


Unit II : ASSOCIATION RULES

Basic Concepts - Market Basket Analysis -
Frequent Itemsets, Closed Itemsets and
Association Rules - Frequent Itemset Mining
Methods – Apriori Algorithm – Generating
Association Rules - Frequent pattern growth
- Mining Various Kinds of Association Rules

Mining Frequent Patterns, Association and Correlations

- Basic concepts and a road map 
- Efficient and scalable frequent itemset mining methods
- Mining various kinds of association rules
- From association mining to correlation analysis
- Constraint-based association mining
- Summary

What Is Frequent Pattern Analysis?

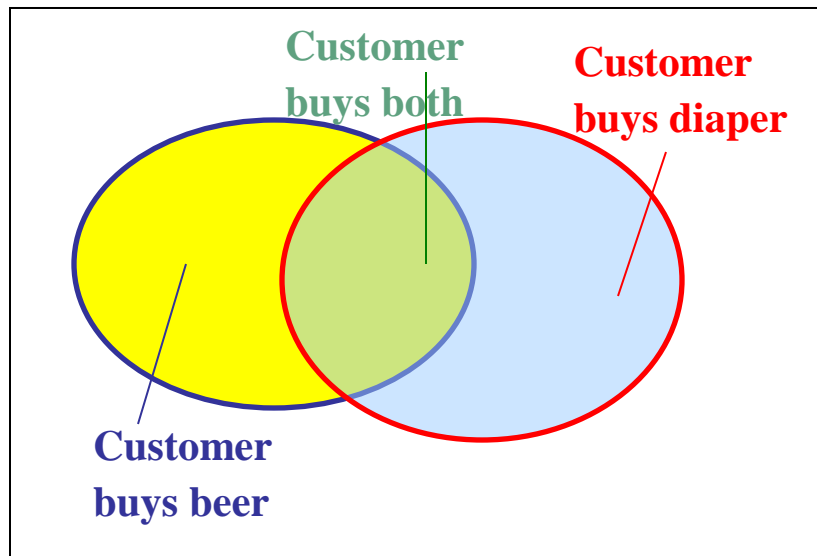
- **Frequent pattern**: a pattern (a set of items, subsequences, substructures, etc.) that occurs frequently in a data set
- First proposed by Agrawal, Imielinski, and Swami [AIS93] in the context of **frequent itemsets** and **association rule mining**
- Motivation: Finding inherent regularities in data
 - What products were often purchased together?— Beer and diapers?!
 - What are the subsequent purchases after buying a PC?
 - What kinds of DNA are sensitive to this new drug?
 - Can we automatically classify web documents?
- Applications
 - Basket data analysis, cross-marketing, catalog design, sale campaign analysis, Web log (click stream) analysis, and DNA sequence analysis.

Why Is Freq. Pattern Mining Important?

- Discloses an intrinsic and important property of data sets
- Forms the foundation for many essential data mining tasks
 - Association, correlation, and causality analysis
 - Sequential, structural (e.g., sub-graph) patterns
 - Pattern analysis in spatiotemporal, multimedia, time-series, and stream data
 - Classification: associative classification
 - Cluster analysis: frequent pattern-based clustering
 - Data warehousing: iceberg cube and cube-gradient
 - Semantic data compression: fascicles
 - Broad applications

Basic Concepts: Frequent Patterns and Association Rules

Transaction-id	Items bought
10	A, B, D
20	A, C, D
30	A, D, E
40	B, E, F
50	B, C, D, E, F



- Itemset $X = \{x_1, \dots, x_k\}$
- Find all the rules $X \rightarrow Y$ with minimum support and confidence
 - **support**, s , **probability** that a transaction contains $X \cup Y$
 - **confidence**, c , **conditional probability** that a transaction having X also contains Y

Let $sup_{min} = 50\%$, $conf_{min} = 50\%$
 Freq. Pat.: $\{A:3, B:3, D:4, E:3, AD:3\}$

Association rules:

$A \rightarrow D$ (60%, 100%)

$D \rightarrow A$ (60%, 75%)


Closed Patterns and Max-Patterns

- A long pattern contains a combinatorial number of sub-patterns, e.g., $\{a_1, \dots, a_{100}\}$ contains $\binom{100}{1} + \binom{100}{2} + \dots + \binom{100}{100} = 2^{100} - 1 = 1.27 \times 10^{30}$ sub-patterns!
- Solution: Mine *closed patterns* and *max-patterns* instead
- An itemset X is **closed** if X is *frequent* and there exists *no super-pattern* $Y \supset X$, with the same support as X (proposed by Pasquier, et al. @ ICDT'99)
- An itemset X is a **max-pattern** if X is frequent and there exists no frequent super-pattern $Y \supset X$ (proposed by Bayardo @ SIGMOD'98)
- Closed pattern is a lossless compression of freq. patterns
 - Reducing the # of patterns and rules

Closed Patterns and Max-Patterns

- Exercise. $DB = \{ \langle a_1, \dots, a_{100} \rangle, \langle a_1, \dots, a_{50} \rangle \}$
 - $Min_sup = 1.$
- What is the set of **closed itemset**?
 - $\langle a_1, \dots, a_{100} \rangle: 1$
 - $\langle a_1, \dots, a_{50} \rangle: 2$
- What is the set of **max-pattern**?
 - $\langle a_1, \dots, a_{100} \rangle: 1$
- What is the set of **all patterns**?
 - !!

Mining Frequent Patterns, Association and Correlations

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Scalable Methods for Mining Frequent Patterns

- The **downward closure** property of frequent patterns
 - Any subset of a frequent itemset must be frequent
 - If **{beer, diaper, nuts}** is frequent, so is **{beer, diaper}**
 - i.e., every transaction having {beer, diaper, nuts} also contains {beer, diaper}
- Scalable mining methods: Three major approaches
 - Apriori (Agrawal & Srikant@VLDB'94)
 - Freq. pattern growth (FPgrowth—Han, Pei & Yin @SIGMOD'00)
 - Vertical data format approach (Charm—Zaki & Hsiao @SDM'02)

Apriori: A Candidate Generation-and-Test Approach

- Apriori pruning principle: If there is **any** itemset which is infrequent, its superset should not be generated/tested!
(Agrawal & Srikant @VLDB'94, Mannila, et al. @ KDD' 94)
- Method:
 - Initially, scan DB once to get frequent 1-itemset
 - **Generate** length $(k+1)$ **candidate** itemsets from length k **frequent** itemsets
 - **Test** the candidates against DB
 - Terminate when no frequent or candidate set can be generated

The Apriori Algorithm—An Example

$Sup_{min} = 2$

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1st scan

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

2nd scan

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

L_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

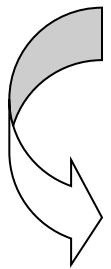
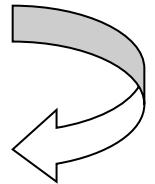
C_3

Itemset
{B, C, E}

3rd scan

L_3

Itemset	sup
{B, C, E}	2



The Apriori Algorithm

- Pseudo-code:

C_k : Candidate itemset of size k

L_k : frequent itemset of size k

$L_1 = \{\text{frequent items}\};$

for ($k = 1; L_k \neq \emptyset; k++$) **do begin**

C_{k+1} = candidates generated from L_k ;

for each transaction t in database do

increment the count of all candidates in C_{k+1}

that are contained in t

L_{k+1} = candidates in C_{k+1} with min_support

end

return $\cup_k L_k$;

Important Details of Apriori

- How to generate candidates?
 - Step 1: self-joining L_k
 - Step 2: pruning
- How to count supports of candidates?
- Example of Candidate-generation
 - $L_3 = \{abc, abd, acd, ace, bcd\}$
 - Self-joining: $L_3 * L_3$
 - $abcd$ from abc and abd
 - $acde$ from acd and ace
 - Pruning:
 - $acde$ is removed because ade is not in L_3
 - $C_4 = \{abcd\}$

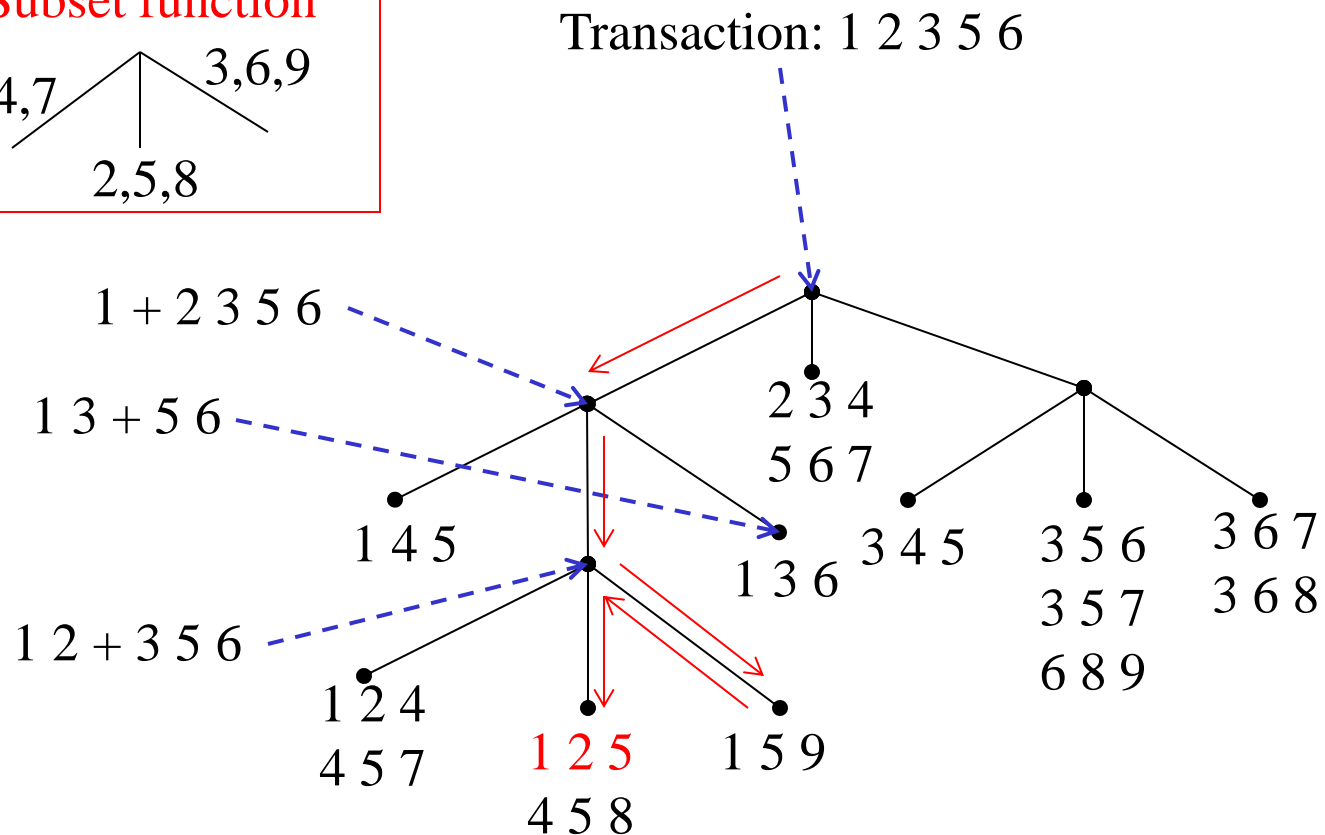
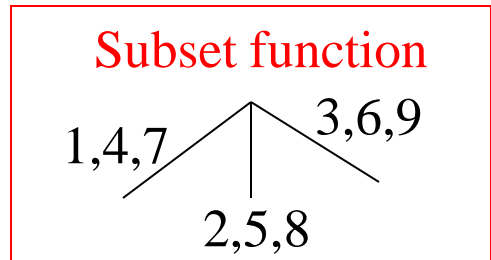
How to Generate Candidates?

- Suppose the items in L_{k-1} are listed in an order
- Step 1: self-joining L_{k-1}
 - insert into C_k
 - select ***p.item₁, p.item₂, ..., p.item_{k-1}, q.item_{k-1}***
 - from ***L_{k-1} p, L_{k-1} q***
 - where ***p.item₁=q.item₁, ..., p.item_{k-2}=q.item_{k-2}, p.item_{k-1} <***
q.item_{k-1}
- Step 2: pruning
 - forall ***itemsets c in C_k*** do
 - forall ***(k-1)-subsets s of c*** do
 - if (*s is not in L_{k-1}*) then delete *c* from *C_k***

How to Count Supports of Candidates?

- Why counting supports of candidates a problem?
 - The total number of candidates can be very huge
 - One transaction may contain many candidates
- Method:
 - Candidate itemsets are stored in a *hash-tree*
 - *Leaf node* of hash-tree contains a list of itemsets and counts
 - *Interior node* contains a hash table
 - *Subset function*: finds all the candidates contained in a transaction

Example: Counting Supports of Candidates



Efficient Implementation of Apriori in SQL

- Hard to get good performance out of pure SQL (SQL-92) based approaches alone
- Make use of object-relational extensions like UDFs, BLOBs, Table functions etc.
 - Get orders of magnitude improvement
- S. Sarawagi, S. Thomas, and R. Agrawal. [Integrating association rule mining with relational database systems: Alternatives and implications](#). In SIGMOD'98

Challenges of Frequent Pattern Mining

- Challenges
 - Multiple scans of transaction database
 - Huge number of candidates
 - Tedious workload of support counting for candidates
- Improving Apriori: general ideas
 - Reduce passes of transaction database scans
 - Shrink number of candidates
 - Facilitate support counting of candidates

Partition: Scan Database Only Twice

- Any itemset that is potentially frequent in DB must be frequent in at least one of the partitions of DB
 - Scan 1: partition database and find local frequent patterns
 - Scan 2: consolidate global frequent patterns
- A. Savasere, E. Omiecinski, and S. Navathe. *An efficient algorithm for mining association in large databases*. In *VLDB'95*

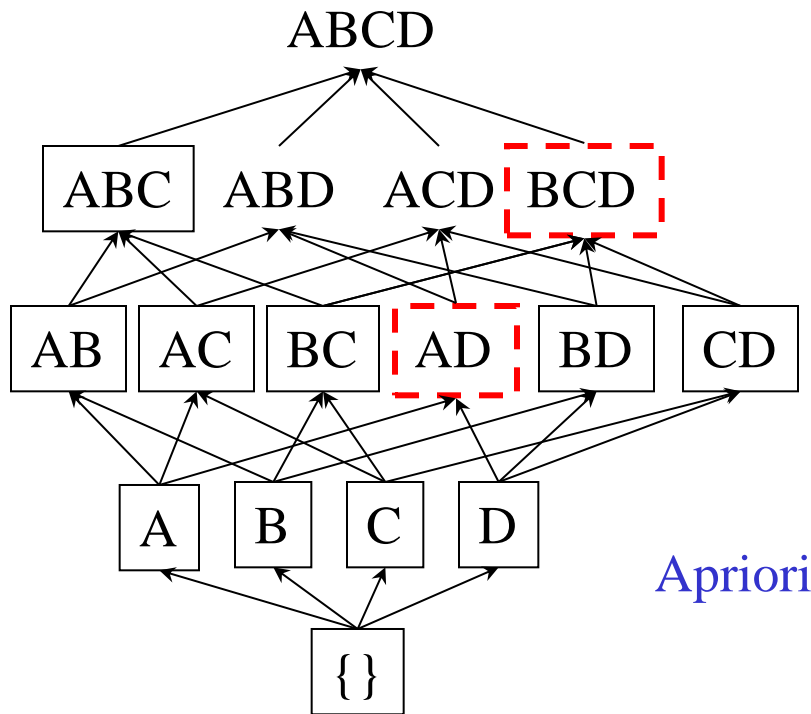
DHP: Reduce the Number of Candidates

- A k -itemset whose corresponding hashing bucket count is below the threshold cannot be frequent
 - Candidates: a, b, c, d, e
 - Hash entries: {ab, ad, ae} {bd, be, de} ...
 - Frequent 1-itemset: a, b, d, e
 - ab is not a candidate 2-itemset if the sum of count of {ab, ad, ae} is below support threshold
- J. Park, M. Chen, and P. Yu. *An effective hash-based algorithm for mining association rules.*

Sampling for Frequent Patterns

- Select a sample of original database, mine frequent patterns within sample using Apriori
- Scan database once to verify frequent itemsets found in sample, only *borders* of closure of frequent patterns are checked
 - Example: check *abcd* instead of *ab, ac, ..., etc.*
- Scan database again to find missed frequent patterns
- H. Toivonen. [Sampling large databases for association rules.](#)

DIC: Reduce Number of Scans



Itemset lattice

Dynamic itemset counting and implication rules for market basket data.

- Once both A and D are determined frequent, the counting of AD begins
- Once all length-2 subsets of BCD are determined frequent, the counting of BCD begins



1-itemsets

2-itemsets

...

1-itemsets

2-items

3-items

DIC

Bottleneck of Frequent-pattern Mining

- Multiple database scans are **costly**
- Mining long patterns needs many passes of scanning and generates lots of candidates
 - To find frequent itemset $i_1 i_2 \dots i_{100}$
 - # of scans: **100**
 - # of Candidates: $\binom{100}{1} + \binom{100}{2} + \dots + \binom{100}{100} = 2^{100} - 1 = \mathbf{1.27 * 10^{30} !}$
- Bottleneck: candidate-generation-and-test
- Can we avoid candidate generation?