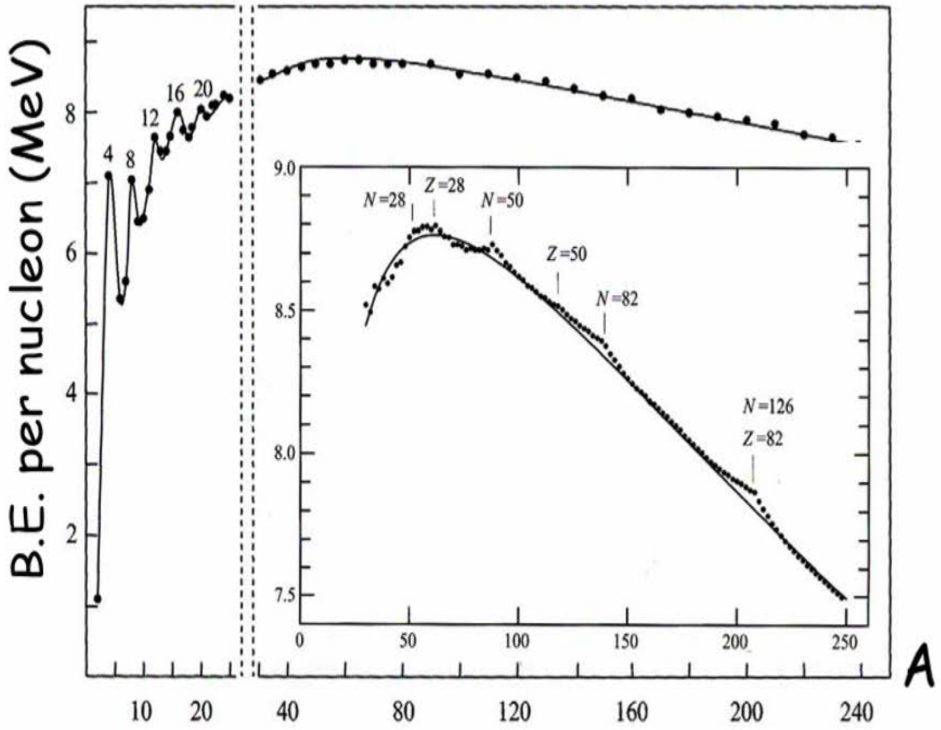
Single Particle Shell Model

Magic Numbers

Nuclei with values of Z and/or N = 2,8,20,28,50, 82,126… are very stable and show significant departures from the average nucleus behaviour. They represent the effects of the filled major shells analogous to the atomic shell model. The binding energy per nucleon is large for magic numbers.

Properties of Magic Number Nuclei

- Doubly magic nuclei extremely stable (where Z and N are magic)
- Energies in alpha and beta decay high when daughter nucleus is magic
- Nuclear radius is not changed much with Z, N at magic numbers
- 1st excited states for magic numbers higher than neighbours
- Spontaneous neutron emitters have magic number +1
- Terrestrial nuclear abundances for Z or N magic are greater than those for nonmagic elements.
- Elements with Z/N magic have many more isotopes than with Z/N non-magic
- Odd A nuclei have small quadrupole moment when magic, etc, etc



Need for Shell Model

The atomic theory based on the shell model has provided remarkable clarification of the complicated details of the atomic structure. Nuclear physicists therefore attempted to use a similar theory to solve the problem of nuclear structure. A major difference is that in the atomic case the potential is supplied by the Coulomb field of the nucleus; the orbits are established by external agent.

In nucleus there is no such external agent; the nucleons move in a potential that they themselves create. This is overcome by the fundamental assumption of the shell model: the motion of a single nucleon is governed by a potential caused by all of the other nucleons. Treating each nucleon individually allows for the nucleons to be occupying the energy levels of a series of sub-shells. Another difficulty is that electrons move in orbits free of collisions with other electrons. Nucleons on the other hand have relatively large diameter compared to the size of the nucleus. However, the existence of spatial orbits depends on the Pauli principle. For example in a heavy nucleus a collision between nucleons in a state near the bottom of the potential well will result in a transfer of energy to one another.

Need for Shell Model

But if all the energy levels are filled up the level of the valence nucleon, there is no way for one of the nucleons to gain energy except to move to the valence level as other low lying levels are filled. The transition thus requires more energy than the nucleons are likely to transfer in a collision. Thus collisions cannot occur and nucleons orbit as if they were transparent to one another.

Assumptions in Shell Model

Each nucleon moves in an averaged potential

- neutrons see average of all nucleon-nucleon nuclear interactions
- protons see same as neutrons plus proton-proton electric repulsion
- the two potentials for n and p are wells of some form (nucleons are bound)

Each nucleon moves in single particle orbit corresponding to its state in the potential

- We are making a single particle shell model
- Q: why does this make sense if nucleus full of nucleons and typical mean free paths of nuclear scattering projectiles = O(2fm)
- A: Because nucleons are fermions and stack up. They can not loose energy in collisions since there is no state to drop into after collision

Use Schrödinger Equation to compute energies (i.e. non-relativistic), justified by simple infinite square well energy estimates

Aim to get the correct magic numbers (shell closures)

Potential as experienced by an individual nucleon

The Saxon-Woods potential can be used to approximate the potential as experienced by an individual nucleon.

$$V(r) = \frac{-V_0}{1 + e^{(r-R)/a}}$$

The parameters R and a give, respectively, the mean radius and skin thickness. Their values are chosen with accordance to measurements such that: R = 1.25A1/3 fm and a = 0.524 fm. V0 is adjusted to give the proper separation energies and is of order 50 MeV. This potential is then substituted into the Schroedinger Equation and the energy levels found. However the central potential alone cannot reproduce the magic numbers, need to account for the spin-orbit interactions.

Spin-orbit potential

In atomic physics, the spin-orbit interaction causes the observed fine structure of spectral lines, comes about because of the electromagnetic interaction of the electron’s magnetic moment with the magnetic field generated by its motion about the nucleus. This concept is adopted in nuclear physics. From scattering experiments there is strong evidence of nucleon-nucleon spin-orbit force. The potential is altered such that:

$$V(r) \rightarrow V(r) + W(r)\mathbf{L.S}$$

where L and S are orbital and spin angular momentum operators and W(r) is a function of radial position

where: $$W(r) = -|V_{LS}|\left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{r}\frac{dV}{dr}$$

and $$V_{LS} = V_{LS}\left(E_{nucleon}\right)$$

V(r) is the Saxon-Woods potential of Eq.1. As with atomic physics, the total angular momentum operator is defined as J=L+S. The eigenvalue of L.S for a stationary state with good quantum numbers l, j and s (=1/2) is

$$\frac{\hbar^2}{2}\left[j(j+1) - l(l+1) - s(s+1)\right]$$

Spin-orbit potential

The potential for j = l + ½ is:

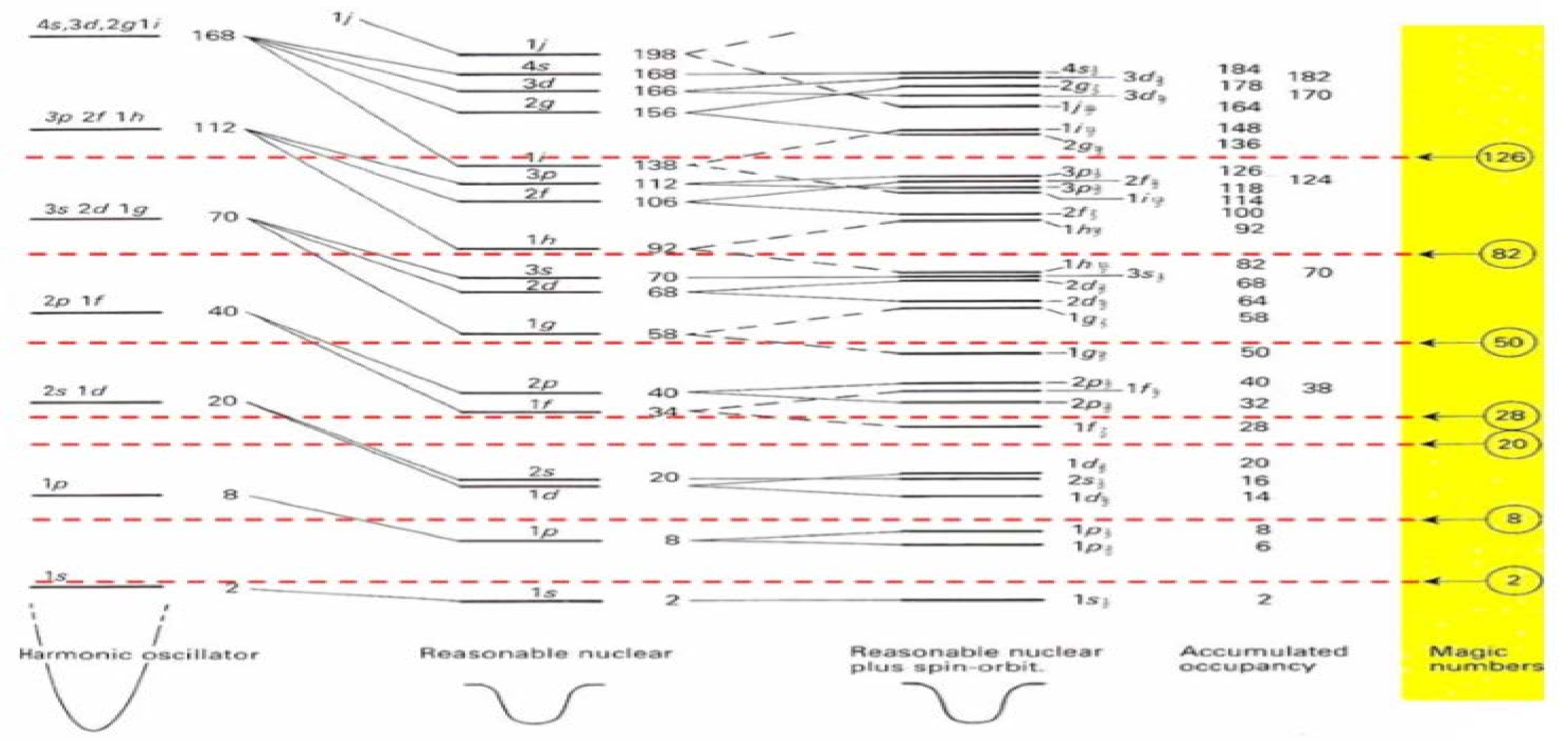
$$V(r)+\frac{1}{2}l\hbar^2W(r)$$

and for j = l - ½ :

$$V(r)-\frac{1}{2}l(l+1)\hbar^2W(r)$$

Since W(r) is negative (to obtain agreement with observation), the j = l + ½ level will be below that with j = l – ½. The resultant energy structure is shown here.

Spin-orbit potential



Summary

Successful Predictions of Shell Model

Origin of magic numbers

Spins and parities of ground states

Trend in magnetic moments

Some excited states near closed shells, small excitations in odd A nuclei

In general not good far from closed shells and non-spherically symmetric potentials

Collective properties of nuclei can be incorporated into the nuclear shell model by replacing the spherically symmetric potential by a deformed one. This improves description for

- Even A excited states
- Electric quadrupole and magnetic dipole moments

Shortcomings of the Shell Model

Cannot predict spin or parity for odd-odd nuclei – do not have a very good model for the LS interactions

A consequence of the above is that the shell model predictions for nuclear magnetic moments are very imprecise

Cannot predict accurate energy levels because:

- we only use one “well” to suit all nuclei
- we ignore the fact that n and p should have separate wells of different shape

As a consequence of the above we cannot reliably predict much (configuration, excitation energy) about excited states other then an educated guess of the configuration of the lowest excitation

References

Williams pg. 131; Cottingham 2nd ed. pg. 56; Krane pg. 116

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