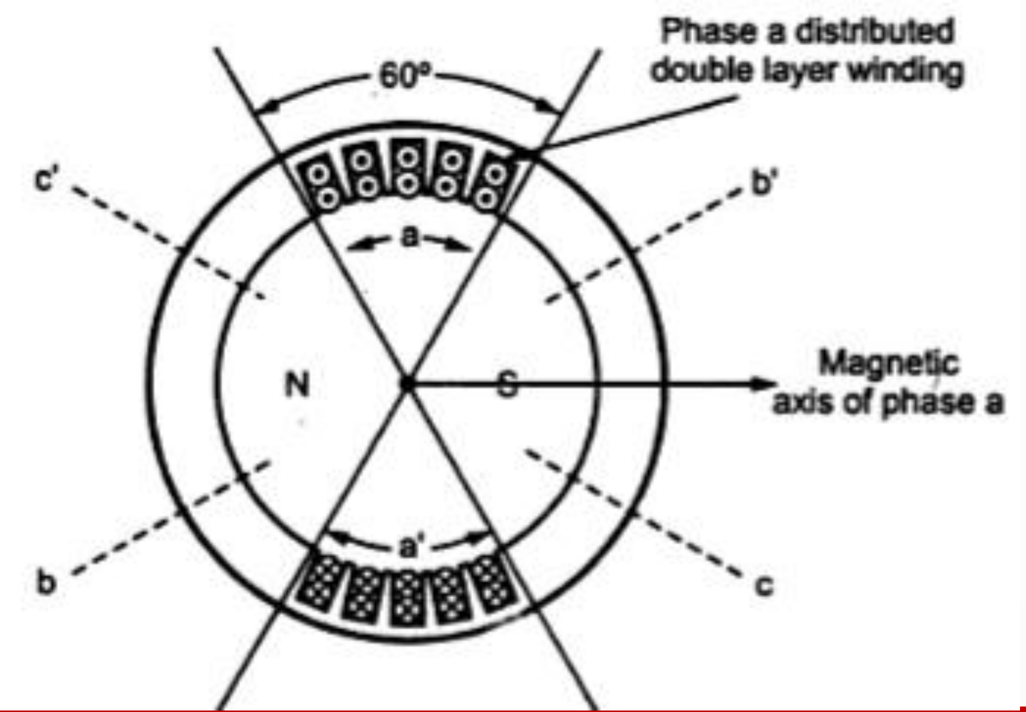
- Consider a 2 pole, cylindrical rotor with,

$$m = \text{slots/pole/phase} = 5$$

$$n = \text{slots/pole} = 5\text{x}3 = 15$$

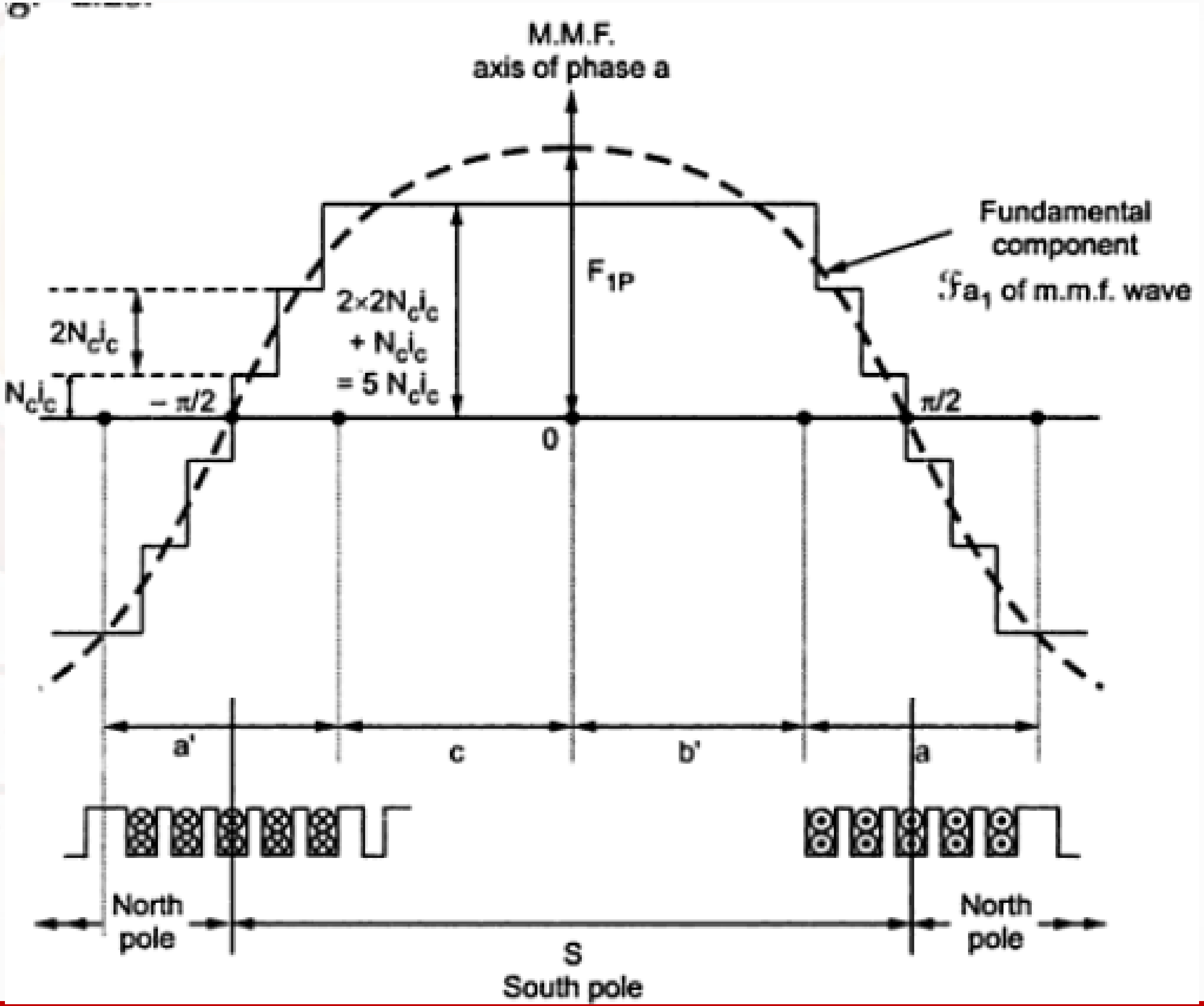
- The distributed winding for phase A, occupying 5 slots per pole is shown below.

- Let N_C = turns in a coil
- i_C = conductor current
- M.M.F in 1 slot = 2. $N_C.i_C$
- As the No. of slots are odd, half of the ampere conductors produce south pole and remaining half produce north pole on stator.



MMF Force and Torque

- At each slot, the MMF wave has a step jump of 2 $N_C i_C$.
- Total MMF produced in 5 slots is $10N_C i_C$.
- Half of this total MMF is used to set up flux from rotor to stator and remaining half is used to create flux from stator to rotor.
- Now F_{1P}, the peak of fundamental waveform has to be determined.



MMF Force and Torque

Let

- T_{ph} = series turns per parallel path of a phase.
- A = Number of parallel paths.

$$Ampere\ turn\ per\ parallel\ path\ = T_{ph} \times i_c$$

$$Ampere\ turn\ per\ phase\ = A[T_{ph} \times i_c]$$

$$Total\ current\ in\ one\ phase, i_a = A \times i_c$$

$$Ampere\ turn/phase = T_{ph} \times i_a$$

$$Ampere\ turn/pole/phase = \frac{T_{ph} \times i_a}{P}$$

MMF Force and Torque

- Using fourier analysis, the equation for mmf wave is given by,

$$\mathcal{F}_{a1} = \frac{4}{\pi}\frac{T_{ph} \times i_a}{P} \cos\theta$$

- Because of short pitched and distributed winding, the mmf gets reduced by a factor K_p and K_d. Hence the equation for mmf wave is given by,

$$\mathcal{F}_{a1} = \left(\frac{4}{\pi} K_p K_d \frac{T_{ph} \times i_a}{P}\right) \cos\theta = F_{1p} \cos\theta$$

Summary

- Torque in Rotational System
- Multiply Excited Magnetic System
- MMF of Distributed AC Windings