Electrodynamics

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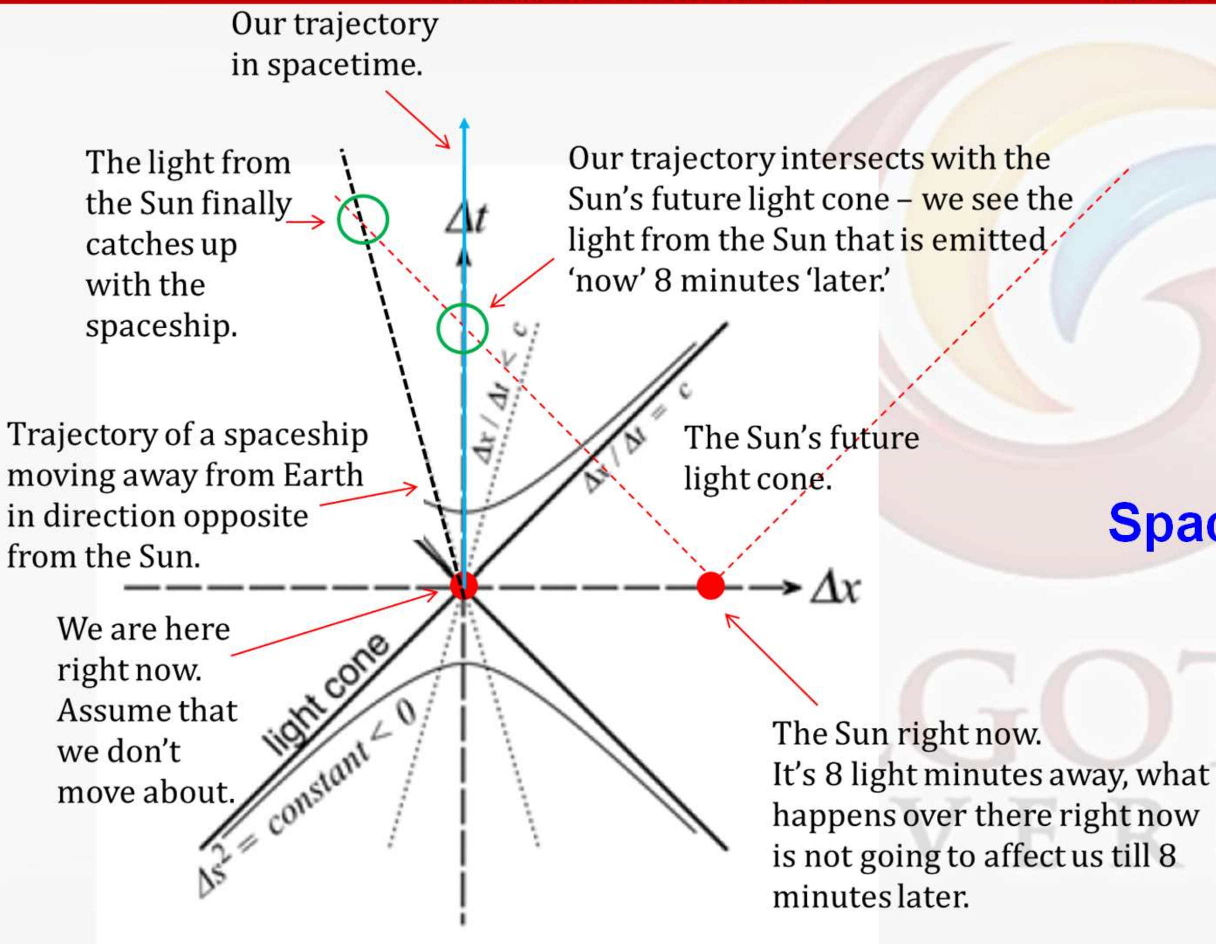


Spacetime diagrams

4-vectors

The Lorentz Transforms were used for transforming the 4-displacement (i.e. coordinates in 4D) in-between different inertial frames of reference.

Therefore, we can define a class of objects called '4-vectors' written as $A^{\mu}$to have the property : 4-vectors follow the same transform as the coordinates transform.

The most basic 4-vector is of course $x^{\mu} = (ct, x, y, z)$. It obviously transforms from one coordinate to another by means of Lorentz Transforms as we've found.

A simple extension would be to define $U^{\alpha} \equiv \frac{\mathrm{d}x^{\alpha}}{\mathrm{d}\tau}$, which we call the '4-velocity' and $a^{\alpha} \equiv \frac{\mathrm{dU}^{\alpha}}{\mathrm{d}\tau}$, which we call '4-acceleration'.

Both of them also transform in-between coordinates like the 4-displacement x^{μ}. This is because we have defined dτ, the proper time to be a scalar quantity, i.e. it is a quantity that doesn't change with coodinates.

4-vectors: Example

Defining $U^{\alpha} \equiv \frac{\mathrm{d}x^{\alpha}}{\mathrm{d}\tau}$ and $a^{\alpha} \equiv \frac{\mathrm{d}U^{\alpha}}{\mathrm{d}\tau}$, it would be useful to see what they look like in 4-form.

Consider a moving spaceship with const. velocity
Then, for people on it, they would consider themselves as stationary, meaning that their displacement is only $\overline{\mathrm{d}x^{\mu}} = (c\mathrm{d}\tau, 0,0,0)$

*the bar symbol is for moving frame

$$L = \begin{pmatrix} \gamma & +\beta\gamma & 0 & 0 \\ +\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Therefore, $\overline{U^{\alpha}} = (c, 0,0,0)$ and $\overline{a^{\alpha}} = (0,0,0,0)$

If we transform $\overline{U^{\alpha}}$ to U^{α} by using $U^{\beta} = \Lambda^{\beta}{}_{\alpha}\overline{U^{\alpha}}$, then we find

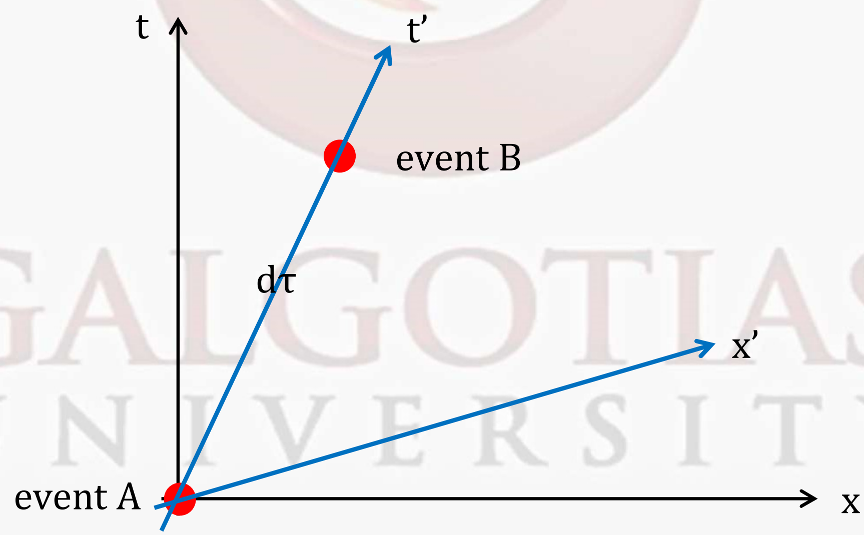
$$U^{\alpha} = (\gamma c, \gamma v, 0,0)$$

Which looks familiar… except with some extra γ s in there. Where are they from?

4-vectors: Example

Remember that $U^{\alpha} \equiv \frac{dx^{\alpha}}{d\tau}$ but our classical velocity is $v = \frac{dx}{dt}$! So we need to find the relation between dt and dτ. From time-dilation, that would be $dt = \gamma d\tau$

Thus, $\frac{dx^{\alpha}}{dt} = U^{\alpha}\frac{d\tau}{dt} = (c, v, 0,0)$ which is exactly the trajectory that we drew for a moving spaceship on the spacetime diagram.



The magnitude of 4-velocity

Similar as in 3D case, the magnitude of 4-vectors can be found by $g_{\alpha\beta}U^{\alpha}U^{\beta} = -c^2$

$$g_{\alpha\beta} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$ is the metric in flat spacetime

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