Conditional Operators, Converse, Inverse and Contrapositive

By
Dr. Varsha Gautam
Galgotias University, Greater Noida(U.P.)

In this presentation we will discuss following things:

- Conditional Operator
- Biconditional Operator
- Converse ,Inverse and Contrapositive

Implication /Conditional Operator

The *implication* $p \rightarrow q$ states that p implies q.

I.e., If p is true, then q is true; but if p is not true, then q could be either true or false.

E.g., let p = "You study hard."

q = "You will get a good grade."

$p \rightarrow q$ = "If you study hard, then you will get a good grade."

$p \rightarrow q$ is **false** only when
p is true but q is **not** true.
$p \rightarrow q$ does **not** say
that p causes q!
$p \rightarrow q$ does **not** require
that p or q **are ever true**!
E.g. "(1=0) $\rightarrow$ pigs can fly" is TRUE!

p	q	$p \rightarrow q$	
F	F	T	
F	T	T	
T	F	**F**	The only False case!
T	T	T	

Examples

"If this lecture ends, then the sun will rise tomorrow." *True* or *False*?

"If Tuesday is a day of the week, then I am a penguin." *True* or *False*?

"If 1+1=6, then Bush is president."

True or *False*?

"If the moon is made of green cheese, then I am richer than Bill Gates." *True* or *False*?

Biconditional Operator

The *biconditional* $p \leftrightarrow q$ states that p is true *if and only if (IFF)* q is true.

p = "Trump wins the 2020 election."

q = "Trump will be president for all of 2021."

$p \leftrightarrow q$ = "If, and only if, Trump wins the 2020 election, Trump will be president for all of 2021."

Biconditional truth table

$p \leftrightarrow q$ means that p and q have the **same** truth value.

Note this truth table is the exact **opposite** of $\oplus$'s!

$p \leftrightarrow q$ means $\neg(p \oplus q)$

$p \leftrightarrow q$ does **not** imply p and q are true, or cause each other.

p	q	$p \leftrightarrow q$
F	F	T
F	T	F
T	F	F
T	T	T

Converse, Inverse, Contrapositive

Some terminology, for an implication $p \rightarrow q$:

Its *converse* is: $q \rightarrow p$.

Its *inverse* is: $\neg p \rightarrow \neg q$.

Its *contrapositive*: $\neg q \rightarrow \neg p$.

One of these three has the *same meaning* (same truth table as $p \rightarrow q$. Can you figure out which?

Contrapositive

Assignment

1.State the converse,contapositive and inverse of conditional statement"If it snows today,I will ski tomorrow".

2. Determine whether following biconditional are true or false

a) 1 + 1 if and only if monkeys can fly.

b) 0 > 1 if and only if 2 > 1.

Video link:1.https://www.youtube.com/watch?v=F3544ZyO-eU&list=PLHXZ9OQGMqxersk8fUxiUMSIx0DBqsKZS&index=16

2.https://www.youtube.com/watch?v=BNhp6LXncI0&list=PLHXZ9OQGMqxersk8fUxiUMSIx0DBqsKZS&index=20

References: Discrete Mathematics and its application by Kenneth H Rosen

Thank You!!!!!