Project report

on

Elastodynamic Response of Time Harmonic Sources in Three-Phase-Lag Orthotropic Thermoelastic Material

Submitted in Partial Fulfilment of the Requirement for the Degree of
M.Sc. Mathematics

Submitted by

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CERTIFICATE

This is to Certify that ***Mr. Lalit Goyal*** has carried out his project work entitled "Elastodynamic Response of time harmonic sources in a three-phase-lag orthotropic thermoelastic material" under our supervision. This work is fit for submission for the award of Master Degree in Mathematics.

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CANDIDATE DECLARATION

I hereby declare that the dissertation entitled "Elastodynamic Response of time harmonic sources in a three- phase- lag orthotropic thermoelastic material" submitted by me in partial fulfillment for the degree of M.Sc. in Applied Mathematics to the Division of Mathematics, School of Basic and Applied Science, Galgotias University, Greater Noida, Uttar Pradesh, India is my original work. It has not been submitted in part or full to this University of any other Universities for the award of diploma or degree.

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ABSTRACT

Three-phase-lag model of a homogenous thermally conducting orthotropic thermoelastic material subjected to mechanical and thermal source has been studied. For solving the present problem, we have applied finite element method. For that system of partial differential equations has been converted into weak form for single element. For system level, we have assembled these equations for n-number of elements. The components of displacements, stresses and temperature distribution obtained in the physical domain are computed by gauss elimination technique, analytically. Comparative study is possible by different value of relaxation times for various theories of thermoelasticity.

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1. Introduction

Thermoelasticity contains the theory of heat conduction and the theory of strain and stresses due to the flow of heat, when coupling of temperature and deformation fields occurs.

The classical uncoupled theory of thermoelasticity predicts two phenomena not compatible with the physical observations. First, the equation of heat conduction of this theory does not contain any elastic terms; second, the heat equation is of a parabolic type, predicting infinite speeds of propagation for heat waves. The coupling between the strain and temperature fields was first postulated by Duhamel (1837) who derived equations for the distribution of strains in an elastic medium subjected to temperature gradients.

The coupling between thermal and strain fields gives rise to the coupled theory of thermoelasticity. For static problems this coupling vanishes and the two fields become independent of each other.

Chadwick and Sneddon (1958) discussed in detail the influence of volume and thermal changes, coupled with each other in the form of plane harmonic waves. Ignaczak (1960) investigated a plane problem of dynamic thermal distortion in thermoelasticity.

Boley and Tolins (1962) investigated the transient coupled thermoelastic boundary-value problems in the half-space. A list of Nowacki's papers on the coupled theory of thermoelasticity can be found in his monumental books (1962, 1975). Hetnarski (1964) investigated the coupled thermoelastic problem for the half-space.

The equations of coupled thermoelasticity consist of two equations: the first, governing the displacement vector, is a wave type equation; the second, governing the temperature field, is a diffusion type equation. Due to the nature of the second equation, if the elastic medium extending to infinity is subjected to mechanical or thermal disturbance, the effect will be felt instantaneously at infinity; this implies that a part of disturbance has an infinite velocity of propagation which is physically impossible. This paradox in the existing coupled theory of thermoelasticity has also been discussed by Boley (1964). To overcome this drawback need was felt to develop the theories of generalized thermoelasticity.

The first is due to Lord and Shulman (1967), who introduced the theory of generalized thermoelasticity with one relaxation time by postulating a new law of heat conduction to replace the classical Fourier's Law. This law contains the heat flux vector as well as its time derivative. It contains also a new constant that acts as a relaxation time. The heat equation of this theory is of the wave-type, it automatically ensuring finite speeds of propagation for heat and elastic waves. The remaining governing equations for this theory, namely, the equations of motion and constitutive relations, remain the same as those for the coupled and uncoupled theories of thermoelasticity.

Fox (1969) derived the constitutive equations of generalized thermoelasticity, which are valid for finite deformation and temperature variations, on the basis of modified Fourier's law to heat conduction. Grimado (1970) obtained thermal stress induced in a semi-infinite body as a result of a suddenly applied step heat input.

The second generalization to the coupled theory of thermoelasticity is what is known as the theory of thermoelasticity with two relaxation times or the theory of temperature-rate-dependent thermoelasticity. Müller (1971) in a review of thermodynamics of thermoelastic solids has proposed an entropy production inequality, with the help of which, he considered restrictions on a class of constitution equations. A generalization of this inequality was proposed by Green and Laws (1972). Green and Lindsay obtained an explicit version of the constitutive equations in (1972). These equations were also obtained independently by Suhubi (1975). This theory contains two constants that act as relaxation times and modifies all the equations of the coupled theory not the heat equation only.

The basic differences between the two theories of generalized thermoelasticity are as under:

I. The Lord and Shulman (L-S) theory modifies only the energy equation of the coupled theory by taking into account the time needed for acceleration of heat flow, whereas the Green and Lindsay (G-L) theory modifies both constitutive equations and the energy equation, accordingly, the L-S theory involves only one relaxation time of the thermoelastic process and the G-L theory involves two relaxation times.
II. The energy equation of L-S theory depends on both, the strain velocity and strain acceleration whereas the corresponding equation of G-L theory depends only on the strain velocity.
III. In the linearized case, according to the G-L theory, the heat can not propagate with a finite speed unless the stresses depend on the temperature velocity. According to L-S theory, heat can propagate with a finite speed even though the stresses are independent of temperature velocity

Roberts (1972) studied the effects of stresses and displacements caused by delta type function of temperature or stress line pulse moving with a constant speed over the surface of a coupled thermoelastic half-space.

Bahar and Hetnarski (1978) pointed out the disadvantages in using a potential function to solve the coupled one-dimensional boundary value problem of thermoelasticity. They used the methods of matrix exponential, which constitutes the basis of the state space approach, to avoid the difficulties of the potential function formulation.

Bahar and Hetnarski (1979) presented a connection between thermoelastic potential and state space approach of elasticity. Dhaliwal and Sherief (1980) derived the governing field equations of generalized thermoelasticity for anisotropic media and also developed a variational principle for these equations.

Chandrasekharaiah (1981) investigated one dimensional dynamical disturbance in a thermoelastic half-space plane boundary due to the application of a step in strain or temperature on the boundary, in the context of the linearized Green and Lindsay (1972) theory of thermoelasticity.Das, Das and Das (1983) employed the eigen value approach to discussed boundary value problems in one-dimensional coupled thermoelasticity.

Hyer and Cooper (1986) investigated the stress and deformations in composites tubes due to circumferential temperature gradient.Sharma (1986) discussed the problem of instantaneous heat sources in temperature rate dependent thermoelasticity. Chandrasekhariah and Srikantiah (1987) discussed temperature rate dependent thermoelastic interactions in an infinite solid due to a point heat source.

Dhaliwal and Rokne (1989) investigated the one dimensional thermal shock problem with two relaxation times. Kumar (1989) studied the coupled thermoelastic wave problem of an infinitely extended elastic plate of finite thickness subjected to an axially symmetric hydrostatic tension.Sharma and Chand (1990) studied the distribution of temperature and stresses in an elastic plate resulting from a suddenly punched hole.

Li (1992) formulated a generalized theory of thermoelasticity for an anisotropic medium using a form of the heat transport equation, which includes the time needed for acceleration of the heat flow. The variation principle corresponding to basic equations of generalized thermoelasticity for an anisotropic medium has been derived. Green and Naghdi (1993) proposed a new theory of thermoelasticity without energy dissipation and presented the derivation of a complete set of governing equations of the linearized version of the theory for homogenous and isotropic materials in terms of displacement and temperature fields and proved the uniqueness of the solutions of the corresponding initial mixed boundary value problem. An important feature of this theory, which is not present in other theories, is that this theory does not accommodate dissipation of thermal energy.

Sherief (1994) studied a thermomechanical shock problem for themoelasticity with two relaxation times. Ezzat (1995) determined the stress and temperature distributions with a continuous line source of heat in an infinite elastic body governed by the equations of generalized thermoelasticity with two relaxation times by using the Laplace and Hankel transform technique.

Another generalization to the coupled theory is known as low-temperature thermoelasticity, introduce by Hetnarski and Ignaczak (1996) (called H-T model). This model is characterized by a system of non-linear equations. Das, Lahiri and Dutta (1996) obtained the thermal stresses in a transversely isotropic elastic medium due to instantaneous heat source. Li and Dhaliwal (1996) discussed the thermal shock problem in thermoelasticity without energy dissipation.

Chandrasekhariah and Srinath (1997a) studied the problem of thermoelastic plane wave without energy dissipation in a half space due to time dependent heating of the boundary.Hata (1997) investigated the problem of stress focusing effects due to an instantaneous concentrated heat source in a sphere.

Chandrasekharaiah (1998) and Tzou (1995) proposed another generalization to coupled theory is known as dual-phase-lag thermoelasticity, in which Fourier law is replaced by an approximation to a modification of the Fourier law with two different translations for the heat flux and temperature gradient

Chandrasekharaiah and Srinath (1998a, 1998b) investigated the thermoelastic interactions without energy dissipation due to the (i) line heat source, and (ii) point heat source. Stefaniak (1998) presented the method of concentrated sources in solving thermal stress problems.

Hetnarski and Ignaczak (1999) in their survey article examined five generalizations to the coupled thermoelasticity, namely Lord and Shulman(1967), Green and Lindsay(1972), Green and Naghdi(1993), Hetnarski and Ignaczak(1996), Chandrasekharaiah and Tzou(1998), and obtained a number of interesting results. Chandrasekhariah (1999) studied one-dimensional disturbance in a half-space due to a thermal impulse on the boundary based on the theories of generalized thermoelasticity (Lord and Shulman, Green and Lindsay). Sharma and Chauhan (1999) investigated the problems of body forces and heat sources in thermoelasticity without energy dissipation.

Royer and Chenu (2000) studied an analytical model developed for the generation of surface acoustic waves in an isotropic solid by a thermoelastic laser line source. Lykotrafitis, Georgidias, and Brock (2001) studied the three-dimensional thermoelastic wave motions in a half-space under the action of a buried source. Das and Lahiri (2001) employed the eigen value approach to determine the thermal stress in an orthotropic elastic slab due to prescribed surface temperatures.

Han and Hasebe (2002) studied the problem of Green's functions of point heat source in various thermoelastic boundary value problems. Chao and Chen (2004) obtained the thermal stresses in an isotropic trimaterial interacted with a pair of point heat source and heat sink. Bakshi, Bera and Debnath (2004) employed the eigen value approach to study the effect of rotation and relaxation time in two dimensional problems of generalized thermoelasticity. Lin (2004) studied thermoelastic problems in anisotropic half-plane. Awrejcewicz and Pyryev (2005) proposed a thermo-mechanical model of frictional self-excited vibrations.

Roy Choudhuri (2007) introduced three-phase-lag model in which heat conduction law has been replaced by an approximation to a modification of the Fourier law with the introduction of three different phase lags for the heat flux vector, the temperature gradient and the thermal gradient.

Kumar and Rani (2007) considered a two-dimensional problem of thermoelasticity and discussed the effects of mechanical and thermal sources in generalized orthorhombic thermoelastic material.

Wirth and Jens (2008) discussed the application of anisotropic thermoelasticity.Yi (2009) studied 2D Green's function for semi-infinite orthotropic thermoelastic plane. Chirita and Ciarletta (2010) obtained the reciprocal and variational principles in linear thermoelasticity without energy dissipaton. Chirita (2011) studied the harmonic vibration in linear theory of thermoelasticity of type III.

Quintanilla (2012) discussed on uniqueness and continuous dependence in type III thermoelasticity. Beom (2013) considered thermoelastic in-plane problems in linear anisotropic solid. Abbas (2014) discussed eigenvalue approach in three dimensional generalized thermoelastic interactions with temperature dependent material properties.

Sharma and Kaur (2015) investigated response of anisotropic thermoelastic miro beam resonators under dynamic loads. Ramp type heating in thermally conducting cubic crystal has been studied by Abbas, Kumar and Rani (2015). Karamany and Ezzat (2016) discussed phase- lag- Green–Naghdi thermoelasticity theories. Bockstal and Marin (2017) studied recovery of a space dependent vector source in anisotropic thermoelastic system.

Hwu (2018) discussed analysis of 2D anisotropic thermoelasticity involving constant volume heat source by directly transformed boundary integral equation. Rani and Singh(2018) studied thermal disturbances in twinned orthotropic thermoelastic material. Rani and Shekhar(2020) discussed response of ramp-type heating in a monoclinic generalized thermoelastic material. Han (2020) investigated three-dimensional Green's functions for transient heat conduction problems in anisotropic bimaterial.

The exact solution for the time dependent problems for coupled and linear/nonlinear systems exists only for very special and simple initial and boundary conditions. Most of the deformation problems can be solved analytically with the help of Laplace and Fourier transform technique but finding the inversion of these methods is quite complicated. For avoiding these complications finite element method is preferable over Laplace/Fourier transform techniques as it can directly solve the problems in time-domain. Procedure for solving the deformation related problems by finite element method has been given in some of the books (1989,1993,2000,2005). It is a powerful technique, developed for numerical solution of complex problems in structural mechanics.

Shekhar and Parvez (2015) studied finite element analysis of the generalized magneto-thermoelastic inhomogeneous orthotropic solid cylinder. Neuman and Casique (2008) discussed Laplace-transform finite element solution of nonlocal and localized stochastic moment equations of transport. Othman and Abbas(2011) studied effect of rotation on plane waves at the free surface of a fibre-reinforced thermoelastic half-space using the finite element method. Abbas and Youssef (2013) discussed Two-temperature generalized thermoelasticity under Ramp-type heating by finite element method. Mukhopadhyay and Shivay (2019) studied a complete Galerkin's type approach of finite element for the solution of a problem on modified Green-Lindsay thermoelasticity for a functionally graded hollow disk.Mukhopadhyay and Shivay (2019) investigate on the solution of a problemof extended thermoelasticity theory by using a complete finite element approach.

2. <u>Proposed Problem</u>

Elastodynamic response of time harmonic source in a three -phase -lag orthotropic thermoelastic material.

3. <u>Work Done</u>

Elastodynamic response of time harmonic source in a three- phase- lag- orthotropic thermoelastic material.

3.1<u>Introduction</u>

Padovan (1974) discussed about Thermoelasticity of an Anisotropic Half Space. Takeuti and Noda (1977) described a Plane Thermoelastic Problem in a Multiply Connected Orthotropic Body. Wu (1984) studied about the Plane anisotropic Thermoelastcity. Lin and Ovaert (2004) discuss about Thermoelastic Problems for the Anisotropic Elastic Half-Plane. Beom (2013) analyised about the Thermoelastic in-plane fields in a linear anisotropic solid. Abbas, Kumar and Rani (2014) investgated about the Response of Thermal Source in Transversely Isotropic Thermoelastic materials without energy dissipation and with two temperatures. Ghosh and Lahiri (2018) emphasized A Study on the Generalized thermoelastic Problem for an Anisotropic Medium .Biswas (2019) concerned with a Three-dimensional Thermoelastic Problem in Orthotropic Medium.

In the present problem, we have considered three-phase-lag Orthotropic Thermoelastic Material. In which harmonic thermal shock has been considered. Elastodynamic equations have been written for four different relaxation times. Equations have been converted from time domain to the frequency domain. For solving these equations, we have used finite element technique, for that we formulate the weak form of these equations for single isoparametric quadrilateral element. These equations have been converting into the system of equations. For complete solution, we have made assembly of system of equations. These equations can be solved by Gauss elimination method.

3.2<u>Basic Equations</u>

The constitutive relations for orthotropic thermoelastic medium following Dhaliwal and Sherief (1980) and Green and Lindsay (1972) are given by

$$\mathrm{t_{ij}} = c_{ijkl}e_{kl} - \beta_{ij}\left(1+\tau_a \frac{\partial}{\partial t}\right)T_{,i}, \qquad \beta_{ij} = c_{ijkl}\alpha_{kl} \quad (\mathrm{i, j, k, l} = 1,2,3) \tag{1}$$

Equation of motion for an orthotropic thermoelastic medium in the absence of body force is given by

$$t_{ij,j} = \rho \ddot{u}_i , \tag{2}$$

The heat conduction equation following Green and Nadhdi (1993) and Choudhuri (2007) is

$$K_{ij}\left(1+\tau_T \frac{\partial}{\partial t}\right)\dot{T}_{,\mathrm{ij}} + K^*{}_{ij}\left(1+\tau_v \frac{\partial}{\partial t}\right)T_{,\mathrm{ij}} = \left(1+\tau_q \frac{\partial}{\partial t} + \tau_q^2 \frac{\partial^2}{\partial t^2}\right)(\rho c_e \ddot{T} + \mathrm{T_o}\beta_{ij}\ddot{u}_{i,j}) \tag{3}$$

$\vec{u} = (u,v,w)$ - displacement vector, T(x,y,z,t) -temperature change, c_{ijkl} - isothermal elastic parameters, t- time, t_{ij} -stress tensor, e_{ij} -strain tensor,T_0 -uniform temperature, ρ - density, τ_T, τ_a, τ_v *and* τ_q -thermal relaxation times, β_{ij} - thermal modulli, α_{kl} -linear thermal expansion tensor.

$K^*{}_{ij} = \dfrac{c_e c_{11}}{4}$ -the material characteristic constant of the theory.

The comma notation is used for spatial derivatives and dot notation represents time differentiation.

c_{ijkl} satisfies the (Green) symmetry conditions:

$c_{ijkl} = c_{klij} = c_{ijlk} = c_{jikl}$.

3.3 <u>Formulaion and solution of the problem</u>

We consider a homogenous, orthotropic thermoelastic half-space in the undeformed state at uniform temperature T_0. The rectangular Cartesian co-ordinate system (x,y,z) having origin on the plane surface z=0 with z-axis pointing vertically into medium is introduced. The boundary of the half-space is affected by mechanical and thermal loading, which depends on time t and spatial coordinate z (- $\infty < z < \infty$).

For plane strain two-dimensional problem, we take displacement vector

$$\vec{u} = (u,0,w) \tag{4}$$

and T(x, z, t) as temperature change.

Using the contracting subscript notations in equation (3) as

$1\to11$, $2\to22$, $3\to33$, $4\to23$, $5\to13$, $6\to12$ to relate c_{ijkl} t o c_{ij}

(i,j,k,l=1,2,3 and p,q=1,2…….,6).

Equation of Motion and equation of heat conduction can be written as

$$c_{11}u_{1,11}+(c_{13}+c_{55})u_{3,31}+c_{55}u_{1,33}-\beta_1(1+\tau_a\frac{\partial}{\partial t})T_{,1}=\rho\ddot{u}_1 \quad (5)$$

$$c_{55}u_{3,11}+(c_{13}+c_{55})u_{1,13}+c_{33}u_{3,33}-\beta_3(1+\tau_a\frac{\partial}{\partial t})T_{,3}=\rho\ddot{u}_3 \quad (6)$$

$$k_1(1+\tau_T\frac{\partial}{\partial t})\dot{T}_{,11}+k_3\left(1+\tau_T\frac{\partial}{\partial t}\right)\dot{T}_{,33}+k_1^*\left(1+\tau_\nu\frac{\partial}{\partial t}\right)T_{,11}+k_3^*\left(1+\tau_\nu\frac{\partial}{\partial t}\right)T_{,33}$$
$$=\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q{}^2}{2}\frac{\partial^2}{\partial t^2}\right)\rho c_e\ddot{T}+\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q{}^2}{2}\frac{\partial^2}{\partial t^2}\right)T_0\left[\beta_{1,}\ddot{u}_{1,1}+\beta_3\ddot{u}_{3,3}\right] \quad (7)$$

Equations (5),(6),(7) can be written as:

$$c_{11}\frac{\partial^2 u}{\partial x^2}+(c_{13}+c_{55})\frac{\partial^2 w}{\partial x\partial z}+c_{55}\frac{\partial^2 u}{\partial z^2}-\beta_1(1+\tau_a\frac{\partial}{\partial t})\frac{\partial T}{\partial x}=\rho\ddot{u} \quad (8)$$

$$c_{55}\frac{\partial^2 w}{\partial x^2}+(c_{13}+c_{55})\frac{\partial^2 u}{\partial x\partial z}+c_{33}\frac{\partial^2 w}{\partial z^2}-\beta_3(1+\tau_a\frac{\partial}{\partial t})\frac{\partial T}{\partial z}=\rho\ddot{w} \quad (9)$$

$$k_1\left(1+\tau_T\frac{\partial}{\partial t}\right)\frac{\partial^2\dot{T}}{\partial x^2}+k_3\left(1+\tau_T\frac{\partial}{\partial t}\right)\frac{\partial^2\dot{T}}{\partial z^2}+k_1^*\left(1+\tau_\nu\frac{\partial}{\partial t}\right)\frac{\partial^2 T}{\partial x^2}+k_3^*\left(1+\tau_\nu\frac{\partial}{\partial t}\right)\frac{\partial^2 T}{\partial z^2}$$
$$=\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q{}^2}{2}\frac{\partial^2}{\partial t^2}\right)\left[\rho c_e\ddot{T}+T_0\left(\beta_1\frac{\partial\ddot{u}}{\partial x}+\beta_3\frac{\partial\ddot{w}}{\partial z}\right)\right] \quad (10)$$

We define the time harmonic behavior as:

$$u=u(x,z)e^{i\omega t}$$
$$w=w(x,z)e^{i\omega t} \quad (11)$$
$$T=T(x,z)e^{i\omega t}$$

Using equation defined by (11), Equation (8), (9), (10) can be written as:

$$c_{11}\frac{\partial^2 u}{\partial x^2}+(c_{13}+c_{55})\frac{\partial^2 w}{\partial x\partial z}+c_{55}\frac{\partial^2 u}{\partial z^2}-\beta_1(1+\tau_a i\omega)\frac{\partial T}{\partial x}=-\omega^2\rho u \quad (12)$$

$$c_{55}\frac{\partial^2 w}{\partial x^2}+(c_{13}+c_{55})\frac{\partial^2 u}{\partial x\partial z}+c_{33}\frac{\partial^2 w}{\partial z^2}-\beta_3(1+\tau_a i\omega)\frac{\partial T}{\partial z}=-\omega^2\rho w \quad (13)$$

$$\left[k_1(1+\tau_T i\omega)(i\omega)+k_1^*(1+\tau_v i\omega)\right]\frac{\partial^2 T}{\partial x^2}+\left[k_3(1+\tau_T i\omega)(i\omega)+k_3^*(1+\tau_v i\omega)\right]\frac{\partial^2 T}{\partial z^2}$$
$$=-\left(1+\tau_q i\omega-\frac{\tau_q^{\,2}}{2}\omega^2\right)\omega^2\left[\rho c_e T+T_0\left(\beta_1\frac{\partial u}{\partial x}+\beta_3\frac{\partial w}{\partial z}\right)\right] \tag{14}$$

The initial and regularity conditions are given by

$$u(x,z,0)=0=\dot{u}(x,z,0),$$
$$w(x,z,0)=0=\dot{w}(x,z,0),$$
$$T(x,z,0)=0=\dot{T}(x,z,0) \quad for\ z\geq 0,\quad -\infty<x<\infty,$$
$$and\ u(x,z,t)=w(x,z,t)=T(x,z,t)=0 \quad for\, t>0 \quad \text{when} \quad z\rightarrow\infty.$$

We define the quantities:

$$x'=\frac{\omega_1^* x}{v_1},\quad z'=\frac{\omega_1^* z}{v_1},\quad t'=\omega_1^* t,\quad c_1=\frac{c_{33}}{c_{11}},\quad c_2=\frac{c_{55}}{c_{11}},\quad c_4=\frac{(c_{13}+c_{55})}{c_{11}}$$
$$u'=\frac{\rho v_1\omega_1^* u}{\beta_1 T_0},\quad w'=\frac{\rho v_1\omega_1^* w}{\beta_1 T_0},\quad T'=\frac{T}{T_0},\quad \bar{\beta}=\frac{\beta_3}{\beta_1},\quad \bar{k}=\frac{k_3}{k_1}$$
$$\tau_T{}'=\omega_1^*\tau_T,\quad \tau_q{}'=\omega_1^*\tau_q,\quad \tau_v{}'=\omega_1^*\tau_v,\quad v_1^{\,2}=\frac{c_{11}}{\rho},\quad \omega_1^*=\frac{c_e c_{11}}{k_1},\quad \omega'=\frac{\omega}{\omega_1^*}$$
$$\sigma'_{zz}=\frac{\sigma_{zz}}{\beta_1 T_0},\quad \sigma'_{zx}=\frac{\sigma_{zx}}{\beta_1 T_0},\quad \sigma'_{xx}=\frac{\sigma_{xx}}{\beta_1 T_0},\quad \frac{\partial x'}{\partial x}=\frac{\omega_1^*}{v_1},\quad \frac{\partial z'}{\partial z}=\frac{\omega_1^*}{v_1},\quad \frac{\partial t'}{\partial t}=\omega_1^* \tag{15}$$

Using the quantities defined by equation (15), the equations (12),(13) and (14) can be written as

$$\frac{\partial^2 u}{\partial x^2}+\frac{(c_{13}+c_{55})}{c_{11}}\frac{\partial^2 w}{\partial x\partial z}+\frac{c_{55}}{c_{11}}\frac{\partial^2 u}{\partial z^{\,2}}-(1+\tau_a i\omega)\frac{\partial T}{\partial x}=-\omega^2 u \tag{16}$$

$$\frac{c_{55}}{c_{11}}\frac{\partial^2 w}{\partial x^2}+\frac{(c_{13}+c_{55})}{c_{11}}\frac{\partial^2 u}{\partial x\partial z}+\frac{c_{33}}{c_{11}}\frac{\partial^2 w}{\partial z^2}-\frac{\beta_3}{\beta_1}(1+\tau_a i\omega)\frac{\partial T}{\partial z}=-\omega^2 w \tag{17}$$

$$\left[(1+\tau_T i\omega)(i\omega)+\bar{k}_1\left(1+\tau_v i\omega\right)\right]\frac{\partial^2 T}{\partial x^2}+\left[\bar{k}(1+\tau_T i\omega)(i\omega)+\bar{k}^*\left(1+\tau_v i\omega\right)\right]\frac{\partial^2 T}{\partial z^2}$$

$$=-\left(1+\tau_q i\omega-\frac{\tau_q^{\ 2}}{2}\omega^2\right)\left[\omega^2\omega_1^{\ *}T+\frac{v_1^{\ 2}\omega_1^{\ 2}\omega_1^{\ *}\beta_1 T_0}{c_e\rho^2}\left(\frac{\partial u}{\partial x}+\bar{\beta}\frac{\partial w}{\partial z}\right)\right] \tag{18}$$

Let

$$L_1=\left(1+\tau_a i\omega\right)$$
$$L_2=\bar{\beta}L_1$$
$$L_3=(1+\tau_T i\omega)(i\omega)+\bar{k}_1\left(1+\tau_v i\omega\right)$$
$$L_4=\bar{k}(1+\tau_T i\omega)(i\omega)+\bar{k}^*\left(1+\tau_v i\omega\right) \tag{19}$$
$$L_5=-\left(1+\tau_q i\omega-\frac{\tau_q^{\ 2}}{2}\omega^2\right)$$
$$L_6=L_5\omega^2\omega_1^{\ *}$$
$$L_7=L_5\frac{v_1^{\ 2}\omega_1^{\ 2}\omega_1^{\ *}\beta_1 T_0}{c_e\rho^2}$$

Using (15) and (19), Equations (16),(17) and(18) can be written as:

$$\frac{\partial^2 u}{\partial x^2}+c_4\frac{\partial^2 w}{\partial x\partial z}+c_2\frac{\partial^2 u}{\partial z^2}-L_1\frac{\partial T}{\partial x}=-\omega^2 u \tag{20}$$

$$c_2\frac{\partial^2 w}{\partial x^2}+c_4\frac{\partial^2 u}{\partial x\partial z}+c_1\frac{\partial^2 w}{\partial z^2}-L_2\frac{\partial T}{\partial z}=-\omega^2 w \tag{21}$$

$$L_3\frac{\partial^2 T}{\partial x^2}+L_4\frac{\partial^2 T}{\partial z^2}=\left[L_6 T+L_7\left(\frac{\partial u}{\partial x}+\bar{\beta}\frac{\partial w}{\partial z}\right)\right] \tag{22}$$

Stress components can be written as:

$$\tau_{xx}=c_{11}\frac{\partial u}{\partial x}+c_{13}\frac{\partial w}{\partial z}-\beta_1\left(1+\tau_a\frac{\partial}{\partial t}\right)T \tag{23}$$

$$\tau_{zx}=c_{55}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right) \tag{24}$$

$$\tau_{zz}=c_{13}\frac{\partial u}{\partial x}+c_{33}\frac{\partial w}{\partial z}-\beta_3\left(1+\tau_a\frac{\partial}{\partial t}\right)T \tag{25}$$

Using (11), (15) and (19), Equation (23), (24) and (25) can be written as

$$\tau_{xx} = \frac{\partial u}{\partial x} + (c_4 - c_2)\frac{\partial w}{\partial z} - L_1 T \quad (26)$$

$$\tau_{zx} = c_2\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) \quad (27)$$

$$\tau_{zz} = (c_4 - c_2)\frac{\partial u}{\partial x} + c_1 \frac{\partial w}{\partial z} - L_2 T \quad (28)$$

Heat flux

$$q = k_1\left(1+\tau_T \frac{\partial}{\partial t}\right)\frac{\partial \dot{T}}{\partial x} + k_3\left(1+\tau_T \frac{\partial}{\partial t}\right)\frac{\partial \dot{T}}{\partial z} + k_1^*\left(1+\tau_v \frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x} + k_3^*\left(1+\tau_v \frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z} \quad (29)$$

Using (7), (11) and (15), equation (25) can be written as:

$$q = L_3 \frac{\partial T}{\partial x} + L_4 \frac{\partial T}{\partial z} \quad (30)$$

3.4 Boundary conditions

The boundary conditions at the plane surface are

$$\begin{aligned}
&t_{zz} = 0, \quad t_{zx} = 0, && \text{at } z = 0 \\
&\frac{\partial T}{\partial z}(x, z = 0) = r(x,t), && \text{for the temperature gradient boundary,} \\
&or && \\
&T(x, z = 0) = r(x,t), && \text{for the temperature input boundary,.} \quad (31)
\end{aligned}$$

where r(x,t)= $\eta(x)\,F(t)$

$$\eta(x) = \begin{cases} 1 & \text{if } |x| \le a, \\ 0 & \text{if } |x| > a, \end{cases}$$

Case 1: Instantaneous **strip loading:**

The plane boundary z=0 is assumed to be traction free and is subjected to an instantaneous input in temperature,i.e.

$$F(t) = F_0 \delta(t)$$

where F_0 is a constant representing the magnitude of constant temperature applied on the boundary, $\delta(t)$ is the Dirac delta function.

Case 2: Continuous **strip loading:**

The plane boundary z=0 is assumed to be traction free and is subjected to an Continuous input in temperature, i.e.

$F(t) = F_0 H(t)$

where $H(t)$ is the Heaviside unit step function

3.5 Finite element formulation

To investigate the effect of the thermal shock, which applied on the surface at x=0. The analytical solution of the given problem is not possible or quite tough to obtain. So, we use finite element method to obtain the numerical solution of the problem. The weak formulations of the non-dimensional governing equations derived for dependent variables u, w and T for the given boundary conditions, respectively. We assume eight shape function N= [N1, N2, N3,……..N8] in two dimensions, respectively. The displacement components (*u,w*) and temperature (*T*) are related to the corresponding nodal values by

$$u = \sum_{i=1}^{n} N_k u_k, \quad w = \sum_{i=1}^{n} N_k w_k, \quad T = \sum_{i=1}^{n} N_k T_k$$

where *n* denotes the number of nodes per element, and *N* represents the shape functions. For the present study, we use eight-noded isoparametric quadrilateral element.

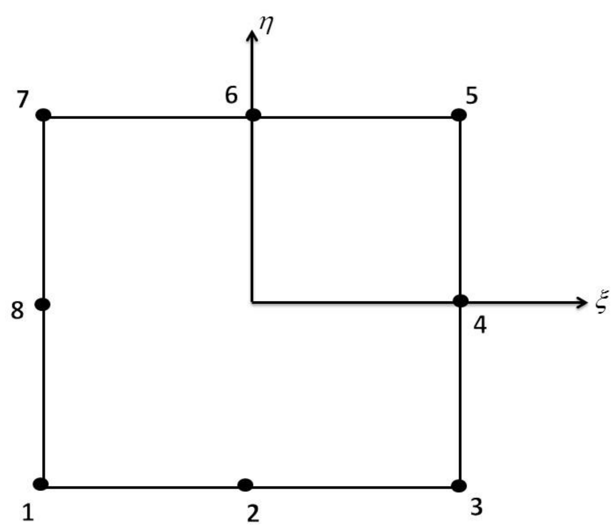

Fig. 1. Eight-noded isoparametric quadrilateral element

With the help of previous equation, weak form of equation of (16), (17) and (18) is written as:

$$F_u = \int_\Omega \left\{ \left[\frac{\partial N^T}{\partial x}\frac{\partial N}{\partial x} + \frac{c_{55}}{c_{11}}\frac{\partial N^T}{\partial z}\frac{\partial N}{\partial z} - \omega^2 N^T N \right]\{\hat{u}\} + \left[\frac{c_{13}}{c_{11}}\frac{\partial N^T}{\partial x}\frac{\partial N}{\partial z} + \frac{c_{55}}{c_{11}}\frac{\partial N^T}{\partial z}\frac{\partial N}{\partial x} \right]\{\hat{w}\} - \left(1+\tau_a \frac{\partial}{\partial t}\right)\frac{\partial N^T}{\partial x} N\{\hat{T}\} \right\} d\Omega \tag{32}$$

$$F_w = \int_\Omega \left\{ \left[\frac{c_{13}}{c_{11}}\frac{\partial N^T}{\partial z}\frac{\partial N}{\partial x} + \frac{c_{55}}{c_{11}}\frac{\partial N^T}{\partial x}\frac{\partial N}{\partial z} \right]\{\hat{u}\} + \left[\frac{c_{33}}{c_{11}}\frac{\partial N^T}{\partial z}\frac{\partial N}{\partial z} + \frac{c_{55}}{c_{11}}\frac{\partial N^T}{\partial x}\frac{\partial N}{\partial x} - \omega^2 N^T N \right]\{\hat{w}\} - \left(1+\tau_a \frac{\partial}{\partial t}\right)\frac{\beta_3}{\beta_1}\frac{\partial N^T}{\partial x} N\{\hat{T}\} \right\} d\Omega \tag{33}$$

$$F_T = \int_\Omega \left\{ \begin{array}{l} \left[-\left(1+\tau_q i\omega - \frac{\tau_q{}^2}{2}\omega^2\right)\frac{v_1{}^2\omega_1{}^2\omega_1{}^*\beta_1 T_0}{c_e\rho^2} N^T \frac{\partial N}{\partial x}\right]\{\hat{u}\} + \left[-\left(1+\tau_q i\omega - \frac{\tau_q{}^2}{2}\omega^2\right)\frac{v_1{}^2\omega_1{}^2\omega_1{}^*\beta_1 T_0}{c_e\rho^2} \bar{\beta} N^T \frac{\partial N}{\partial z}\right]\{\hat{w}\} \\ + \left[\begin{array}{l} \left[(1+\tau_T i\omega)(i\omega) + \bar{k}_1(1+\tau_v i\omega)\right]\frac{\partial N^T}{\partial x}\frac{\partial N}{\partial x} + \left[\bar{k}(1+\tau_T i\omega)(i\omega) + \bar{k}^*(1+\tau_v i\omega)\right]\frac{\partial N^T}{\partial z}\frac{\partial N}{\partial z} \\ + \left\{ -\left(1+\tau_q i\omega - \frac{\tau_q{}^2}{2}\omega^2\right)\omega^2\omega_1{}^* \right\} N^T N \end{array} \right]\{\hat{T}\} \end{array} \right\} d\Omega \tag{34}$$

And

$$\begin{cases} F_u = \int_\Gamma \{N[n_x\tau_{xx} + n_z\tau_{zx}]\} d\Gamma \\ F_w = \int_\Gamma \{N[n_z\tau_{zz} + n_x\tau_{zx}]\} d\Gamma \\ F_T = \int_\Gamma \{N[n_x q_x + n_z q_z]\} d\Gamma \end{cases} \tag{35}$$

Where $n_x\tau_{xx}, n_z\tau_{zx}, n_z\tau_{zz}$ are the Cauchy surface traction boundary condition on Γ; n_x and n_z are the direction cosines between the normal and the x and z directions, respectively; Ω is the domain and Γ is the boundary of the physical domain. q_x and q_z represents heat flux in x and z directions.

Using (15) and (19), Equations (32), (33) and (34) can be written as:

$$F_u = \int_\Omega \left\{ \left[\frac{\partial N^T}{\partial x}\frac{\partial N}{\partial x} + c_2\frac{\partial N^T}{\partial z}\frac{\partial N}{\partial z} - \omega^2 N^T N \right]\{\hat{u}\} + \left[(c_4 - c_2)\frac{\partial N^T}{\partial x}\frac{\partial N}{\partial z} + c_2\frac{\partial N^T}{\partial z}\frac{\partial N}{\partial x} \right]\{\hat{w}\} - L_1\frac{\partial N^T}{\partial x} N\{\hat{T}\} \right\} d\Omega \tag{36}$$

$$F_w = \int_\Omega \left\{ \left[(c_4 - c_2) \frac{\partial N^T}{\partial z} \frac{\partial N}{\partial x} + c_2 \frac{\partial N^T}{\partial x} \frac{\partial N}{\partial z} \right] \{\hat{u}\} + \left[c_1 \frac{\partial N^T}{\partial z} \frac{\partial N}{\partial z} + c_2 \frac{\partial N^T}{\partial x} \frac{\partial N}{\partial x} - \omega^2 N^T N \right] \{\hat{w}\} - L_2 \frac{\partial N^T}{\partial x} N \{\hat{T}\} \right\} d\Omega \tag{37}$$

$$F_T = \int_\Omega \left\{ L_7 N^T \frac{\partial N}{\partial x} \{\hat{u}\} + L_7 \bar{\beta} N^T \frac{\partial N}{\partial z} \{\hat{w}\} + \left[L_3 \frac{\partial N^T}{\partial x} \frac{\partial N}{\partial x} + L_4 \frac{\partial N^T}{\partial z} \frac{\partial N}{\partial z} + L_6 N^T N \right] \{\hat{T}\} \right\} d\Omega \tag{38}$$

Let

$$\begin{cases} [K_{11}] = \int_\Omega \left[\frac{\partial N^T}{\partial x} \frac{\partial N}{\partial x} + c_2 \frac{\partial N^T}{\partial z} \frac{\partial N}{\partial z} - \omega^2 N^T N \right] d\Omega \\ [K_{12}] = \int_\Omega \left[(c_4 - c_2) \frac{\partial N^T}{\partial x} \frac{\partial N}{\partial z} + c_2 \frac{\partial N^T}{\partial z} \frac{\partial N}{\partial x} \right] d\Omega \\ [K_{13}] = \int_\Omega \left[-L_1 \frac{\partial N^T}{\partial x} N \right] d\Omega \end{cases} \tag{39}$$

Using (39), Equation (36) can be written as:

$$F_u = [K_{11}]\{u\} + [K_{12}]\{w\} + [K_{13}]\{T\} \tag{40}$$

Let

$$\begin{cases} [K_{21}] = \int_\Omega \left[(c_4 - c_2) \frac{\partial N^T}{\partial z} \frac{\partial N}{\partial x} + c_2 \frac{\partial N^T}{\partial x} \frac{\partial N}{\partial z} \right] d\Omega \\ [K_{22}] = \int_\Omega \left[c_1 \frac{\partial N^T}{\partial z} \frac{\partial N}{\partial z} + c_2 \frac{\partial N^T}{\partial x} \frac{\partial N}{\partial x} - \omega^2 N^T N \right] d\Omega \\ [K_{23}] = \int_\Omega \left[-L_2 \frac{\partial N^T}{\partial x} N \right] d\Omega \end{cases} \tag{41}$$

Using (41), Equation (37) can be written as:

$$F_w = [K_{21}]\{u\} + [K_{22}]\{w\} + [K_{23}]\{T\} \tag{42}$$

Let

$$
\begin{cases}
[K_{31}] = \int_{\Omega} \left[L_7 N^T \frac{\partial N}{\partial x} \right] d\Omega \\
[K_{32}] = \int_{\Omega} \left[L_7 \bar{\beta} N^T \frac{\partial N}{\partial z} \right] d\Omega \\
[K_{33}] = \int_{\Omega} \left[L_3 \frac{\partial N^T}{\partial x} \frac{\partial N}{\partial x} + L_4 \frac{\partial N^T}{\partial z} \frac{\partial N}{\partial z} + L_6 N^T N \right] d\Omega
\end{cases} \tag{43}
$$

Using (43), equation (38) can be written as:

$$
F_T = [K_{31}]\{u\} + [K_{32}]\{w\} + [K_{33}]\{T\} \tag{44}
$$

On combining Equations (40), (42) and (44) we get

$$
\begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \begin{bmatrix} u \\ w \\ T \end{bmatrix} = \begin{bmatrix} F_u \\ F_w \\ F_T \end{bmatrix} \tag{45}
$$

This equation is written for single element. For finding the solution for complete domain, we have to make assembly up to n-elements. Final system of equations will be solved analytically by Gauss-elimination method and find the values of all variables u, v and T throughout the domain.

4. Conclusion

Deformation of time harmonic source in three-phase-lag orthotropic thermoelastic material has been studied and finite element method technique has been used to solve the problem. Due to unavailability of the MatLab software, we are not able to produce the numerical results. This study may give good information in the field of thermoelasticity.

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