Periodic Functions

Periodic Functions

The Mathematic Formulation:

Any function that satisfies

$$f(t) = f(t+T)$$

where T is a constant and is called the *period* of the function.

Periodic Functions

Definition:

If the value of each ordinate $f(t)$ repeats itself at equal intervals in the abscissa, then $f(t)$ is said to be a periodic function.

If $f(t) = f(t + T) = f(t + 2T) = \ldots$ then T is called the period of the function $f(t)$.

Example:

$$f(t) = \cos\frac{t}{3} + \cos\frac{t}{4}$$

Find its period.

$$f(t) = f(t+T) \Rightarrow \cos\frac{t}{3} + \cos\frac{t}{4} = \cos\frac{1}{3}(t+T) + \cos\frac{1}{4}(t+T)$$

Fact: $\cos\theta = \cos(\theta + 2m\pi)$

$$\frac{T}{3} = 2m\pi \Rightarrow T = 6m\pi$$

$$\frac{T}{4} = 2n\pi \Rightarrow T = 8n\pi$$

$$\Rightarrow T = 24\pi$$ smallest T

Example:

$$f(t) = \cos 10t + \cos(10+\pi)t$$

Is this function a periodic one?

$$\frac{\omega_1}{\omega_2} = \frac{10}{10+\pi}$$

not a rational number

Example:

The period of $\sin x$, $\cos x$, $\sec x$, and $\operatorname{cosec} x$ is 2π.

The period of $\tan x$ and $\cot x$ is π.

- $\sin x = \sin(x + 2\pi) = \sin(x + 4\pi) = \ldots$ so $\sin x$ is a periodic function with the period 2π.
- $\sin 5x = \sin(5x + 2\pi) = \sin 5\left(x + \frac{2\pi}{5}\right)$, Period $= \frac{2\pi}{5}$

Example:

- $\cos 3x = \cos(3x + 2\pi) = \cos 3\left(x + \frac{2\pi}{3}\right)$, Period $= \frac{2\pi}{3}$

- $$\cos\frac{2n\pi x}{k} = \cos\left(\frac{2n\pi x}{k} + 2\pi\right) = \cos\frac{2n\pi}{k}\left(x + \frac{2\pi k}{2n\pi}\right)$$

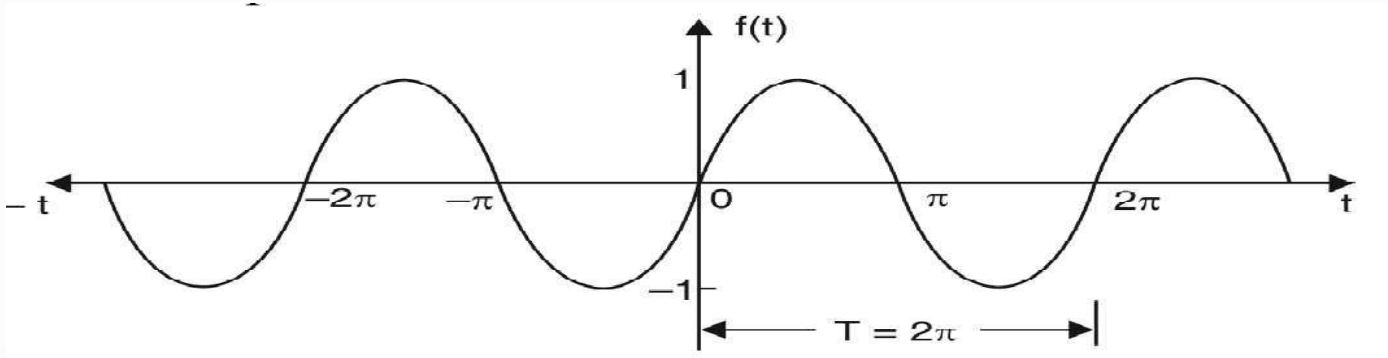
$$= \cos\frac{2n\pi}{k}\left(x + \frac{k}{n}\right), \text{Period} = \frac{k}{n}$$

Example:

$$\tan 2x = \tan (2x + \pi) = \tan 2\left(x + \frac{\pi}{2}\right), \quad \text{Period} = \frac{\pi}{2}$$

Graphical representation :

This is also periodic Function



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