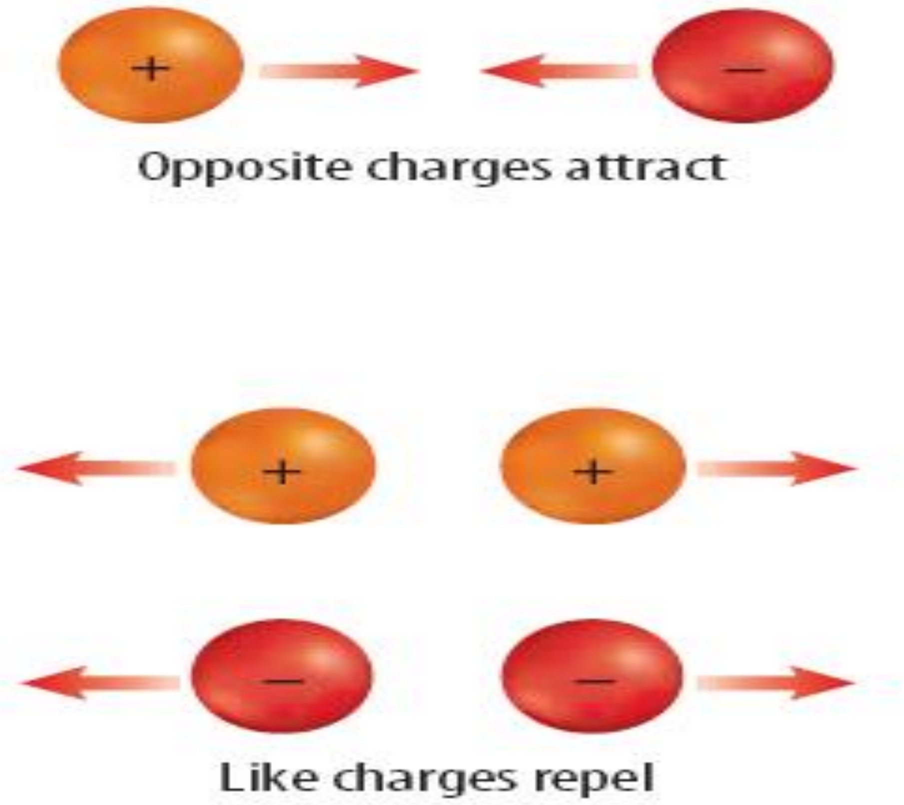
# Electricity and Magnetism

## Topic: Electric Field and Potential

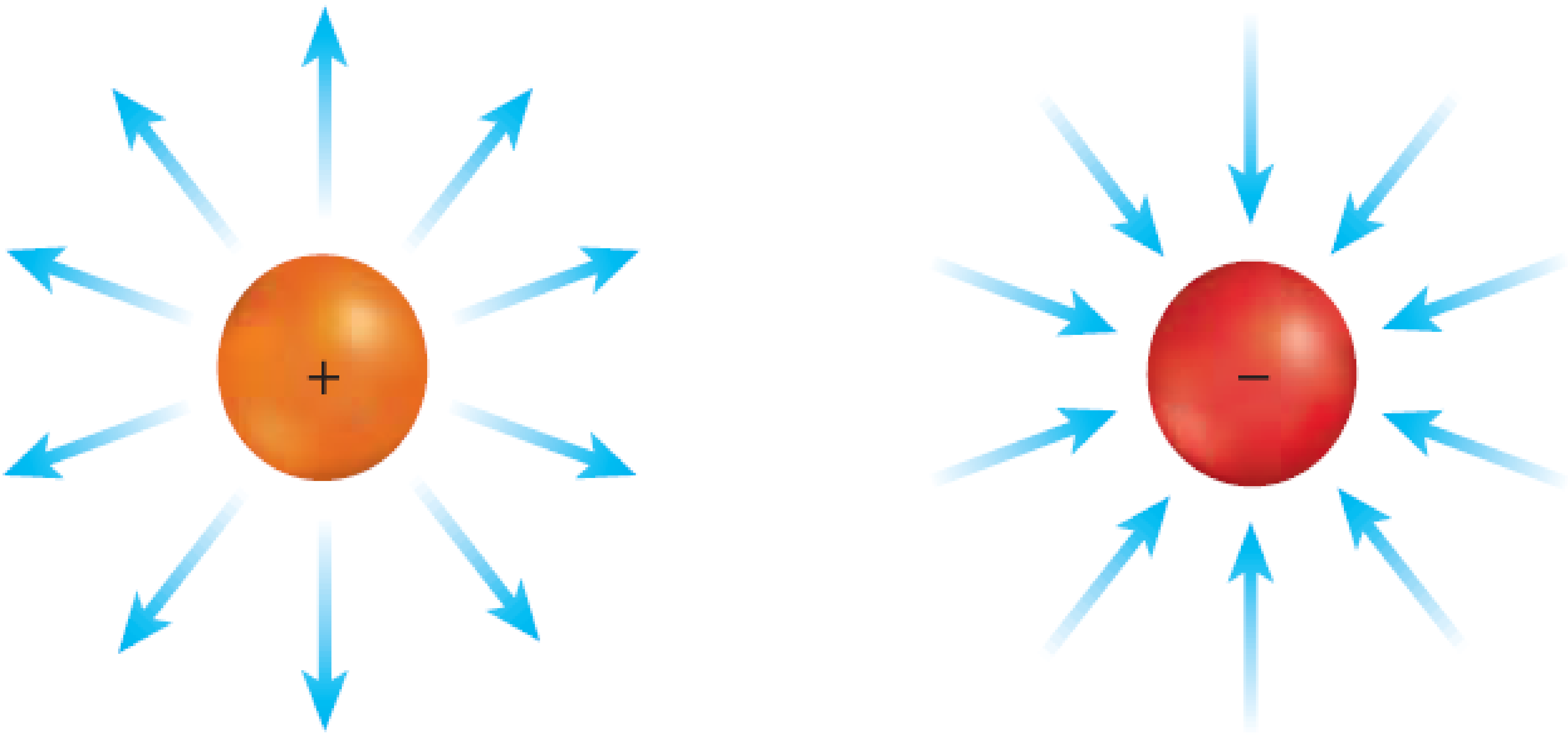
## Electric Field and Electric Potential

As electrons collect on an object, it becomes negatively charged. As electrons leave an object it attains a positive charges. Charges interact with each other:




Often when you remove clothes from the clothes dryer, they seem to stick together. This is because some of the clothes have gained electrons by rubbing against other clothes. The clothes losing electrons become positive. The negative clothes are attracted to the positive clothes.

# Electric Field and Electric Potential




Surrounding every charge is an electric field. Through the electric field, a charge is able to push or pull on another charge.

## Electric Field and Electric Potential

# Check out these static electricity video clips

- Static electricity at a gas station
- Van de Graaf Generator's effect on human hair
- Static on Baby's hair
- Kid gets static going down a slide
- "Cat abuse" by static electricity

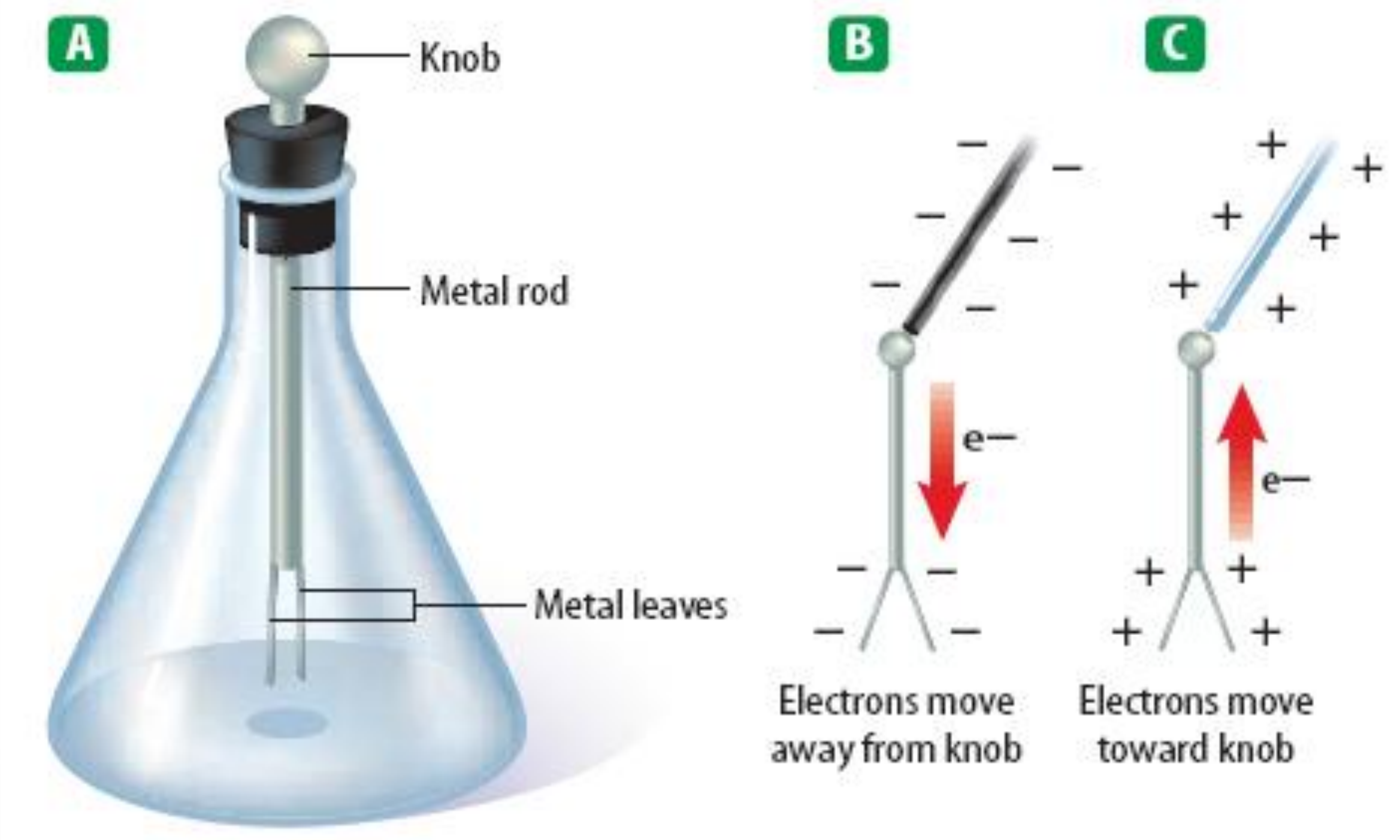
What is a conductor and insulator?

A conductor is a material which allows an electric current to pass. Metals are good conductors of electricity.

An insulator is a material which does not allow an electric current to pass. Nonmetals are good conductors of electricity. Plastic, glass, wood, and rubber are good insulators

## Electric Field and Electric Potential

# How are static charges detected?




Notice the position of the leaves on the electroscope when they are A uncharged, B negatively charged, and C positively charged.

# Potential Due to a Point Charge

Start with (set $V_f$=0 at $\infty$ and $V_i$=V at R)

$$\Delta V = V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s} = -\int_i^f (E\cos 0°)ds = -\int_R^\infty E dr$$

We have

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Then

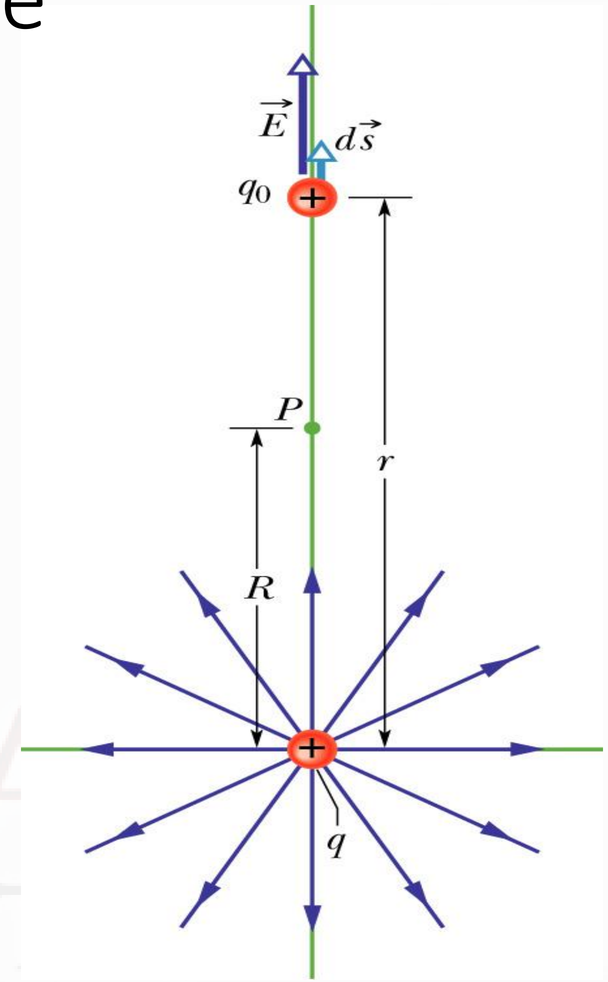
$$0 - V = -\frac{q}{4\pi\varepsilon_0}\int_R^\infty \frac{1}{r^2}dr = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_R^\infty = -\frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

So

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

A positively charged particle produces a positive electric potential.

A negatively charged particle produces a negative electric potential



# Potential due to a group of point charges

Use superposition

$$V = -\int_{\infty}^{r} \vec{E} \cdot d\vec{s} = -\sum_{i=1}^{n} \int_{\infty}^{r} \vec{E}_i \cdot d\vec{s} = \sum_{i=1}^{n} V_i$$

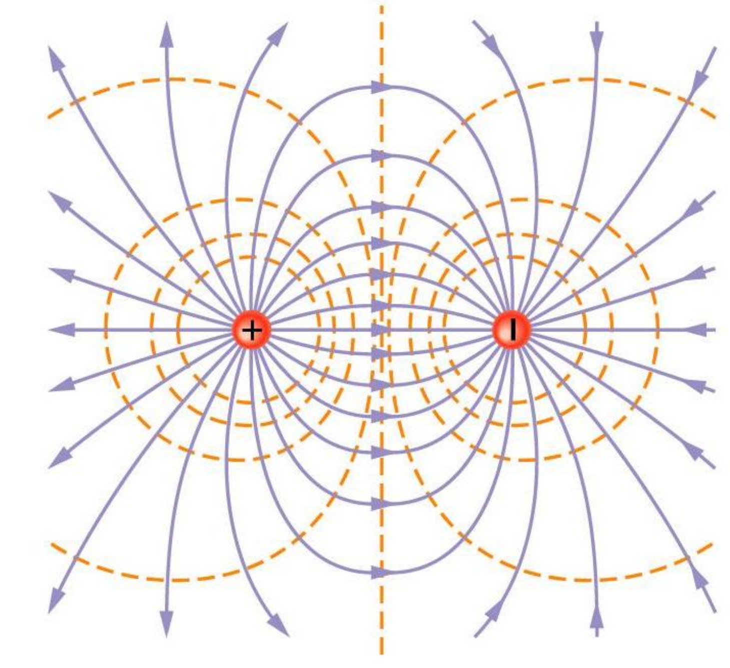
For point charges

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i}$$

The sum is an algebraic sum, not a vector sum.

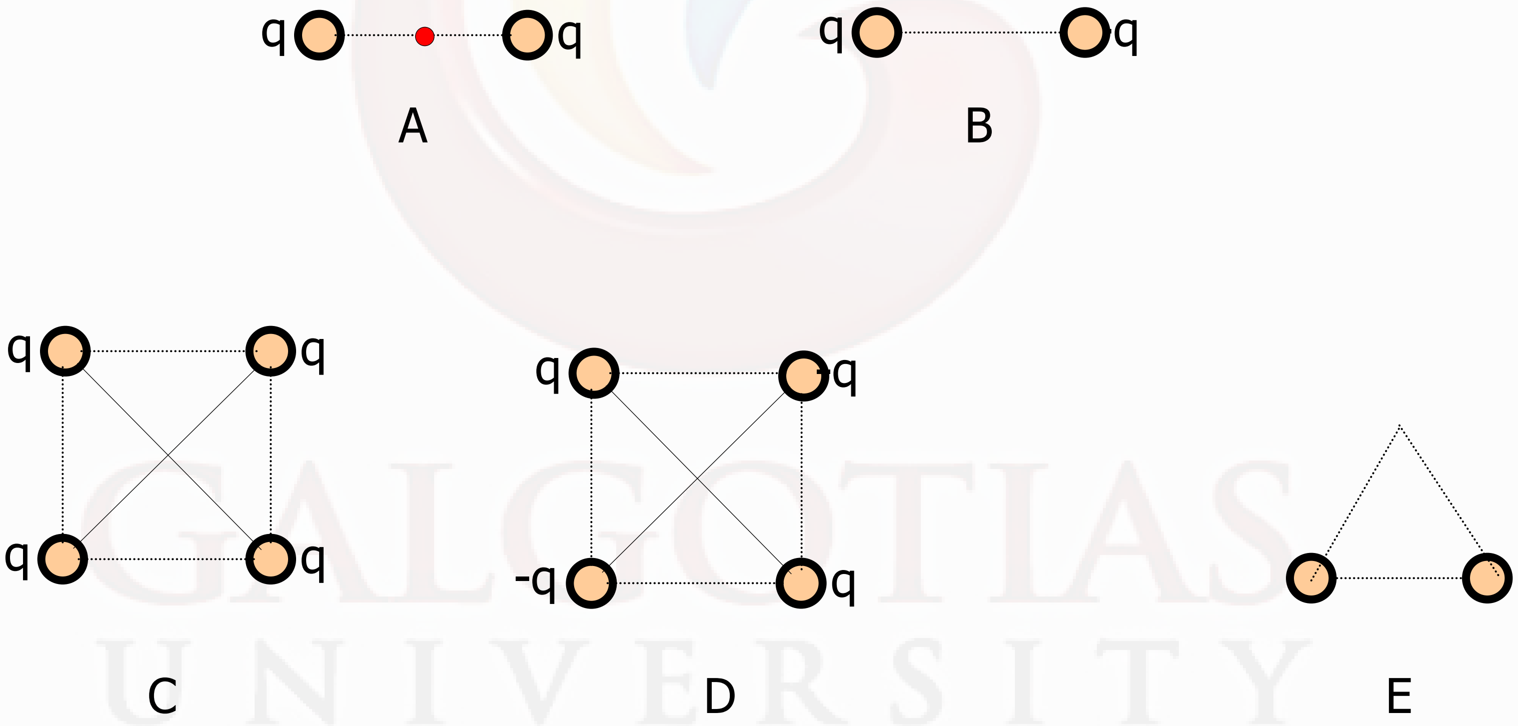
E may be zero where V does not equal to zero.

V may be zero where E does not equal to zero.

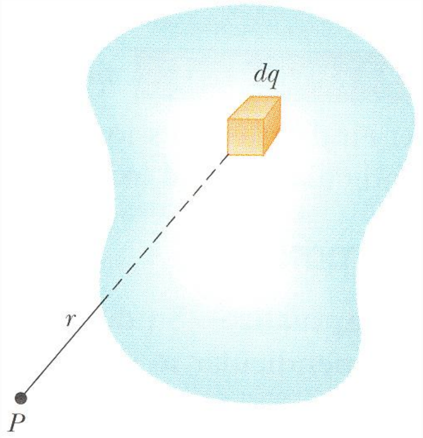


# Electric Field and Electric Potential

4. Which of the following figures have V=0 and E=0 at red point?



# Potential due to a Continuous Charge Distribution




- Find an expression for dq:
  - dq = λdl for a line distribution
  - dq = σdA for a surface distribution
  - dq = ρdV for a volume distribution
- Represent field contributions at P due to point charges dq located in the distribution.

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$$

- Integrate the contributions over the whole distribution, varying the displacement as needed,

$$V = \int dV = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$$

# Example: Potential Due to a Charged Rod

- A rod of length L located along the x axis has a uniform linear charge density λ. Find the electric potential at a point P located on the y axis a distance d from the origin.
- Start with

$$dq = \lambda dx$$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{(x^2+d^2)^{1/2}}$$

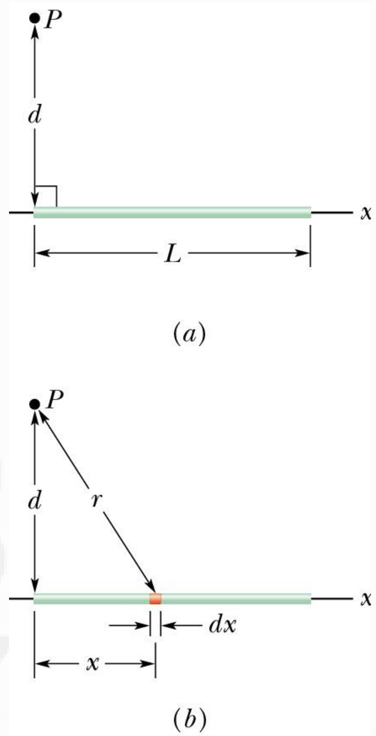
- then,

$$V = \int dV = \int_0^L \frac{\lambda}{4\pi\varepsilon_0}\frac{dx}{(x^2+d^2)^{1/2}} = \frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(x+(x^2+d^2)^{1/2}\right)\right]_0^L$$

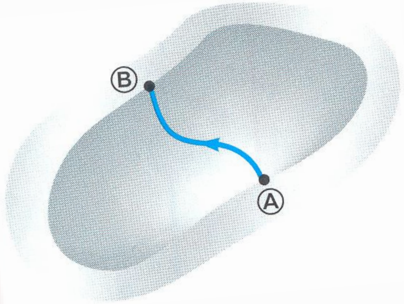
$$= \frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(L+(L^2+d^2)^{1/2}\right)-\ln d\right]$$

- So

$$V = \frac{\lambda}{4\pi\varepsilon_0}\ln\left[\frac{L+(L^2+d^2)^{1/2}}{d}\right]$$



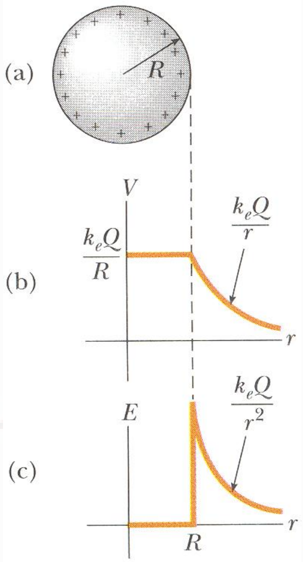
# Potential Due to a Charged Isolated Conductor




- According to Gauss' law, the charge resides on the conductor's outer surface.
- Furthermore, the electric field just outside the conductor is perpendicular to the surface and field inside is zero.
- Since

$$V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s} = 0$$

- Every point on the surface of a charged conductor in equilibrium is at the same electric potential.
- Furthermore, the electric potential is constant everywhere inside the conductor and equal to its value to its value at the surface.



# Calculating the Field from the Potential

- Suppose that a positive test charge $q_0$ moves through a displacement ds from on equipotential surface to the adjacent surface.
- The work done by the electric field on the test charge is $W = -dU = -q_0\,dV$.
- The work done by the electric field may also be written as
- Then, we have

$$-q_0 dV = q_0 E(\cos\theta)ds$$

$$E\cos\theta = -\frac{dV}{ds}$$

- So, the component of E in any direction is the negative of the rate at which the electric potential changes with distance in that direction.

$$E_s = -\frac{\partial V}{\partial s}$$

- If we know V(x, y, z),

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

# Electric Potential Energy of a System of Point Charges

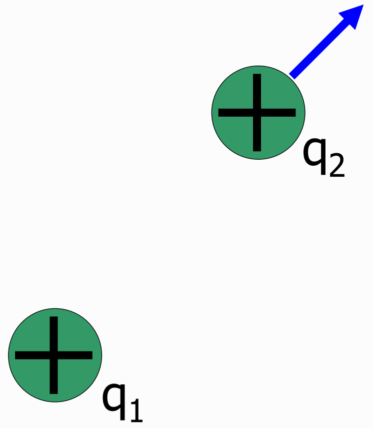
$$\Delta U = U_f - U_i = -W$$

$$W = \vec{F}\cdot\Delta\vec{r} = q\vec{E}\cdot\Delta\vec{r}$$

$$W_{app} = -W$$

$$\Delta U = U_f - U_i = W_{app}$$

Start with (set $U_i$=0 at ∞ and $U_f$=U at r)




$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}$$

We have

$$U = q_2 V = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

If the system consists of more than two charged particles, calculate U for each pair of charges and sum the terms algebraically.

$$U = U_{12} + U_{13} + U_{23} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

# Summary

Electric Potential Energy: a point charge moves from *i* to *f* in an electric field, the change in electric potential energy is

$$\Delta U = U_f - U_i = -W$$

Electric Potential Difference between two points *i* and *f* in an electric field:

$$\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q}$$

Equipotential surface: the points on it all have the same electric potential. No work is done while moving charge on it. The electric field is always directed perpendicularly to corresponding equipotential surfaces.

Finding V from E:

$$\Delta V \equiv \frac{\Delta U}{q_0} = -\int_i^f \vec{E} \cdot d\vec{s}$$

Potential due to point charges:

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

Potential due to a collection of point charges:

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$$

Potential due to a continuous charge distribution:

$$V = \int dV = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$$

Potential of a charged conductor is constant everywhere inside the conductor and equal to its value to its value at the surface.

Calculatiing E from V:

$$E_s = -\frac{\partial V}{\partial s} \quad E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$

Electric potential energy of system of point charges:

$$U = q_2 V = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$