Magnetic Energy Circuits

Acknowledgement: *The materials presented in this lecture has been taken from open source, reference books etc. This can be used only for student welfare and academic purpose.*

Prerequisite

- Electromagnetic Field Theory
- Network Theory

Lecture-1 Objectives

- Design of a magnetic circuit
- Significant of magnetic reluctance
- Its components and analogy with electrical circuit
- Faraday's law and Lenz law
- Right hand rule

Magnetic Energy Circuit

Electromagnetic system is an important element of all rotating electric machinery and static devices like transformer.

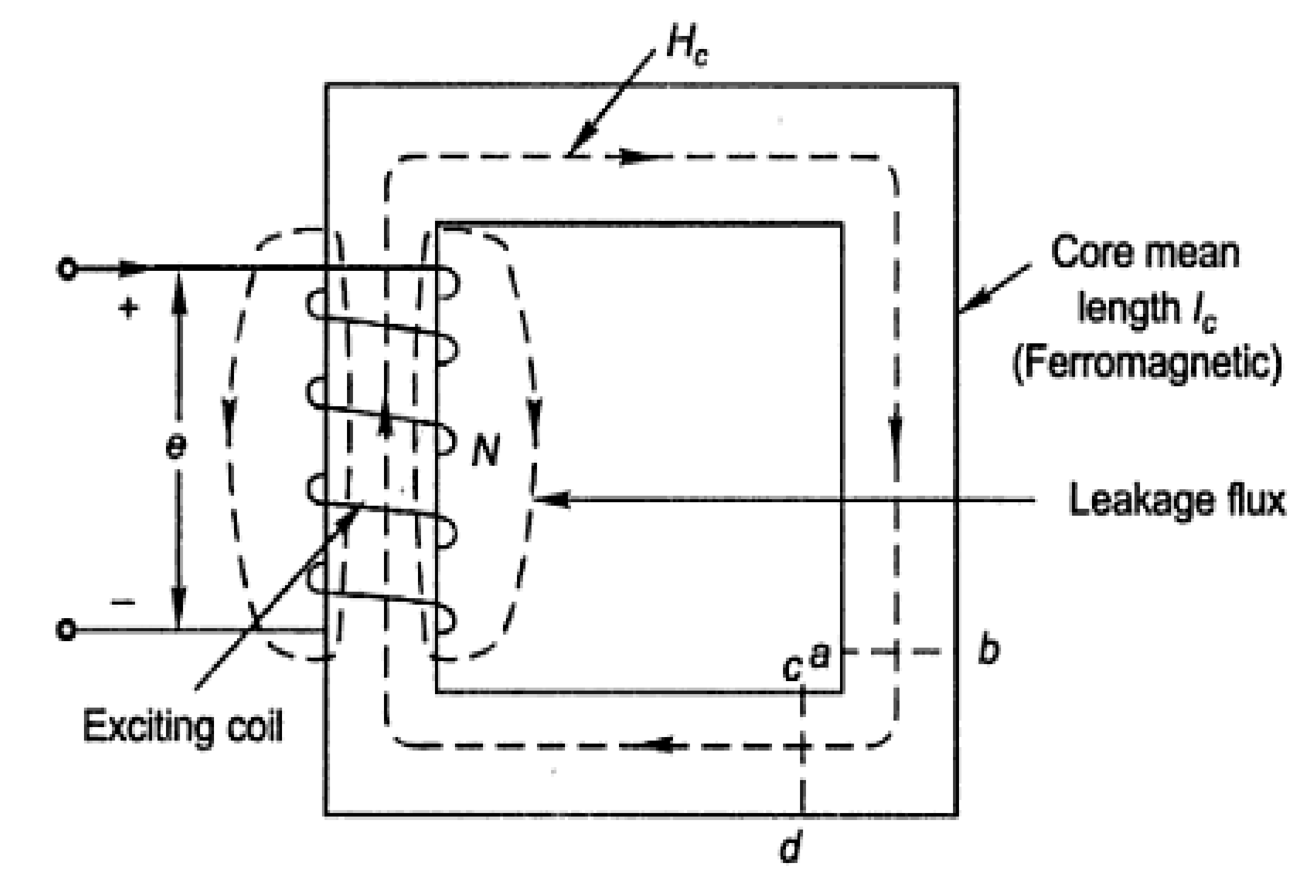
Role is to create & control electromagnetic fields for electromechanical energy conversion (EMEC) process.

EMEC happens with the help of magnetic field as a coupling medium.

The closed path followed by the magnetic flux is called a magnetic circuit.

Made up of materials having high permeability such as iron, soft steel etc.

Magnetic Energy Circuit



Electromagnetic system

- Ferromagnetic core
- Exciting coil
- Coil has N turns
- Coil carries a current of I amps
- Magnetic field established
- Magnetic flux flows through the core
- Small flux leaks through air

The magnetic field intensity produced in the core is ***H*** and from ampere circuital law,

$$\oint H.dl = N\,I$$

$$H.l = N\,I$$

$$H = \frac{N\,I}{l}\ AT/_m \ \text{------}\ -1$$

Magnetic field intensity ***H*** causes a flux density ***B*** to be set up in the magnetic core. It is given by,

$$B = \mu\,H \ \text{------}\ -2$$

$$B = \mu_0\mu_r H$$

Sub equation 1 in equation 2,

$$B = \mu \frac{N\,I}{l} \text{ ------ } -3$$

Flux flowing through the core is given by,

$$\phi = B\,.\,A \text{ ------ } -4$$

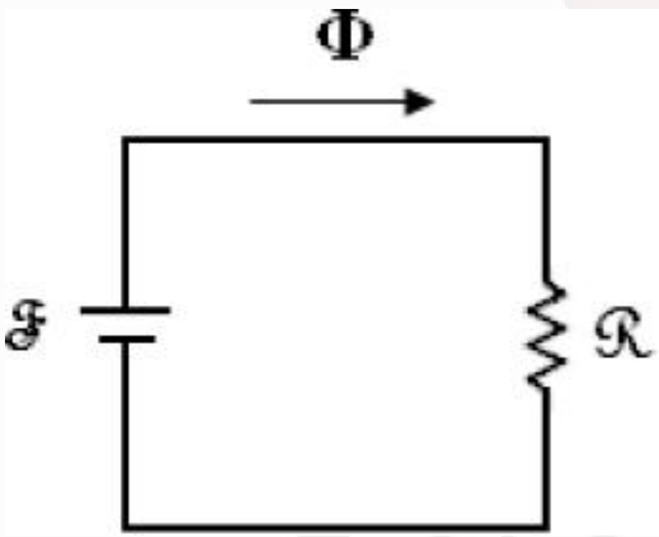
Where ***B*** is the average flux density and ***A*** is the area of cross section of the core.

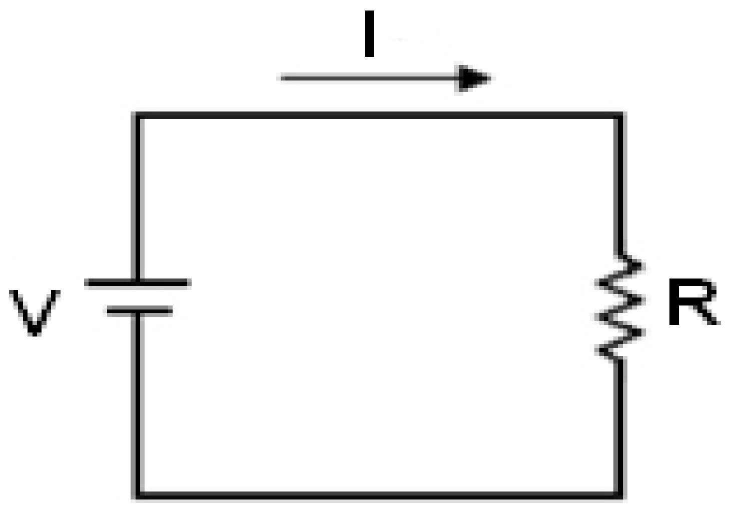
Substituting equation 3 in equation 4, we get,

$$\phi = \mu \frac{N\,I}{l} A$$

$$\phi = \frac{N\,I}{\left({}^{l}/_{\mu.A}\right)} = \frac{N\,I}{\mathcal{R}} = \frac{\mathcal{F}}{\mathcal{R}}$$

Magnetic Circuit and Electric Circuit



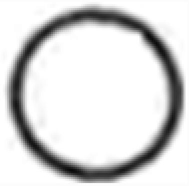


Comparison of Magnetic and Electric Circuits

Magnetic Circuit	Electric Circuit
Hopkinson's Law $\left(\phi = \frac{\mathcal{F}}{\mathcal{R}}\right)$	Ohm's Law $\left(I = \frac{V}{R}\right)$
$\text{Reluctance}, \mathcal{R} = \frac{\ell}{\mu.\text{A}}$	$\text{Resistance}, \text{R} = \frac{\ell}{\sigma.\text{A}}$
Flux (ϕ)	Current (I)
MMF ($\mathcal{F}$)	EMF (V)
Permeability (μ)	Conductivity (σ)
Permeance ($\mathcal{P}$)	Conductance (G)

Direction of Current in a Conductor

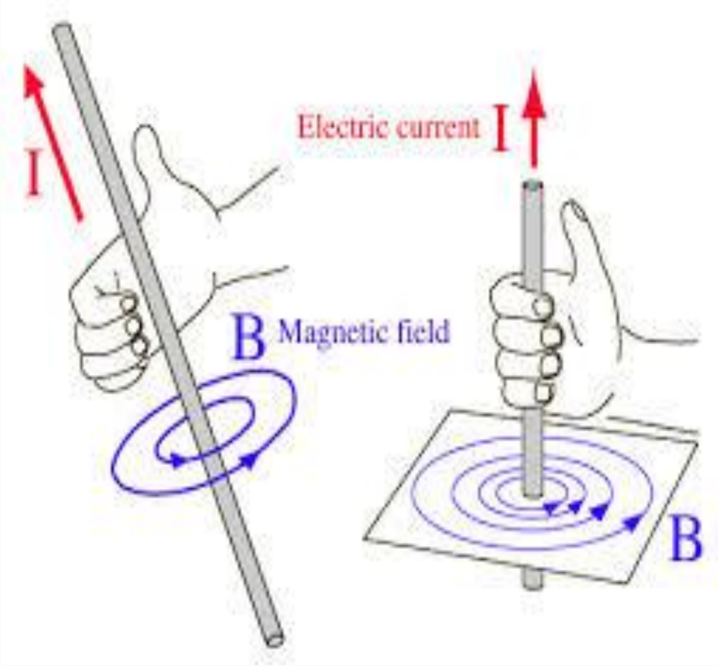
- No current through the conductor.
- Conductor carries current away from the reader.
- Conductor carries current towards the reader.

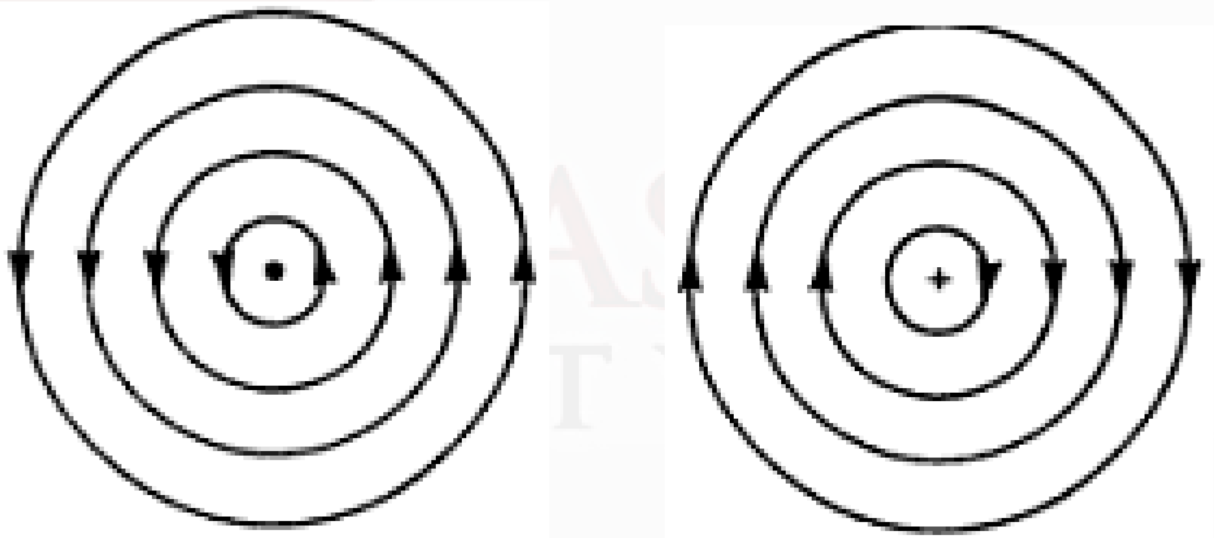


Right Hand Rule

The direction of magnetic flux is found by using **right hand rule**.

Rule says that if one holds the conductor in such a way that the thumb points in the direction of current, then the closed fingers give the direction of flux produced.





Faradays Law

Whenever there is a variation of magnetic flux linking with a coil, an EMF is induced in that coil.

The magnitude of this EMF is proportional to the rate of change of flux linkages.

$$Induced\ EMF, e = -N\frac{d\phi}{dt} = -\frac{d\lambda}{dt}$$

Lenz’s Law

Lenz’s law states that the induced EMF in a coil will induce a current whose direction is such that it opposes the cause producing the EMF.

Given Data

$$A = A_g = A_A = A_B = A_C = 0.001 \text{ m}^2$$

$$l_A = 0.3 \text{ m}, \; l_B = 0.2 \text{ m}, \; l_C = 0.1 \text{ m} \quad l_g = 0.1 \text{ mm} = 0.0001 \text{ m}$$

$$\mu_{r_A} = 5000, \quad \mu_{r_B} = 1000, \quad \mu_{r_C} = 10000$$

$$\phi = 7.5 \times 10^{-4} \text{ Wb}$$

Magnetic Energy Circuit

Air-gap and three sections form a series magnetic circuit.

Flux in the air-gap is same as that of the three sections.

Hence total mmf is the sum of mmf for each part of the magnetic circuit.

$$\text{(i) Total mmf} = H_g l_g + H_A l_A + H_B l_B + H_C l_C$$

$$= \frac{B}{\mu_0} l_g + \frac{B_A}{\mu_0 \mu_{r_A}} l_A + \frac{B_B}{\mu_0 \mu_{r_B}} l_B + \frac{B_C}{\mu_0 \mu_{r_C}} l_C$$

$$= \frac{\phi}{\mu_0 A_g} l_g + \frac{\phi}{\mu_0 \mu_{r_A} A_A} l_A + \frac{\phi}{\mu_0 \mu_{r_B} A_B} l_B + \frac{\phi}{\mu_0 \mu_{r_C} A_C} l_C$$

Solution

$$= \frac{\phi}{\mu_0 A}\left(l_g + \frac{l_A}{\mu_{r_A}} + \frac{l_B}{\mu_{r_B}} + \frac{l_C}{\mu_{r_C}}\right)$$

$$= \frac{7.5\times10^{-4}}{4\pi\times10^{-7}\times0.001}\left(0.0001 + \frac{0.3}{5000} + \frac{0.2}{1000} + \frac{0.1}{10000}\right)$$

$$= 220.83 \text{ AT}$$

(ii) Exciting current

$$\text{Total mmf} = NI$$

$$220.83 = 100 \times I$$

$$I = 2.2 \text{ A}$$

(iii) Reluctances of each section

$$S_A = \frac{l_A}{\mu_0 \mu_{r_A} A_A} = \frac{0.3}{4\pi \times 10^{-7} \times 5000 \times 0.001} = 47.75 \times 10^3 \text{ AT/Wb}$$

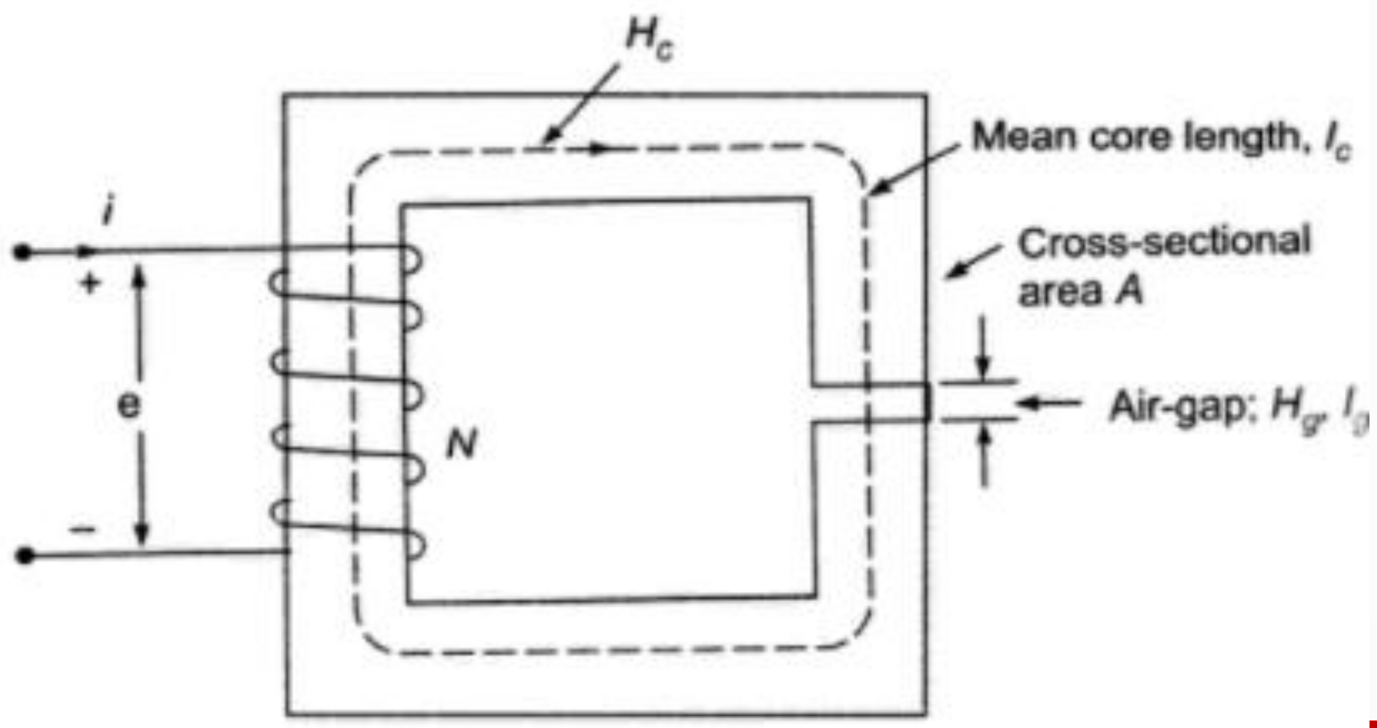
$$S_B = \frac{l_B}{\mu_0 \mu_{r_B} A_B} = \frac{0.2}{4\pi \times 10^{-7} \times 1000 \times 0.001} = 159.15 \times 10^3 \text{ AT/Wb}$$

$$S_C = \frac{l_C}{\mu_0 \mu_{r_C} A_C} = \frac{0.1}{4\pi \times 10^{-7} \times 10000 \times 0.001} = 7.96 \times 10^3 \text{ AT/Wb}$$

$$S_g = \frac{l_g}{\mu_0 \mu_r A_g} = \frac{0.1 \times 10^{-3}}{4\pi \times 10^{-7} \times 1 \times 0.001} = 79.58 \times 10^3 \text{ AT/Wb}$$

Magnetic Energy Circuit

The magnetic circuit has dimensions: $A_C = 4 \times 4\ cm^2$, $l_g = 0.06$ cm, $l_c = 40$ cm and N = 600 turns. Assume the value of $\mu_r = 6000$ for iron. Find the exciting current for $B_C = 1.2$ T and the corresponding flux and flux linkages.



Solution

$$Ni = \frac{B_c}{\mu_0 \mu_r} l_c + \frac{B_g}{\mu_0} l_g$$

Neglecting fringing

$A_c = A_g$ therefore $B_c = B_g$

$$i = \frac{B_c}{\mu_0 N}\left(\frac{l_c}{\mu_r} + l_g\right)$$

$$= \frac{1.2}{4\pi \times 10^{-7} \times 600}\left(\frac{40}{6000} + 0.06\right) \times 10^{-2}$$

$$= 1.06 \text{ A}$$

$$\phi = B_c A_c = 1.2 \times 16 \times 10^{-4} = 19.2 \times 10^{-4} \text{ Wb}$$

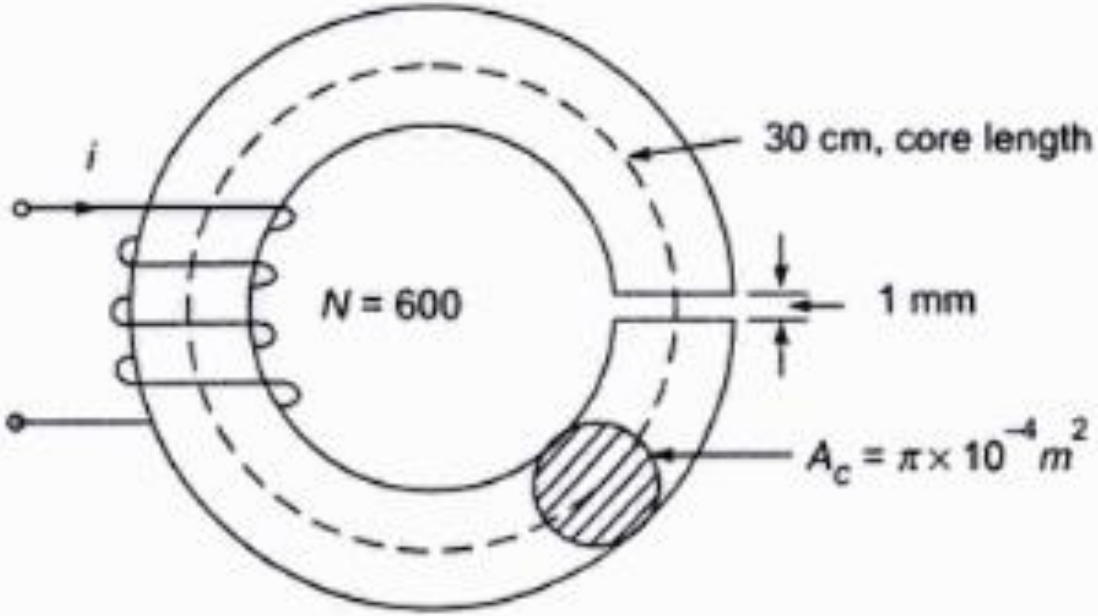
$$\lambda = N\phi = 600 \times 19.2 \times 10^{-4} = 1.152 \text{ Wb-turns}$$

A wrought iron bar 30 cm long and 2 cm in diameter is bent into a circular shape as shown in figure below. It is then wound with 600 turns of wire. Calculate the current required to produce a flux of 0.5 mWb in the magnetic circuit in the following cases:

(i) no air – gap

(ii) with an air-gap of 1 mm

(μ_r of iron = 4000)



(i) No Air-Gap

$$N.i = H_c l_c$$

$$N.i = \frac{B}{\mu_0 \mu_r} l_c$$

$$N.i = \frac{\phi . l_c}{A.\mu_0 .\mu_r}$$

$$i = \frac{\phi . l_c}{N.A.\mu_0 .\mu_r}$$

$$i = \frac{0.5 \times 10^{-3} \times 30 \times 10^{-2}}{600.\pi.(1 \times 10^{-2})^2 (4\pi \times 10^{-7}) \times 4000}$$

$$i = 0.158\ A$$

(ii) With Air-Gap

$$N.i = H_c l_c + H_g l_g$$

$$N.i = \frac{B}{\mu_0 \mu_r} l_c + \frac{B}{\mu_0} l_g$$

$$N.i = \frac{B}{\mu_0}\left(\frac{l_c}{\mu_r} + l_g\right)$$

$$N.i = \frac{\phi}{A.\mu_0}\left(\frac{l_c}{\mu_r} + l_g\right)$$

$$i = \frac{\phi}{N.A.\mu_0}\left(\frac{l_c}{\mu_r} + l_g\right)$$

$$i = \frac{0.5 \times 10^{-3}}{600.\pi.(1 \times 10^{-2})^2 (4\pi \times 10^{-7})}\left(\frac{30 \times 10^{-2}}{4000} + 1 \times 10^{-3}\right)$$

$$i = 2.2\ A$$

The magnetic circuit shown below has steel core with dimensions as shown.

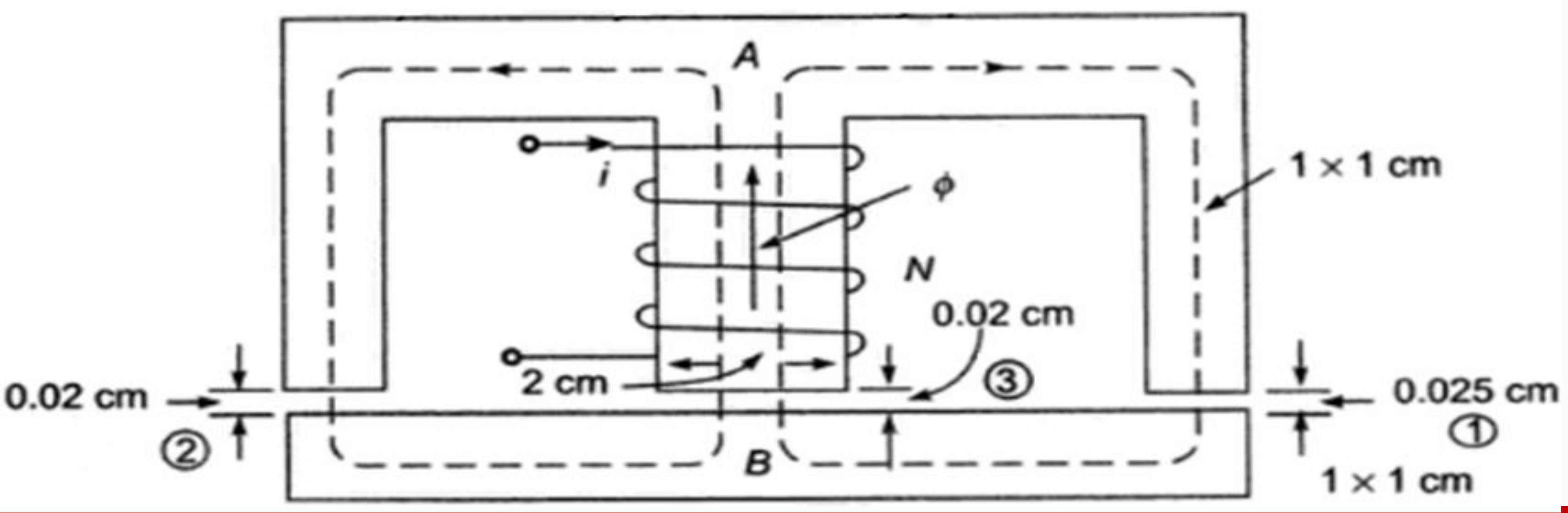
Mean length from A to B through either outer limb = 0.5 m

Mean length from A to B through central limb = 0.2 m

It is required to establish a flux of 0.75 mWb in the air-gap of the central limb. Determine the mmf of the exciting coil if the core material has

$$(a)\ \mu_r = \infty \qquad (b)\ \mu_r = 5000$$

Neglect fringing.

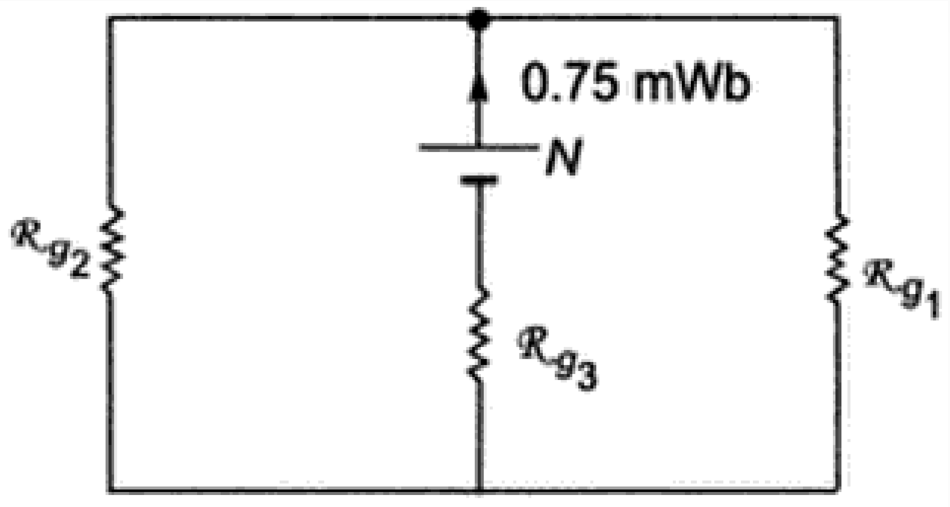


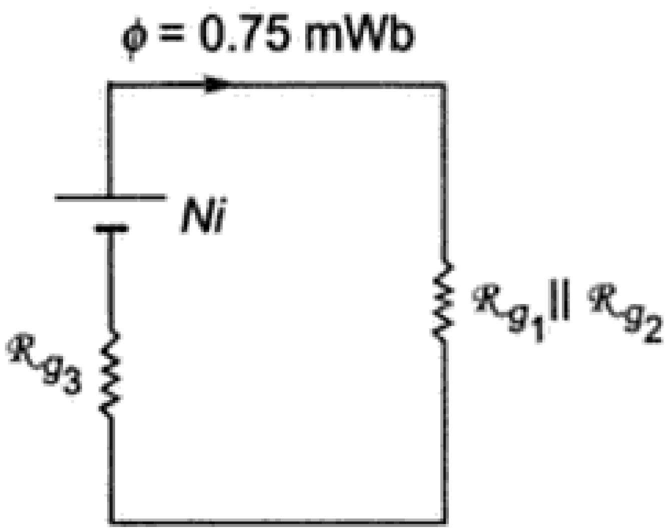
(a) $\mu_r = \infty$

$$\mathcal{R}_{g1} = \frac{0.025 \times 10^{-2}}{4\pi \times 10^{-7} \times 1 \times 10^{-4}} = 1.99 \times 10^{6}$$

$$\mathcal{R}_{g2} = \frac{0.02 \times 10^{-2}}{4\pi \times 10^{-7} \times 1 \times 10^{-4}} \quad 1.592 \times 10^{6}$$

$$\mathcal{R}_{g3} = \frac{0.02 \times 10^{-2}}{4\pi \times 10^{-7} \times 2 \times 10^{-4}} = 0.796 \times 10^{6}$$



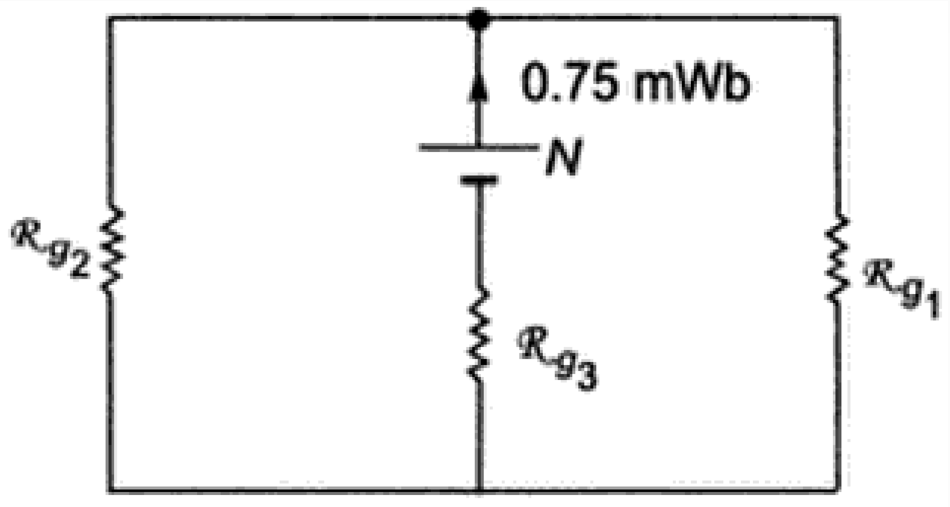


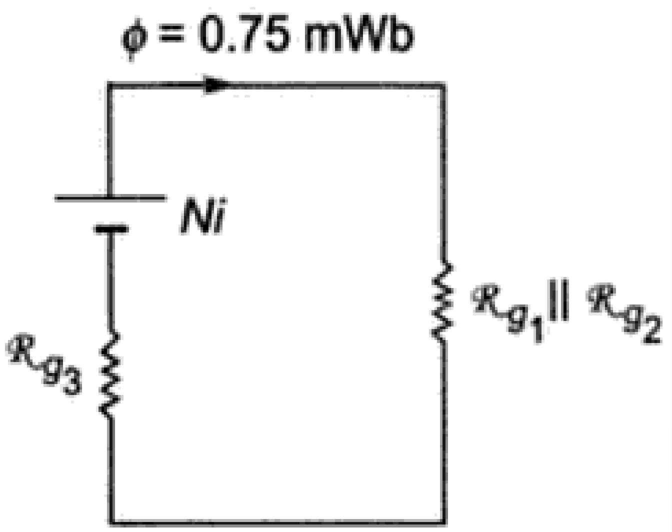
$(a)\ \mu_r = \infty$

$$Ni = 0.75 \times 10^{-3} (\mathcal{R}_{g3} + \mathcal{R}_{g1} \parallel \mathcal{R}_{g2})$$

$$= 0.75 \times 10^{-3} (0.796 + 0.844) \times 10^{6}$$

$$= 1230 \text{ AT}$$





$(b)\ \mu_r = 5000$

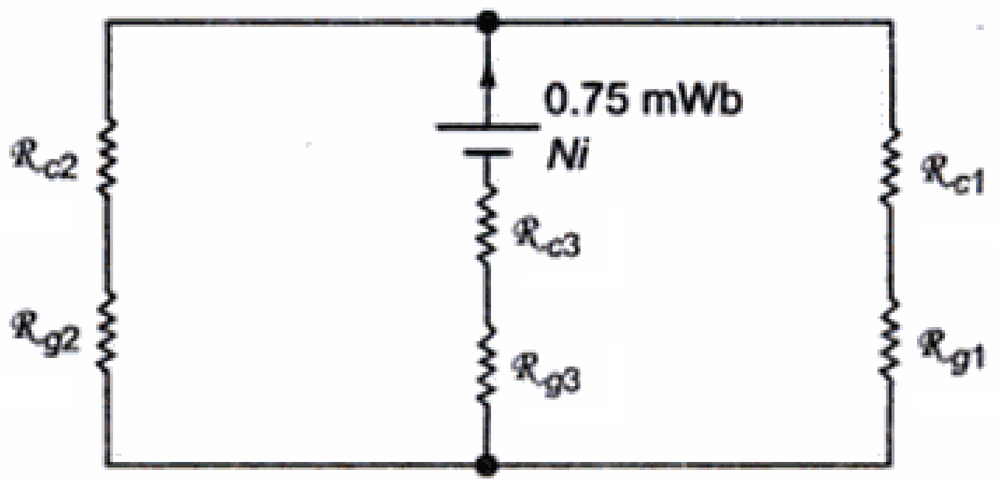
$$\mathcal{R}_{eq} = (\mathcal{R}_{c1} + \mathcal{R}_{g1}) \| (\mathcal{R}_{c2} + \mathcal{R}_{g2}) + \mathcal{R}_{c3} + \mathcal{R}_{g3})$$

$$\mathcal{R}_{c1} = \frac{0.5}{4\pi \times 10^{-7} \times 5000 \times 1 \times 10^{-4}} = 0.796 \times 10^6$$

$$\mathcal{R}_{c2} = \mathcal{R}_{c1} = 0.796 \times 10^6$$

$$\mathcal{R}_{c3} = \frac{0.2}{4\pi \times 10^{-7} \times 5000 \times 2 \times 10^{-4}} = 0.159 \times 10^6$$

$$\mathcal{R}_{eq} = \frac{27.86 \times 23.86}{51.72} \times 10^6 + 0.955 \times 10^6 = 1.955 \times 10^6$$



$$Ni = \phi \mathcal{R}_{eq}$$
$$= 0.75 \times 10^{-3} \times 1.955 \times 10^6$$
$$= 1466 \text{ AT}$$

Summary

Analogy of a magnetic circuit

Significant of magnetic reluctance

Its components and analogy with electrical circuit

Faraday’s law and Lenz law

Right hand rule