Inductances and Induced EMF

Acknowledgement: *The materials presented in this lecture has been taken from open source, reference books etc. This can be used only for student welfare and academic purpose.*

Recap

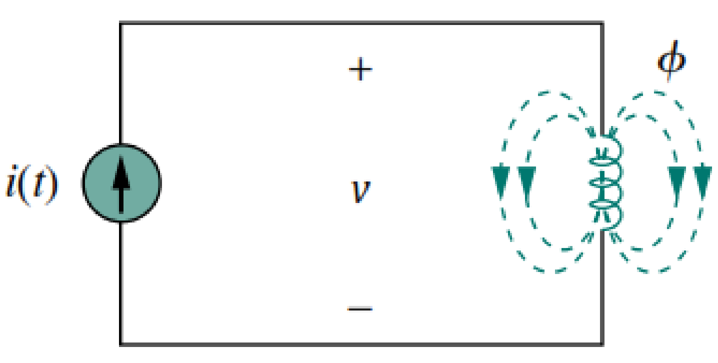
- Design of a magnetic circuit
- Significant of magnetic reluctance
- Its components and analogy with electrical circuit
- Faraday’s law and Lenz law
- Right hand rule

Lecture-2 Objectives

- Needs of magnetic coils
- Self inductance and Mutual inductances
- Dot convention in magnetic circuits
- Induced EMF and its types in Magnetic link circuits

Self Inductance

- Consider a coil with N turns.
- When current ***i*** flows through it, a flux $\boldsymbol{\phi}$ will be produced.
- As per Faraday's law,



$$V = N\frac{d\phi}{dt}$$

- Flux $\boldsymbol{\phi}$ is produced by current ***i*** and hence any change in $\boldsymbol{\phi}$ is caused by changes in ***i***. Therefore,

$$V = N\frac{d\phi}{di}\cdot\frac{di}{dt} \qquad V = L.\frac{di}{dt}$$

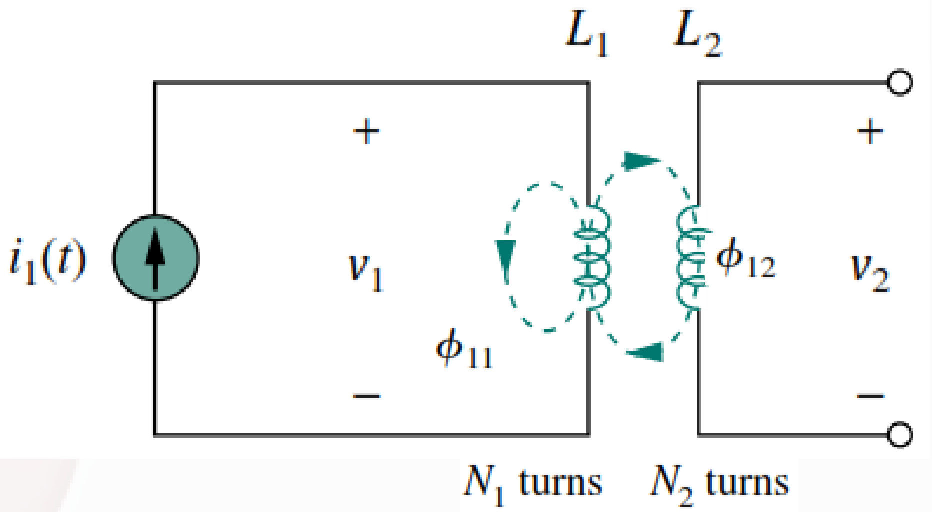
- The inductance ***L*** of the inductor is,

$$L = N\frac{d\phi}{di}$$

- This is called the ***Self Inductance*** of the coil.

Mutual Inductance

- Consider 2 coils with self inductances L_1 & L_2 are kept close together.
- Coil 1 has N_1 turns and coil 2 has N_2 turns.



- Current $\boldsymbol{i_1}$ creates a flux $\boldsymbol{\phi_1}$ in coil 1.
- This flux has got 2 components.
  - $\boldsymbol{\phi_{11}}$ links with coil 1 only.
  - $\boldsymbol{\phi_{12}}$ links with both the coils.

$$\phi_1 = \phi_{11} + \phi_{12}$$

- Although both coils are physically separated, they are magnetically coupled.

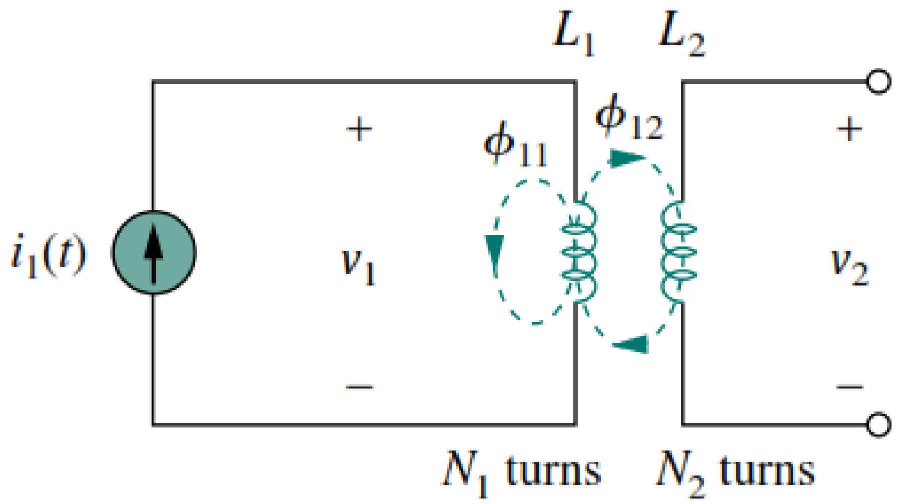
Mutual Inductance

- Voltage induced in coil 1 is,

$$V_1 = N_1 \frac{d\phi_1}{dt}$$

$$V_1 = N_1 \frac{d\phi_1}{di_1} \cdot \frac{di_1}{dt} \quad = L_1 \frac{di_1}{dt}$$

$$L_1 = N_1 \frac{d\phi_1}{di_1}$$



- L_1 is the self inductance of the coil.

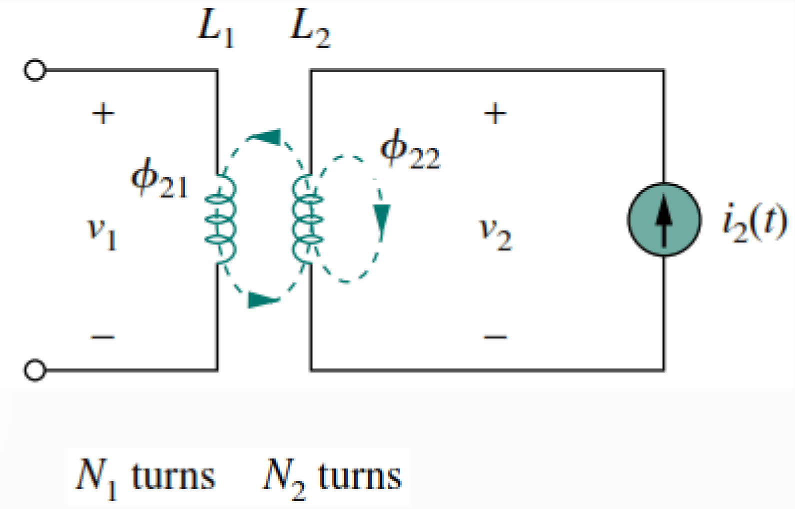
- Voltage induced in coil 2 is, $V_2 = N_2 \frac{d\phi_{12}}{dt}$

$$V_2 = N_2 \frac{d\phi_{12}}{di_1} \cdot \frac{di_1}{dt} \quad = M \frac{di_1}{dt} \qquad M = N_2 \frac{d\phi_{12}}{di_1}$$

- M is the mutual inductance between the 2 coils.

Mutual Inductance

- Now consider a current i_2 flows through coil 2 and produces a flux ϕ_2.

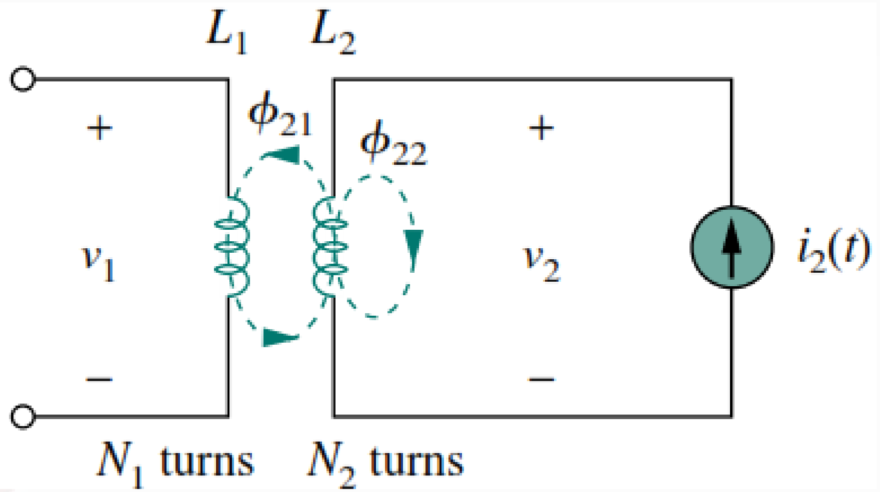


- This flux has got 2 components.
  - ϕ_{22} links with coil 1 only.
  - ϕ_{21} links with both the coils.

$$\phi_2 = \phi_{22} + \phi_{21}$$

Mutual Inductance

- Voltage induced in coil 2 is,



$$V_2 = N_2 \frac{d\phi_2}{dt}$$

$$V_2 = N_2 . \frac{d\phi_2}{di_2} . \frac{di_2}{dt} \quad = L_2 . \frac{di_2}{dt}$$

$$L_2 = N_2 . \frac{d\phi_2}{di_2}$$

- L_2 is the self inductance of the coil.

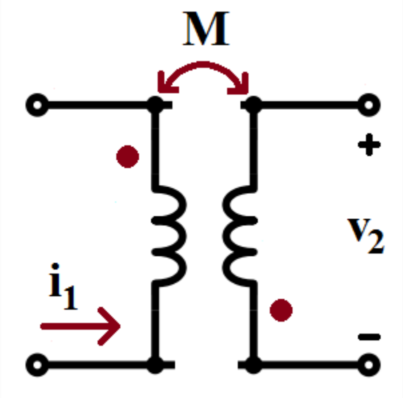
- Voltage induced in coil 1 is, $V_1 = N_1 \frac{d\phi_{21}}{dt}$

$$V_1 = N_1 . \frac{d\phi_{21}}{di_2} . \frac{di_2}{dt} \quad = M . \frac{di_2}{dt} \qquad M = N_1 . \frac{d\phi_{21}}{di_2}$$

- M is the mutual inductance between the 2 coils.

Mutual Inductance

- Mutual inductance is the ability of one inductor to induce a voltage across a neighboring inductor.
- It is measured in henrys (H).
- The polarity of mutual voltage is determined by using dot covention.
- A dot is placed at one end of each coupled coils to indicate the direction of the magnetic flux if current enters that dotted terminal of the coil.



Types of Induced EMF

- According to Faraday's law of electromagnetic induction, an EMF is induced by changing the flux linkages in a coil. It can happen in two ways.
- EMF is induced either "moving the coil and keeping the magnetic field stationary" or "moving the magnetic field and keeping the coil stationary".
- EMF is induced by changing the flux linking with a coil without moving either coil or magnetic field system.

Dynamically Induced EMF

- “Moving the coil and keeping the magnetic field stationary” or “moving the magnetic field and keeping the coil stationary”.

- EMF induced by this way is called dynamically induced emf.

Dynamically Induced EMF

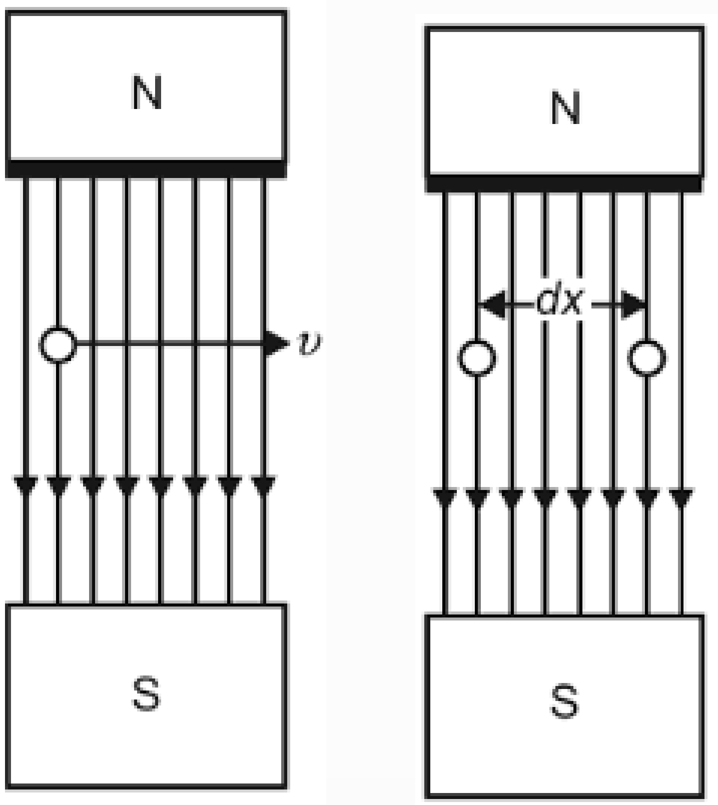
$$\text{Area swept by the conductor}, A = l \times dx$$

$$\text{Flux cut by the conductor}, d\phi = B \times A = B.l.dx$$

- According to Faraday's law,

$$\text{Induced emf}, e = \frac{d\phi}{dt} = \frac{B.l.dx}{dt} = B.l.v$$

- Since dx/dt=velocity



Dynamically Induced EMF

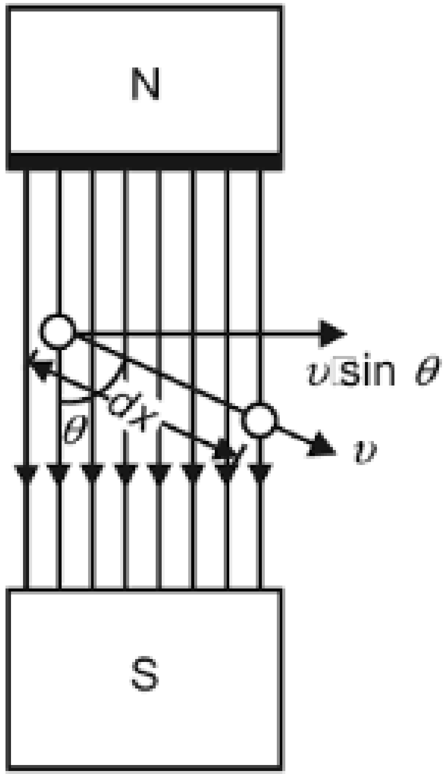
- Now the conductor moves at an angle ϴ with the direction of magnetic field.

$$Distance\ covered\ by\ the\ conductor\ in\ dt\ seconds\ is = dx$$

$$The\ component\ of\ distance\ perpendicular\ to\ the\ magnetic\ field\ is = dx.sin\theta$$

$$Area\ swept\ by\ the\ conductor, A = l \times dx.sin\theta$$

$$Flux\ cut\ by\ the\ conductor, d\phi = B \times A = B.l.dx.sin\theta$$



- According to Faraday's law,

$$Induced\ emf, e = \frac{d\phi}{dt} = \frac{B.l.dx.sin\theta}{dt} = B.l.v.sin\theta$$

Statically Induced EMF

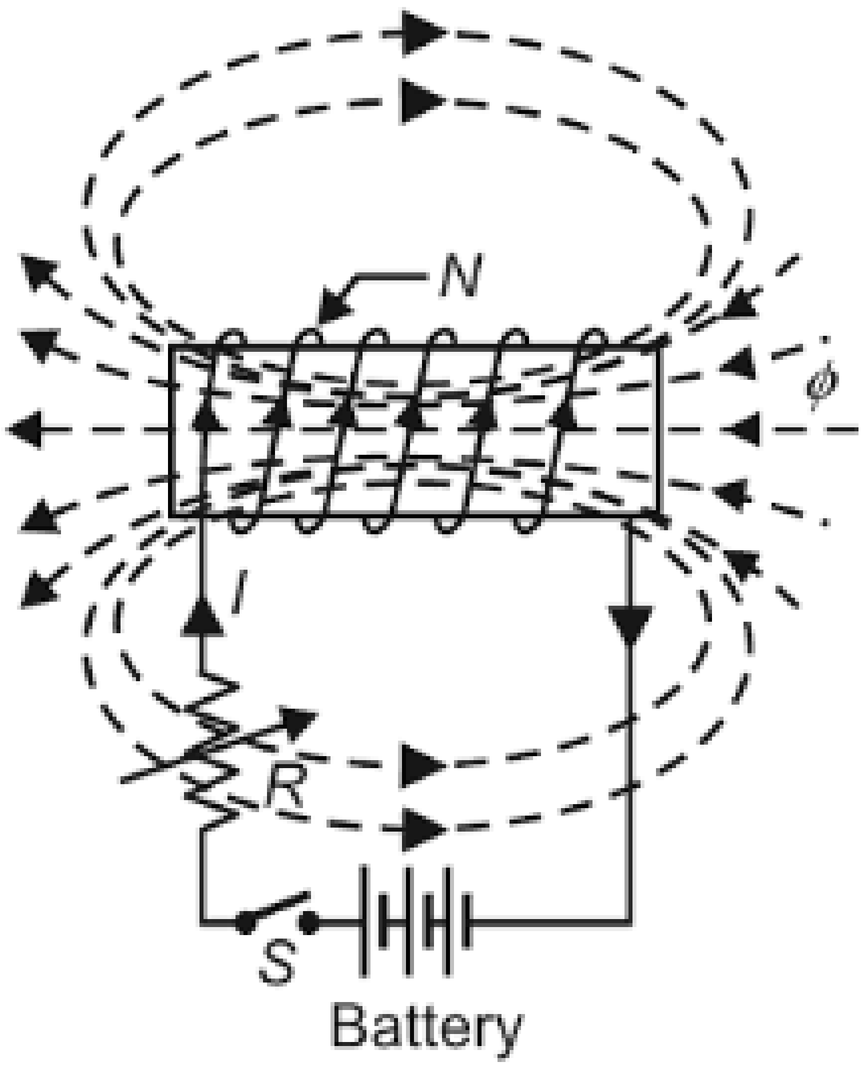
- EMF induced in a coil when both the coil and magnetic field system are stationary but the magnetic flux linking with the coil changes is called statically induced emf.

Types of Statically Induced EMF

- Self – Induced e.m.f.
- Mutually Induced e.m.f.

Self Induced EMF

- Self-induced e.m.f. is the e.m.f. induced in a coil due to its own changing flux linked with it.



Self Induced EMF

- Direction of induced voltage is such that it opposes the cause producing it.

- Rate of change of flux depends on rate of change of current.

$$e \propto \frac{di}{dt} \quad or \quad e = L\frac{di}{dt}$$

- L is the self inductance of the coil.

Mutually Induced EMF

- Mutually induced e.m.f. is the e.m.f. induced in a coil due to the change of flux produced by another coil (kept close) linking with it.