### School of Basic and Applied Sciences<br>
Course Code : MSCP6001 Course Name: ELECTRODYNAMICS<br>
Electrodynamics<br>
Topic Covered **Electrodynamics** Topic Covered

- $\square$  Spacetime diagrams
- □ 4-vectors
- □ 4-vectors: Example
- $\Box$  The magnitude of 4-velocity
- □ References

# U operators Example<br>
□ 4-vectors: Example<br>
□ The magnitude of 4-velocity<br>
□ References<br>
□ References<br>
□ N I V E R S I T Y<br>
Name of the Faculty: Dr. ASHUTOSH KUMAR<br>
Program Name: M.Sc. Physics



### 4-vectors

School of Basic and Applied Sciences<br>
Course Code : MSCP6001 Course Name: ELECTRODYNAMICS<br>
The Lorentz Transforms were used for transforming the 4-displacement (i.e.<br>
coordinates in 4D) in-between different inertial frames The Lorentz Transforms were used for transforming the 4-displacement (i.e. coordinates in 4D) in-between different inertial frames of reference.

Therefore, we can define a class of objects called '4-vectors' written as  $A^\mu$ to have the property : 4-vectors follow the same transform as the coordinates transform. **School of Basic and Applied Sciences**<br>
Course Code : MSCP6001 **Course Name: ELECTRODYNAMICS**<br>
The Lorentz Transforms were used for transforming the 4-displacement (i.e.<br>
coordinates in 4D) in-between different inertial f

one coordinate to another by means of Lorentz Transforms as we've found.

The most basic 4-vector is of course  $x^{\mu} = (ct, x, y, z)$ . It obviously transforms from<br>one coordinate to another by means of Lorentz Transforms as we've found.<br>A simple extension would be to define  $U^{\alpha} \equiv \frac{dx^{\alpha}}{dt}$ , whi A simple extension would be to define  $U^{\alpha} \equiv \frac{dx^{\alpha}}{dz}$ , which we call the '4-velocity' and **Name: ELECTRODYNAMICS**<br>
orming the 4-displacement (i.e.<br>
ial frames of reference.<br>
led '4-vectors' written as  $A^{\mu}$  to have<br>
sform as the coordinates transform.<br>  $(x, y, z)$ . It obviously transforms from<br>
tz Transforms as  $a^{\alpha} \equiv \frac{dU^{\alpha}}{dx}$ , which we call '4-acceleration'.  $\frac{10}{d\tau}$ , which we call '4-acceleration'.

Both of them also transform in-between coordinates like the 4-displacement  $x^{\mu}$ . This is because we have defined  $d\tau$ , the proper time to be a scalar quantity, i.e. it is a quantity that doesn't change with coodinates.

### 4-vectors: Example

**School of Basic and Applied Sciences**<br>
Course Code : MSCP6001 Course Name: ELECTRODYNAMICS<br>
4-vectors: Example<br>  $U^{\alpha} \equiv \frac{dx^{\alpha}}{d\tau}$  and  $a^{\alpha} \equiv \frac{dU^{\alpha}}{d\tau}$ , it would be useful to see what they look like in 4-form. Defining  $U^\alpha \equiv \frac{dx^\alpha}{d\tau}$  and  $a^\alpha \equiv \frac{dU^\alpha}{d\tau}$ , it would be useful to see what they  $\frac{d\sigma}{d\tau}$ , it would be useful to see what they look like in 4-form.

Consider a moving spaceship with const. velocity Then, for people on it, they would consider themselves as stationary, meaning that their **School of Basic and Applied Sciences**<br>
Course Code : MSCP6001 Course Name: ELECTRODYNAMICS<br>  $4\text{-vectors: Example}$ <br>
Defining  $U^{\alpha} \equiv \frac{dx^{\alpha}}{d\tau}$  and  $a^{\alpha} \equiv \frac{dU^{\alpha}}{d\tau}$ , it would be useful to see what they look like in 4-for

displacement is only  $\overline{dx^{\mu}} = (c\overline{d}\tau, 0,0,0)$ <br>
\*the bar symbol is for moving frame<br>  $L = \begin{pmatrix} \gamma & +\beta\gamma & 0 & 0 \\ +\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ <br>
Therefore,  $\overline{U^{\alpha}} = (c, 0,0,0)$  and  $\overline{a^{\alpha}} = (0,0,0,0)$ <br> **Course Code : MSCP6001 Course Name: ELECTRODYNAMICS**<br> **A-vectors: Example**<br>
Defining  $U^{\alpha} \equiv \frac{dx^{\alpha}}{dt}$  and  $a^{\alpha} \equiv \frac{du^{\alpha}}{dt}$ , it would be useful to see what they look like in 4-form.<br>
Consider a moving spaceshi **botage is: Little Example:**<br> **botage is: Container themselves** as stationary, meaning that their<br>
(0,0)<br>
(0,0,0,0)<br> **botage is:**  $L = \begin{pmatrix} \gamma & +\beta\gamma & 0 & 0 \\ +\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ <br>
(0,0,0,0)<br>  $\beta = A$ **Example 20** and the term and y to take the state of the polyon of the state of  $L = \begin{pmatrix} \gamma$  = **TRODYNAMICS**<br>
y look like in 4-form.<br>
y, meaning that their<br>  $\gamma + \beta \gamma = 0$ <br>  $\beta \gamma = \gamma = 0$ <br>
0 0 1 0<br>
0 0 1 0<br>
0 0 1 +βγ 0 0 **TRODYNAMICS**<br>y look like in 4-form.<br>y, meaning that their<br> $\gamma$  +  $\beta \gamma$  0 0<br> $\beta \gamma$   $\gamma$  0 0<br>0 0 1 0<br>0 0 1 0 **TRODYNAMICS**<br>y look like in 4-form.<br>y, meaning that their<br> $\gamma$  +  $\beta \gamma$  0 0<br> $\beta \gamma$   $\gamma$  0 0<br>0 0 1 0<br>0 0 1 0 \*the bar symbol is for moving frame<br> $L = \begin{pmatrix} r & r & r \\ +\beta \gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ 

If we transform  $\overline{U^{\alpha}}$  to  $U^{\alpha}$  by using  $U^{\beta} = \Lambda^{\beta}{}_{\alpha} \overline{U^{\alpha}}$ , then we find

 $U^{\alpha} = (\gamma c, \gamma v, 0.0)$ Which looks familiar... except with some extra  $\gamma$  s in there. Where are they from?

### 4-vectors: Example

**School of Basic and Applied Sciences**<br>
Course Code : MSCP6001 Course Name: ELECTRODYNAMICS<br>
4-**vectors: Example**<br>
Remember that  $U^{\alpha} \equiv \frac{dx^{\alpha}}{dt^{\alpha}}$  but our classical velocity is  $v = \frac{dx}{dt}$ ! So we need to find the re Remember that  $U^\alpha \equiv \frac{dx^\alpha}{d\tau}$  but our classical velocity is  $v = \frac{dx}{dt}$ ! So we need to find the **Basic and Applied Sciences**<br> **ourse Name: ELECTRODYNAMICS**<br> **ectors: Example**<br>  $\frac{dx^{\alpha}}{dx}$  but our classical velocity is  $v = \frac{dx}{dt}$ ! So we need to find the<br>
d d T. From time-dilation, that would be dt =  $\gamma d\tau$ <br>
,  $v$ **School of Basic and Applied Sciences**<br> **Course Code : MSCP6001 Course Name: ELECTRODYNAMICS**<br> **A-Vectors: Example**<br>
Remember that  $U^{\alpha} = \frac{dx^{\alpha}}{dt}$  but our classical velocity is  $v = \frac{dx}{dt}$ ! So we need to find the<br>
r **ol of Basic and Applied Sciences**<br> **SECPEGODE CORE CONSTREMENT CONSTREMENT ACT CORE ASSEMBLE UT ALLOCATE:** Example<br>  $U^{\alpha} \equiv \frac{dx^{\alpha}}{d\tau}$  but our classical velocity is  $v = \frac{dx}{dt}$ ! So we need to find the n dt and dr. Fr

Thus,  $\frac{dx^{\alpha}}{dt} = U^{\alpha} \frac{d\tau}{dt} = (c, v, 0, 0)$  which is exactly th moving spaceship on the spacetime diagram.



## School of Basic and Applied Sciences School of Basic and Applied Sciences<br>
Course Code : MSCP6001 Course Name: ELECTRODYNAMICS<br>
The magnitude of 4-velocity

### The magnitude of 4-velocity

Similar as in 3D case, the magnitude of 4-vectors can be found by  $g_{\alpha\beta}U^\alpha U^\beta=-c^2$  $\mathcal{S}$ <br> $\beta = -c^2$ 

**School of Basic and Applied Sciences**  
\n**Course Code : MSCP6001**  
\n**Course Name: ELECTRODYNAMIC**  
\n**The magnitude of 4-velocity**  
\nSimilar as in 3D case, the magnitude of 4-vectors can be found by 
$$
g_{\alpha\beta}U^{\alpha}U
$$
  
\n
$$
g_{\alpha\beta} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$
is the metric in flat spacetime

$$
g_{\alpha\beta} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$
 is the metric in flat spacetime  
\n  
\nName of the Faculty: Dr. ASHUTOSH KUMAR  
\nProgram Name: M.Sc. Physics

## School of Basic and Applied Sciences School of Basic and Applied Sciences<br>
Course Code : MSCP6001 Course Name: ELECTRODYNAMICS<br>
References

### **References**

- D.J. Griffiths, Introduction to Electrodynamics,4th ed.,Pearson, USA, 2013.
- 
- 
- 
- **School of Basic and Applied Sciences**<br> **Course Code : MSCP6001 Course Name: ELECTRODYNAMICS**<br> **References**<br>
 D.J. Griffiths, Introduction to Electrodynamics, 4th ed.,Pearson, USA, 2013.<br>
 J.D. Jackson, Classical Elect **• School of Basic and Applied Sciences**<br>
• D.J. Griffiths, Introduction to Electrodynamics, 4th ed., Pearson, USA, 2013.<br>
• J.D. Jackson, Classical Electrodynamics, 3rd ed., New Age, New Delhi, 2009<br>
• R.K. Patharia ,Theo Saunders college Publishing House, 1995.

# Name of the Faculty: Dr. ASHUTOSH KUMAR<br>
Name of the Faculty: Dr. ASHUTOSH KUMAR<br>
Program Name: M.Sc. Physics<br>
Program Name: M.Sc. Physics<br>
Program Name: M.Sc. Physics