# Tautology,Contradiction, Satisfiability, Equivalence

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In this presentation we will discuss following things:

- Tautology and Contradiction
- Satisfiability
- Equivalence

## Tautology and Contradiction

A *tautology* is a compound proposition that is **true** *no matter what* the truth values of its atomic propositions are!

*Ex.* $p \vee \neg p$ [What is its truth table?]

A *contradiction* is a compound proposition that is **false** no matter what! *Ex.* $p \wedge \neg p$ [Truth table?]

Other compound props. are *contingencies*.

## Satisfiability

A compound proposition is *satisfiable* if it is true for at least one interpretation.

A statement is *unsatisfiable* if it is false for every interpretation.

Note: A tautology and contradiction are satisfiable.

A *contradiction* is unsatisfiable.

The compound propositions $p$ and $q$ are called *logically equivalent* if $p \leftrightarrow q$ is a tautology. The notation $p \equiv q$ denotes that $p$ and $q$ are logically equivalent.
Compound propositions that have the same truth values in all possible cases are called logically equivalent.
Example: Show that $\neg p \vee q$ and $p \rightarrow q$ are logically equivalent.

*Ex.* Prove that $p \vee q \Leftrightarrow \neg(\neg p \wedge \neg q)$.

| $p$ | $q$ | $p \vee q$ | $\neg p$ | $\neg q$ | $\neg p \wedge \neg q$ | $\neg(\neg p \wedge \neg q)$ |
|---|---|---|---|---|---|---|
| F | F | F | T | T | T | F |
| F | T | T | T | F | F | T |
| T | F | T | F | T | F | T |
| T | T | T | F | F | F | T |

**Assignment**

1. Show that $(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$ is a **tautology.**
2. Show that $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are not logically equivalent**.**

Video link: https://www.youtube.com/watch?v=Ji2vr-9duPI&list=PLHXZ9OQGMqxersk8fUxiUMSIx0DBqsKZS&index=14

References: Discrete Mathematics and its application by Kenneth H Rosen

Thank you!!!!!