BLACK-SCHOLES OPTION PRICING MODEL

Name of the Faculty: GIRISH GARG
Program Name: BBA (FIA)

The Black-Scholes Option Pricing Model

- The B-S option pricing model for a call is:

$$C = S_0 - Xe^{-rT} + P$$

$$C = S_0 N(d_1) - Xe^{-rT} N(d_2)$$

where

$d_1 = [ln(S/X)+(r+ ½\sigma^2)T]/\sigma\sqrt{T}$

$d_2 = d_1 - \sigma\sqrt{T}$

$N(d)$ = cumulative normal distribution

Black-Scholes Put Price

- Price of a European put is:

$$P = C - S_0 + Xe^{-rT}$$

$$= S_0[N(d_1)-1] - Xe^{-rT}[N(d2)-1]$$

where d_1, d_2, and N(d) are defined as before.

Black-Scholes Pricing Example

- Assume:
  - $S_0 = \$100$
  - $X = \$100$
  - $r = 5\%$
  - $\sigma = 22\%$
  - $T = 1$ year
- Then:
  - $d_1 = 0.34$, $N(d_1) = 0.6331$
  - $d_2 = 0.12$ $N(d_2) = 0.5478$

$$C = S_0 N(d_1) - Xe^{-rT} N(d_2)$$

$$d_1 = [ln(S/X)+(r+ ½\sigma^2)T]/\sigma\sqrt{T}$$

$$d_1 = [ln(100/100)+(.05+ ½(0.22)^2)1]/(0.22)\sqrt{1}$$

$$d_1 = 0 + .0742/.22 = .337274$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$d_2 = .33727 - 0.22/\sqrt{1} = .117273$$

Call Option Example

- Price of a call is then:

  $C = S_0N(d_1) - Xe^{-rT}N(d_2)$

  $C = 100(0.6331) - 100(0.9512)(0.5478)$

  $= \$11.20$

- Price of a put is then:

  $P = S_0[N(d_1)-1] - Xe^{-rT}[N(d2)-1]$

  $P = 100[.6331 - 1] - 100(1/e^{(.05*1)})(.5478-1)$

  $P = 100(-0.3669) - 100(0.9512)(-0.4522)$

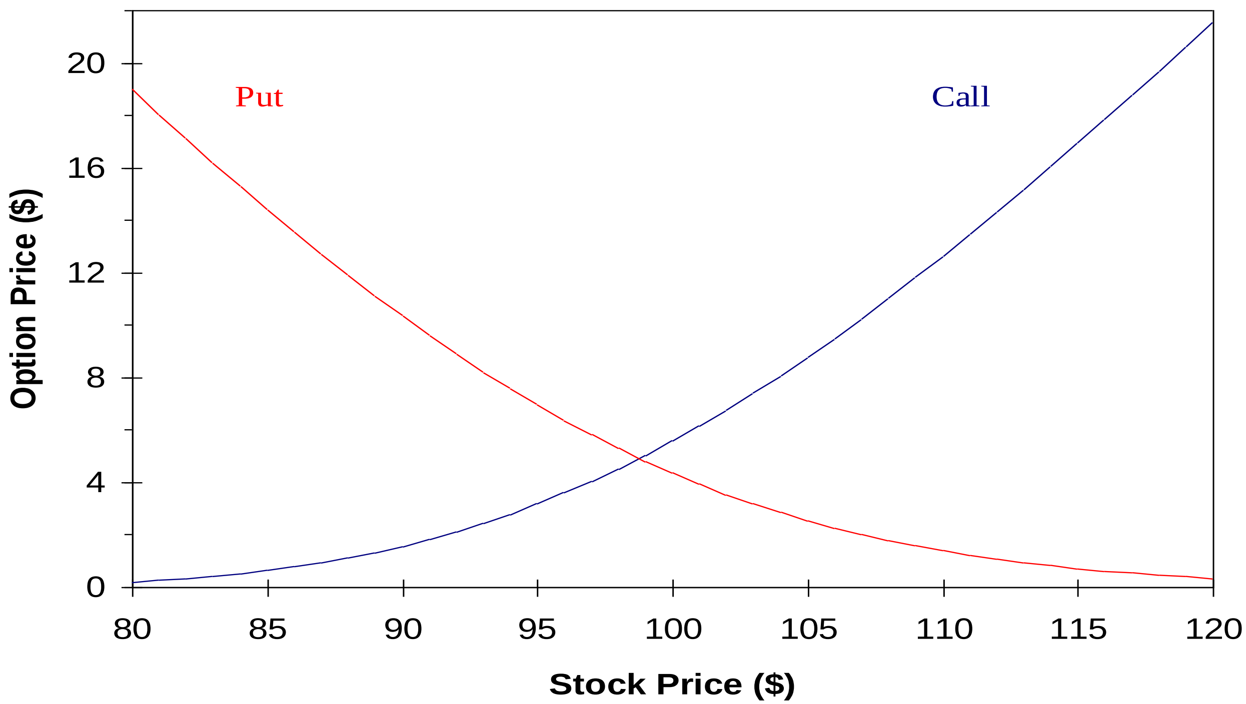
  $= \$6.32$

- Double check through Put-Call Parity:

  $P = C - S_0 + Xe^{-rT}$

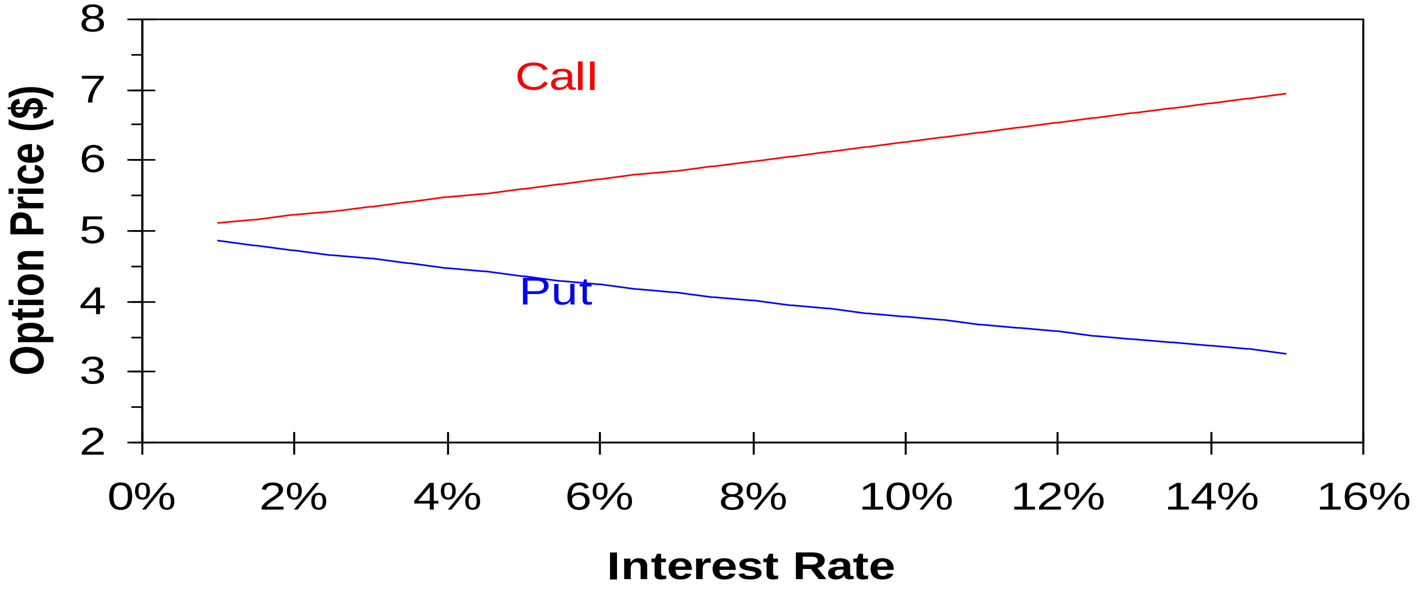
  $6.32 = 11.20 – 100 + 100(0.9512)$

Relationship of Option and Security Prices



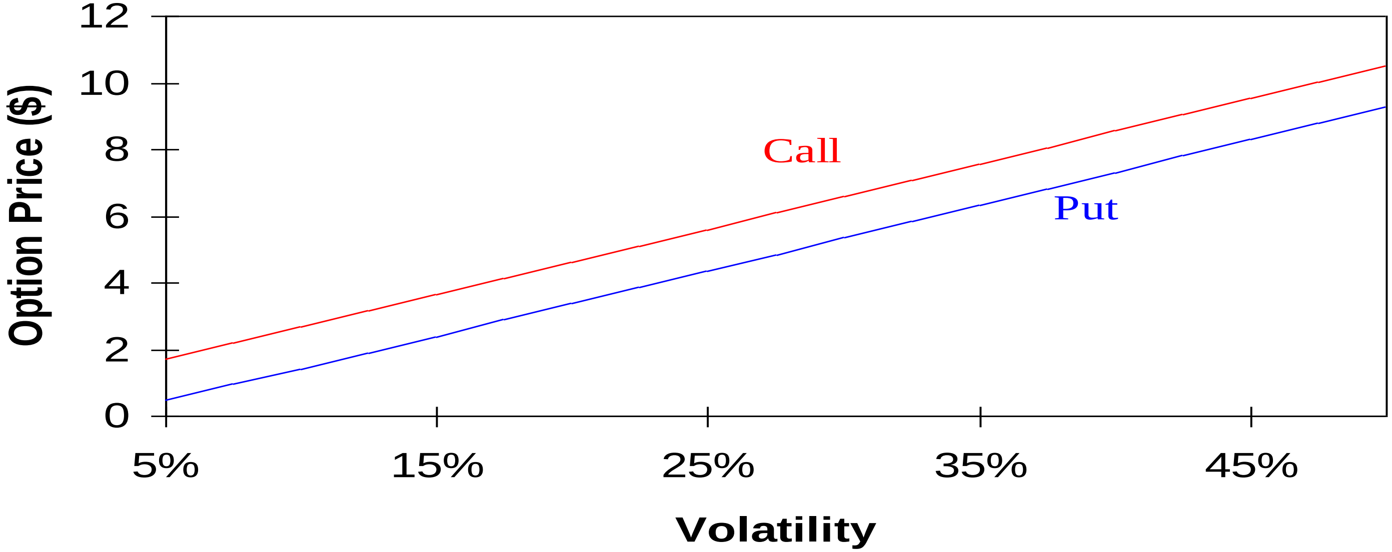
Parameters: X = \$100, T = 3 months, *r* = 5%, and σ = 25%
Changing S

Relationship of Option Prices to Interest Rates



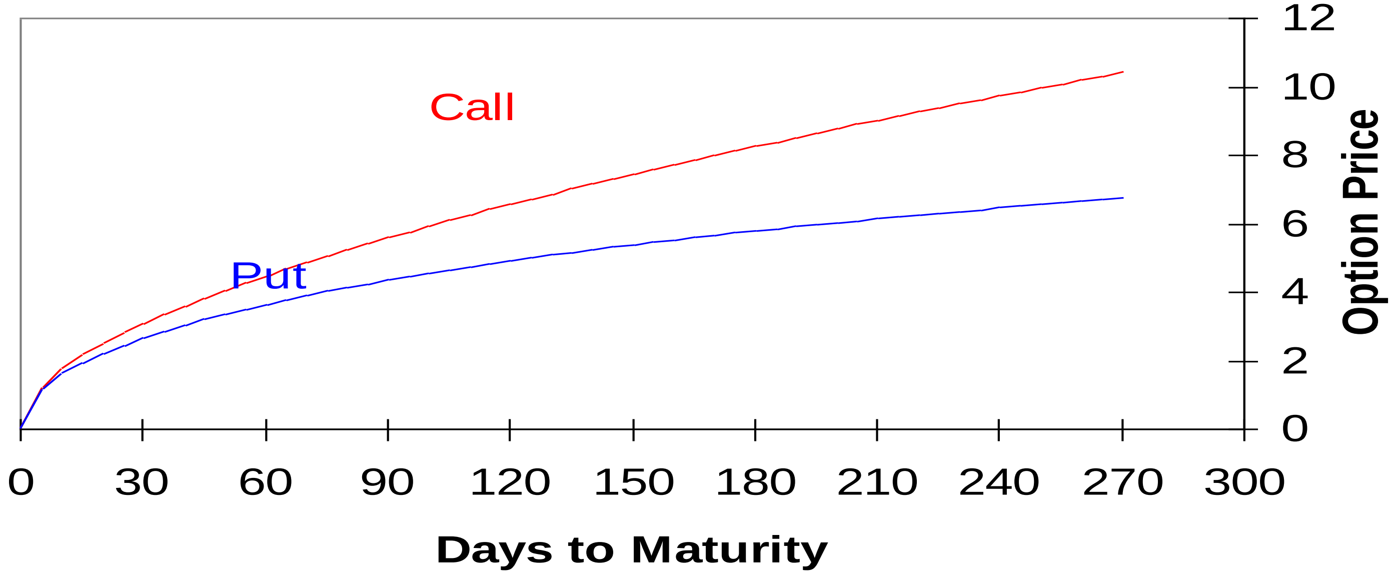
Parameters: S=$100, X = $100, T = 3 months, and σ = 25%
Changing r

Relationship of Option Prices to Volatility



Parameters: S=$100, X = $100, T = 3 months, and *r* = 5%
Changing σ

Relationship of Option Prices to Time to Expiration



Parameters: *S* = \$100, *X* = \$100, *r* = 5%, and σ = 25%
Changing t

Parameters of the Black-Scholes Model

- Need to know:
  - S, X, r, T, σ.
- All readily observable, except the last.
- The interest rate should be a continuously compounded rate
  - To convert simple annualized rate to continuously compounded rate:

$$r = \ln(1+R)$$

Volatility as a Parameter

- In pricing options, analysts usually use some measure of historical volatility of the underlying security.
- Volatility obtained from other than annualized returns must be converted to annualized volatility.
  - *e.g.*, Variance of weekly returns must be multiplied by 52.
  - *e.g.*, Standard deviation of weekly returns must be multiplied by $\sqrt{52}$.

Implied Volatility

- Alternatively, can use all the other inputs, and infer a volatility estimate from the current option price.
  - Is called the implied volatility.
- Can then compare implied volatility with recent historical volatility.
  - Higher implied than historical may indicate the option is expensive.
  - Lower implied than historical may indicate the option is cheap.

Implied Volatility Using the Black-Scholes Model

http://www.numa.com/derivs/ref/calculat/option/calc-opa.htm

Volatility Assumptions	*Put Price*	*Call Price*
15%	$1.41	$2.04
20%	1.98	2.61
25%	2.55	3.18
30%	3.11	3.74
35%	3.68	4.31

Volatility implied by option prices

Given Information

S_0 = $100, X = 100

r = 8%, *T* = 30 days,

P = $3.10, and C = $3.73

Assumptions In Original Option Pricing Model

- Underlying returns log normally distributed.
- Variance is constant over time.
- The interest rate is constant over time.
- No sudden jumps in underlying price.
- No dividends.
- No early exercise (*i.e.*, European option).

Enhancing Firm Value through Hedging

- Reducing Volatility of cash flows does not guarantee increased value.
- Hedging has transaction costs, so hedging is not free.
- Hedging can add value if
  - Taxes are reduced
  - Transaction costs (like default risk) is reduced
  - When it aligns incentives to take positive NPV projects

Un-hedged

Outcome	Probability	Value of the Firm in Period 1
Price of oil high	0.5	1000
Price of oil low	0.5	200

Capital Structure	Book Values	Price of Oil High Market Value at t=1	Price of Oil Low Market Value at t=1	Market Value
Debt	500	500	200	350
Equity	500	500	0	250
		1000	**200**	**600**

Does hedging this company's risk increase value?

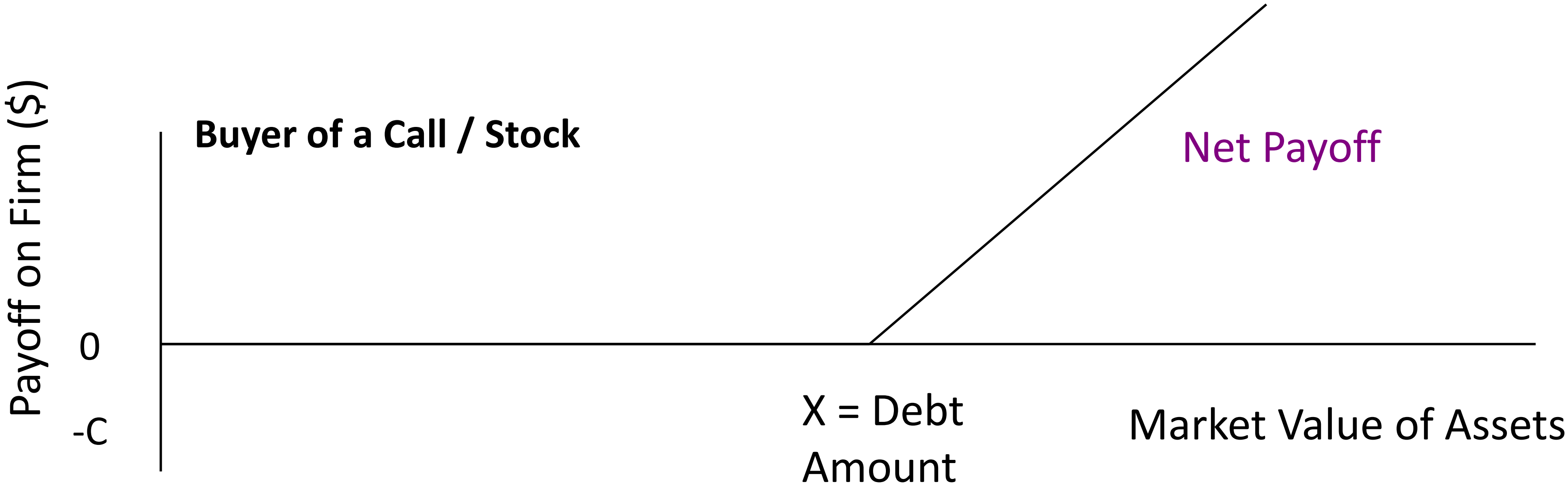
Hedged

Outcome	Probability	Value of the Firm in Period 1
Price of oil high	0.5	600
Price of oil low	0.5	600

Capital Structure	Book Values	Price of Oil High Market Value at t=1	Price of Oil Low Market Value at t=1	Market Value
Debt	500	500	500	500
Equity	500	100	100	100
		600	**600**	**600**

The total market value is not affected (both are $600); however the distribution is affected. The Stockholder value was decreased from $250 to $100 with hedging, showing that there is a transfer of wealth to bondholders. This is due to the fact that the firm is on the brink of insolvency.

Note the similarities between the payoff on stock and a call option.



In our prior example, stockholders only get paid after the debt holders receive their value. Therefore, the value of the debt is like the exercise price on a call option. If the value of the firm is less than the value of the debt, stockholders will walk away and leave the firm to the debt holders. If the value of the firm is greater than the value of the debt, the stockholders remain in control of the firm.

This also shows why reducing volatility (through hedging) does not guarantee an increase in the value of the firm. In fact, as shown in the Black Scholes formula, decreasing volatility can reduce the value of the firm to equity holders (see the hedging example several slides earlier.

Will the Un-hedged firm add a risk-free project when new capital must be added by equity holders

Outcome	Probability	Value of the Firm in Period 1	Value of the Firm in Period 1 w/Investment
Price of oil high	0.5	1000	1300
Price of oil low	0.5	200	500
New Investment	200		
Cash Flow at t=1	300	Should the investment be taken?	

Capital Structure	Book Values	Price of Oil High Market Value at t=1	Price of Oil Low Market Value at t=1	Market Value
Debt	500	500	500	500
Equity	700	800	0	400
		1300	**500**	**900**

Equityholders have a value of $400, compared to a value of $250 if no project is taken. But remember, that the equityholders added $200 to make the investment. So they gained $150 but it cost them $200 to obtain this gain. Only the bondholders have benefited.

Would New Bondholders add the new capital?

Bondholders generally enter as subordinate to the old bonds.

Outcome	Probability	Value of the Firm in Period 1	Value of the Firm in Period 1 w/Investment	
Price of oil high	0.5	1000	1300	
Price of oil low	0.5	200	500	
New Investment	200			
Cash Flow at t=1	300	Should the investment be taken?		

Capital Structure	Book Values	Price of Oil High Market Value at t=1	Price of Oil Low Market Value at t=1	Market Value
Senior Debt	500	500	500	500
Sub. Debt	200	200	0	100
Equity	500	600	0	300
		1300	**500**	**900**

New debtholders will not enter into this transaction, it has a guaranteed loss for the new debtholders.

Will the hedged firm add take a risk-free project?

Outcome	Probability	Value of the Firm in Period 1	Value of the Firm in Period 1 w/Investment
Price of oil high	0.5	600	900
Price of oil low	0.5	600	900
New Investment	200		
Cash Flow at t=1	300		

Capital Structure	Book Values	Price of Oil High Market Value at t=1	Price of Oil Low Market Value at t=1	Market Value
Debt	500	500	500	500
Equity	700	400	400	400
		900	**900**	**900**

When the firm does not have concerns about market value falling below the debt outstanding, then the firm will take any positive NPV projects.

Note: From our original example, we would only choose to hedge the firm if the NPV of the project was greater than $150 (the amount of value lost from the decision to hedge in the prior slide).

References:

- srivastava, r. (2017). *financial derivative and risk management.* new delhi: oxford university press.
- hull, j. c. (1988). *options, futures and other derivatives.* (9th, Ed.) pearson.

Thank you