

School of Basic and Applied Sciences

Course Code : MSCM303

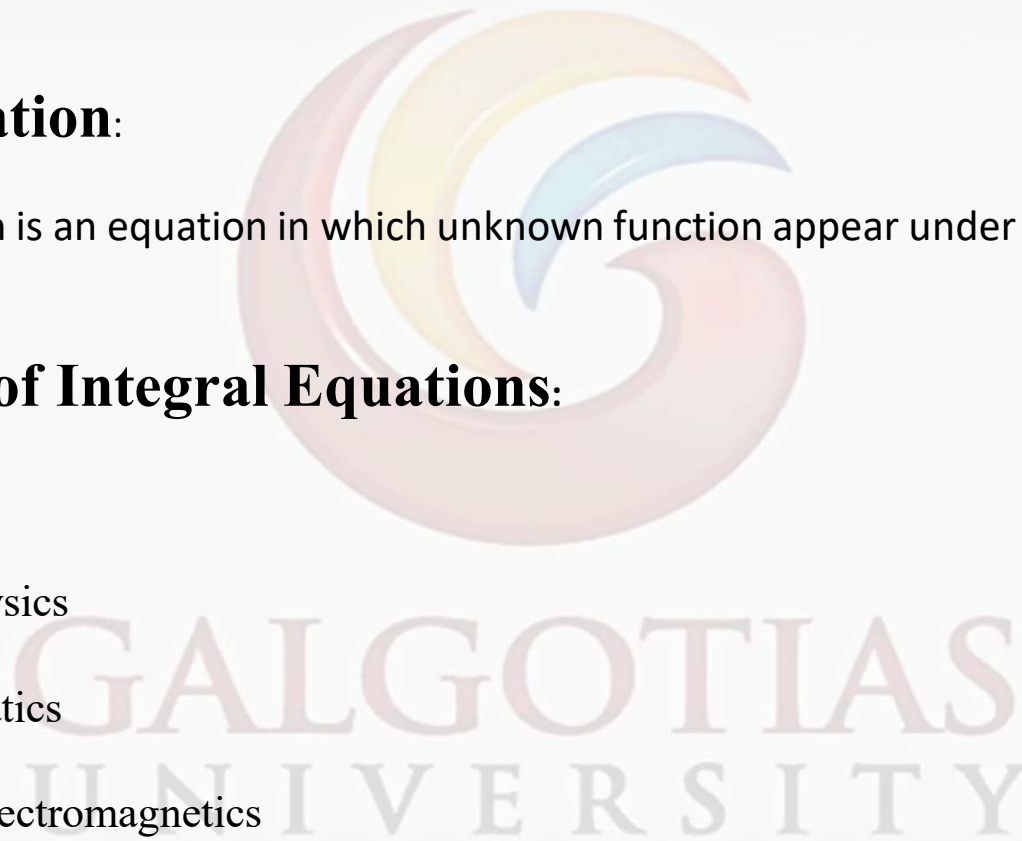
Course Name: Integral equations and calculus of variation

Integral Equation:

An integral equation is an equation in which unknown function appear under integral sign.

Applications of Integral Equations:

1. Mechanics
2. Mathematical physics
3. Applied Mathematics
4. Computational Electromagnetics



Name of the Faculty: Dr. Leena Rani

Program Name: M.Sc(Mathematics)

Abel's Problem:

Let us consider a smooth curve in a vertical plane. Let a particle starts from rest at any point P with co-ordinate x under the influence of gravity along the curve.

Let $t=f(x)$ where $f(x)$ is the given curve. The absolute velocity of the particle at any point Q whose ordinate is η is given by

$$v = \sqrt{2g(x-\eta)}$$

Let β be the angle of inclination of the tangent to the curve at Q with ξ axis.

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then
$$\frac{d\eta}{dt} = -\sqrt{2g(x-\eta)} \sin \beta$$

or
$$dt = -\frac{d\eta}{\sin \beta \sqrt{2g(x-\eta)}} .$$

Now, let $\frac{1}{\sin \beta} = \phi(\eta)$ then

$$\sqrt{2g} dt = -\frac{\phi(\eta) d\eta}{\sqrt{(x-\eta)}} .$$

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Hence

$$t\sqrt{2g} = -\int_P^R \frac{\phi(\eta)d\eta}{\sqrt{x-\eta}}$$

$$\sqrt{2g} f_1(x) = \int_0^x \frac{\phi(\eta)d\eta}{\sqrt{x-\eta}}$$

$$f(x) = \int_0^x \frac{\phi(\eta)d\eta}{\sqrt{x-\eta}}.$$



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where $f(x) = f_1(x)\sqrt{2g}$ is known and $\phi(\eta)$ is the unknown function.

After finding $\phi(\eta)$, We obtain the equation of the curve as

$$\phi(\eta) = \frac{1}{\sin \beta}$$

$$\eta = \Phi(\beta) .$$

$$\frac{d\eta}{d\xi} = \tan \beta$$



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or

$$\frac{\Phi'(\beta)d\beta}{d\xi} = \tan\beta$$

⇒

$$\xi = \int \frac{\Phi'(\beta)}{\tan\beta} d\beta = \Phi_1(\beta) .$$



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Thus the required curve is given in parametric form as

$$\xi = \Phi_1(\beta), \eta = \Phi(\beta).$$

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Hence the Abel's Problem reduces to a solution of Volterra Integral equation of First kind.

$$f(x) = \int_0^x K(x,t)\phi(t)dt,$$

where $\phi(x)$ is an unknown function, $K(x,t) = \frac{1}{\sqrt{x-t}}$ and $f(x)$ are given functions.

Reference:

<https://nptel.ac.in/courses/111/107/111107103/>

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