STUDENT SOLUTIONS MANUAL

United States Edition

FOR

Digital and Analog Communication Systems

8th Edition, United States Edition

Leon W. Couch, II

2013

PEARSON Prentice Hall

Pearson Education, Inc.
One Lake Street
Upper Saddle River, NJ 07458

PREFACE and ACKNOWLEDGEMENTS

This **Student Solutions Manual** for *Digital and Analog Communication Systems, 8th Edition (United States)* contains complete solutions for the homework problems in the 8th Edition that are marked with a ★. If the problem is designed for a MATLAB computer solution (as denoted by a computer symbol), then a MATHCAD printed solution is shown in this solutions manual. (MATHCAD solutions are shown since they clearly display the algorithms used and the output takes up less space.)

MATLAB m files for these problems can be downloaded from the internet websites maintained by the author located at

http://lcouch.us
or
http://www.couch.ece.ufl.edu

In the textbook, a computer symbol is used to indicate that MATLAB solutions are provided for that material, although those homework problems marked with a computer symbol but not including a ★ are available only to the instructor from Pearson/Prentice Hall. This website is located at
http://www.pearsonhighered.com/educator/catalog/index.page

This solutions manual was prepared by Leon W. Couch, II, with the help and valuable suggestions of many undergraduate and graduate electrical engineering students at the University of Florida. Their assistance is greatly appreciated. Several graduate students worked out solutions or contributed problems with solutions; they are:

Samel Celebi	Lawrence K. Thompson
Brady E. Gaughan	Yeong-Cheng Wang
Charles S. Prewitt	Ching-Jang Wu

Thanks also to Ronald F. Smith who wrote the original code for many of the MATLAB M files.

The author values your comments and suggestions. Also, for future editions, new problems and problems with computer solutions are welcomed. Please send them to:

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October 12, 2011

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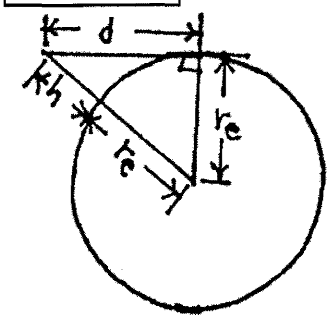
Chapter 1

1-3



$(h+r_e)^2 = d^2 + r_e^2$

$\Rightarrow h^2 + 2hr_e = d^2$ where $h \ll r_e$

$\Rightarrow d^2 \approx 2hr_e$ and $r_e = \frac{4}{3}(3960 \text{ miles}) = 5280 \text{ miles}$

Let h = antenna height in feet, and d in miles.

$\Rightarrow d^2 (\text{miles})^2 = 2h\,(\cancel{\text{feet}})\left(\frac{1 \text{ mile}}{5280\,\cancel{\text{feet}}}\right)(5280 \text{ miles})$

$\Rightarrow d^2 = 2h$ or $\underline{\underline{d = \sqrt{2h}}}$

1-6

$\{m_i\} = \{-1.0, 0.0, 3.0, 4.0\}$ $i = 1,4$; $P_1 = P_2 = 0.2$; $P_3 = P_4 = 0.3$

$\Rightarrow I_1 = I_2 = \frac{-\ln(0.2)}{\ln 2} = 2.322$ bits, $I_3 = I_4 = \frac{-\ln(0.3)}{\ln 2} = 1.737$ bits

$$H = \sum_{j=1}^{M} P_j I_j = 2[(0.2)(2.322) + (0.3)(1.737)] = \underline{\underline{1.971 \text{ bits}}}$$

1-9

Let p_1 = prob. of sending a binary 1

(a) p_2 = prob. of sending a binary 0 = $1 - p_1$

$$H = \sum_{i=1}^{2} P_i I_i = p_1 \log_2\left(\frac{1}{p_1}\right) + (1-p_1)\log_2\left(\frac{1}{1-p_1}\right)$$

$$H = \frac{1}{\ln 2}\left[-p_1 \ln(p_1) - (1-p_1)\ln(1-p_1)\right]$$

$$\frac{\partial H}{\partial p_1} = 0 \Rightarrow -(\ln p_1 + 1) - \left((-1)\ln(1-p_1) + \frac{1-p_1}{1-p_1}(-1)\right) = 0$$

$$\Rightarrow -\ln p_1 \cancel{-1} + \ln(1-p_1) \cancel{+1} = 0$$

or $\ln\left(\frac{1-p_1}{p_1}\right) = 0 = \ln 1$

thus $\frac{1}{p_1} - 1 = 1 \Rightarrow \underline{\underline{p_1 = \frac{1}{2} = p_2}}$

(b) $H_{max} = \frac{1}{2}\log_2 2 + \left(1 - \frac{1}{2}\right)\log_2 2 = \underline{\underline{1 \text{ bit}}}$

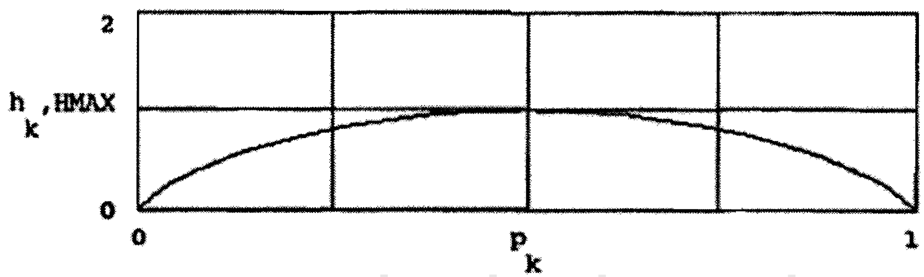
Math CAD Solution

LET p=The probability for sending a binary 1, then the probability for sending a binary 0 is (1-p). From the entropy formula for H(p), we can draw the figure of H(p) ,and from this figure , we can find the maximum entropy and the p.

H(p)=(p*ln(P)+(1-p)*ln(1-p))/(-ln(2))

$k \equiv 0 \,..50 \qquad p_k := \frac{k}{50} \qquad HMAX := 1$

$$h_k := \frac{-1}{\ln(2)}\left[p_k \ln\left[p_k\right] + \left[1 - p_k\right]\cdot\ln\left[1 - p_k\right]\right]$$



From the above figure, we know the maximum entropy is 1 where the probability for sending 1 or 0 is 0.5.

1-10 $P_1 = 0.25$; $P_2 = P_3 = 0.15$;

$P_4 = P_5 = P_6 = P_7 = P_8 = P_9 = 0.07$

$0.25 + 2(0.15) + 6(0.07) = 0.97$

$\therefore\ P_{10} = 0.03 \quad \text{since} \sum_{j=1}^{10} P_j = 1.0$

$$H = \sum_{j=1}^{M} P_j \log_2\left(\frac{1}{P_j}\right) = \left[\frac{-1}{\ln 2}\right]\sum_{j=1}^{10} P_j \ln P_j$$

$$= \left[\frac{-1}{\ln 2}\right]\left[.25 \ln .25 + (2).15 \ln .15 + (6).07 \ln .07 + .03 \ln .03\right]$$

$$\underline{\underline{H = 3.084 \text{ bits}}}$$

1-12

$M = 10 \qquad P_j = \frac{1}{10} \quad j = 1, 10 \qquad R = \frac{H}{T} = 3\,\frac{b}{s}$

$$H = \frac{-10(.1)\ln .1}{\ln 2} = 3.322 \text{ bits}$$

$$T = \frac{H}{R} = \frac{3.322 \text{ bits}}{3 \text{ bits/sec}} = \underline{\underline{1.11 \text{ sec.} = T}}$$

1-15

(a) chars := 110 Number of characters available

$b := \text{ceil}\left[\frac{\log(\text{chars})}{\log(2)}\right]$ Number of bits required to represent a character

$\Longrightarrow$ b = 7 bits

(b) B := 3200 Hz Channel bandwidth
SNRdB := 20 dB Signal to noise ratio

$SNR := 10^{\frac{SNRdB}{10}}$ $\Longrightarrow$ SNR = 100 (Absolute power ratio)

$C := B\cdot\left[\frac{\log(1 + SNR)}{\log(2)}\right]$ $\Longrightarrow$ $C = 2.131\cdot 10^{4}$ Channel capacity (bits/sec)

$c := \frac{C}{b}$ $\Longrightarrow$ $c = 3.044\ 10^{3}$ Channel capacity (chars/sec)

(c) Assuming equally likely characters,
information content of each character is:

$P := \frac{1}{\text{chars}}$ Probability of each character

$I := \frac{\log\left[\frac{1}{P}\right]}{\log(2)}$ $\Longrightarrow$ I = 6.781 bits

1-18

$x_0 := 1$ $x_1 := 0$ $x_2 := 1$ $x_3 := 1$ $x_4 := 1$ Input vector

$ga_0 := 1$ $ga_1 := 0$ $ga_2 := 0$ $ga_3 := 1$ Gain vector, mod2 adder

$gb_0 := 1$ $gb_1 := 1$ $gb_2 := 1$ $gb_3 := 1$ Gain vector, mod2 adder

$k := 0 \,..\, \text{length}(ga) - 2$

$v_k := 0$ $k := 0 \,..\, \text{length}(x) - 1$ $v_{k+\text{length}(ga)-1} := x_k$

$k := \text{length}(x) + \text{length}(ga) - 1 \,..\, \text{length}(x) + 2\,\text{length}(ga) - 3$

$v_k := 0$ $i := 0 \,..\, \text{length}(v) - \text{length}(ga)$

$j := 0 \,..\, \text{length}(ga) - 1$

$$sa_i := \sum_j \left[ga_{\text{length}(ga)-j-1}\, v_{j+i} \right]$$

$$sa_i := \text{mod}\left[sa_i, 2 \right]$$

$$sb_i := \sum_j \left[gb_{\text{length}(gb)-j-1}\, v_{j+i} \right]$$

$sb_i := \text{mod}\left[sb_i, 2 \right]$ $s_{2\,i} := sa_i$ $s_{2\,i+1} := sb_i$

$i := 0 \,..\, 2\,\text{length}(x) - 1$

$out_i := s_i$

For $x^T = (1 \;\; 0 \;\; 1 \;\; 1 \;\; 1)$

====> $out^T = (1 \;\; 1 \;\; 0 \;\; 1 \;\; 1 \;\; 0 \;\; 0 \;\; 1 \;\; 1 \;\; 1)$

Chapter 2

2-1 $v(t) = A\sin\omega_0 t$; $V_{rms}^2 = \langle v^2(t)\rangle$

$$\langle v^2(t)\rangle = \frac{1}{T_0}\int_0^{T_0} A^2 \sin^2\omega_0 t\,dt = \frac{A^2}{T_0}\int_0^{T_0}[1-\underbrace{\cos^2(\omega_0 t)}_{\frac{1}{2}[1+\cos(2\omega_0 t)]}]\,dt$$

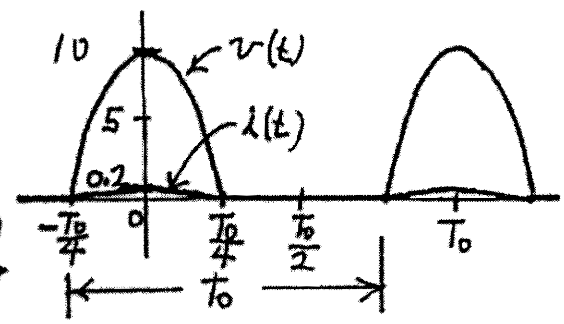
$$= \frac{A^2}{T_0}\left[T_0 - \frac{T_0}{2} - \frac{1}{2}\underbrace{\int_0^{T_0}\cos(2\omega_0 t)\,dt}_{0}\right] = \frac{A^2}{T_0}\left(\frac{T_0}{2}\right)$$

$$\Rightarrow V_{rms} = \sqrt{\langle v^2(t)\rangle} = \sqrt{\frac{A^2}{2}} = \frac{A}{\sqrt{2}}$$

2-4

(a.) $i(t) = \frac{v(t)}{R} = \frac{v(t)}{50}$

$$i(t) = \begin{cases} 0.2\cos(\omega_0 t), & |t-nT_0| < \frac{T_0}{2} \\ 0, & \text{elsewhere} \end{cases}$$



(b.)

$$V_{DC} = \langle v(t)\rangle = \frac{V_p}{T_0}\int_{-\frac{T_0}{4}}^{\frac{T_0}{4}}\cos(\omega_0 t)\,dt = \frac{2V_p}{T_0}\,\frac{\sin(\omega_0\frac{T_0}{4})}{\omega_0}$$

$$= \frac{2V_p}{T_0}\,\frac{\sin\left(\frac{2\pi}{T_0}\frac{T_0}{4}\right)}{\frac{2\pi}{T_0}} = \frac{2}{2\pi}V_p\sin\left(\frac{\pi}{2}\right)$$

$$\Rightarrow V_{DC} = \frac{V_p}{\pi} \underset{V_p=10}{=} \frac{10}{\pi} = \underline{\underline{3.183\text{ volts}}}$$

$$\Rightarrow I_{DC} = \frac{I_p}{\pi} \underset{I_p=0.2}{=} \frac{0.2}{\pi} = \underline{\underline{0.064\text{ Amps}}}$$

2-4. Cont'd (c.) $V_{rms}^2 = \langle v^2(t) \rangle = \frac{1}{T_0}\int_0^{T_0/2} v^2(t)\,dt = \frac{V_p^2}{T_0}\int_{-T_0/4}^{T_0/4} \cos^2\omega_0 t\,dt$

$$\Rightarrow V_{rms}^2 = \frac{V_p^2}{T_0}\int_{-T_0/4}^{T_0/4} \frac{1}{2}\left[1+\cos(2\omega_0 t)\right]dt = \frac{V_p^2}{2T_0}\left[2\frac{T_0}{4} + \frac{\sin(2\omega_0 t)}{2\omega_0}\Big|_{-T_0/4}^{T_0/4}\right]$$

$$= \frac{V_p^2}{2T_0}\,\frac{2T_0}{4} = \frac{V_p^2}{4} = V_{rms}^2$$

$$\Rightarrow V_{rms} = \frac{V_p}{2} = \frac{10}{2} = \underline{\underline{5 \text{ volts rms}}}$$

$$I_{rms} = \frac{I_p}{2} = \frac{0.2}{2} = \underline{\underline{0.1 \text{ amps}}}$$

(d) $P = \langle p(t) \rangle = V_{rms} I_{rms} = (5)(0.1) = \underline{\underline{0.5 \text{ watts}}}$

2-10 $P_{in} = I_{rms}^2 R_{in} = (0.5\times 10^{-3})^2(2\times 10^3) = 5.0\times 10^{-4}\,W$

$$P_{out} = \frac{V_{rms}^2}{R_{Load}} = \frac{100}{50} = 2\,W$$

$$dB = 10\log_{10}\left(\frac{P_{out}}{P_{in}}\right) = 10\log\left(\frac{2}{5.0\times 10^{-4}}\right) = \underline{\underline{36\text{ dB}}}$$

2-11 (a.) $P_{in} = \frac{V_{rms}^2}{R_{in}} = \frac{(3.5\times 10^{-6})^2}{300} = \underline{\underline{4.083\times 10^{-14}\,W}}$

(b.) $dBm = 10\log_{10}\left(\frac{P}{10^{-3}}\right) = 10\log_{10}\left(\frac{4.08\times 10^{-14}}{10^{-3}}\right) = \underline{\underline{-103.9\text{ dBm}}}$

(c.) $P_{in} = \frac{V_{rms}^2}{75} = 4.08\times 10^{-14}$

$$\Rightarrow V_{rms} = \sqrt{75(4.08\times 10^{-14})} = \underline{\underline{1.75\,\mu\text{volts}}}$$

2-15

$$W(f) = \int_{-\infty}^{\infty} w(t) e^{-j\omega t}\, dt = \int_{1}^{\infty} e^{-(\alpha + j\omega)t}\, dt$$

$$= \left. \frac{e^{-(\alpha+j\omega)t}}{-(\alpha+j\omega)} \right|_{1}^{\infty} = \underline{\underline{\frac{e^{-\alpha} e^{-j2\pi f}}{\alpha + j2\pi f} = W(f)}}$$

2-18

$$S(f) = \int_{-\infty}^{\infty} s(t)\, e^{-j\omega t}\, dt = \int_{0}^{T_0} A t\, e^{-j\omega t}\, dt$$

$$= A\left[e^{-j\omega t}\left(\frac{t}{-j\omega} + \frac{1}{\omega^2} \right) \right] \Bigg|_{0}^{T_0}$$

$$\int x e^{ax} dx = e^{ax}\left[\frac{x}{a} - \frac{1}{a^2} \right]$$

$$= A\left\{ e^{-j\omega T_0}\left(\frac{T_0}{-j\omega} + \frac{1}{\omega^2} \right) - \frac{1}{\omega^2} \right\}$$

$$= \frac{A}{(2\pi f)^2}\left\{ e^{-j2\pi f T_0} - 1 \right\} + \frac{A T_0\, e^{-j2\pi f T_0}}{-j2\pi f}$$

$$\Rightarrow \underline{\underline{S(f) = \frac{-A}{(2\pi f)^2} + A e^{-j2\pi f T_0}\left(\frac{1}{(2\pi f)^2} + j \frac{T_0}{2\pi f} \right)}}$$

2-24

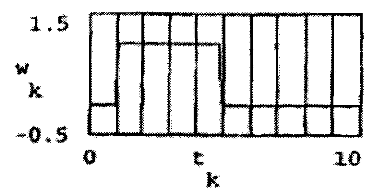
(a.)

$M := 8 \qquad N := 2^M \qquad N = 256 \qquad k := 0 \,..N - 1 \qquad T := 40$

$dt := \frac{T}{N} \qquad t_k := k\,dt - 10$

$w_k := \Phi\left[t_k - 1.0\right] - \Phi\left[t_k - 5.0\right]$

WAVEFORM



$w_0 = 0 \qquad dt = 0.156$

$f4 := 4$

$n := 0 \,..N - 1$

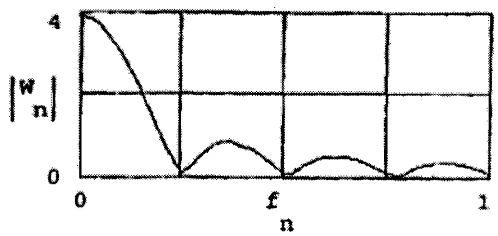
$W := dt\left[\sqrt{N}\right] \text{icfft}(w)$

$f_n := \frac{n}{T} \qquad fs := \frac{1}{dt}$

$W_0 = 3.906 \qquad fs = 6.4$

$f_1 = 0.025 \qquad f4 = 4$

MAGNITUDE SPECTRUM out to 4th null

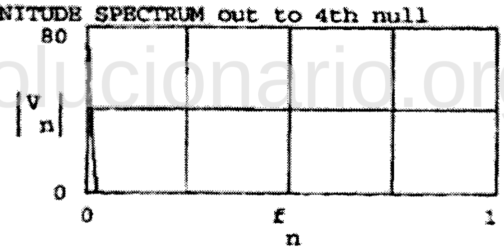


(b.) $v_k := 2.0$

$V := dt\left[\sqrt{N}\right] \text{icfft}(v)$

$V_0 = 80$

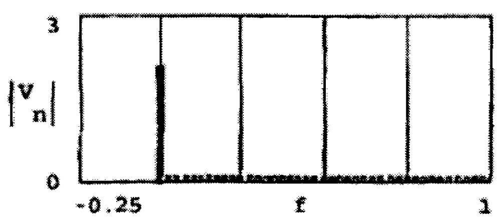
MAGNITUDE SPECTRUM out to 4th null



NOTE: The FFT cannot give the correct amplitude value for a delta function since the delta function has an infinite amplitude. However the area under the FFT result that approximates the delta function will be approximately the correct weight for the delta function. The value for the weight of the delta function may be calculated directily via the FFT by using (2-187). This is shown below.

$V := \left[\frac{1}{\sqrt{N}}\right] \text{icfft}(v)$

$V_0 = 2$ <--Weight of δ

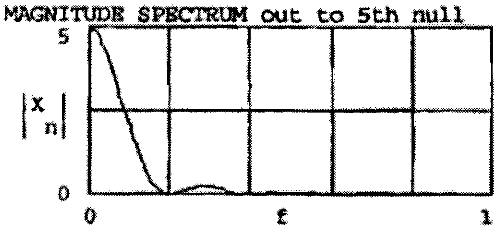


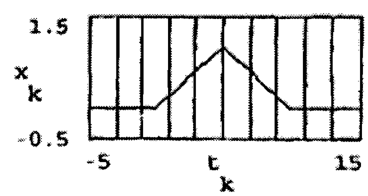
(c.)

$x_k := 0.2\left[t_k\left[\Phi\left[t_k\right] - \Phi\left[t_k - 5\right]\right] - \left[t_k - 10\right]\left[\Phi\left[t_k - 5\right] - \Phi\left[t_k - 10\right]\right]\right]$

$X := dt\left[\sqrt{N}\right] \text{icfft}(x)$

MAGNITUDE SPECTRUM out to 5th null

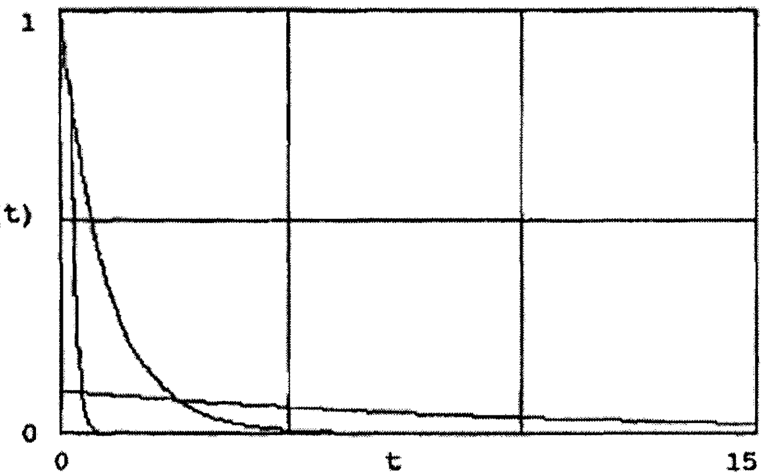




$X_0 = 5$

2-32 (a) $t := 0, 0.05 \,..\, 15$

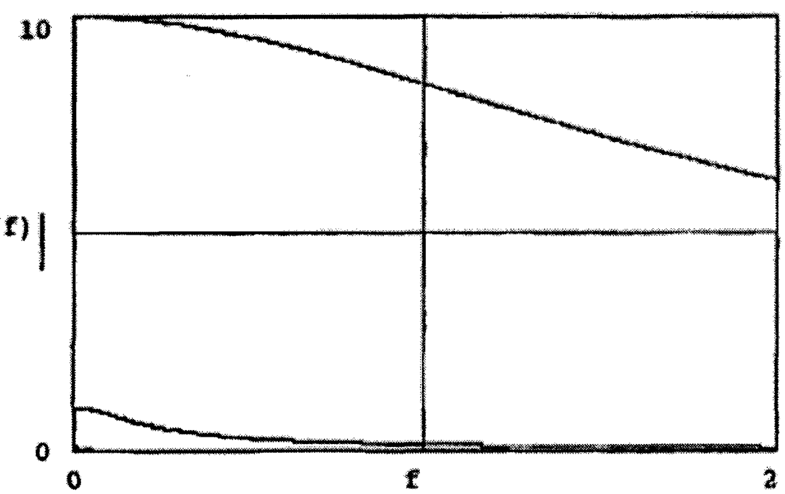
$v_1(t) := 0.1 \cdot e^{-0.1\,t}$ $\quad v_2(t) := e^{-t}$ $\quad v_3(t) := 10 \cdot e^{-10 \cdot t}$



(b) $f := 0, 0.001 \,..\, 2$

$V_1(f) := \dfrac{0.1}{1 + j \cdot 20\,\pi \cdot f}$ $\quad V_2(f) := \dfrac{1}{1 + j\ 2 \cdot \pi \cdot f}$ $\quad V_3(f) := \dfrac{10}{1 + j \cdot 0.2 \cdot \pi \cdot f}$

$V_1(0) = 0.1$ $\quad V_2(0) = 1$ $\quad V_3(0) = 10$



2-37

$$w(t) = w_1(t)\,w_2(t)$$

$$W(f) = \int_{-\infty}^{\infty} w(t)\,e^{-j\omega t}\,dt = \int_{-\infty}^{\infty} w_1(t)\,w_2(t)\,e^{-j\omega t}\,dt$$

$$= \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} W_1(\lambda)\,e^{j2\pi\lambda t}\,d\lambda\right] w_2(t)\,e^{-j2\pi f t}\,dt$$

$$= \int_{-\infty}^{\infty} W_1(\lambda)\underbrace{\int_{-\infty}^{\infty} w_2(t)\,e^{-j2\pi(f-\lambda)t}\,dt}_{W_2(f-\lambda)}\,d\lambda = \int_{-\infty}^{\infty} W_1(\lambda)\,W_2(f-\lambda)\,d\lambda = W(f)$$

2-42

(a.) $$\int_{-\infty}^{\infty} \frac{\sin 4\lambda}{4\lambda}\,\delta(t-\lambda)\,d\lambda = \underline{\underline{\frac{\sin(4t)}{4t}}}$$

(b.) $$\int_{-\infty}^{\infty} (\lambda^3 - 1)\,\delta(2-\lambda)\,d\lambda = 2^3 - 1 = \underline{\underline{7}}$$

2-48 (a.) $$V_{DC} = \frac{1}{T}\int_0^T s(t)\,dt$$

$$= \frac{1}{3}\left[-2A + \int_2^3 A\sin(\pi(t-2))\,dt\right]$$

let $t_1 = t-2$

$$= \frac{1}{3}\left[-2A + A\int_0^1 \sin \pi t_1\,dt_1\right]$$

$$= \frac{A}{3}\left[-2 - \frac{\cos \pi t_1}{\pi}\Big|_0^1\right] = \frac{A}{3}\left[-2 + \frac{2}{\pi}\right]$$

$$= -\frac{2A}{3\pi}[\pi - 1] = \underline{\underline{-0.454\,A = V_{DC}}}$$

2-48 Cont'd

(b.) $V_{rms}^2 = \frac{1}{T}\int_0^T s^2(t)\,dt$

$$= \frac{1}{3}\left[(-A)^2 2 + \int_0^1 [A\sin \pi t_1]^2 dt_1\right]$$

$$= \frac{A^2}{3}\left[2 + \int_0^1 \sin^2(\pi t_1)\,dt_1\right]$$

$$= \frac{A^2}{3}\left[2 + \frac{1}{2}\int_0^1 (1-\cos 2\pi t_1)\,dt_1\right]$$

$$= \frac{A^2}{3}\left[2 + \frac{1}{2} - \frac{1}{2}\underbrace{\cancel{\frac{\sin 2\pi t_1}{2\pi}}\Big|_0^1}_{0}\right] = \frac{A^2}{3}\left(\frac{5}{2}\right)$$

$$V_{rms} = \sqrt{\frac{5}{6}}\,A = \underline{\underline{0.913\,A = V_{rms}}}$$

(c.) $s(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$; $\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{3}$

$$c_n = \frac{1}{T}\int_0^{T_0} s(t)\,e^{-jn\omega_0 t}\,dt$$

$$= \frac{1}{3}\left[\int_0^2 -A e^{-jn\omega_0 t}dt + \int_2^3 A\sin \pi(t-2)\,e^{-jn\omega_0 t}dt\right.$$

2-48(c). Cont'd **Aside:**

① $\int_0^2 -A e^{-jn\omega_0 t}\,dt = \left.\frac{-A e^{-jn2\pi t/3}}{-jn2\pi/3}\right|_0^2$

$$= \frac{3A}{jn2\pi}\left(e^{-jn4\pi/3}-1\right) = \frac{-j3A}{n2\pi}\left(e^{-jn4\pi/3}-1\right)$$

② $\int_2^3 A\sin[\pi(t-2)]\, e^{-jn2\pi t/3}\,dt =$

$$= A\int_2^3 \sin(\pi t)\, e^{-jn2\pi t/3}\,dt$$

From Sec. A-5

Let $ax = -jn2\pi t/3 \;\therefore\; a = -jn2/3$
$x = \pi t$

② $= \frac{A}{\pi}\,\frac{e^{a\pi t}}{1+a^2}\left[a\sin(\pi t)-\cos(\pi t)\right]\Big|_2^3$

$$= \frac{A}{\pi(1+a^2)}\left[e^{3a\pi}\left(a\,\overset{0}{\cancel{\sin 3\pi}}-\cos 3\pi\right) - e^{2a\pi}\left(a\,\overset{0}{\cancel{\sin 2\pi}}-\cos 2\pi\right)\right]$$

$$= \frac{A}{(1+a^2)\pi}\left[e^{3a\pi}+e^{2a\pi}\right]$$

$$= \frac{A}{\pi(1-4n^2/9)}\left[\overset{1}{\cancel{e^{-jn2\pi}}} + e^{-jn4\pi/3}\right]$$

$c_n = ① + ②$

$$= \frac{A}{3\pi}\left[\frac{-j3}{2n}\left(e^{-jn4\pi/3}-1\right) + \frac{\left(1+e^{-jn4\pi/3}\right)}{(1-4n^2/9)}\right]$$

$$\therefore\; c_n = \frac{A}{\pi}\left[\frac{-j}{2n}\left(e^{-jn4\pi/3}-1\right) + \frac{\left(1+e^{-jn4\pi/3}\right)}{(3-4n^2/3)}\right]$$

$c_0 = V_{DC} = -0.454A$

(d) $S(f) = \sum_{n=-\infty}^{\infty} c_n\,\delta(f-nf_0)$

Note: The DFT Computer Solution is given in P2-95.

2-50 $s(t) = A_1 \cos(\omega_1 t + \phi_1) + A_2 \cos(\omega_2 t + \phi_2)$

$\phi_2 = 0$ for simplicity

(a.) $\omega_1 = \omega_2$; $\phi_1 = \phi_2 = 0$

$$s(t) = (A_1 + A_2)\cos \omega_1 t$$

$$s_{rms}(t) = \left[(A_1+A_2)^2 \frac{1}{T}\int_0^T \cos^2(\omega_1 t)\,dt\right]^{1/2}$$

$$= (A_1+A_2)\left[\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{1}{2} + \frac{1}{2}\cos(2\theta)\right)d\theta\right]^{1/2}$$

$\omega t = \theta$

$dt = \frac{d\theta}{\omega_1} = \frac{d\theta\, T}{2\pi}$

$$= (A_1+A_2)\left[\frac{1}{2\pi}\left(\frac{1}{2}\right)2\pi\right]^{1/2} = \frac{(A_1+A_2)}{\sqrt{2}}$$

(b.) $\omega_1 = \omega_2$; $\phi_1 = \phi_2 + \pi/2 = \pi/2$

$$s(t) = A_1 \cos(\omega t + \pi/2) + A_2 \cos(\omega t)$$

$$= A_1(0 - \sin\omega t \sin \pi/2) + A_2 \cos \omega t$$

$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

$$= -A_1 \sin\omega t + A_2 \cos\omega t$$

0 odd

$$\langle s^2(t)\rangle = \langle A_1^2 \sin^2(\omega t)\rangle - \langle A_1 A_2 \sin(\omega t)\cos(\omega t)\rangle + \langle A_2^2 \cos^2(\omega t)\rangle = \frac{A_1^2 + A_2^2}{2}$$

$$\therefore s_{rms}(t) = \frac{\sqrt{(A_1^2 + A_2^2)}}{\sqrt{2}}$$

2-50. Cont'd

(c.) $\omega_1 = \omega_2$; $\phi_1 = \phi_2 + \pi = \pi$

$$s(t) = A_1 \cos(\omega t + \pi) + A_2 \cos \omega t$$

$$= (A_2 - A_1) \cos \omega t$$

$$\underline{\underline{s_{rms}(t) = \frac{(|A_2 - A_1|)}{\sqrt{2}}}} \quad \text{from (a.) above}$$

(d.) $\omega_1 = 2\omega_2$; $\phi_1 = \phi_2 = 0$

$$s(t) = A_1 \cos(2\omega_2 t) + A_2 \cos(\omega_2 t)$$

$$\langle s^2(t) \rangle = \langle A_1^2 \cos^2(2\omega_2 t) \rangle \qquad 0$$

$$+ A_1 A_2 \cancel{\langle \cos(2\omega_2 t) \cos(\omega_2 t) \rangle} \rightarrow 0$$

$$+ \langle A_2^2 \cos^2(\omega_2 t) \rangle$$

$$\therefore \underline{\underline{s_{rms}(t) = \frac{\sqrt{(A_1^2 + A_2^2)}}{\sqrt{2}}}}$$

(e.) $\omega_1 = 2\omega_2$; $\phi_1 = \phi_2 + \pi = \pi$

$$s(t) = A_1 \cos(2\omega_2 t + \pi) + A_2 \cos(\omega_2 t)$$

$$= -A_1 \cos(2\omega_2 t) + A_2 \cos(\omega_2 t)$$

$$\langle s^2(t) \rangle = \left(\frac{A_1^2 + A_2^2}{2} \right)$$

$$\underline{\underline{s_{rms}(t) = \frac{\sqrt{(A_1^2 + A_2^2)}}{\sqrt{2}}}}$$

2-52 (a.) Over interval $(-4,4)$ $\perp$

$$\int_{-4}^{4} \phi_1^2(t)\,dt = 8$$

$$\int_{-4}^{4} \phi_2^2(t)\,dt = 8$$

$$\int_{-4}^{4} \phi_3^2(t)\,dt = 8$$

$$\int_{-4}^{4} \phi_1(t)\phi_2(t)\,dt = 4-4 = 0$$

$$\int_{-4}^{4} \phi_1(t)\phi_3(t)\,dt = -2+4-2 = 0$$

$$\int_{-4}^{4} \phi_2(t)\phi_3(t)\,dt = -2+2-2+2 = 0$$

(b.) $$\int_{-4}^{4} \phi_j(t)\phi_j^*(t)\,dt = 8 = K_j \quad j=1,3$$

$$\therefore \phi_j'(t) = \left\{\frac{\phi_j(t)}{\sqrt{8}}\right\} = \underline{\underline{\left\{\frac{\phi_j(t)}{2\sqrt{2}}\right\}}} \quad j=1,3$$

2-53 $$w(t) = \frac{1}{2}\phi_1(t) - \frac{1}{2}\phi_2(t)$$

$$\underline{\underline{= \sqrt{2}\,\phi_1'(t) - \sqrt{2}\,\phi_2'(t)}}$$

2-54 $$\epsilon = \int_{-4}^{4}\left[w(t) - \sum_{j=1}^{3} a_j\phi_j(t)\right]^2 dt$$

$$= \int_{-4}^{4}\left[w(t) - \frac{1}{2}\phi_1(t) + \frac{1}{2}\phi_2(t)\right] dt$$

$$\epsilon = \underline{\underline{0}}$$

2-55 (e.) Cont'd $\quad a_j = \frac{1}{K_j}\int_a^b w(t)\,\phi_j^*(t)\,dt$

$$a_1 = \frac{1}{8}\int_{-4}^{4}\cos(\pi t/4)\,dt = \frac{1}{2\pi}\sin(\pi t/4)\Big|_{-4}^{4} = \underline{\underline{0}}$$

$$a_2 = \frac{1}{8}\left[\int_{-4}^{0}\cos(\pi t/4)\,dt - \int_{0}^{4}\cos(\pi t/4)\,dt\right]$$

$$= \frac{1}{2\pi}\left[\sin(\pi t/4)\Big|_{-4}^{0} - \sin(\pi t/4)\Big|_{0}^{4}\right]$$

$$= \frac{1}{2\pi}\left\{0 - 0\right\} = \underline{\underline{0}}$$

$$a_3 = \frac{1}{8}\left\{\int_{-4}^{-2} -\cos\left(\frac{\pi t}{4}\right)dt + \int_{-2}^{2}\cos\left(\frac{\pi t}{4}\right)dt - \int_{2}^{4}\cos\left(\frac{\pi t}{4}\right)dt\right]$$

$$= \frac{1}{2\pi}\left\{-\sin\left(\frac{\pi t}{4}\right)\Big|_{-4}^{-2} + \sin\left(\frac{\pi t}{4}\right)\Big|_{-2}^{2} - \sin\left(\frac{\pi t}{4}\right)\Big|_{2}^{4}\right]$$

$$= \frac{1}{2\pi}\left[1 + 2 + 1\right] = \underline{\underline{\frac{2}{\pi} = a_3}}$$

$$\therefore\ \underline{\underline{w(t) = \frac{2}{\pi}\,\phi_3(t)}} = \underline{\underline{\frac{4\sqrt{2}}{\pi}\,\phi_3'(t)}}$$

2-56

$$\epsilon = \int_{-4}^{4}\left[w(t) - \sum_{j=1}^{3} a_j\,\phi_j(t)\right]^2 dt$$

Normalized (orthonormal)

2-56. Cont'd

$$\epsilon = \underbrace{\int_{-4}^{-2}\left[\cos\left(\frac{\pi t}{4}\right)+\frac{2}{\pi}\right]^2 dt}_{①} + \underbrace{\int_{-2}^{2}\left[\cos\left(\frac{\pi t}{4}\right)-\frac{2}{\pi}\right]^2 dt}_{②} + \underbrace{\int_{2}^{4}\left[\cos\left(\frac{\pi t}{4}\right)+\frac{2}{\pi}\right]^2 dt}_{③}$$

From Symmetry ② = ① and ③ = 2 · ①

$\Rightarrow \epsilon = 4 \cdot ①$

$$① = \int_{-4}^{-2}\left[\cos^2\left(\frac{\pi t}{4}\right) + \frac{4}{\pi}\cos\left(\frac{\pi t}{4}\right) + \frac{4}{\pi^2}\right]dt$$

$$= \frac{1}{2}\int_{-4}^{-2}\left[1+\cos\left(\frac{\pi t}{2}\right)\right]dt + \frac{16}{\pi^2}\sin\left(\frac{\pi t}{4}\right)\Bigg|_{-4}^{-2} + \frac{4}{\pi^2}t\Bigg|_{-4}^{-2}$$

$$= \frac{t}{2}\Bigg|_{-4}^{-2} + \frac{1}{\pi}\cancelto{0}{\sin\left(\frac{\pi t}{2}\right)}\Bigg|_{-4}^{-2} - \frac{16}{\pi^2} + \frac{8}{\pi^2} = 1 - \frac{8}{\pi^2}$$

$\Rightarrow ① = 0.189 \quad \Rightarrow \quad \epsilon = 4 \cdot ① = \underline{\underline{0.756 = \epsilon}}$

$\{\phi_j(t)\}$ do not form a complete orthonormal set since they can represent only a subclass of possible waveforms.

2-61

$$c_n = \frac{1}{T}\int_{\tau_0}^{\tau_0+b} A\, e^{-jn\omega t}\, dt$$

$$= \frac{-A}{T}\,\frac{1}{jn\omega}\, e^{-jn\omega t}\Big|_{\tau_0}^{\tau_0+b}$$

$$= \frac{-A}{jn\omega T}\left(e^{-jn\omega(\tau_0+b)} - e^{-jn\omega\tau_0}\right)$$

$$= \frac{-A}{jn\omega T}\, e^{-jn\omega(\tau_0+\frac{b}{2})}\left(e^{-jn\frac{\omega b}{2}} - e^{jn\frac{\omega b}{2}}\right)$$

$$= \frac{2A}{n\omega T}\, e^{-jn\omega(\tau_0+\frac{b}{2})}\left(\frac{e^{jn\frac{\omega b}{2}} - e^{-jn\frac{\omega b}{2}}}{j2}\right)$$

$\omega = \frac{2\pi}{T}$

$$= \frac{A}{n\pi}\, e^{-jn\omega(\tau_0+\frac{b}{2})}\sin\left(\frac{n\pi b}{T}\right)$$

$$c_n = \frac{Ab}{T}\, e^{-jn\omega(\tau_0+\frac{b}{2})}\,\frac{\sin\left(\frac{n\pi b}{T}\right)}{n\pi b/T}$$

2-64 Use (2-110) and (2-112)

(a.) $c_n = f_0\, P(f)\Big|_{f=nf_0}$

where $P(f) = \mathcal{F}[p(t)] = \int_{-\infty}^{\infty} p(t)\, e^{-j\omega t}\, dt$

For $f=0$
$$P(0) = \int_0^T A\, t\, dt = \frac{At^2}{2}\Big|_0^T = \frac{AT^2}{2}, \quad f=0$$

For $f \neq 0$
$$P(f) = \int_0^T A\, t\, e^{-j\omega t}\, dt$$

2-64. (a.) Cont'd

Let $u = At \quad dv = e^{-j\omega t}$
$du = A\,dt \quad v = e^{-j\omega t}/-j\omega$

$$P(f) = \left.\frac{A t e^{-j\omega t}}{-j\omega}\right|_0^T + \frac{A}{j\omega}\int_0^T e^{-j\omega t}\,dt$$

$$= \frac{jATe^{-j\omega T}}{\omega} + \frac{A}{\omega^2}\left(e^{-j\omega T} - 1\right)$$

$$P(f) = \frac{A\left[e^{-j\omega T} + j\omega T e^{-j\omega T} - 1\right]}{\omega^2}, \quad f \neq 0$$

$$c_n = \frac{1}{T_0} P\left(\omega = \frac{n2\pi}{T_0}\right) = f_0 P(\omega = n2\pi f_0)$$

$$c_n = \begin{cases} \dfrac{AT^2}{2T_0}, & n = 0 \\ \dfrac{A\left[e^{-j2\pi n f_0 T}(1 + jn2\pi f_0 T) - 1\right]}{T_0\,\omega^2}, & n \neq 0 \end{cases}$$

(b.) $x_n = Re\{c_n\}$; $y_n = Im\{c_n\}$

$$c_n = \frac{A\left\{\left[\cos(n2\pi f_0 T) - j\sin(n2\pi f_0 T)\right] \cdot \left[1 + jn2\pi f_0 T\right] - 1\right\}}{T_0\,\omega^2}$$

2-64 (b.) Cont'd

$$x_n = \begin{cases} \dfrac{AT^2}{2T_0}, & n=0 \\ A\left\{ \dfrac{\cos(n2\pi f_0 T) + n2\pi f_0 T \sin(n2\pi f_0 T) - 1}{T_0 \omega^2} \right\}, & n \neq 0 \end{cases}$$

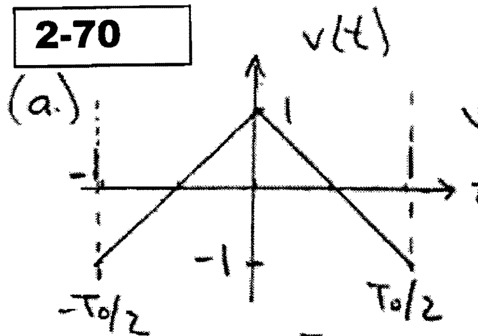
$$y_n = \begin{cases} 0, & n=0 \\ A\left\{ \dfrac{n2\pi f_0 T \cos(n2\pi f_0 T) - \sin(n2\pi f_0 T)}{T_0 \omega^2} \right\}, & n \neq 0 \end{cases}$$

(c.)

$$D_n = \begin{cases} c_0, & n=0 \\ 2\sqrt{x_n^2 + y_n^2}, & n \geq 1 \end{cases}, \qquad \phi_n = \begin{cases} 0, & n=0 \\ \tan^{-1}\left(\dfrac{y_n}{x_n}\right), & n \geq 1 \end{cases}$$

2-70

(a.)



$$v(t) = \begin{cases} \dfrac{4}{T_0}\left(t + T_0/4\right), & -\frac{T_0}{2} < t < 0 \\ -\dfrac{4}{T_0}\left(t - T_0/4\right), & 0 < t < \frac{T_0}{2} \end{cases}$$

$$c_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} v(t)\, e^{-jn\omega_0 t}\, dt \qquad \underline{T_0 = 2}$$

$$= \frac{4}{T_0^2}\left[\int_{-T_0/2}^{0} \left(t + \frac{T_0}{4}\right) e^{-jn\omega_0 t}\, dt - \int_{0}^{T_0/2} \left(t - \frac{T_0}{4}\right) e^{-jn\omega_0 t}\, dt \right]$$

2-70(a) Cont'd

$$C_n = \frac{-8}{4n^2\pi^2}\left[\cos(n\omega_0 t) + n\omega_0 t \sin(n\omega_0 t)\right]\Bigg|_0^{T_0/2}$$

$$\ni C_n = \frac{2}{n^2\pi^2}\left\{1-(-1)^n\right\} = \begin{Bmatrix} 0, & n = \text{even} \\ \frac{4}{n^2\pi^2}, & n = \text{odd} \end{Bmatrix}$$

(b) $$\langle v^2(t)\rangle = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} v^2(t)\,dt \underset{\text{For } v(t) = \text{even}}{=} \frac{2}{T_0}\int_0^{T_0/2} v^2(t)\,dt$$

$$= \frac{2}{T_0}\int_0^{T_0/2}\left[\frac{-4}{T_0}\left(t-\frac{T_0}{4}\right)\right]^2 dt = \frac{32}{T_0^3}\frac{\left(t-\frac{T_0}{4}\right)^3}{3}\Bigg|_0^{T_0/2}$$

$$= \frac{2\cdot 4^2}{3T_0^3}\left[\frac{2T_0^3}{4^3}\right] = \underline{\underline{\frac{1}{3} \text{ watt}}}$$

Computer Solution and comparision of results follows.

2-70. Cont'd

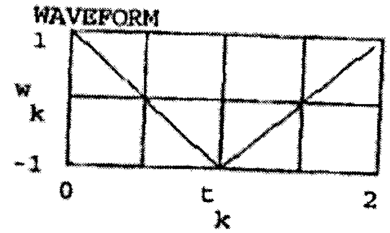
$M := 5 \qquad N := 2^M \qquad N = 32 \qquad k := 0 \,..\, N-1 \qquad T := 2$

$dt := \frac{T}{N} \qquad t_k := k \cdot dt$

$w_k := \mathrm{if}\left[t_k < 1, -2\left[t_k - 0.5\right], 2\left[t_k - 1.5\right]\right]$

$w_0 = 1 \qquad dt = 0.063$

WAVEFORM



(a.) Find the complex Fourier series.

$n := 0 \,..\, N-1 \qquad f_n := \frac{n}{T} \qquad fo := \frac{1}{T}$

From analytical computation,

$c_n := \mathrm{if}\left[\mathrm{mod}(n,2) \neq 0, \frac{4}{(n\,\pi)^2}, 0\right]$

Analytical values

FFT values

Alternately, computing FS using the FFT via (2-187),

$cc := \left[\frac{1}{\sqrt{N}}\right] \cdot \mathrm{icfft}(w)$

n	c_n
0	0
1	0.405
2	0
3	0.045
4	0
5	0.016
6	0
7	0.008

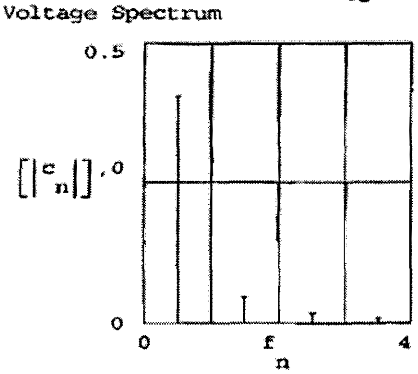
$cc = \begin{bmatrix} 0 \\ 0.407 \\ 0 \\ 0.046 \\ 0 \\ 0.018 \\ 0 \\ 0.01 \end{bmatrix}$

(b)

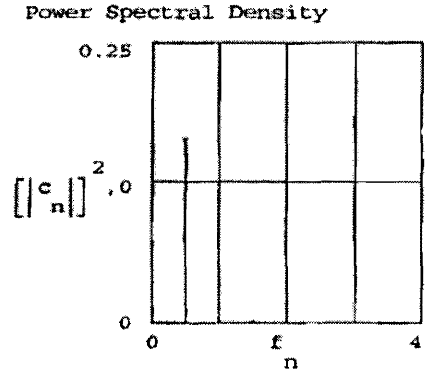
$P := 2 \sum_n c_n^{\,2} - c_0^{\,2} \qquad P = 0.333$

(c) and (d) $|V(f)| = \sum_{-\infty}^{\infty} |c_n|\,\delta(f - nf_0)\,, \quad \mathcal{P}(f) = \sum_{-\infty}^{\infty} |c_n|^2\,\delta(f - nf_0)$

Voltage Spectrum



Power Spectral Density



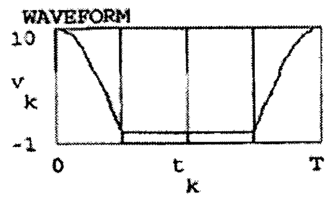
2-72

$M := 5 \qquad N := 2^M \qquad N = 32 \qquad k := 0 ..N-1 \qquad T := 1$

$dt := \frac{T}{N} \qquad t_k := k\,dt \qquad Sa(x) := \text{if}\left[x \neq 0, \frac{\sin(x)}{x}, 1\right] \qquad \omega o := 2\frac{\pi}{T}$

$v_k := \text{if}\left[\left|t_k - 0.5\,T\right| < 0.25\,T, 0, 10\cos\left[\omega o\, t_k\right]\right]$

$v_0 = 10 \qquad dt = 0.031$



Find the complex Fourier series.

$n := 0 ..N-1 \qquad f_n := \frac{n}{T} \qquad fo := \frac{1}{T}$

From analytical computation,

$c_n := 2.5\,(Sa(0.5\,(n+1)\,\pi) + Sa(0.5\,(n-1)\,\pi))$

Analytical results

Alternately, computing FS using the FFT via (2-187),

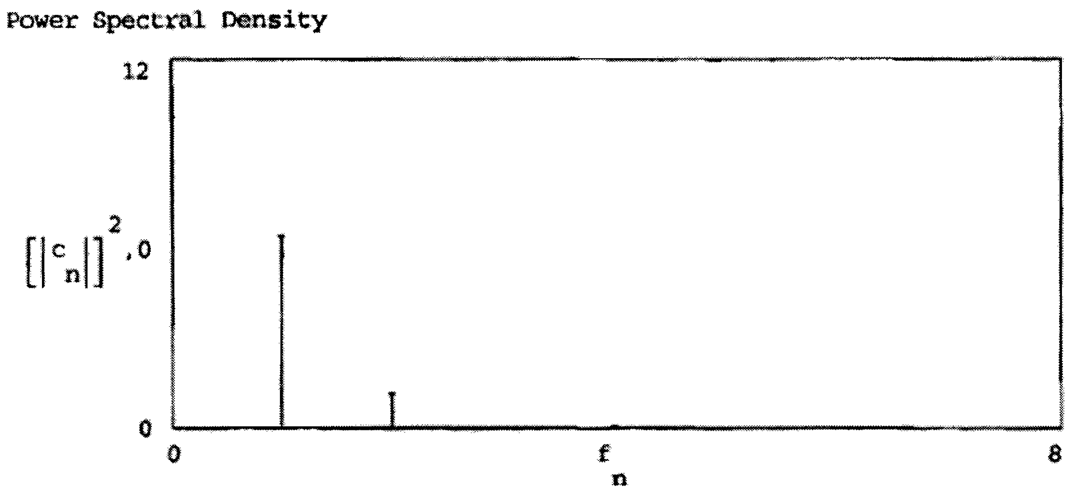
FFT results

$cc := \left[\frac{1}{\sqrt{N}}\right] \text{icfft}(v)$

n	c_n
0	3.183
1	2.5
2	1.061
3	0
4	-0.212
5	0
6	0.091
7	0
8	-0.051
9	0
10	0.032
11	0
12	-0.022
13	0
14	0.016
15	0
16	-0.012
17	0
18	0.01
19	0
20	-0.008
21	0
22	0.007
23	0
24	-0.006
25	0
⋮	⋮

$$cc = \begin{bmatrix} 3.173 \\ 2.5 \\ 1.071 \\ 1.203\cdot 10^{-13} + 3.216\cdot 10^{-14}\,i \\ -0.223 \\ 6.336\cdot 10^{-14} + 3.226\cdot 10^{-14}\,i \\ 0.102 \\ -2.175\cdot 10^{-12} - 1.086\cdot 10^{-12}\,i \\ -0.062 \\ 2.705\cdot 10^{-14} + 3.207\cdot 10^{-14}\,i \\ 0.045 \\ 1.767\cdot 10^{-14} + 3.256\cdot 10^{-14}\,i \\ -0.036 \\ 9.774\cdot 10^{-15} + 3.232\cdot 10^{-14}\,i \\ 0.032 \\ -5.456\cdot 10^{-13} - 1.634\cdot 10^{-12}\,i \\ -0.031 \\ -3.22\cdot 10^{-15} + 3.235\cdot 10^{-14}\,i \\ \vdots \end{bmatrix}$$

2-72. cont'd $\mathcal{P}(f) = \sum_{-\infty}^{\infty} |c_n|^2\, \delta(f - nf_0)$



2-80

$$x(t) = e^{-400\pi t} \longleftrightarrow X(f) = \frac{1}{400\pi + j2\pi f}$$

$$\text{Energy in } x(t) = E_x = \int_{\infty}^{\infty} x^2(t)\,dt = \int_0^{\infty} e^{-800\pi t}\,dt = \frac{1}{800\pi} \text{ Joules}$$

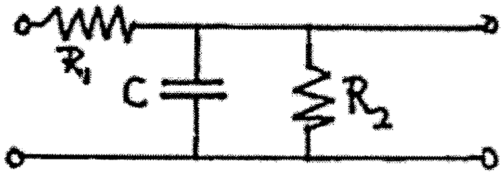
$$E_{out} = \frac{1}{2}E_x = \frac{1}{1600\pi} = 2\int_0^B |X(f)|^2 df = 2\int_0^B \frac{\frac{1}{4\pi^2}}{4\times10^4 + f^2}\,df = \frac{1}{2\pi}\left[\frac{1}{200}\tan^{-1}\left(\frac{B}{200}\right)\right]$$

$$\Rightarrow \frac{400\pi^2}{1600\pi} = \tan^{-1}\left(\frac{B}{200}\right) \Rightarrow \frac{B}{200} = \tan\left(\frac{\pi}{4}\right) = 1$$

$$\Rightarrow \underline{\underline{B = 200\ \text{Hz}}}$$

2-84

$$C \| R_1 \Rightarrow Z_{\|} = \frac{R_2 \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}} = \frac{R_2}{1 + j\omega R_2 C}$$



$$\Rightarrow H(f) = \frac{\frac{R_2}{1 + j\omega R_2 C}}{R_1 + \frac{R_2}{1 + j\omega R_2 C}} = \frac{R_2}{R_1 + R_2 + j\omega R_1 R_2 C}$$

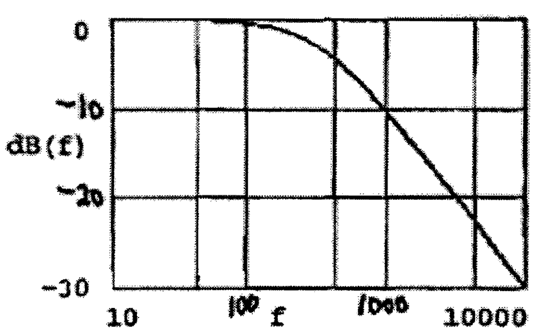
$R1 := 7.5 \cdot 10^{3}$ $\quad R2 := 15 \cdot 10^{3}$ $\quad C := 100\ 10^{-9}$

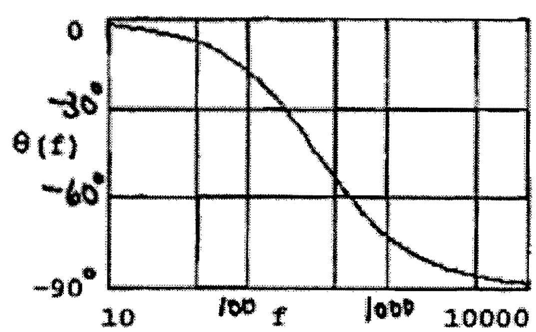
$f := 10, 20 .. 10000$ $\quad j := \sqrt{-1}$ $\quad Hm := \frac{R2}{R1 + R2}$

$$H(f) := \frac{R2}{(R1 + R2) + j \cdot 2 \cdot \pi \cdot f \cdot R1 \cdot R2 \cdot C}$$

$Hm = 0.667$

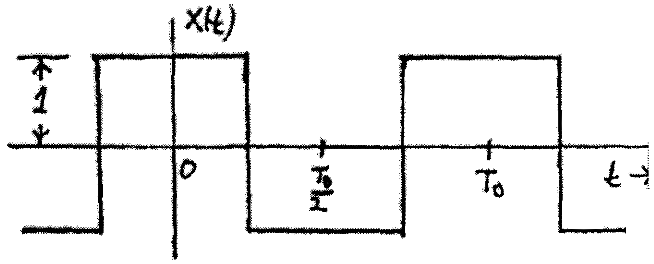
$$dB(f) := 20 \cdot \log\left[\frac{|H(f)|}{Hm}\right] \qquad \theta(f) := \left[\frac{180}{\pi}\right] \cdot \arg(H(f))$$





$$f3dB := \frac{R1 + R2}{2 \cdot \pi \cdot R1\ R2 \cdot C} \qquad f3dB = 318.31$$

2-87



Let the input square wave be represented by the Fourier series:

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

where
$$c_n = \begin{cases} \frac{2\sin(n\frac{\pi}{2})}{n\pi} & , n \neq 0 \\ 0 & , n = 0 \end{cases}$$

for the waveform shown above.

Then the output waveform is, using (2-140),

$$y(t) = \sum_{-\infty}^{\infty} H(nf_0) c_n e^{jn\omega_0 t} = \sum_{-\infty}^{\infty} d_n e^{jn\omega_0 t}$$

where $d_n \triangleq H(nf_0)\, c_n$, $\quad H(f) = \frac{1}{1 + j\left(\frac{f}{f_1}\right)}$

and $f_1 = 1{,}500$ Hz for the RC low-pass filter.

We also know that $d_{-n} = d_n^*$ since $x(t)$ is real and the impulse response of the filter is real.

Now reduce the output Fourier series to a form that can be easily plotted. Using (2-103),

$$y(t) = D_0 + \sum_{n=1}^{n=\infty} D_n \cos(n\omega_0 t + \phi_n)$$

where $D_0 = 0$ since $c_0 = 0$

and $D_n = 2|d_n| = 2|H(nf_0)c_n|$ $\quad , n > 0$

2-87. Cont'd

or

$$\Rightarrow D_n = 2\left|\frac{1}{1+j\left(\frac{nf_0}{f_1}\right)}\right|\begin{Bmatrix}\frac{2}{n\pi}, & n=\text{odd}\\ 0, & n=\text{even}\end{Bmatrix} = \underline{\frac{4}{\sqrt{1+\left(\frac{nf_0}{f_1}\right)^2}\,(n\pi)},\quad n=\text{odd}}$$

$$\phi_n = \underline{/d_n} = \underline{/2H(nf_0)c_n} = \underline{-\tan^{-1}\left(\frac{nf_0}{f_1}\right)+\pi\left(\frac{1-\sin\left(\frac{n\pi}{2}\right)}{2}\right),\quad n=\text{odd}}$$

$$\Rightarrow y(t) = \underline{\underline{\sum_{\substack{n=1\\ n=\text{odd}}}^{\infty} D_n\cos(n\omega t+\phi_n)}}$$

The following MathCAD program plots this $y(t)$.

```
fo := 300        f1 := 1500        n := 1,3 . 11

                                   t := 0,0.00005 .0.004
```

$$D_n := \frac{4}{n\,\pi\sqrt{1+\left[n\,\frac{fo}{f1}\right]^2}}$$

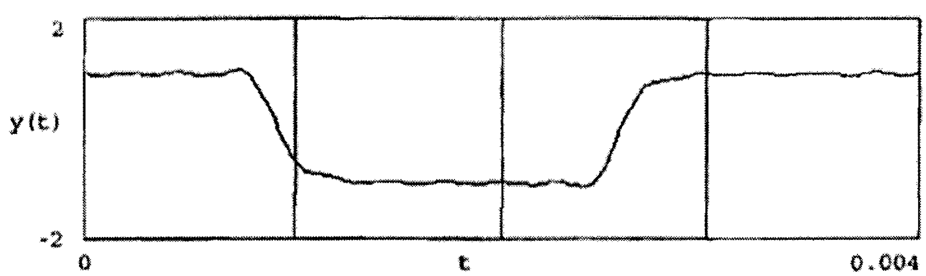
2-87. Cont'd.

$$\phi_n := \pi \left[\frac{1 - \sin\left[n \frac{\pi}{2}\right]}{2} \right] - \operatorname{atan}\left[n \frac{fo}{f1}\right]$$

$$y(t) := \sum_n D_n \cos\left[n\, 2\, \pi\, fo\, t + \phi_n\right]$$

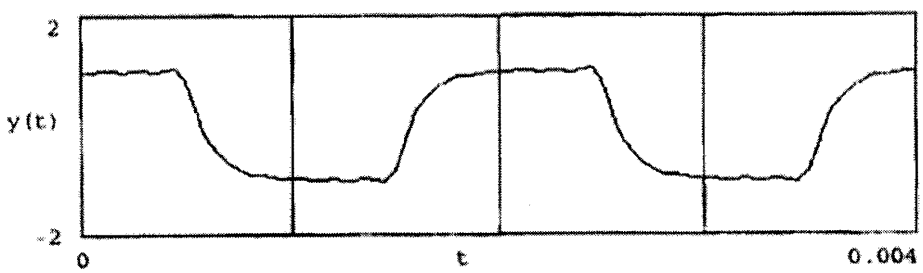
(a)

fo = 300



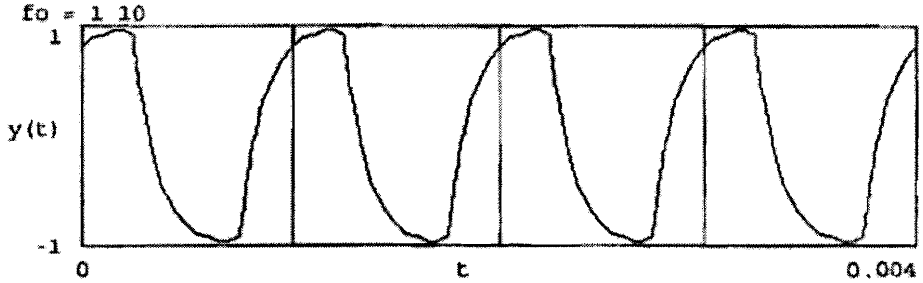
(b)

fo = 500



(c)

$fo = 1 \cdot 10^3$



2-90

$$\omega_0 = 2\pi f_0 = 500 \Rightarrow f_0 = \frac{500}{2\pi}$$

$$f_s > 2f_0 = \frac{2(500)}{2\pi} = \frac{500}{\pi}$$

(a.) $T_s = \frac{1}{f_s} \le \frac{\pi}{500} = \underline{\underline{6.28 \text{ msec}}}$

(b.) $N = \frac{1 \text{ sec}}{6.28 \times 10^{-3} \text{ sec/sample}} = \underline{\underline{160 \text{ samples}}}$

2-92

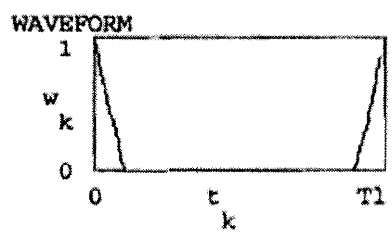
$M := 6 \quad N := 2^M \quad N = 64 \quad k := 0 .. N-1 \quad T1 := 10 \quad T := 1$

$dt := \frac{T1}{N} \qquad t_k := k\,dt$

NOTE: In FFT time domain, points for negative time are the same as those measured from the end of the data span-length T1 for positive time.

$$w_k := \text{if}\left[t_k < T, \left[\frac{-1}{T}\right]\left[t_k - T\right], 0\right] + \text{if}\left[t_k > (T1 - T), \frac{t_k - (T1 - T)}{T}, 0\right]$$

$w_0 = 1 \qquad dt = 0.156$

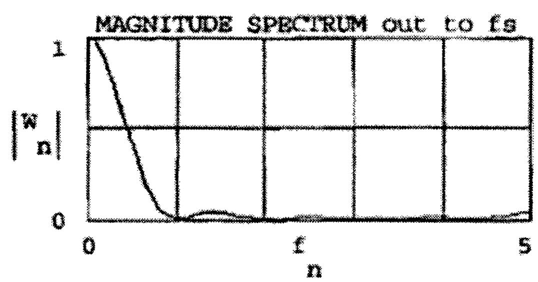


$n := 0 .. N-1$

$W := dt\left[\sqrt{N}\right]\ \text{icfft}(w)$

$f_n := \frac{n}{T1} \qquad fs := \frac{1}{dt}$

$f_1 = 0.1 \qquad fs = 6.4$



2-100 $s(t) = \Lambda\left(\frac{t}{T_0}\right) \xleftrightarrow[\text{Table 2-2}]{} S(f) = T_0\left[S_a(\pi f T_0)\right]^2$

(a.) Using results in 2-61 (1.) above $\Rightarrow$ $\underline{\underline{B_{abs} = \infty}}$

(b.) $S(f_{3dB}) = \frac{T_0}{\sqrt{2}} = T_0\left[S_a(\pi f_{3dB} T_0)\right]^2$

$\Rightarrow \pi f_{3dB} T_0 \approx (2)^{1/4} \Rightarrow \underline{\underline{B_{3dB} = f_{3dB} = \frac{1.19}{\pi T_0} = 0.38/T_0}}$

(c.) $B_{eq} = \frac{1}{|H(f_0)|^2}\int_0^\infty |H(f)|^2 df = \frac{1}{T_0^2}\int_0^\infty T_0^2\left[S_a(\pi f T_0)\right]^4 df$

$= \frac{1}{\pi T_0}\left(\frac{\pi}{3}\right) = \frac{1}{3T_0} \Rightarrow \underline{\underline{B_{eq} = \frac{1}{3T_0}}}$

(d.) $B_{zero\ cross} = \frac{1}{T_0}$ (Similar to 2-90 (4) above)

Chapter 3

3-3

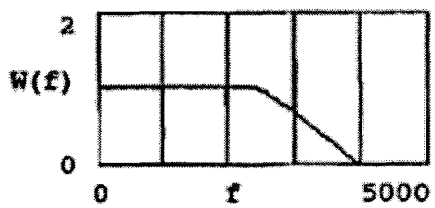
$A := 1 \qquad f1 := 2500 \qquad f2 := 4000$

$f := 0, 200 .. 5000$

$W1(x) := \text{if}(|x| < f1, A, 0)$

$$W2(x) := \left[\frac{-A}{f2 - f1}\right] \cdot (|x| - f2) \cdot (\Phi(|x| - f1) - \Phi(|x| - f2))$$

$W(x) := W1(x) + W2(x)$



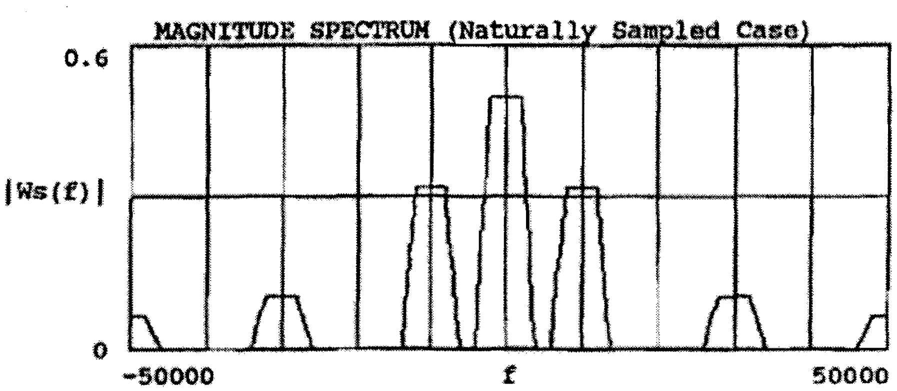
$fs := 10000 \qquad \tau := 50\ 10^{-6} \qquad Ts := \dfrac{1}{fs} \qquad d := \dfrac{\tau}{Ts}$

Naturally-sampled PAM

$n := -5, -4 .. 5 \qquad Sa(x) := \text{if}\left[x \neq 0, \dfrac{\sin(x)}{x}, 1\right]$

$f := -50000, -48000 .. 50000$

$$Ws(f) := d \cdot \sum_{n} (Sa(\pi\ n\ d)) \cdot W(f - n \cdot fs)$$



3-4

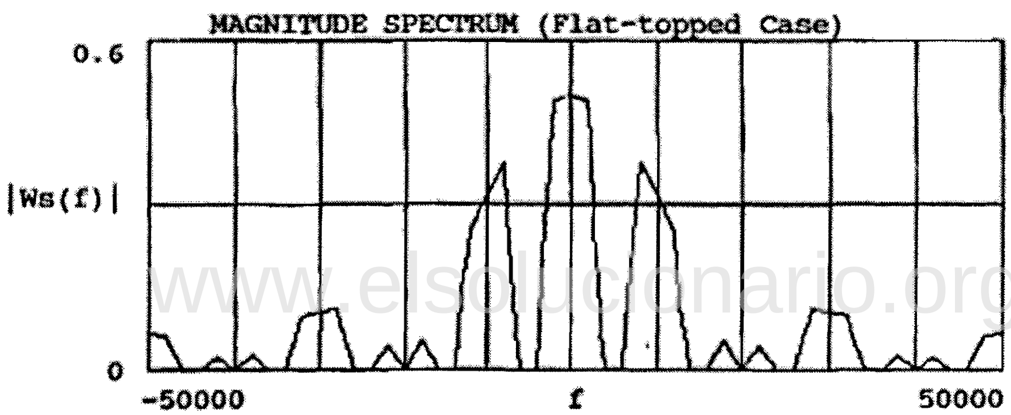
Use the parameters given in P 3-3 above.

Flat-topped PAM

$$H(f) := \tau\, Sa(\pi \cdot \tau\, f)$$

$$Ws(f) := \left[\frac{1}{Ts}\right] \cdot H(f) \cdot \sum_{n} W(f - n\, fs)$$

See next screen for plot

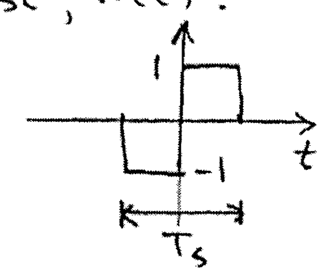


3-7

Using (3-10)

$$W_s(f) = \frac{H(f)}{T_s} \sum_{k=-\infty}^{\infty} W(f - kf_s)$$

where $H(f)$ is the spectrum of the Manchester encoded pulse, $h(t)$.



Thus

$$H(f) = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-T_s/2}^{0} (-1)\, e^{-j\omega t}\, dt + \int_{0}^{T_s/2} (1)\, e^{-j\omega t}\, dt$$

3-7 Cont'd.

$$= \frac{j}{\omega}\left[-e^{-j\omega t}\Big|_{-T_s/2}^{0} + e^{-j\omega t}\Big|_{0}^{T_s/2} \right]$$

$$= \frac{-j}{\omega}\left[2 - 2\left(\underbrace{\frac{e^{j\omega T_s/2} + e^{-j\omega T_s/2}}{2}}_{\cos \frac{\omega T_s}{2}} \right)\right]$$

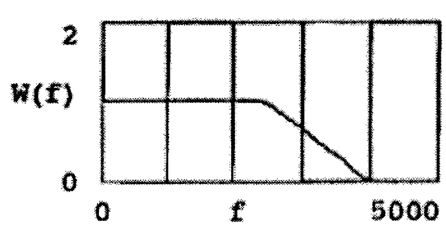
$$H(f) = -j T_s \frac{\left(1 - \cos \omega T_s/2\right)}{\omega T_s/2}$$

```
A := 1        f1 := 2500      f2 := 4000

                                            f := 0,200 ..5000
W1(x) := if(|x| < f1,A,0)

          [   -A   ]
W2(x) :=  [--------]  (|x| - f2) (Φ(|x| - f1) - Φ(|x| - f2))
          [f2 - f1 ]

W(x)  := W1(x) + W2(x)
```



3-7 Cont'd.

$fs := 10000$ $\qquad Ts := \dfrac{1}{fs}$

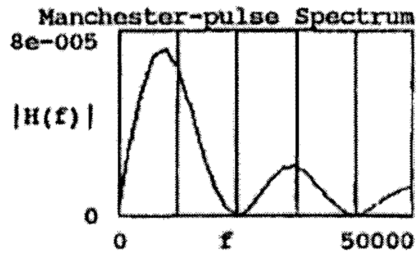
$j := \sqrt{-1}$

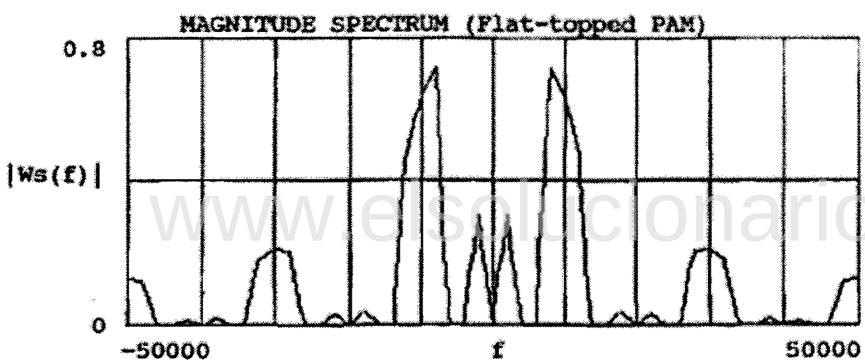
$n := -5, -4 \,..\, 5$ $\qquad Sa(x) := \text{if}\left[x \neq 0, \dfrac{\sin(x)}{x}, 1\right]$

$f := -50000, -48000 \,..\, 50000$

$H(f) := j \cdot Ts \cdot Sa\left[\pi \cdot f \dfrac{Ts}{2}\right] \cdot \sin\left[2\,\pi \cdot f \cdot \dfrac{Ts}{4}\right]$

$Ws(f) := \left[\dfrac{1}{Ts}\right] \cdot H(f) \cdot \sum_{n} W(f - n\,fs)$





3-9 (a.) $f_s = 2B = 2(100) = \underline{\underline{200 \text{ samples/sec}}}$

(b.) Using the results given in prob. 3-8.

$n \geq 3.32 \log_{10}\left(\dfrac{50}{P}\right) = 3.32 \log_{10}\left(\dfrac{50}{0.1}\right)$

$= 8.96$

$\underline{\underline{n = 9 \text{ bits/word}}}$

(c.) $R = \left(\dfrac{n \text{ bits}}{\text{word}}\right)\left(\dfrac{f_s \text{ words}}{\text{sec}}\right) = 200(9) = \underline{\underline{1.8 \dfrac{\text{kbits}}{\text{sec}}}}$

(d.) For binary PCM $\quad D = R$

eq. (3-74) $\quad D = \dfrac{2B}{1+r}$, for B_{min}, $r = 0$

$\Rightarrow \quad B = \dfrac{D}{2} = \underline{\underline{900 \text{ Hz}}}$

3-12

(a.) $f_s \geq 2\,B_{analog} = 2(20\text{ kHz}) = 40\ \frac{\text{k samples}}{\text{sec}}$

For 8X oversampling of the recovered PCM signal (used to increase f_s 8X and simplify LPF requirements)

$\ni f_{8x} = 8 f_s = 320$ k samples/sec

$$B_{null} = R = n f_{8x} = \left(16\frac{\text{bits}}{\text{sample}}\right)\left(320\frac{\text{k samples}}{\text{sec}}\right) = \underline{\underline{5.12\text{ MHz}}}$$

(b) Using (3-18)

$$\left(\frac{S}{N}\right)_{peak} = 6.02n + 4.77\text{dB} = 6.02(15)+4.77 = \underline{\underline{94.77\text{dB}}}$$

3-16 (a.) $P_e = 10^{-4}$ $\quad \frac{S}{N} \geq 30\text{dB}$

Eq. (3-16)

$$\left(\frac{S}{N}\right)_{dB} = 10\log_{10}\left[\frac{3M^2}{1+4(M^2-1)P_e}\right]$$

$M = 2^n$ levels

for $n=4$: $\left(\frac{S}{N}\right)_{out} = 28.4\text{dB}$

for $\underline{\underline{n=5}}$: $\left(\frac{S}{N}\right)_{out} = 33.4\text{dB}$

$\longrightarrow \quad M = 2^5 = \underline{\underline{32}}$ levels

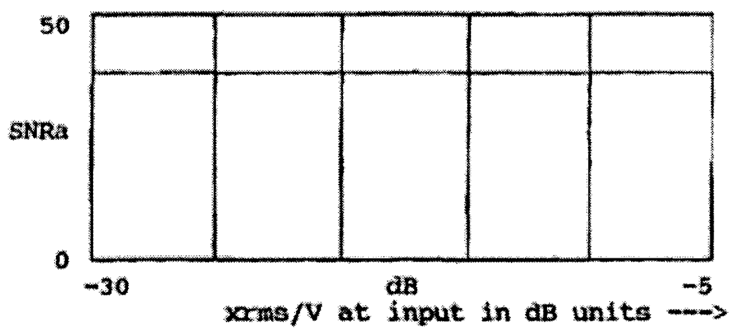
(b.) $f_s = 2(2.7\text{ kHz}) = 5.4\text{ k}\ \frac{\text{samples}}{\text{sec}}$

The first zero-crossing of the $\frac{\sin x}{x}$ type spectrum is :

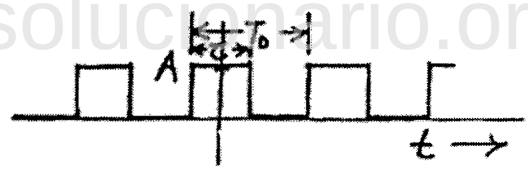
$$B = \frac{n}{T_s} = n f_s = 5(5.4\text{k}) = \underline{\underline{27\text{ kHz} = B}}$$

3-20

```
dB := -30,-29 ..-5        M := 256                n := log(M)/log(2)
     μ := 255              n = 8
SNRa := 6.02 n + 4.77 - 20 log(ln(1 + μ))
```



3-24 For alternating data the waveform is periodic where $T_0 = 2T_b$.



From (2-109) the spectrum is

$$W(f) = \sum_{-\infty}^{\infty} c_n\, \delta(f - nf_0)$$

Where
$$c_n = \frac{1}{T_0}\int_a^{a+T_0} w(t)\, e^{-jn\omega_0 t}\, dt = \frac{1}{T_0}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} A\, e^{-jn\omega_0 t}\, dt$$

$$= \frac{A}{T_0}\, \frac{e^{-jn\omega_0 t}}{-jn\omega_0}\Bigg|_{-\frac{\tau}{2}}^{\frac{\tau}{2}} = \frac{2A}{T_0}\left(\frac{e^{-jn\omega_0\tau/2} + e^{+jn\omega_0\tau/2}}{-2jn\omega_0}\right)$$

$\sin(n\omega_0\tau/2)$

$$\Rightarrow c_n = \frac{\cancel{2}A\tau}{T_0\cancel{2}}\, \frac{\sin(\omega_0 n\tau/2)}{\frac{n\omega_0\tau}{2}} = \frac{A}{2}\left(\frac{\tau}{T_b}\right)\frac{\sin\left(\frac{n\pi}{2}\frac{\tau}{T_b}\right)}{\left(\frac{n\pi}{2}\frac{\tau}{T_b}\right)}$$

$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2T_b}$

$$\Rightarrow W(f) = \sum_{n=-\infty}^{\infty} \frac{A}{2}\left(\frac{\tau}{T_b}\right)\left(\frac{\sin\left(\frac{n\pi}{2}\frac{\tau}{T_b}\right)}{\frac{n\pi}{2}\frac{\tau}{T_b}}\right)\delta\left(f - \frac{n}{2}R\right) \qquad (A)$$

where $R = \frac{1}{T_b}$ = bit rate

3-24. Cont'd

(a) Using (A) for NRZ signaling with $\tau = T_b$

$$|W(f)| = \sum_{-\infty}^{\infty} \frac{A}{2} \left| \frac{\sin\left(\frac{n\pi}{2}\right)}{\left(\frac{n\pi}{2}\right)} \right| \delta\left(f - \frac{n}{2}R\right)$$ Unipolar NRZ (alternating data)

If the data are a sequence of four "1"'s followed by four "0"'s, the waveform would have the same shape except T_0 would be 4 times as large. i.e. $T_0 = 8T_b$.

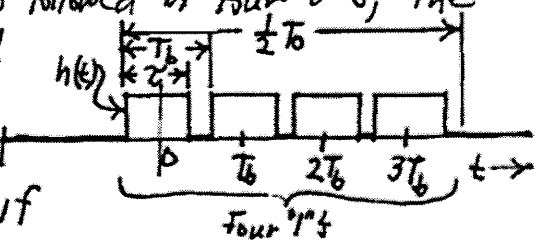
$$\Rightarrow |W(f)| = \sum_{-\infty}^{\infty} \frac{A}{2} \left| \frac{\sin\left(\frac{n\pi}{2}\right)}{\left(\frac{n\pi}{2}\right)} \right| \delta\left(f - \frac{n}{8}R\right)$$ Unipolar NRZ 4"1"'s & 4"0"'s

3-25

Using (A) for RZ signaling with $\tau = \frac{3}{4}T_b$,

$$|W(f)| = \sum_{-\infty}^{\infty} \frac{3}{8}A \left| \frac{\sin\left(\frac{3}{8}n\pi\right)}{\left(\frac{3}{8}n\pi\right)} \right| \delta\left(f - \frac{n}{2}R\right)$$ Unipolar RZ (alternating data)

For RZ with four "1"'s followed by four "0"'s, the periodic waveform would appear as shown where $T_0 = 8T_b$. The mathematical calculations are simplified if (2-112) is used



$$c_n = f_0\, H(nf_0)$$

where $h(t)$ is the basic waveform that is repeated to create the periodic waveform (as shown in the figure). $h(t)$ consists of the superposition of four rectangular pulses. Using the time delay theorem of Table 2-1

3-25 Cont'd.

and the rectangular pulse spectrum of Table 2-2

$$H(f) = A\tau \frac{\sin(\pi f \tau)}{\pi f \tau}\left[1 + e^{-j\omega T_b} + e^{-j\omega 2T_b} + e^{-j\omega 3T_b}\right]$$

Or

$$c_n = \frac{A\tau}{8T_b}\,\frac{\sin\left(\frac{n\pi}{8}\frac{\tau}{T_b}\right)}{\left(\frac{n\pi}{8}\frac{\tau}{T_b}\right)}\left[1 + e^{-j\frac{n\pi}{4}} + e^{-j\frac{n\pi}{2}} + e^{-j\frac{3}{4}n\pi}\right]$$

$$f = nf_0 = \frac{n}{T_0} = \frac{n}{8T_b}$$

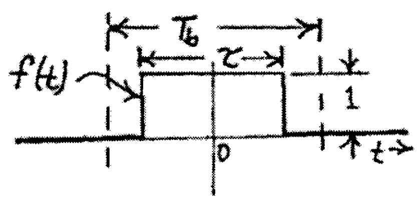
For RZ with $\tau = \frac{3}{4}T_b$, this becomes

$$c_n = \frac{3}{32}A\left(\frac{\sin\left(\frac{3}{32}n\pi\right)}{\left(\frac{3}{32}n\pi\right)}\right)\left[1 + e^{-j\frac{n\pi}{4}} + e^{-j\frac{n\pi}{2}} + e^{-j\frac{3}{4}n\pi}\right]$$

Thus, the spectrum for Unipolar RZ with four alternate "1" and "0"'s is

$$|W(f)| = \sum_{n=-\infty}^{\infty}\frac{3}{32}A\left|\frac{\sin\left(\frac{3}{32}n\pi\right)}{\left(\frac{3}{32}n\pi\right)}\right|\left|1 + e^{-j\frac{n\pi}{4}} + e^{-j\frac{n\pi}{2}} + e^{-j\frac{3}{4}n\pi}\right|\delta\left(f - \frac{n}{8}R\right)$$

3-29 (a) Substituting (3-40) into (3-36a) the PSD for Polar RZ signaling is



$$\mathcal{P}(f) = \frac{A^2}{T_b}\,|F(f)|^2$$

where the pulse shape, $f(t)$, is shown in the figure. Thus,

$$F(f) = \mathfrak{F}[f(t)] = \tau\frac{\sin(\pi f \tau)}{(\pi f \tau)}$$

and

$$\mathcal{P}(f) = \frac{A^2\tau^2}{T_b}\left[\frac{\sin(\pi f \tau)}{(\pi f \tau)}\right]^2$$

3-29 Cont'd.

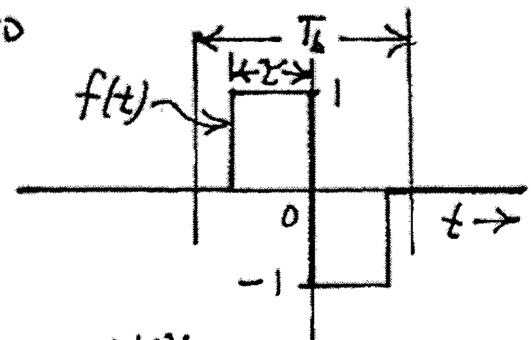
For the case of $\tau = \frac{1}{2} T_b$, this becomes

$$P(f) = \frac{A^2 T_b}{4} \left[\frac{\sin(\frac{\pi}{2} f T_b)}{(\frac{\pi}{2} f T_b)} \right]^2$$

The first-null bandwidth is $B_{null} = \frac{2}{T_b} = 2R$

and the bandwidth efficiency is $\eta = \frac{1}{2}$ (bit/sec)/Hz.

(b) Equation (3-36) can also be used to evaluate the PSD for RZ Manchester signaling where the pulse shape is shown in the figure.



$$F(f) = \tau \left(\frac{\sin(\pi f \tau)}{(\pi f \tau)} \right) \left[e^{j\omega\frac{\tau}{2}} - e^{-j\omega\frac{\tau}{2}} \right]$$

$$\Rightarrow F(f) = j2\tau \left(\frac{\sin(\pi f \tau)}{(\pi f \tau)} \right) \sin\left(\frac{\omega \tau}{2}\right)$$

Using (3-40) and (3-36), the PSD for Manchester signaling is

$$P(f) = \frac{4A^2\tau^2}{T_b} \left[\frac{\sin(\pi f \tau)}{(\pi f \tau)} \right]^2 [\sin(\pi f \tau)]^2$$

If $\tau = \frac{1}{4} T_b$, this becomes

$$P(f) = \frac{1}{4} A^2 T_b \left[\frac{\sin(\frac{\pi}{4} f T_b)}{(\frac{\pi}{4} f T_b)} \right]^2 \left[\sin\left(\frac{\pi}{4} f T_b\right)\right]^2$$

The first-null bandwidth is $B_{null} = \frac{4}{T_b} = 4R$

and the spectral efficiency is $\eta = \frac{1}{4}$ (bits/sec)/Hz.

3-31

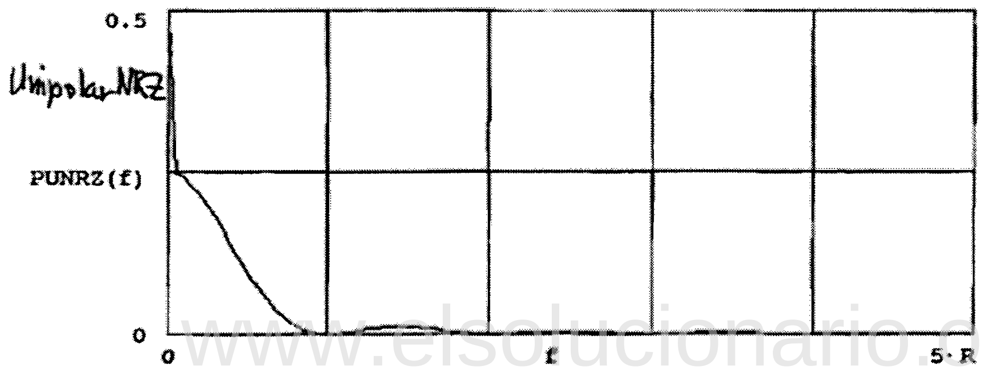
$A := 1 \qquad R := 1 \qquad f := 0, 0.05 \,..\, 5 \qquad Tb := \frac{1}{R}$

$$Sa(x) := \mathrm{if}\left[x \neq 0, \frac{\sin(x)}{x}, 1\right]$$

The PSD for Unipolar NRZ is given by (3-39b) and consists of both a continuous spectrum and a discrete spectrum. The computer cannot plot infinite values for the delta functions, so plot the weights of the delta functions instead. Thus (3-39b) will be broken into two functions, one for the continuous spectral plot and one for the discrete spectral plot.

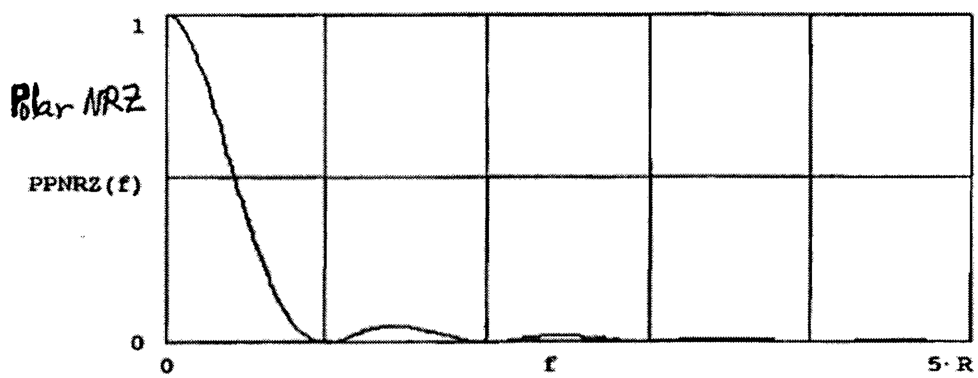
$$PUNRZc(f) := \left[A^2 \cdot \frac{Tb}{4}\right] \cdot (Sa(\pi\ f\ Tb))^2 \qquad PUNRZd(f) := \mathrm{if}\left[f \neq 0, 0, \frac{A^2}{4}\right]$$

$$PUNRZ(f) := PUNRZc(f) + PUNRZd(f)$$



Use (3-41) for Polar NRZ spectrum:

$$PPNRZ(f) := \left[A^2\ Tb\right]\ (Sa(\pi\ f \cdot Tb))^2$$



The PSD for Unipolar RZ is given by (3-43) and consists of both a continuous spectrum and a discrete spectrum. The computer cannot plot infinite values for the delta functions, so plot the weights of the delta functions instead. Thus (3-43) will be broken into two functions, one for the continuous spectral plot and one for the discrete spectral plot.

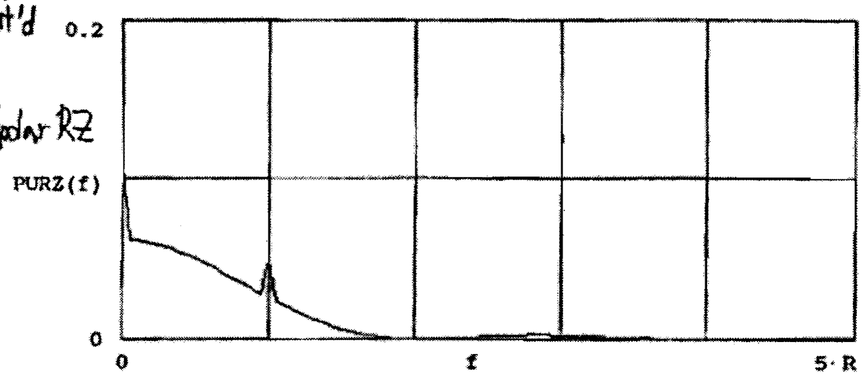
$$PURZc(f) := \left[A^2\ \frac{Tb}{16}\right]\left[Sa\left[\pi\ f\ \frac{Tb}{2}\right]\right]^2$$

$$PURZd(f) := \mathrm{if}\left[\mathrm{mod}(f, R) \neq 0, 0, \frac{A^2}{16} \cdot \left[Sa\left[\pi\ f\ \frac{Tb}{2}\right]\right]^2\right]$$

3-31 Cont'd

$PURZ(f) := PURZc(f) + PURZd(f)$

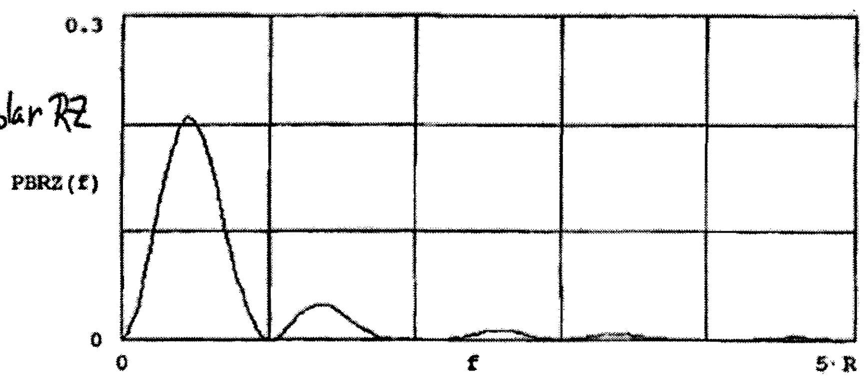
Unipolar RZ



Using (3-45) the PSD for Bipolar RZ is:

$$PBRZ(f) := A^2 \left[\frac{Tb}{4}\right] \cdot \left[Sa\left[\pi\ f \cdot \frac{Tb}{2}\right]\right]^2 (\sin(\pi\ f\ Tb))^2$$

Bipolar RZ



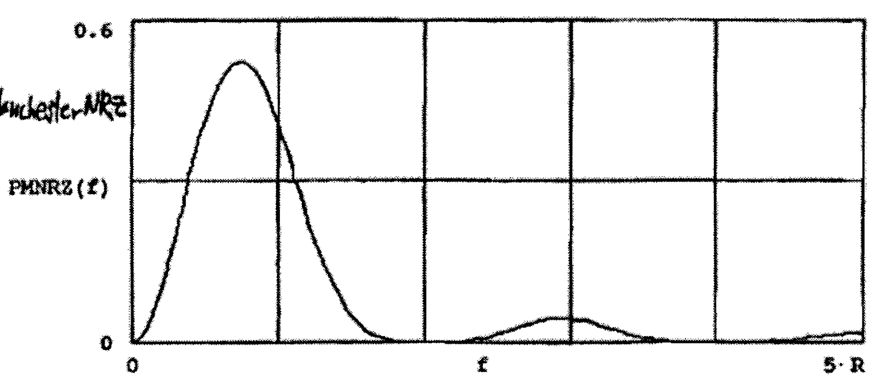
$A := 1$ $R := 1$ $f := 0, 0.05 .. 5$ $Tb := \frac{1}{R}$

$$Sa(x) := \text{if}\left[x \neq 0, \frac{\sin(x)}{x}, 1\right]$$

Use (3-46c) for the Manchester NRZ spectrum:

$$PMNRZ(f) := A^2 \cdot Tb \left[Sa\left[\pi \cdot f \cdot \frac{Tb}{2}\right]^2\right] \cdot \left[\sin\left[\pi\ f \cdot \frac{Tb}{2}\right]^2\right]$$

Manchester NRZ



3-39 Use the result from Prob 3-8.

(a) $n \geq 3.32 \log_{10}\left(\frac{50}{P}\right) = 3.32 \log_{10}(50) = 5.64 \Rightarrow$ Use $n = 6$ bits/word.

$f_s = 2B = 5.4\text{ kHz} \Rightarrow R_{min} = nf_s = 6(5.4\text{ kHz}) = \underline{\underline{32.4\text{ kbits/sec}}}$

(b) $L = 8 = 2^{\ell} \Rightarrow \ell = 3$ bit D/A $\quad D = \frac{R}{\ell} = \frac{32.4\text{ kbit/sec}}{3\text{ bits/symbol}} = \underline{\underline{10.8\text{ ksym/sec}}}$

(c) $D = \frac{2B}{1+r}$ where $r = 0$ for min BW $\Rightarrow B = \frac{D}{2} = \underline{\underline{5.4\text{ kHz}}}$

3-40 $L = 8 = 2^{\ell} \Rightarrow \ell = 3$

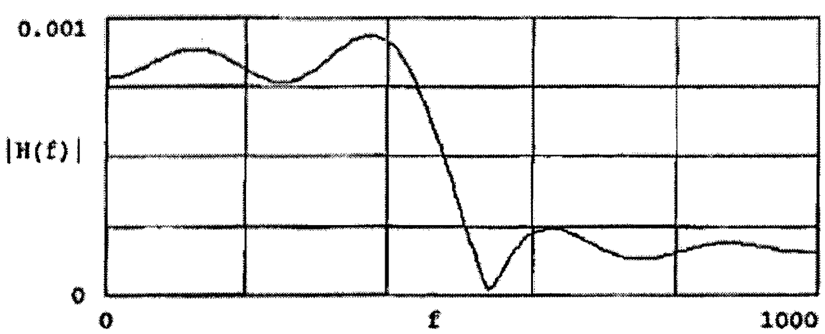
(a) $D = \frac{R}{\ell} = \frac{9600\text{ bits/sec}}{3\text{ bits/symbol}} = \underline{\underline{3.2\text{ k symbol/sec}}}$

(b) $D = \frac{2B}{1+r} = \frac{2(2.4k)}{1+r} = 3.2k \Rightarrow \underline{\underline{r = 0.5}}$

3-43 (a.)

f := 0,10 ..1000 $\qquad$ fs := 1000

$$H(f) := \int_0^{0.008} \frac{\sin(\pi \cdot fs \cdot (t - 0.004))}{\pi\, fs\, (t - 0.004)}\, e^{-2j \cdot \pi f t}\, dt$$



(b.) From the figure above, the bandwidth for the causal approximation is $\underline{B = 540\text{ Hz}}$

The bandwidth for the noncausal filter is $B = \frac{1}{2} f_s = \underline{\underline{500\text{ Hz}}}$

3-47 $h_e(t) = \int_{-\infty}^{\infty} H_e(f)\, e^{j2\pi f t}\, df$

For $t = nT_s \Rightarrow h_e(nT_s) = \int_{-\infty}^{\infty} H_e(f)\, e^{j2\pi n T_s f}\, df$

Break into multiple integrals, each with a $\frac{1}{T_s}$ wide interval.

$$\Rightarrow h_e(nT_s) = \sum_{k=-\infty}^{\infty} \int_{-\frac{1}{2T_s}+\frac{k}{T_s}}^{\frac{1}{2T_s}+\frac{k}{T_s}} H_e(f)\, e^{j2\pi n T_s f}\, df$$

Let $f_1 = f - \frac{k}{T_s}$

Then $$h_e(nT_s) = \sum_{k=-\infty}^{\infty} \int_{-\frac{1}{2T_s}}^{\frac{1}{2T_s}} H_e\left(f_1 + \frac{k}{T_s}\right) e^{j2\pi n T_s \left(f_1 + \frac{k}{T_s}\right)}\, df_1$$

or

$$h_e(nT_s) = \int_{-\frac{1}{2T_s}}^{\frac{1}{2T_s}} \sum_{k=-\infty}^{\infty} H_e\left(f_1 + \frac{k}{T_s}\right) e^{j2\pi n T_s f_1}\, df_1$$

Assume $\sum_{-\infty}^{\infty} H_e\left(f_1 + \frac{k}{T_s}\right) = T_s, \quad |f_1| < \frac{1}{2T_s}$

Then

$$h(nT_s) = \int_{-\frac{1}{2T_s}}^{\frac{1}{2T_s}} T_s\, e^{j2\pi n T_s f_1}\, df_1 = \left. \frac{\cancel{T_s}\, e^{j2\pi n T_s f_1}}{j2\pi n \cancel{T_s}} \right|_{-\frac{1}{2T_s}}^{\frac{1}{2T_s}}$$

or

$$h(nT_s) = \frac{e^{j\cancel{2}\pi n \cancel{T_s} \frac{1}{\cancel{2T_s}}} - e^{-j\cancel{2}\pi n \cancel{T_s} \frac{1}{\cancel{2T_s}}}}{j2\pi n} = \frac{\sin(n\pi)}{n\pi} = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

Q.E.D.

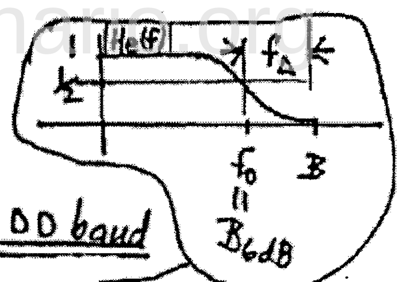
3-50 $M = 16 = 2^4 \Rightarrow n = 4$

(a) Binary PCM $\Rightarrow \ell = 1$, $R = nf_s = 4f_s = D$

$$D = \frac{2B}{1+r} = \frac{2(4\,kHz)}{1+0.5} = \underline{\underline{5.33\ k\,bits/sec}}$$

(b) From (a) $f_s = \frac{D}{4} = \frac{5.33k}{4} = 1.33\,kHz$

$$B_{\substack{analog\\max}} = \frac{f_s}{2} = \frac{1.33k}{2} = \underline{\underline{667\ Hz}}$$



3-52 (a) $L = 2^\ell = 4 \Rightarrow \ell = 2$

$$D = R/\ell = \frac{2400}{2} = \underline{\underline{1200\ baud}}$$

(b) $B = \frac{1}{2}(1+r)D$ where $r = \frac{f_\Delta}{f_0} = 0 \Rightarrow B = f_0 = B_{6dB}$

$$\Rightarrow B_{6dB} = \frac{1}{2}(1+0)D = \frac{1}{2}(1200) = \underline{\underline{600\ Hz}}$$

(c)

$$B_{absolute} = \frac{1}{2}(1+r)D = \frac{1}{2}(1+0.5)(1200) = \frac{3}{4}(1200)$$

$r = 0.5$

$$\Rightarrow \underline{\underline{B_{absolute} = 900\ Hz}}$$

3-59

(a.) From (3-84)

$$\delta = \frac{2\pi f_a A}{f_s} \quad ; \quad f_a = 3.4\,\text{kHz} \ \ \& \ \ A = \frac{1}{2}$$

We need to determine the f_s which the channel can support. Assuming that a $r=0$ roll-off factor is used, then

$$f_s = D = 2B = 2(1\,\text{MHz}) = 2\times10^6 \ \frac{\text{samples}}{\text{sec}}$$

$$\Rightarrow \delta = \frac{2\pi(3.4k)(\frac{1}{2})}{2\times10^6} = \underline{\underline{0.00534}}$$

(Note: The channel has to be equalized with a Nyquist filter.)

(b.)

$$\delta = \frac{2\pi(3.4k)(\frac{1}{2})}{25\times10^3} = \underline{\underline{0.427}}$$

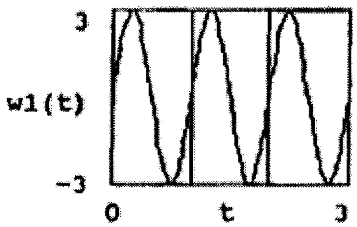
(Note: No channel equalization required.)

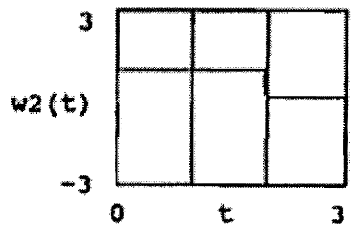
3-61

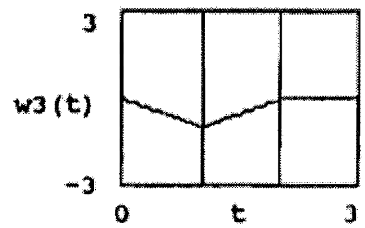
$T := 0.05 \qquad t := 0, 0.05 .. 3$

$w1(t) := 3\cdot\sin(2\cdot\pi\cdot t) \qquad w2(t) := \text{if}((|t| - 1) \le 1, 1, 0)$

$w3(t) := \text{if}(|t-1| \le 1, |t-1| - 1, 0)$



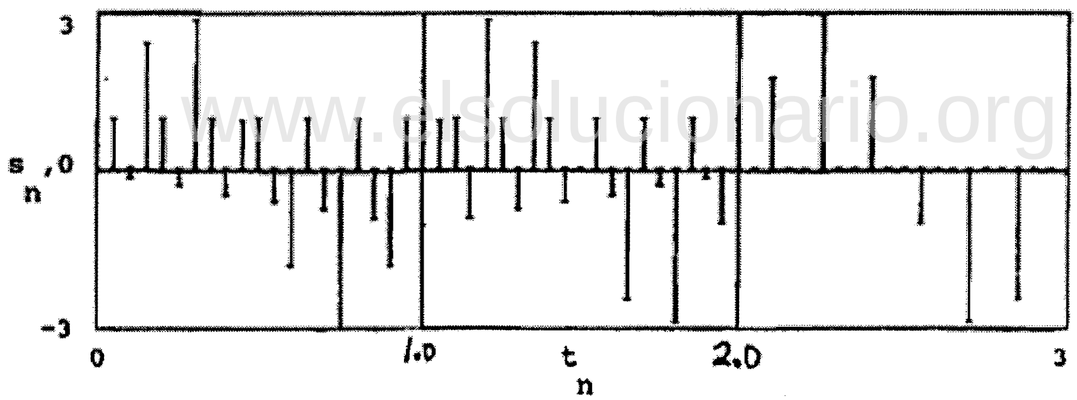




$n := 0 .. 62 \qquad t_n := n\cdot T \qquad k := 0 .. 20$

$k1(k) := 3\cdot k \qquad k2(k) := 3\cdot k + 1 \qquad k3(k) := 3\cdot k + 2$

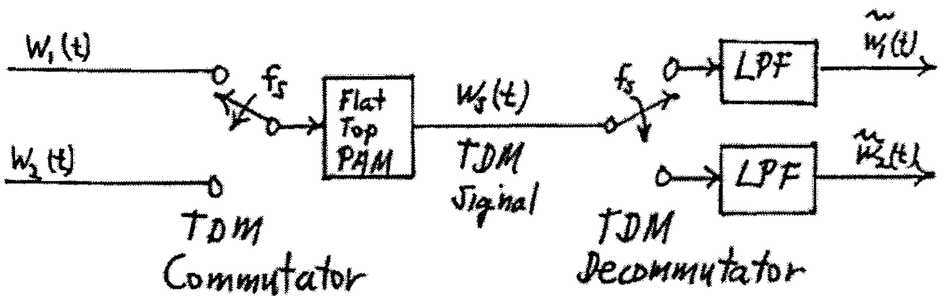
$s_{k1(k)} := w1\left[t_{k1(k)}\right] \qquad s_{k2(k)} := w2\left[t_{k2(k)}\right] \qquad s_{k3(k)} := w3\left[t_{k3(k)}\right]$



3-63 (a.) Each analog signal has a highest frequency of $B = 3\,kHz$

$\Rightarrow$ The minimum sampling frequency for each analog signal is $\underline{\underline{f_s = 2B = 6\,kHz}}$

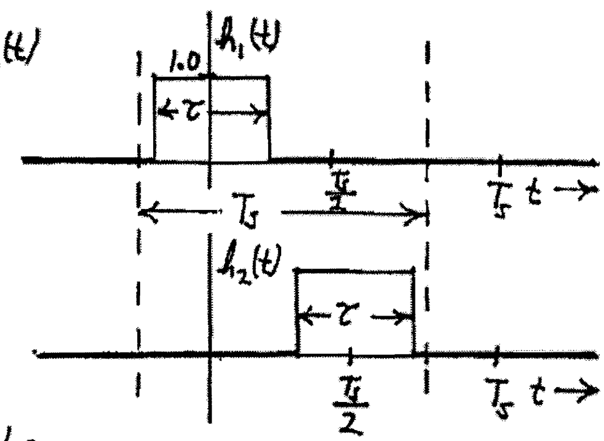
3-63, (a.) Cont'd



(b.) Referring to (3-8), the sampled TDM signal is

$$w_s(t) = \sum_{k=-\infty}^{\infty} w_1(kT_s)h_1(t-kT_s) + \sum_{k=-\infty}^{\infty} w_2(kT_s)h_2(t-kT_s)$$

where $h_1(t)$ and $h_2(t)$ are shown in the figure and $\tau \le \frac{T_s}{2}$ and $f_s \ge 2B$.



Following the same procedure as described in (3-8) thru (3-13), the spectrum of the TDM instantaneously sampled (flat-topped) PAM signal is

$$W_s(f) = \frac{1}{T_s} H_1(f)\sum_{k=-\infty}^{\infty} W_1(f-kf_s) + \frac{1}{T_s} H_2(f)\sum_{k=-\infty}^{\infty} W_2(f-kf_s)$$

where $H_1(f) = \tau\left(\frac{\sin(\pi f\tau)}{\pi f\tau}\right)$ and $H_2(f) = \tau\left(\frac{\sin(\pi f\tau)}{\pi f\tau}\right)e^{-j2\pi f\frac{T_s}{2}}$

$$\Rightarrow W_s(f) = f_s\,\tau\,\frac{\sin(\pi f\tau)}{\pi f\tau}\sum_{k=-\infty}^{\infty}\Pi\left(\frac{f-kf_s}{2B}\right) + 2B\tau\left(\frac{\sin(\pi f\tau)}{(\pi f\tau)}\right)$$

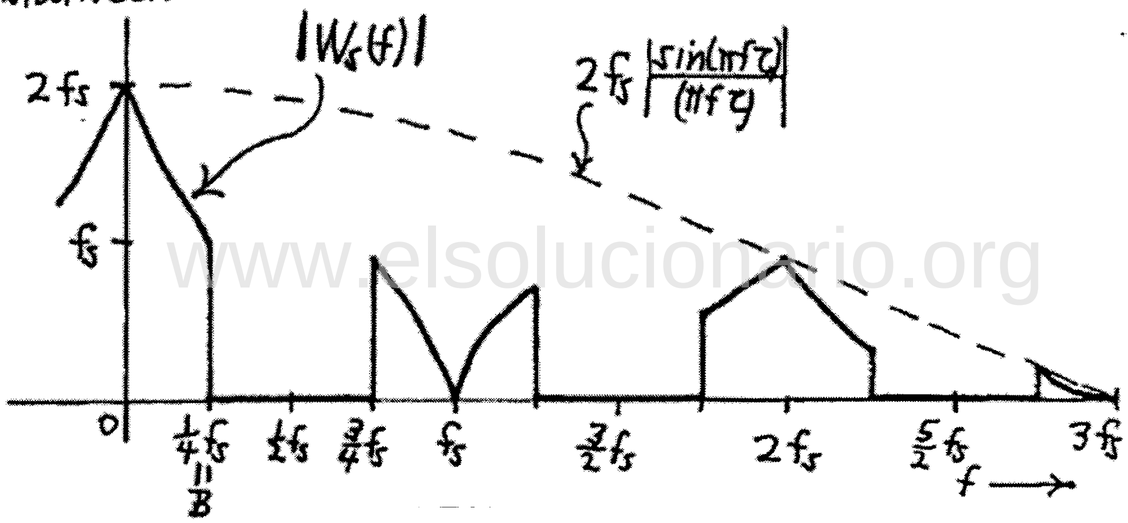
$$+ 2B\tau\left(\frac{\sin(\pi f\tau)}{(\pi f\tau)}\right)e^{-j\pi T_s f}\sum_{k=-\infty}^{\infty}\Lambda\left(\frac{f-kf_s}{B}\right)$$

3-63 (b.) Cont'd

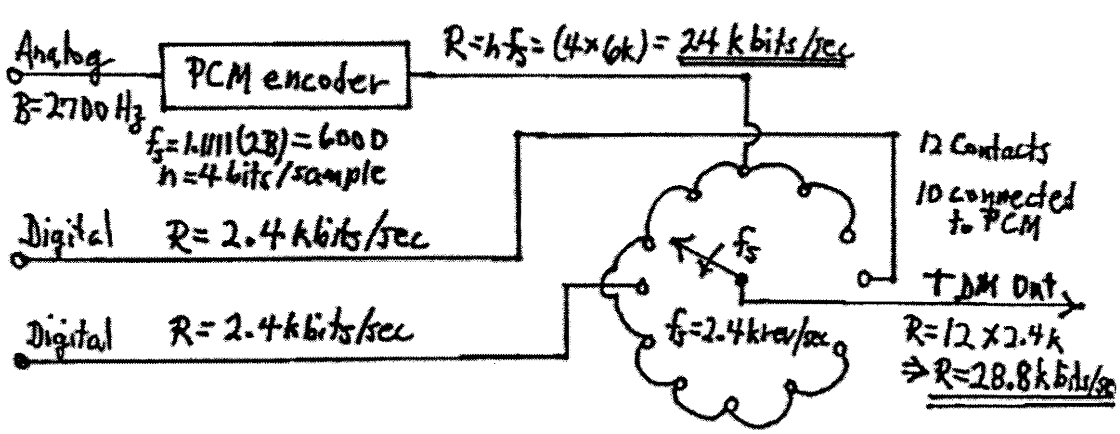
Thus,

$$|W_s(f)| = f_s \left|\frac{\sin(\pi f \tau)}{(\pi f \tau)}\right| \sum_{k=-\infty}^{\infty} \left| \Pi\left(\frac{f-kf_s}{2B}\right) + e^{j\pi T_s f} \Lambda\left(\frac{f-kf_s}{B}\right)\right|$$

For the sketch, let the parameters be the same as those shown in Fig. 3-6. Let $\tau/T_s = 1/3$, $f_s = 4B$. Using a programable calculator, the following sketch is obtained.



3-66



3-70

(a) For PCM a $N=8$ dimensional system is used since any of the 256 messages can be represented by

$$s_i(t) = \sum_{j=1}^{8} s_{ij}\,\varphi_j(t)$$

where $s_{ij} = \pm 1$ for binary PCM.

$$T_0 = \frac{1}{10 \text{ mess/sec.}} = \underline{\underline{0.1 \text{ sec/message}}}$$

$$B = \frac{1}{2}\left(\frac{N}{T_0}\right) = \frac{1}{2}\left(\frac{8}{0.1}\right) = \underline{\underline{40 \text{ Hz}}}$$

(b) For PPM a $N=256$ dimensional system is used:

$$s_i(t) = \sum_{j=1}^{256} s_{ij}\,\varphi_j(t) \quad \text{where } s_{ij} = \delta_{ij}$$

$$B = \frac{1}{2}\left(\frac{N}{T_0}\right) = \frac{1}{2}\left(\frac{256}{0.1}\right) = \underline{\underline{1{,}280 \text{ Hz}}}$$

Chapter 4

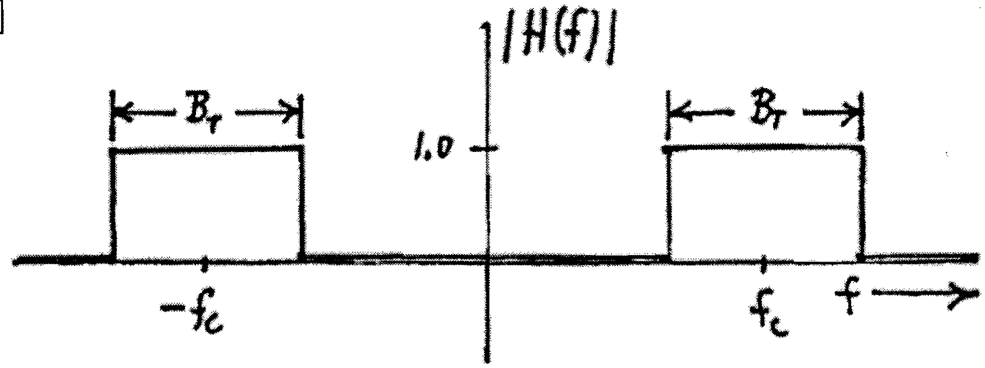
4-3 Using (2-26) with the help of Sec. A-5,

$$G(f) = A_c M(f) = 50[-j\delta(f-1000) + j\delta(f+1000)]$$

Substituting this into (4-15) and using $\delta(-f) = \delta(f)$, the voltage spectrum of this DSB-SC signal is

$$S(f) = -j25\,\delta(f-f_c-1000) + j25\,\delta(f-f_c+1000) - j25\,\delta(f+f_c-1000) + j25\,\delta(f+f_c+1000)$$

4-9



4-9 Cont'd

(b) $v_2(t) = \mathrm{Re}\{g_2(t)e^{j\omega_c t}\}$

where $g_2(t) = \frac{1}{2} g_1(t) * k(t) \leftrightarrow G_2(f) = \frac{1}{2} G_1(f) K(f)$

and $k(t) = \mathcal{F}^{-1}[K(f)]$

Also, $H(f) = \frac{1}{2}[K(f-f_c) + K^*(-f-f_c)]$

$$\Rightarrow \quad K(f) = \begin{cases} 2, & |f| < B_T/2 \\ 0, & f \text{ elsewhere} \end{cases}$$

Evaluate $k(t)$:

$$k(t) = \int_{-\frac{B_T}{2}}^{\frac{B_T}{2}} 2e^{j2\pi f t}\,df = 2B_T \frac{\sin(\pi B_T t)}{(\pi B_T t)}$$

Know that $g_1(t) = A\,\Pi\left(\frac{t}{T}\right) = A\begin{cases} 1, & |t| < T/2 \\ 0, & t \text{ elsewhere} \end{cases}$

$$\Rightarrow g_2(t) = \frac{1}{2} g_1(t) * k(t) = \frac{1}{\cancel{2}} \int_{-\frac{T}{2}}^{\frac{T}{2}} A \cancel{2} B_T \frac{\sin[\pi B_T(t-\lambda)]}{[\pi B_T(t-\lambda)]}\,d\lambda$$

Let $\lambda_1 = \pi B_T(t-\lambda) \Rightarrow d\lambda_1 = -\pi B_T\,d\lambda$

$$\Rightarrow g_2(t) = A \cancel{B_T} \int_{\pi B_T(t+\frac{T}{2})}^{\pi B_T(t-\frac{T}{2})} \frac{\sin\lambda_1}{\lambda_1}\left(-\frac{1}{\pi \cancel{B_T}}\,d\lambda_1\right)$$

$$= \frac{A}{\pi}\left[-\int_{\pi B_T(t+\frac{T}{2})}^{0} \frac{\sin\lambda_1}{\lambda_1}\,d\lambda_1 - \int_{0}^{\pi B_T(t-\frac{T}{2})} \frac{\sin\lambda_1}{\lambda_1}\,d\lambda_1\right]$$

$$\Rightarrow g_2(t) = \frac{A}{\pi}\left\{+\,Si\left[\pi B_T\left(t+\tfrac{T}{2}\right)\right] - Si\left[\pi B_T\left(t-\tfrac{T}{2}\right)\right]\right\}$$

and $v_2(t) = \mathrm{Re}\{g_2(t)e^{j\omega_c t}\}$

4-9 Cont'd (b.)

Thus,

$$v_2(t) = \frac{A}{\pi}\left\{ Si\left[\pi B_T\left(t+\frac{T}{2}\right)\right] - Si\left[\pi B_T\left(t-\frac{T}{2}\right)\right]\right\}\cos(\omega_c t)$$

(c.) When $B_T = \frac{4}{T}$

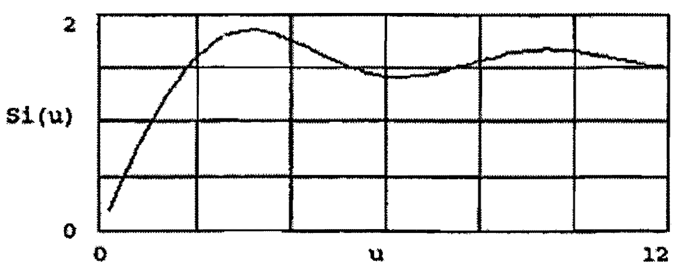
$$v_2(t) = \frac{A}{\pi}\left\{ Si\left[2\pi\left(\frac{2t}{T}+1\right)\right] - Si\left[2\pi\left(\frac{2t}{T}-1\right)\right]\right\}\cos(\omega_c t)$$

This is plotted with the help of the Si(u) function. (See p. 232 of Abramowitz and Stegun for a description of the Si(u) function.)

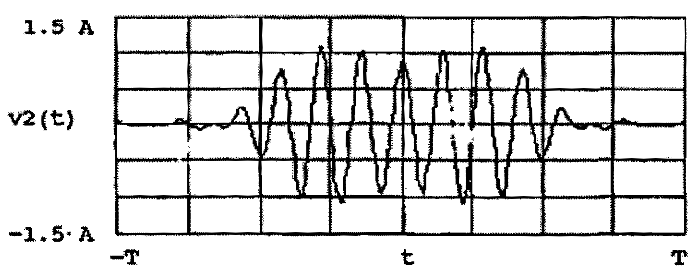
Using MathCAD we get:

$A := 1 \qquad T := 1 \qquad \omega := 2\cdot\pi\cdot 7 \qquad u := 0, 0.2 \,..\, 12$

$$Si(u) := \int_{0.001}^{u} \frac{\sin(x)}{x}\,dx \qquad t := -T, -T + 0.01 \,..\, T$$



$$v2(t) := \left[\frac{A}{\pi}\right]\cdot\left[Si\left[2\,\pi\left[2\cdot\frac{t}{T}+1\right]\right] - Si\left[2\cdot\pi\cdot\left[2\cdot\frac{t}{T}-1\right]\right]\right]\cos(\omega\, t)$$



4-11

$R := 400 \qquad L := 1.583\ 10^{-3} \qquad C := 1 \cdot 10^{-6}$

(a)

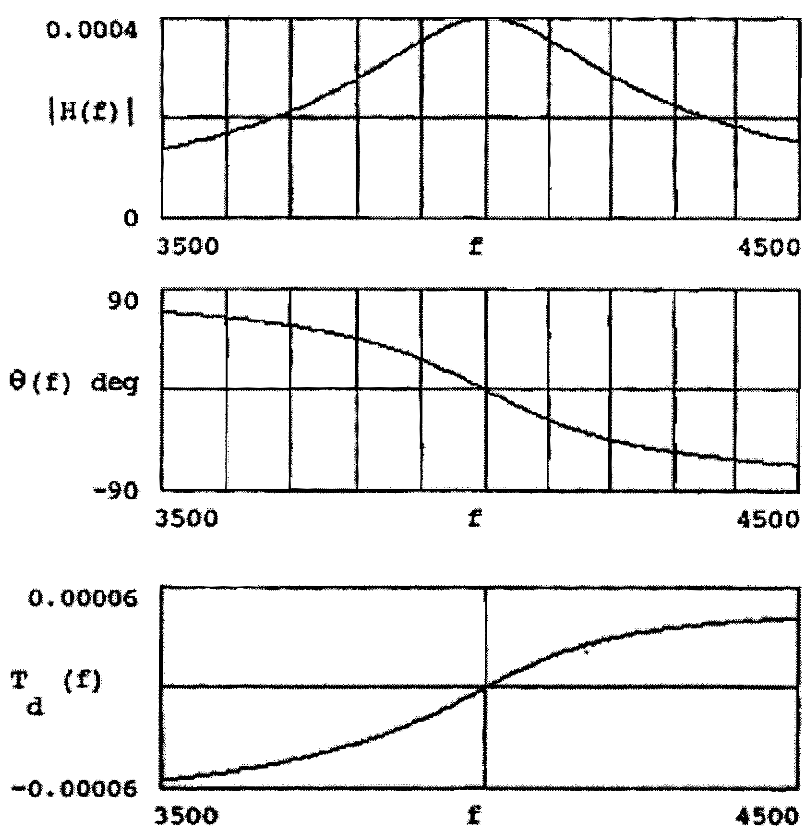
$$f0 := \frac{1}{2\cdot\pi\sqrt{L\cdot C}} \qquad f0 = 4\cdot 10^{3} \qquad Q := R\sqrt{\frac{C}{L}} \qquad Q = 10.054$$

$$B := \frac{f0}{Q} \qquad B = 397.887$$

(b) $f := 3500, 3504 \,..\, 4500 \qquad rad \equiv 1 \qquad deg \equiv \frac{rad}{\pi}\cdot 180$

$$H(f) := \frac{j\ 2\cdot\pi\ f}{(j\ 2\ \pi\ f)^2 + \left[2\cdot\pi\cdot\frac{f0}{Q}\right]\cdot(j\ 2\cdot\pi\cdot f) + (2\cdot\pi\cdot f0)^2}$$

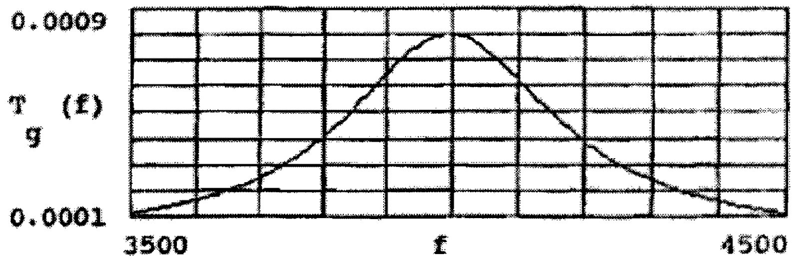
$$\theta(f) := \arg(H(f)) \qquad T_d(f) := \frac{1}{-2\cdot\pi\ f}\cdot\theta(f)$$



4-11 Cont'd

(c)

$$T_g(f) := \frac{1}{-2\cdot\pi}\frac{d}{df}(\theta(f)\ \mathrm{rad})$$



(d.) The group delay variation over the 200 Hz signal bandwidth (about 4 kHz) is about 0.2 msec. The period of a 200 Hz signal is 5 msec. Thus, the group delay variation is negligable and, consequently the distortion is negligable.

4-14

(a.) $s(t) = Re\{500\, e^{j\omega_c t}\} + Re\{-j100\, e^{j(\omega_c+\omega_a)t} + j100\, e^{j(\omega_c-\omega_a)t}\}$

$$s(t) = Re\left\{500\left[1 - j(2j)\frac{100}{500}\left(\frac{e^{j\omega_a t} - e^{-j\omega_a t}}{2j}\right)\right]e^{j\omega_c t}\right\}$$

$$= Re\{500\underbrace{[1 + \tfrac{2}{5}\sin(\omega_a t)]}_{1+m(t)}\, e^{j\omega_c t}\} \qquad AM$$

$\Rightarrow$ $\underline{\underline{g(t) = 500 + 200\sin(\omega_a t)}}$, $\underline{\underline{m(t) = 0.4\sin(\omega_a t)}}$

(b.) $x(t) = Re\{g(t)\} = \underline{\underline{500 + 200\sin(\omega_a t)}}$, $y(t) = Im\{g(t)\} = \underline{\underline{0}}$

(c.) $R(t) = |g(t)| = \underline{\underline{500 + 200\sin(\omega_a t)}}$, $\theta(t) = \angle g(t) = \underline{\underline{0^\circ}}$

(d.) $$P = \frac{\frac{1}{2}\langle |g(t)|^2\rangle}{50} = \frac{1}{100}\left[(500)^2 + 2\times10^5\overbrace{\langle \sin\omega_a t\rangle}^{0} + (200)^2\overbrace{\langle\sin^2(\omega_a t)\rangle}^{\frac{1}{2}}\right]$$

$\Rightarrow$ $\underline{\underline{P = 2,700 \text{ watts}}}$

4-15

(a) $S(f) = 100\mathcal{F}[\sin(\omega_c+\omega_a)t] + 500\mathcal{F}[\cos\omega_c t] - 100\mathcal{F}[\sin(\omega_c-\omega_a)t]$

Aside: We know that

$$\mathcal{F}[\sin(\omega_x t)] = \frac{1}{2j}[\delta(f-f_x) - \delta(f+f_x)]$$

and $\mathcal{F}[\cos(\omega_x t)] = \frac{1}{2}[\delta(f-f_x)+\delta(f+f_x)]$

Thus

$$S(f) = -j50[\delta(f-(f_c+f_a)) - \delta(f+(f_c+f_a))] + 250[\delta(f-f_c)+\delta(f+f_c)] + j50[\delta(f-(f_c-f_a)) - \delta(f+(f_c-f_a))]$$

$$\Rightarrow S(f) = 250[\delta(f-f_c)+\delta(f+f_c)] + j50[\delta(f+f_c+f_a) - \delta(f-f_c-f_a) + \delta(f-f_c+f_a) - \delta(f+f_c-f_a)] \quad (1)$$

(b) Let $g(t) = 500 + 200\sin(\omega_a t)$

Thus, it can be shown that $s(t) = \mathrm{Re}[g(t)e^{j\omega_c t}] = s(t)$ of 4-9

$$G(f) = \mathcal{F}[g(t)] = 500\delta(f) - j100[\delta(f-f_a) - \delta(f+f_a)]$$

Using $S(f) = \frac{1}{2}[G(f-f_c) + G^*(-f-f_c)]$ & $\delta(-f) = \delta(f)$

$$\Rightarrow S(f) = 250[\delta(f-f_c) + \delta(\underbrace{-f-f_c}_{\delta(f+f_c)})] - j50[\delta(f-f_c-f_a) - \delta(\underbrace{-f-f_c-f_a}_{\delta(f+f_c+f_a)})] + j50[\delta(f-f_c+f_a) - \delta(\underbrace{-f-f_c+f_a}_{\delta(f+f_c-f_a)})]$$

$$\Rightarrow S(f) = 250[\delta(f-f_c)+\delta(f+f_c)] + j50[\delta(f-f_c+f_a) - \delta(f-f_c-f_a) + \delta(f+f_c+f_a) - \delta(f+f_c-f_a)] \quad (2)$$

Thus (1) = (2)

4-19

$B := 100 \qquad f := 10, 20 \,..\, 1000 \qquad rad \equiv 1 \qquad deg \equiv \frac{rad}{\pi} 180$

$$H1(f) := \frac{1}{\left[1 - \left[\frac{f}{B}\right]^2\right] + j\left[\sqrt{2}\,\frac{f}{B}\right]}$$

$\theta 1(f) := \arg(H1(f))$

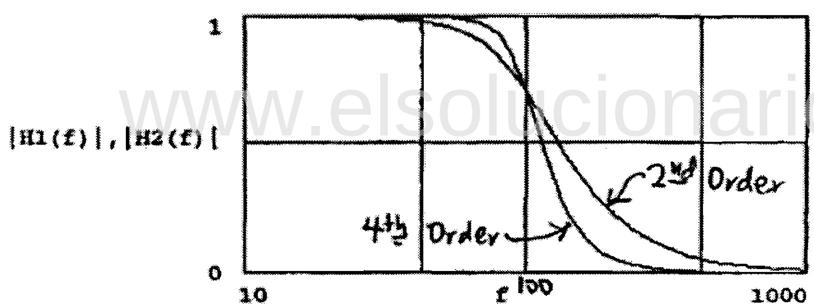
$$Td1(f) := \frac{1}{-2\cdot\pi\cdot f}\cdot\theta 1(f)$$

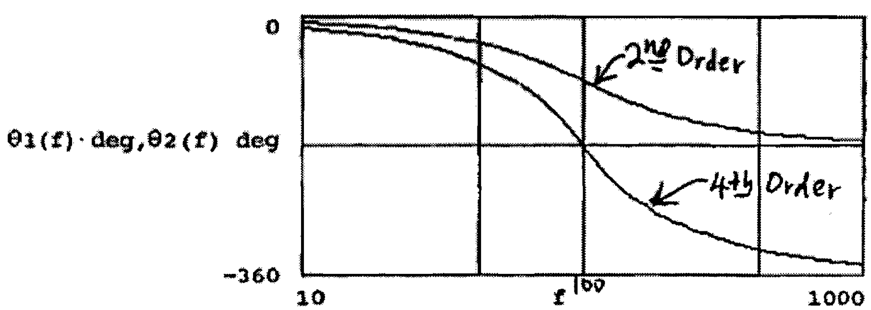
$$H2(f) := \frac{1}{\left[1 - \left[\frac{f}{B}\right]^2 + 0.765j\cdot\frac{f}{B}\right]}\cdot\frac{1}{\left[1 - \left[\frac{f}{B}\right]^2 + 1.848j\cdot\frac{f}{B}\right]}$$

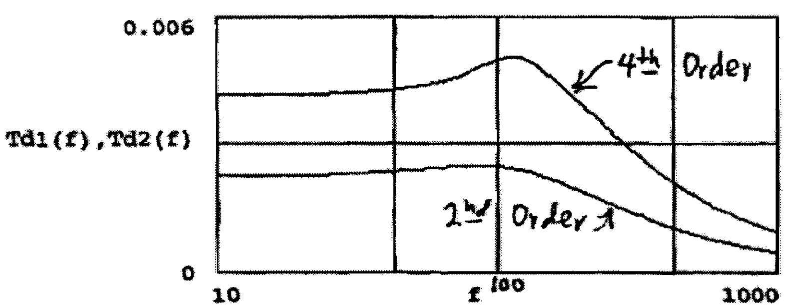
$\theta(f) := \arg(H2(f))$

$\theta 2(f) := \mathrm{if}(f < 100, \theta(f), \theta(f) - 2\cdot\pi)$

$$Td2(f) := \frac{1}{-2\cdot\pi\, f}\cdot\theta 2(f)$$







4-22

$t := 0, 0.1 .. 4 \qquad m := 1 .. 6$

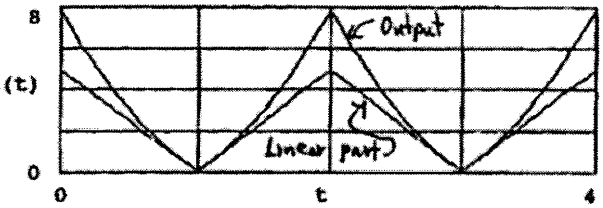
(a.)

$$vin(t) := \frac{1}{2} + \frac{4}{\pi^2}\left[\sum_m \left[\frac{\cos((2\cdot m - 1)\cdot \pi \cdot t)}{(2\, m - 1)^2}\right]\right]$$

Note: vin(t) is the Fourier series for a triangle waveform.

$vout(t) := 5\cdot vin(t) + 1.5\cdot (vin(t))^2 + 1.5\cdot (vin(t))^3$

$vol(t) := 5\cdot vin(t)$ <-----Linear part of the output.



(b.)

$M := 5 \qquad N := 2^M \qquad N = 32 \qquad k := 0 .. N - 1 \qquad T := 2$

$dt := \frac{T}{N} \qquad dt = 0.063 \qquad t_k := k\cdot dt$

$vo_k := vout(k\cdot dt) \qquad vol_k := vol(k\, dt) \qquad n := 0 .. N - 1$

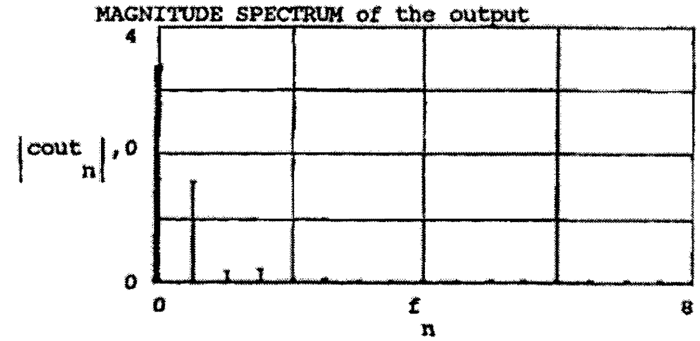
Since the signal is periodic, the spectrum will consist of delta functions (which can't be plotted directly since the delta function has an infinite value). However the weights of the delta functions are finite and can be plotted. The weights may be obtained from the complex Fourier series coefficients. Furthermore, the complex Fourier series coefficients may be calculated using the FFT by substituting (2-178) into (2-186). Thus,

$cout := \left[\frac{1}{\sqrt{N}}\right]\cdot icfft(vo) \qquad col := \left[\frac{1}{\sqrt{N}}\right] icfft(vol)$

$f_n := \frac{n}{T} \qquad f_1 = 0.5 \qquad fs := \frac{1}{dt} \qquad fs = 16$

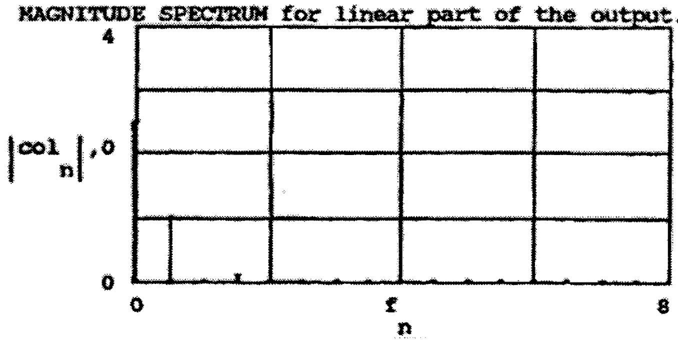
MAGNITUDE SPECTRUM of the output

$cout_0 = 3.375$

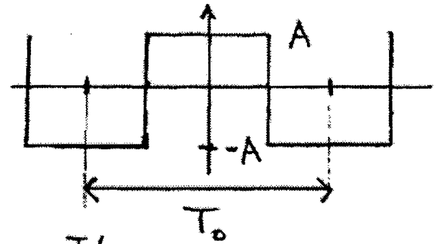


MAGNITUDE SPECTRUM for linear part of the output.

$col_0 = 2.5$



4-25 The output is a square wave as shown

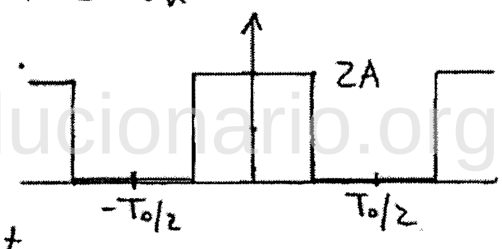


$$v(t) = \sum_{n=0}^{\infty} V_n \cos(n\omega_0 t)$$

where, using (2-96),

$b_n = 0$ and $a_n = V_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} v(t)\cos(n\omega_0 t)\,dt$

Since we are only interested in V_n for $n \geq 1$, we can shift the DC level of the waveform to make the integral easier to evaluate and yet the V_n will remain the same for $n \neq 0$.



$$V_n = \frac{2}{T_0}\int_{-T_0/4}^{T_0/4} 2A\cos(n\omega_0 t)\,dt$$

$$= \frac{2}{T_0}(2A)\left.\frac{\sin(n\omega_0 t)}{n\omega_0}\right|_{-T_0/4}^{T_0/4} = \frac{4A}{n2\pi}\,2\sin\left(\frac{n\pi}{2}\right)$$

$$= \frac{4A}{n\pi}\begin{cases} 0, & n = \text{even} \\ +1, & n = 1, 5, 9, \ldots \\ -1, & n = 3, 7, 11, \ldots \end{cases}$$

$$\Rightarrow V_n^2 = \begin{cases} 0, & n \text{ even} \\ \left(\frac{4A}{n\pi}\right)^2, & n \text{ odd} \end{cases}$$

Using (4-47)

$$\text{THD}\,\% = \sqrt{\frac{\sum_{n=2}^{\infty} V_n^2}{V_1^2}} \times 100 = \sqrt{\frac{\sum_{n=3}^{\infty}\left(\frac{4A}{n\pi}\right)^2}{\left(4A/\pi\right)^2}}$$

4-25. Cont'd

$$THD\% = \sqrt{\sum_{\substack{n=3 \\ n=\text{odd}}}^{\infty} \frac{1}{n^2}} \times 100 = \underline{\underline{48.3\%}}$$

Using programable calculator

<u>Check</u>:

$$THD\% = \sqrt{\frac{\text{Total Power} - \left(\frac{V_1}{\sqrt{2}}\right)^2}{\left(\frac{V_1}{\sqrt{2}}\right)^2}} \times 100$$

$$= \sqrt{\frac{A^2 - \left(\frac{4A}{\pi\sqrt{2}}\right)^2}{\left(\frac{4A}{\pi\sqrt{2}}\right)^2}} \times 100$$

$$= \sqrt{\frac{\pi^2 - 8}{8}}\,(100) = \sqrt{0.2337}\,(100) = \underline{\underline{48.3\%}}$$

4-28 $s(t) = A_c[m(t)\cos\omega_c t \mp \hat{m}(t)\sin\omega_c t]$

$$= \mathrm{Re}\{A_c[m(t) \pm j\hat{m}(t)]\,e^{j\omega_c t}\}$$

$$\Rightarrow g(t) = A_c[m(t) \pm j\hat{m}(t)] = R(t)\,\underline{/\theta(t)}$$

Output of Envelope Detector is

$$v_{out}(t) = KR(t) = k|g(t)|$$

$$\Rightarrow v_{out}(t) = kA_c\sqrt{m^2(t) + \hat{m}^2(t)} \neq k\,m(t)$$

The output is <u>distorted</u>.

4-33

$$\frac{d\theta_e(t)}{dt} = \frac{d\theta_i(t)}{dt} - k_d k_v \theta_e(t) * f(t)$$

$$\Rightarrow \quad s\Theta_e(s) = s\Theta_i(s) - k_d k_v \Theta_e(s) F(s)$$

or

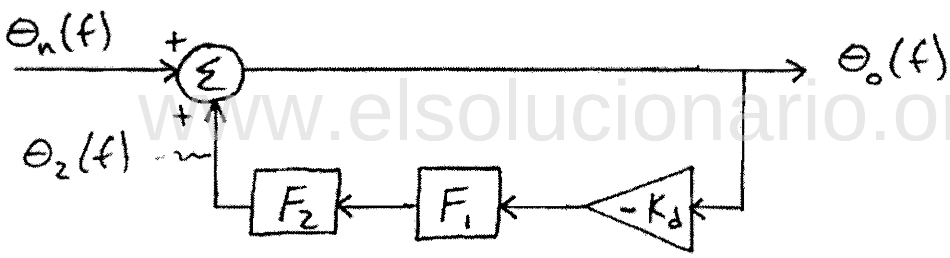
$$\Theta_e(s) = \frac{s\Theta_i(s)}{s + k_d k_v F(s)}$$

Final value theorem:

$$\lim_{t\to\infty} \theta_e(t) = \lim_{s\to 0}[s\Theta_e(s)] = \lim_{s\to 0} \frac{s^2\Theta_i(s)}{s + k_d k_v F(s)}$$

$$\Rightarrow \text{If } F(0) \neq 0 \Rightarrow \underline{\underline{\lim_{t\to\infty} \theta_e(t) = 0}}$$

4-35 (a.)



$$\Theta_o(f) = \Theta_n(f) + \Theta_2(f)$$

where $\Theta_2(f) = \Theta_o(f)[-K_d F_1(f) F_2(f)]$

$$\Rightarrow \Theta_o(f) = \Theta_n(f) - K_d F_1(f) F_2(f) \Theta_o(f)$$

$$\Theta_o(f)[1 + K_d F_1 F_2] = \Theta_n(f)$$

$$\frac{\Theta_o(f)}{\Theta_n(f)} = \frac{1}{1 + K_d F_1 F_2} = \frac{1}{1 + \frac{K_d K_v F_1(f)}{j2\pi f}}$$

$F_2(f) = K_v / j2\pi f$

4-35 (a.) Cont'd

$$\frac{\Theta_o(f)}{\Theta_n(f)} = \frac{j2\pi f}{j2\pi f + K_d K_v F_1(f)}$$

(b.) $F_1(f) = \dfrac{1}{1 + jf/f_1}$; $f_1 = \dfrac{1}{2\pi RC}$

$$\frac{\Theta_o(f)}{\Theta_n(f)} = \frac{1}{1 + \dfrac{K_d K_v}{j2\pi f\,(1 + jf/f_1)}}$$

$$= \frac{j2\pi f\,(1 + jf/f_1)}{j2\pi f\,(1 + jf/f_1) + K_d K_v}$$

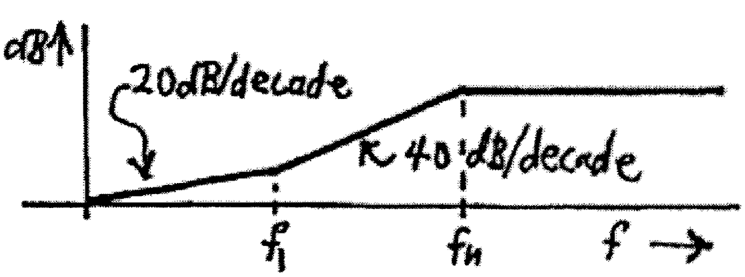
$$= \frac{1}{K_d K_v}\;\frac{j2\pi f\,(1 + jf/f_1)}{1 + j\dfrac{2\pi f}{K_d K_v} - \dfrac{2\pi f^2}{K_d K_v f_1}}$$

$$= \frac{1}{K_d K_v}\;\frac{j2\pi f\,(1 + jf/f_1)}{1 + j2\zeta\dfrac{f}{f_n} - \left(\dfrac{f}{f_n}\right)^2}$$

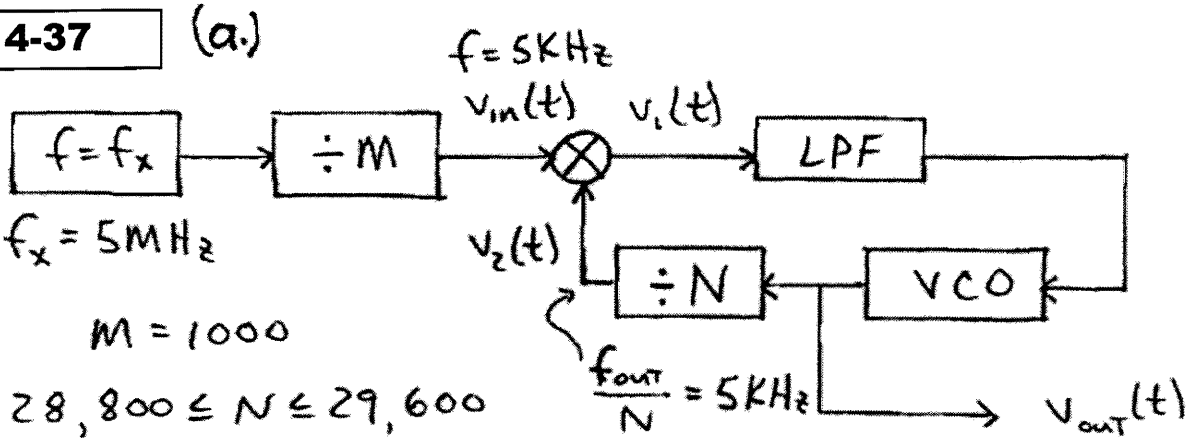
where $f_n = \sqrt{\dfrac{K_d K_v f_1}{2\pi}}$; $\zeta = \dfrac{f_1}{2f_n}$

$$0 \le \zeta \le 1 \implies f_n \ge \frac{f_1}{2}$$

$\left|\dfrac{\Theta_o(f)}{\Theta_n(f)}\right|_{dB}$



4-37 (a.)



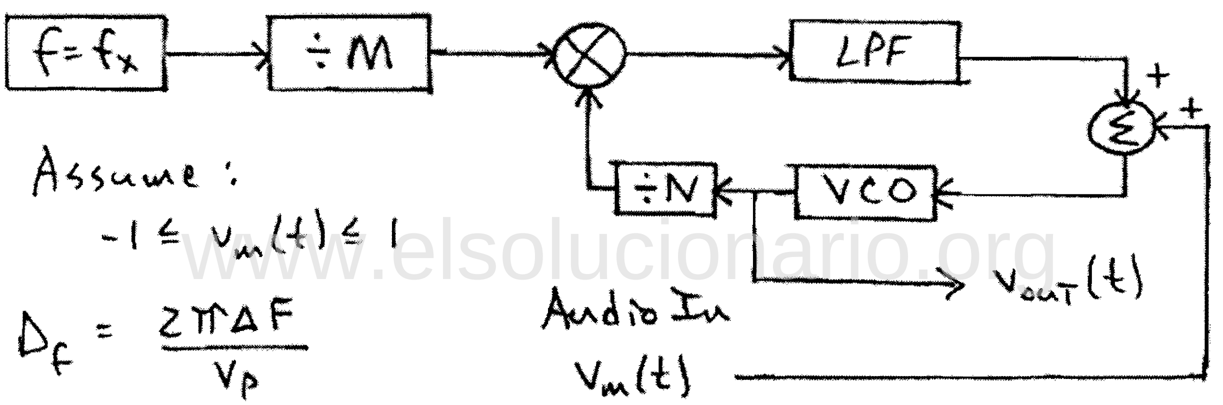
$f_x = 5MHz$

$M = 1000$

$28,800 \le N \le 29,600$

(b.)

$144\ MHz \le f_{out} \le 148 MHz$



Assume:

$-1 \le v_m(t) \le 1$

$$D_f = \frac{2\pi \Delta F}{V_p}$$

$$= 2\pi(5000)\ \frac{\text{rad.}}{\text{volt}\cdot\text{sec}}$$

4-42 (a.) $f_{LO} = 96.9 + 10.7 = \underline{\underline{107.6 MHz}}$

(b.) RF: Flat bandpass over 96.81MHz to 96.99MHz and reject image frequency of 118.3MHz

IF: Flat bandpass over 10.61 MHz to 10.79MHz and reject adjacent channel signals on each side of this bandpass

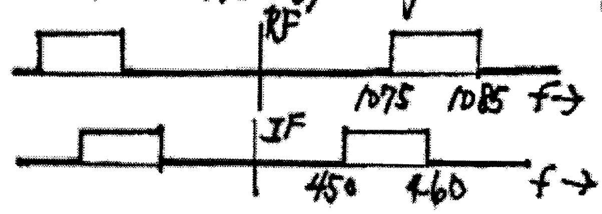
(c.) $f_{image} = f_c + 2f_{if} = 96.9 + 2(10.7) = \underline{\underline{118.3 MHz}}$

4-47

(a.) RF filter $\Rightarrow f_c \pm B/2$; IF filter $\Rightarrow f_{if} \pm B/2$ where $B = 10$ kHz be since the AM channel spacing is 10 kHz.

RF: 1080 ± 5 kHz

IF: 455 ± 5 kHz



(b.) $f_{image} = f_c + 2f_{if} = 1990$ kHz

Chapter 5

5-1

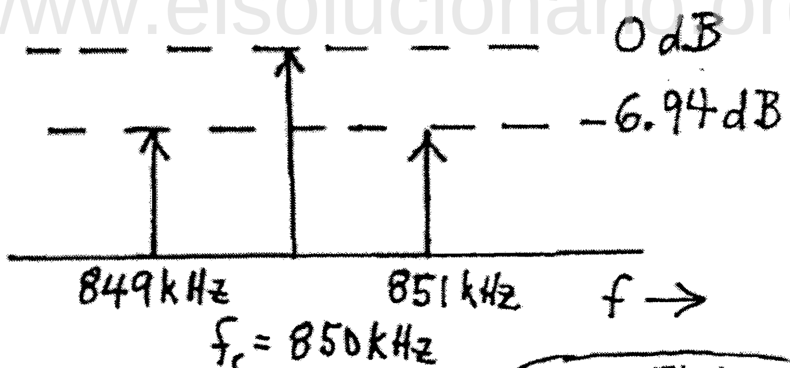
(a.) $dBk = 10\log_{10}\left(\frac{5000}{1000}\right) = \underline{\underline{6.99\ dBk}}$

(b.) $P = \frac{A_c^2}{2R} \Rightarrow A_c = \sqrt{2PR} = \sqrt{2(5000)(50)} = \underline{\underline{707\text{ volts}}}$

$$\underline{\underline{s(t) = 707\,[1 + 0.9\cos(2000\pi t)]\cos(2\pi\, 850{,}000\, t)}}$$

(c.)

$$s(t) = 707\cos\omega_c t + \frac{0.9(707)}{2}\cos[(\omega_c - \omega_m)t] + \frac{0.9(707)}{2}\cos[(\omega_c + \omega_m)t]$$



$20\log_{10}\left(\frac{318}{707}\right) = -6.94$

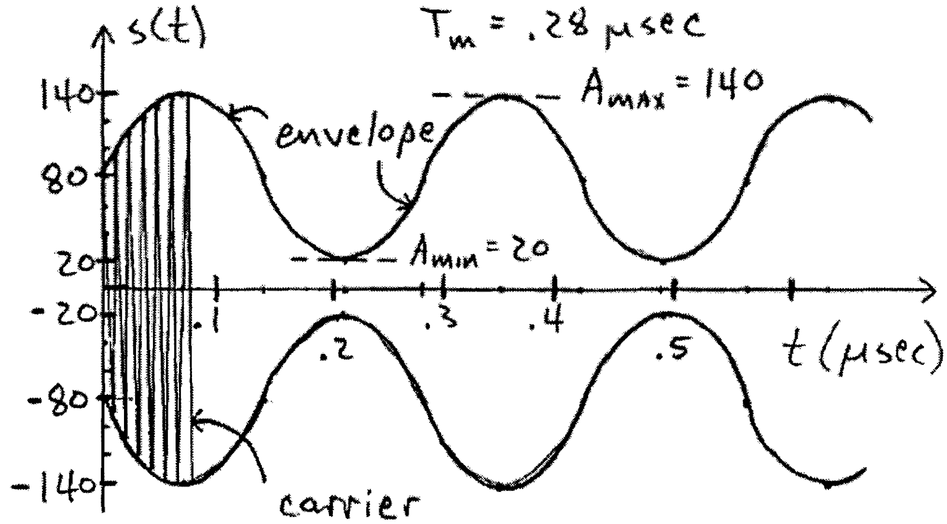
(d.) $P_{AVE} = \frac{(707)^2}{2(50)} + \frac{(318)^2}{2(50)} + \frac{(318)^2}{2(50)} = \underline{\underline{7.021\ kW}}$

(e.) $PEP = \frac{[(707)(1.9)]^2}{2(50)} = \underline{\underline{18.045\ kW}}$

5-4 (a.) $m(t) = -0.2 + 0.6 \sin \omega_1 t$

$$f_m = f_1 = 3.57 \text{ MHz} \;;\; A_c = \underline{\underline{100}}$$

$$s(t) = 100\left(0.8 + 0.6 \sin \omega_1 t\right) \cos \omega_c t$$

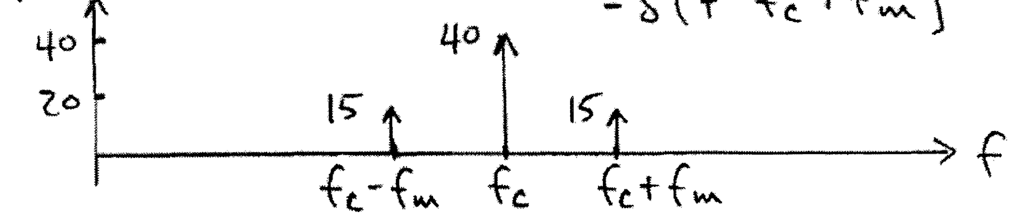


(b.) $$\% \text{ pos. mod.} = \frac{A_{max} - A_c}{A_c}(100) = \frac{140-100}{100}(100) = \underline{\underline{40\%}}$$

$$\% \text{ neg. mod.} = \frac{A_c - A_{MIN}}{A_c}(100) = \frac{100-20}{100}(100) = \underline{\underline{80\%}}$$

(c.) $\underline{f > 0}$

$$S(f) = 40\,\delta(f - f_c) - j15\left[\delta(f - f_c - f_m) - \delta(f - f_c + f_m)\right]$$



5-6 From (5-5a)

$$\% \text{ Pos. Mod.} = \frac{A_{max} - A_c}{A_c}(100) \overset{\text{given}}{=} 120$$

where:

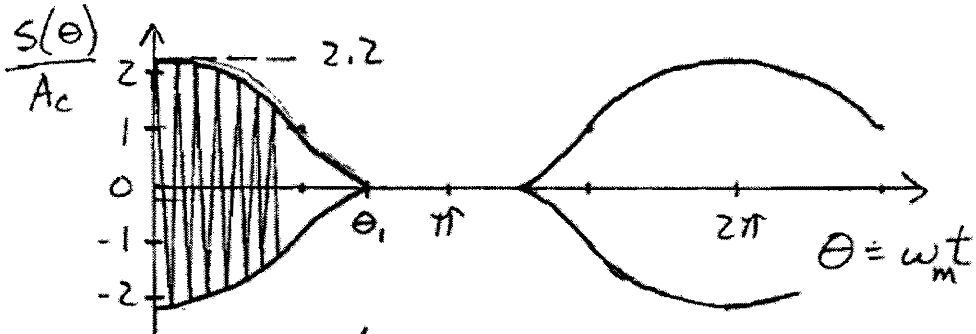
$$s(t) = \begin{cases} A_c\{1 + A_m \cos\omega_m t\}\cos\omega_c t \ ; & m(t) \geq -1 \\ 0 \ ; & m(t) < -1 \end{cases}$$

$$m(t) = A_m \cos\omega_m t \quad ; \quad A_{max} = A_c[1 + A_m]$$

$$\frac{A_{max} - A_c}{A_c} = \underline{\underline{A_m = 1.2}}$$

$$g(t) = \begin{cases} A_c\{1 + 1.2\cos\omega_m t\} \ , & 1.2\cos\omega_m t \geq -1 \\ 0 \ , & 1.2\cos\omega_m t < -1 \end{cases}$$

$$= \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_m t} \Rightarrow G(f) = \sum_{-\infty}^{\infty} c_n \delta(f - nf_m)$$



$$A_m \cos\theta_1 = -1$$

Aside:

$$\theta_1 = \cos^{-1}\left(\frac{-1}{1.2}\right) = \underline{146.4^\circ}$$

$$c_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g(t)\, e^{-jn\omega_m t}\, dt$$

$$= \frac{A_c}{2\pi}\int_{-\theta_1}^{\theta_1} [1 + A_m\cos\theta]\, e^{-jn\theta}\, d\theta$$

5-6, Cont'd

$$c_n = \frac{A_c}{2\pi}\left[\left.\frac{e^{-jn\theta}}{-jn}\right|_{-\theta_1}^{\theta_1} + A_m \int_{-\theta_1}^{\theta_1} (\cos\theta)\, e^{-jn\theta}\, d\theta \right]$$

$$= \frac{A_c}{2\pi}\left[\frac{2}{n}\left(\frac{e^{jn\theta_1} - e^{-jn\theta_1}}{j2}\right) + A_m \frac{e^{-jn\theta}}{(-jn)^2+1} \cdot \left(-jn\cos\theta + \sin\theta\right)\Big|_{-\theta_1}^{\theta_1} \right]$$

Using Sec. A-5 where $a = -jn$

$$= \frac{A_c}{2\pi}\left[\frac{2\sin n\theta_1}{n} + A_m \left\{ \frac{e^{-jn\theta_1}(-jn\cos\theta_1 + \sin\theta_1)}{1-n^2} - \frac{e^{jn\theta_1}(-jn\cos\theta_1 - \sin\theta_1)}{1-n^2} \right\} \right]$$

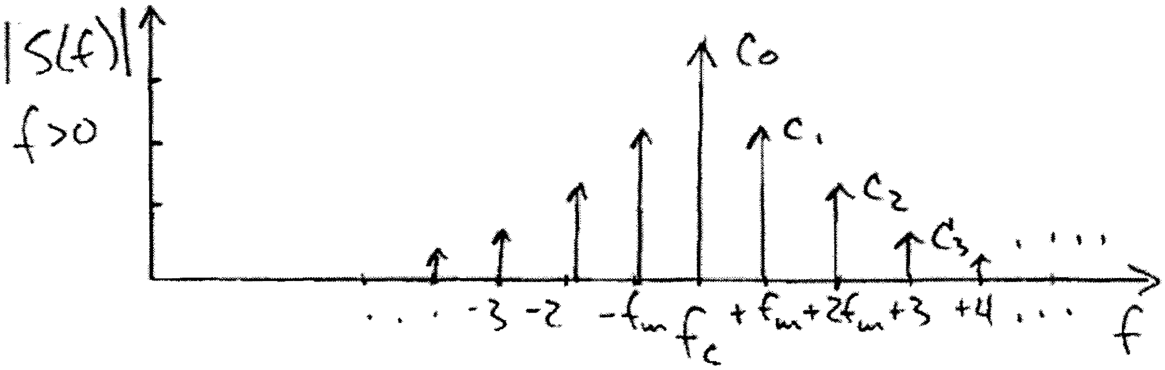
$$= \frac{A_c}{2\pi}\left[2\theta_1\left(\frac{\sin n\theta_1}{n\theta_1}\right) + A_m \left\{ \frac{jn(2j)\left(\frac{e^{jn\theta_1} - e^{-jn\theta_1}}{2j}\right)\cos\theta_1}{1-n^2} + \frac{2\left(\frac{e^{jn\theta_1} + e^{-jn\theta_1}}{2}\right)\sin\theta_1}{1-n^2} \right\} \right]$$

$A_m = 1.2$
$\theta_1 = 146.4°$

$$c_n = \frac{A_c}{2\pi}\left[2\theta_1\left(\frac{\sin(n\theta_1)}{n\theta_1}\right) + 2A_m \left\{ \frac{\cos(n\theta_1)\sin\theta_1 - n\sin(n\theta_1)\cos\theta_1}{1-n^2} \right\} \right]$$

$$S(f) = \frac{1}{2}\left[\sum_{-\infty}^{\infty} c_n\, \delta(f - f_c - nf_m) + \sum_{-\infty}^{\infty} c_n^*\, \delta(-f - f_c - nf_m) \right]$$

5-6 . Cont'd $\quad c_n = c_n^* \quad (c_n \text{ real})$



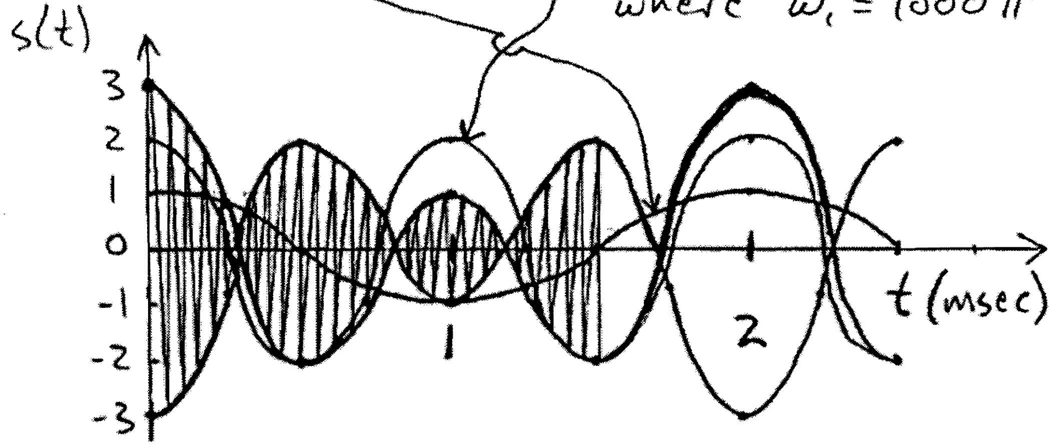
$$c_0 = \frac{A_c}{2\pi}\left[2\theta_1(\text{rad.}) + 2A_m \sin\theta_1\right]$$

$$= \frac{A_c}{2\pi}\left[2(2.56) + 2(1.2)(.553)\right]$$

5-10 (a.) DSB-SC $\quad m(t) = \cos\omega_1 t + 2\cos 2\omega_1 t$

$$s(t) = [\cos\omega_1 t + 2\cos 2\omega_1 t]\cos\omega_c t$$

where $\omega_1 = 1000\pi$

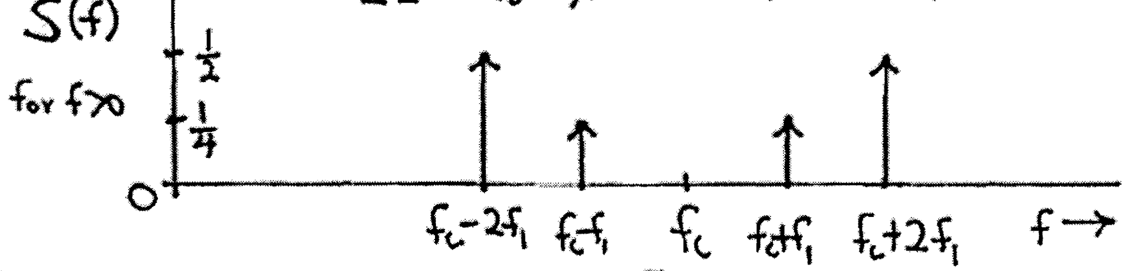


(b.) $s(t) = \frac{1}{2}\left[\cos(\omega_c - \omega_1)t + \cos(\omega_c + \omega_1)t\right.$

$\left. + \cos(\omega_c - 2\omega_1)t + \cos(\omega_c + 2\omega_1)t\right]$

5-10 (b) Cont'd $\quad S(-f) = S(f)$ even

$$S(f) = \mathcal{F}[s(t)] = \tfrac{1}{4}\left[\delta(f-(f_c-f_1)) + \delta(f+(f_c-f_1)) + \delta(f-(f_c+f_1)) + \delta(f+(f_c+f_1))\right]$$
$$+ \tfrac{1}{2}\left[\delta(f-(f_c-2f_1)) + \delta(f+(f_c-2f_1)) + \delta(f-(f_c+2f_1)) + \delta(f+(f_c+2f_1))\right]$$



(c.) $P_{AV_{norm}} = \frac{1}{2}\left[(\tfrac{1}{2})^2 + (\tfrac{1}{2})^2 + (1)^2 + (1)^2\right] = \underline{\underline{1.25\ W}}$

(d.) $A_{max} = 3 \Rightarrow PEP_{norm} = \dfrac{(3)^2}{2} = \underline{\underline{4.5\ W}}$

5-15 $\quad v_1(t) = \cos(2\pi(\tfrac{1}{2}B)t) = \cos(\pi B t)$

$v_3(t) = m(t)v_1(t) = m(t)\cos(\pi B t) \leftrightarrow V_3(f) = \frac{1}{2}\left[M(f-\tfrac{1}{2}B) + M(f+\tfrac{1}{2}B)\right]$

$v_4(t) = m(t)v_2(t) = m(t)\sin(\pi B t) \leftrightarrow V_4(f) = \frac{1}{2j}\left[-M(f-\tfrac{1}{2}B) + M(f+\tfrac{1}{2}B)\right]$

$$V_5(f) = \begin{Bmatrix} V_4(f), & |f| < \tfrac{1}{2}B \\ 0, & f \text{ elsewhere} \end{Bmatrix} = \begin{Bmatrix} \tfrac{1}{2}\left[M(f-\tfrac{1}{2}B) + M(f+\tfrac{1}{2}B)\right], & |f| < \tfrac{1}{2}B \\ 0 & , f \text{ elsewhere} \end{Bmatrix}$$

Likewise
$$V_6(f) = \begin{Bmatrix} \tfrac{1}{2j}\left[-M(f-\tfrac{1}{2}B) + M(f+\tfrac{1}{2}B)\right], & |f| < \tfrac{1}{2}B \\ 0 & , f \text{ elsewhere} \end{Bmatrix}$$

$v_9(t) = v_5(t)\cos[2\pi(f_c + \tfrac{1}{2}B)t]$

$$\Rightarrow V_9(f) = \tfrac{1}{2}\left[V_5(f - f_c - \tfrac{1}{2}B) + V_5(f + f_c + \tfrac{1}{2}B)\right]$$
$$= \frac{1}{4}\begin{Bmatrix} [M(f-f_c-\tfrac{1}{2}B-\tfrac{1}{2}B) + M(f-f_c-\tfrac{1}{2}B+\tfrac{1}{2}B)], & |f-f_c-\tfrac{1}{2}B| < \tfrac{1}{2}B \\ [M(f+f_c+\tfrac{1}{2}B-\tfrac{1}{2}B) + M(f+f_c+\tfrac{1}{2}B+\tfrac{1}{2}B)], & |f+f_c+\tfrac{1}{2}B| < \tfrac{1}{2}B \\ 0 & , f \text{ elsewhere} \end{Bmatrix}$$

5-15 Cont'd.

Aside: $|f-f_c-\frac{1}{2}B|<\frac{1}{2}B \Rightarrow -\frac{1}{2}B<f-f_c-\frac{1}{2}B<\frac{1}{2}B$

$\Rightarrow f_c+\frac{1}{2}B-\frac{1}{2}B<f<f_c+\frac{1}{2}B+\frac{1}{2}B \Rightarrow f_c<f<f_c+B$

Likewise $|f_c+f_c+\frac{1}{2}B|<\frac{1}{2}B \Rightarrow -f_c-B<f<-f_c$

Thus,

$$V_9(f)=\begin{cases}\frac{1}{4}[M(f-f_c-B)+M(f-f_c), & f_c<f<f_c+B\\ \frac{1}{4}[M(f+f_c)+M(f+f_c+B)], & -f_c-B<f<-f_c\\ 0 & ,f \text{ elsewhere}\end{cases}$$

Likewise

$$V_{10}(f)=\frac{1}{2}j\left[-V_6(f-f_c-\tfrac{1}{2}B)+V_6(f+f_c+\tfrac{1}{2}B)\right]$$

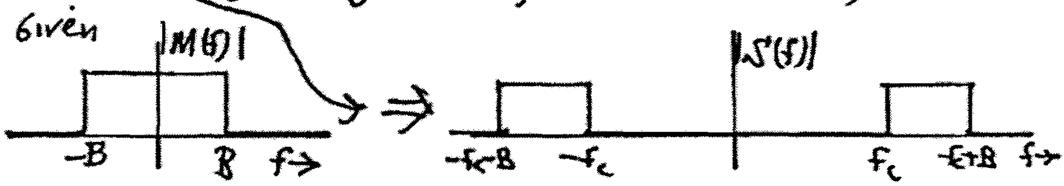
$$=\begin{cases}\frac{1}{4}[-M(f-f_c-\frac{1}{2}B-\frac{1}{2}B)+M(f-f_c-\frac{1}{2}B+\frac{1}{2}B)], & |f-f_c-\frac{1}{2}B|<\frac{1}{2}B\\ \frac{1}{4}[M(f+f_c+\frac{1}{2}B-\frac{1}{2}B)-M(f+f_c+\frac{1}{2}B+\frac{1}{2}B)], & |f+f_c+\frac{1}{2}B|<\frac{1}{2}B\\ 0 & ,f \text{ elsewhere}\end{cases}$$

$$\Rightarrow V_{10}(f)=\begin{cases}\frac{1}{4}[-M(f-f_c-B)+M(f-f_c)], & f_c<f<f_c+B\\ \frac{1}{4}[M(f+f_c)-M(f+f_c+B)], & -f_c-B<f<-f_c\\ 0 & ,f \text{ elsewhere}\end{cases}$$

Output $=s(t)=v_9(t)+v_{10}(t)$

$\Rightarrow S(f)=V_9(f)+V_{10}(f)$

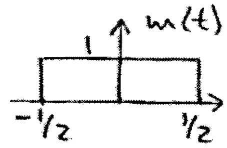
$$\Rightarrow S(f)=\begin{cases}\frac{1}{2}M(f-f_c) & ,\ f_c<f<f_c+B\\ \frac{1}{2}M(f+f_c) & ,\ -f_c-B<f<-f_c\\ 0 & ,\ f \text{ elsewhere}\end{cases}=USSB$$



$\Rightarrow s(t)$ is a USSB signal QED

5-17

$$m(t) = \begin{cases} 1, & |t| < 1/2 \\ 0, & t \text{ elsewhere} \end{cases}$$



(a.) $\hat{m}(t) = m(t) * \frac{1}{\pi t}$

$$= \int_{-1/2}^{1/2} \frac{1}{\pi}\frac{1}{t-\lambda}\, d\lambda = \frac{-1}{\pi}\int_{t+1/2}^{t-1/2} \frac{1}{\lambda_1}\, d\lambda_1$$

$\lambda_1 = t - \lambda$
$d\lambda_1 = -d\lambda$

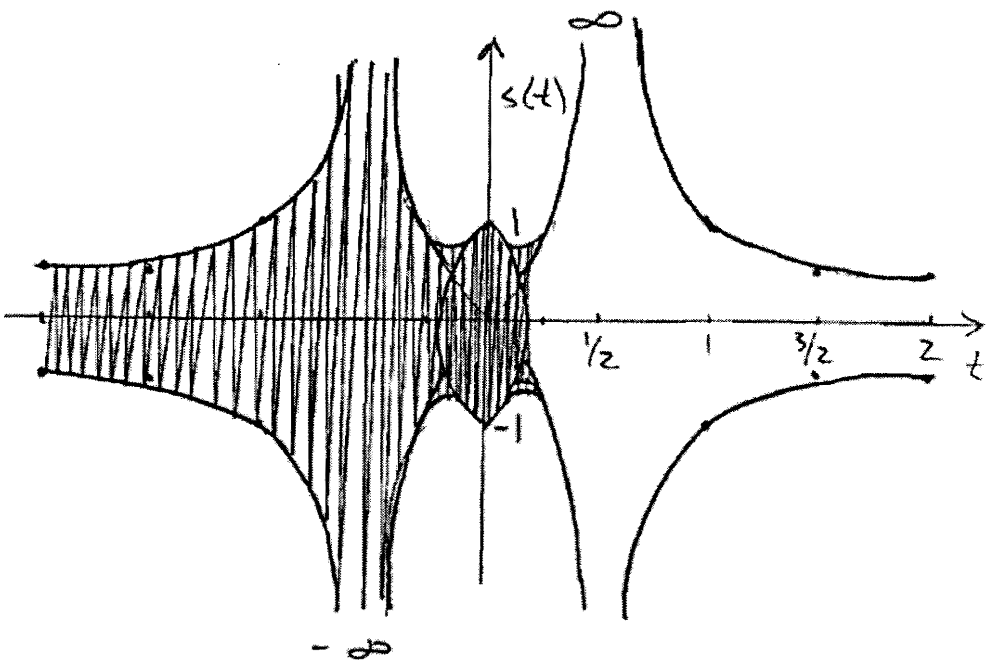
$$= \int_{t-1/2}^{t+1/2} \frac{1}{\pi}\frac{1}{\lambda_1}\, d\lambda_1 = \frac{1}{\pi}\left(\ln|\lambda_1|\right)\Big|_{t-1/2}^{t+1/2}$$

$$\hat{m}(t) = \frac{1}{\pi}\ln\left[\frac{|t+\frac{1}{2}|}{|t-\frac{1}{2}|}\right]$$

(b.) For USSB:

$$s(t) = m(t)\cos\omega_c t - \hat{m}(t)\sin\omega_c t$$

$$= \Pi(t)\cos\omega_c t - \frac{1}{\pi}\ln\left[\frac{|t+1/2|}{|t-1/2|}\right]\sin\omega_c t$$



(c.) $s(t)_{max} = \infty$

5-19 Note: T has units of Hz.

(a.)

$$m(t)=\frac{\sin(\pi T t)}{\pi T t} \longleftrightarrow M(f)=\frac{1}{T}\Pi\left(\frac{f}{T}\right)=\frac{1}{T}\left[\Pi\left(\frac{f-\frac{T}{4}}{T/2}\right)+\Pi\left(\frac{f+\frac{T}{4}}{T/2}\right)\right]$$

$$\Rightarrow \mathcal{F}[\hat{m}(t)]=M(f)\begin{cases}-j, & f>0\\ j, & f<0\end{cases}=\frac{\frac{1}{2}}{\frac{T}{2}}\left[-j\Pi\left(\frac{f-T/4}{T/2}\right)+j\Pi\left(\frac{f+T/4}{T/2}\right)\right]$$

$$\Rightarrow \hat{m}(t)=-j\frac{1}{2}\frac{\sin(\pi\frac{T}{2}t)}{\pi\frac{T}{2}t}e^{j2\pi\frac{T}{4}t}+j\frac{1}{2}\frac{\sin(\pi\frac{T}{2}t)}{\pi\frac{T}{2}t}e^{-j2\pi\frac{T}{4}t}$$

$$=\frac{\sin(\pi\frac{T}{2}t)}{\pi\frac{T}{2}t}\,\frac{e^{j\pi\frac{T}{2}t}-e^{-j\pi\frac{T}{2}t}}{2j}=\frac{\sin(\pi\frac{T}{2}t)}{\pi\frac{T}{2}t}\sin\left(\pi\frac{T}{2}t\right)=\frac{\sin^2(\pi\frac{T}{2}t)}{\pi\frac{T}{2}t}$$

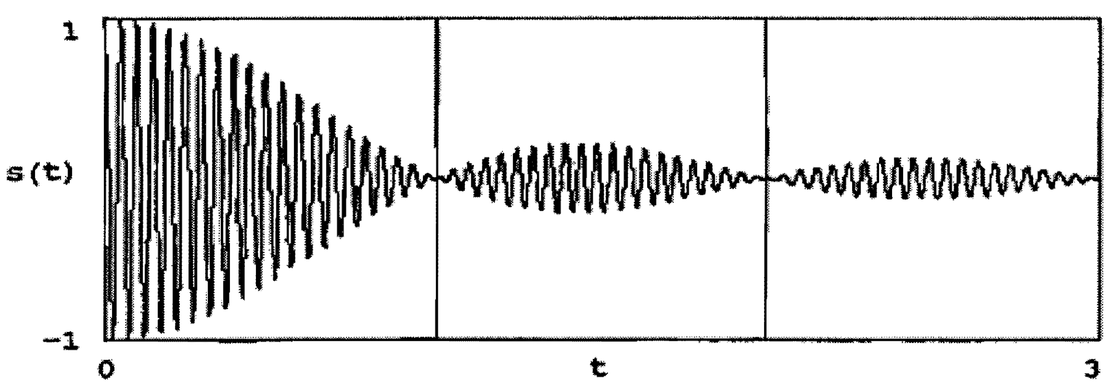
(b)

$t := 10^{-7}, 0.002 .. 3$ $\qquad T := 2$

$f_c := 20$ $\qquad \omega_c := 2\cdot\pi\cdot f_c$

$$m(t) := \frac{\sin(\pi\cdot T\cdot t)}{\pi\cdot T\cdot t} \qquad m_h(t) := \frac{\left[\sin\left[\pi\cdot\frac{T}{2}\cdot t\right]\right]^2}{\pi\cdot\frac{T}{2}\cdot t}$$

$$s(t) := m(t)\cdot\cos\left[\omega_c\cdot t\right]-m_h(t)\cdot\sin\left[\omega_c\cdot t\right]$$



5-27

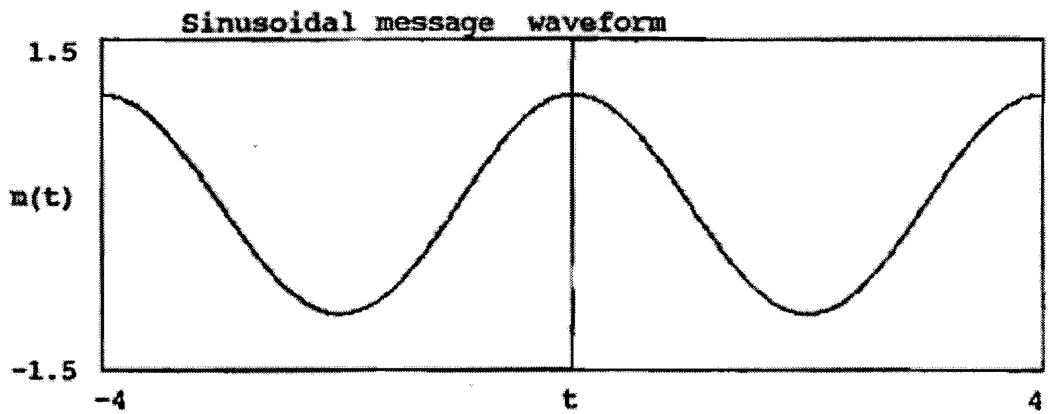
$t := -5, -4.99 .. 5 \quad fc := 1 \quad fm := 0.25\ fc \quad Dp := \pi \quad Df := \pi$

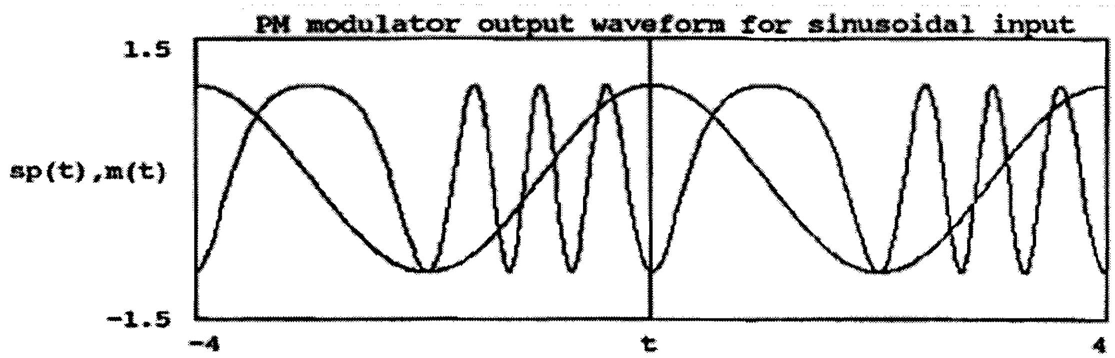
$m(t) := \cos(2 \cdot \pi\ fm \cdot t)$

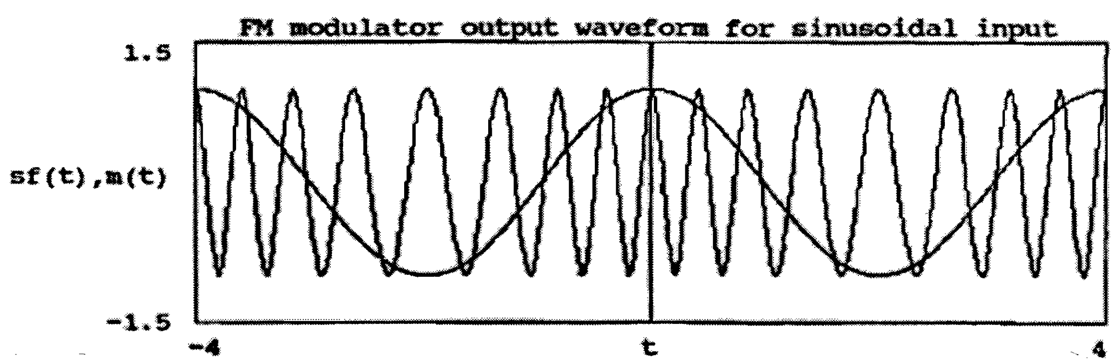
$$\theta(t) := \left[\frac{Df}{2 \cdot \pi \cdot fm}\right] \cdot \sin(2\ \pi\ fm \cdot t)$$

$sp(t) := \cos(2\ \pi \cdot fc \cdot t + Dp \cdot m(t))$

$sf(t) := \cos(2 \cdot \pi \cdot 2 \cdot fc \cdot t + \theta(t))$







5-30 (a.) $f_{BPF} = \frac{103.7}{8}$ MHz $= \underline{\underline{12.96\ \text{MHz}}}$

$$\Delta F_{BPF} = \frac{75\ \text{kHz}}{8} = 9.375\ \text{kHz}$$

$$BW_{BPF} = 2(\Delta F + f_m) = 2(9.375 + 15)\ \text{kHz}$$

$$= \underline{\underline{48.75\ \text{kHz}}}$$

(b.) $f_{BPF} = f_c + f_o \Rightarrow f_o = 12.96 - 5 = \underline{\underline{7.96\ \text{MHz}}}$

$f_{BPF} = f_c - f_o \Rightarrow f_o = 12.96 + 5 = \underline{\underline{17.96\ \text{MHz}}}$

$f_c = 5\ \text{MHz}$

(c.) $\Delta F_{FME} = \frac{75\ \text{kHz}}{8} = \underline{\underline{9.38\ \text{kHz}}}$

5-32 (a.) $\theta(t) = D_p m_p(t) = 20\cos\omega_1 t$

$$\Rightarrow m_p(t) = \frac{20}{D_p}\cos\omega_1 t = \underline{\underline{0.2\cos(2000\pi t)}},\ m_p(t)\big|_{peak} = \underline{\underline{0.2\,V}},\ f_m = \underline{\underline{1\,kHz}}$$

(b) $\theta(t) = D_f \int_{-\infty}^{t} m_f(\lambda)\,d\lambda = 20\cos\omega_1 t$

$$\Rightarrow m_f(t) = \frac{20}{D_p}\frac{d}{dt}[\cos\omega_1 t] = \frac{-20}{10^6}(2000\pi)\sin\omega_1 t$$

$$m_f(t) = -\underline{\underline{0.1257\sin\omega_1 t}},\ m_f(t)\big|_{peak} = \underline{\underline{0.1257\,V}},\ f_m = \underline{\underline{1\ kHz}}$$

(c) $P_{Av} = \frac{V_{rms}^2}{R} = \frac{(500)^2}{2(50)} = \underline{\underline{2.5\,kW}}$

$PEP = \underline{\underline{2.5\,kW}}$

5-37 (a.) $P_T = \frac{1}{2}\langle|g(t)|^2\rangle = \frac{A_c^2}{2} = \frac{(100)^2}{2} = \underline{\underline{5{,}000\ watts}}$

(b.) $P_T = \frac{1}{2}\langle|g(t)|^2\rangle \underset{\text{Using (5-55)}}{=} \frac{1}{2}\left\langle\sum_{n=-N}^{N}|c_n|^2\right\rangle = \frac{1}{2}\sum_{n=-N}^{N}|c_n|^2 \underset{\text{Using (5-57)}}{=} \frac{1}{2}A_c^2\sum_{n=-N}^{N}J_n(\beta)$

$$\Rightarrow P_T = \frac{1}{2}A_c^2\left[J_0^2(\beta) + 2\sum_{n=1}^{N}J_n^2(\beta)\right]$$

where $2Nf_m = B_T \Rightarrow N = \frac{B_T}{2f_m} = \frac{56\,Hz}{2(8)} = 3.5 \Rightarrow$ Use $N=3$

Also, $\beta = \frac{\Delta F}{f_m} = \frac{K_d A_m}{f_m} = \frac{(8\,Hz/volt)(5\,volt)}{8\,Hz} = 5.0$

Using MathCAD: Ac := 100 β := 5.0 N := 3 n := 1 ..N

$$PT := 0.5\cdot Ac^2\left[J0(\beta)^2 + 2\cdot\sum_{n} Jn(n,\beta)^2\right]$$

$\underline{\underline{PT = 2583.485\ \ watts}}$ (normalized for 1Ω)

5-38 $s(t) = A_c \cos[w_c t + \beta \sin w_m t]$

where $A_c = 1$, $w_c = 2\pi(146.52 \times 10^6)$

$w_m = 2\pi(10^3)$, $\beta = 45^\circ \left(\frac{\pi \text{ radians}}{180^\circ}\right) = 0.7854 \text{ rad.}$

$$(5\text{-}60) \quad G(f) = \sum_{n=-\infty}^{\infty} J_n(\beta)\, \delta(f - nf_m)$$

$\delta(-f) = \delta(f)$

$$S(f) = \frac{1}{2}\left[G(f-f_c) + G^*(-f-f_c)\right]$$

$$= \frac{1}{2}\left\{\sum_{n=-\infty}^{\infty} J_n(\beta)\left[\delta(f - nf_m - f_c) + \delta(f - nf_m + f_c)\right]\right\}$$

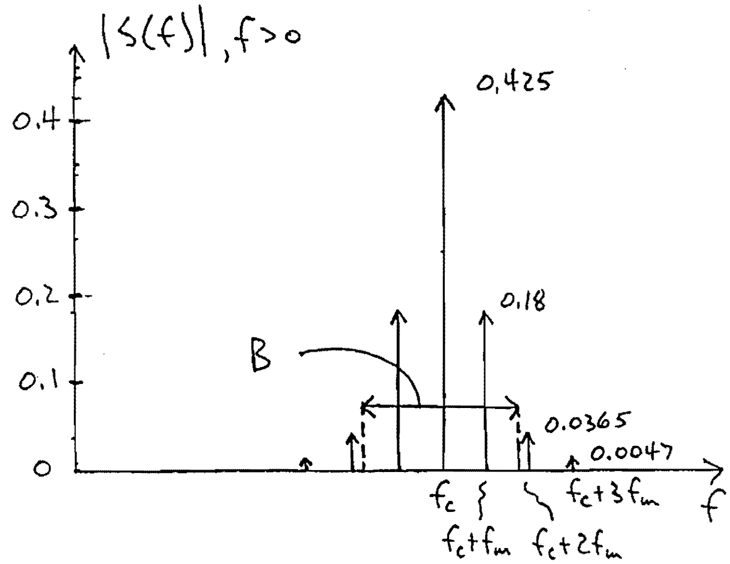
n	$J_n(\beta)$
0	0.85
1	0.36
2	0.073
3	0.0094

Carson's Rule BW:

$B = 2f_m(\beta + 1)$

$= 2(1\text{ KHz})(1.7854)$

$= \underline{\underline{3.57 \text{ KHz}}}$



$$\text{Total } P_{AV} = \frac{A_c^2}{2} = 0.5$$

$$P_{av}(\text{within } B) = \frac{(.85)^2 + 2(.36)^2}{2} = \frac{0.9817}{2} = \underline{\underline{98.17\%}}$$

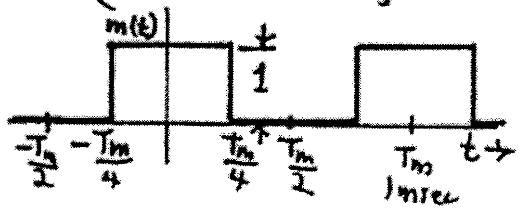
5-42

$$s(t) = \mathrm{Re}\{g(t)\,e^{j\omega_c t}\} = \mathrm{Re}\{10\,e^{j\theta(t)}\,e^{j\omega_c t}\}$$

$$\theta(t) = \beta m(t)$$

$$\beta = 45^\circ\left(\frac{\pi\ \text{rad}}{180^\circ}\right) = 0.785 = \pi/4$$

$$g(t) = \sum_{-\infty}^{\infty} c_n e^{jn\omega_m t}$$



$$c_n = \frac{1}{T_m}\int_{-T_m/2}^{T_m/2} g(t)\,e^{-jn\omega_m t}\,dt = \frac{10}{T_m}\left[\int_{-T_m/2}^{-T_m/4} e^{j0}e^{-jn\omega_m t}\,dt + \int_{-T_m/4}^{T_m/4} e^{j\beta}e^{-jn\omega_m t}\,dt + \int_{T_m/4}^{T_m/2} e^{j0}e^{-jn\omega_m t}\,dt\right]$$

$$c_n = \frac{10}{T_m}\left[\int_{T_m/4}^{T_m/2} e^{jn\omega_m t}\,dt + \int_{-T_m/4}^{T_m/4} e^{j\beta}\,e^{-jn\omega_m t}\,dt + \int_{T_m/4}^{T_m/2} e^{-jn\omega_m t}\,dt\right]$$

$$= \frac{20}{T_m}\int_{T_m/4}^{T_m/2}\Big(\underbrace{\frac{e^{jn\omega_m t}+e^{-jn\omega_m t}}{2}}_{\cos n\omega_m t}\Big)dt + \frac{10e^{j\beta}}{T_m}\left.\frac{e^{-jn\omega_m t}}{-jn\omega_m}\right|_{-\frac{T_m}{4}}^{\frac{T_m}{4}}$$

$$= \frac{20}{T_m}\left[\left.\frac{\sin(n\omega_m t)}{n\omega_m}\right|_{T_m/4}^{T_m/2} + e^{j\beta}\,\frac{e^{j\pi/2}-e^{-j\pi/2}}{2j\,n\omega_m}\right]$$

$$= \frac{20}{T_m}\left[\frac{\overset{0}{\cancel{\sin(n\pi)}} - \sin\left(\frac{n\pi}{2}\right)}{n\omega_m} + e^{j\beta}\,\frac{\sin\left(\frac{n\pi}{2}\right)}{n\omega_m}\right]$$

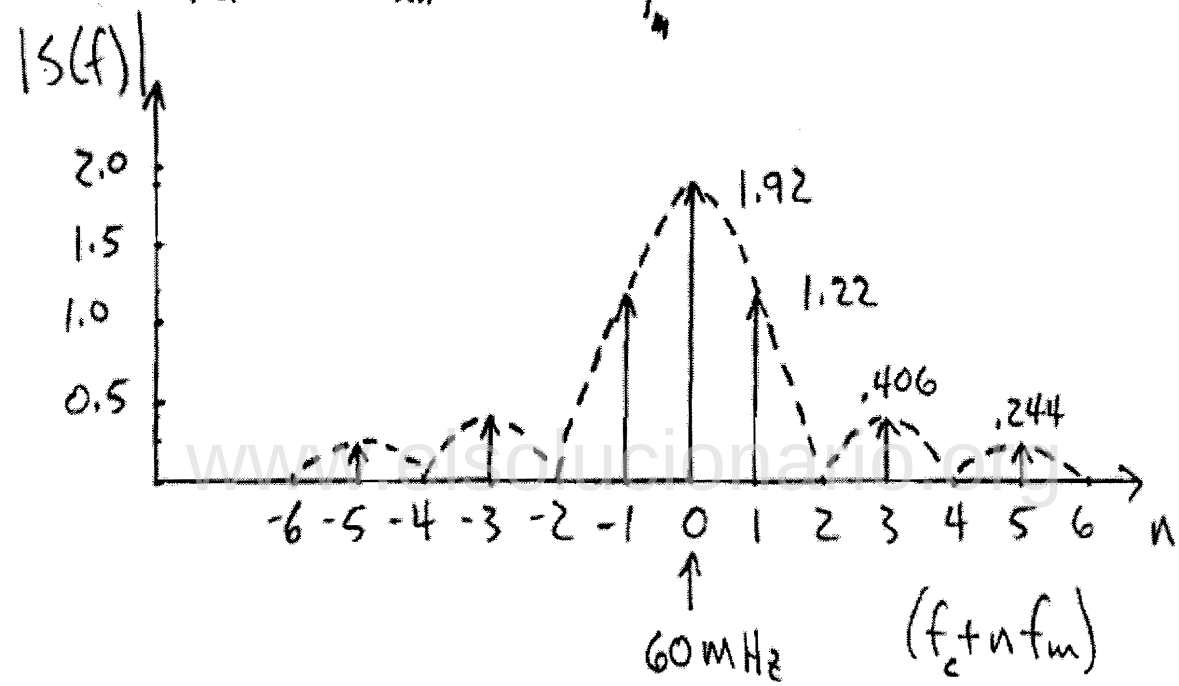
$$c_n = 5\left(e^{j\beta}-1\right)\left[\frac{\sin\left(\frac{n\pi}{2}\right)}{\frac{n\pi}{2}}\right], \quad \beta = \pi/4$$

$$|c_n| = 5\sqrt{(\cos\beta - 1)^2 + (\sin\beta)^2}\,\left|\frac{\sin\left(\frac{n\pi}{2}\right)}{\frac{n\pi}{2}}\right|$$

$$\Rightarrow |c_n| = 3.83\left|\frac{\sin\left(\frac{n\pi}{2}\right)}{\frac{n\pi}{2}}\right|$$

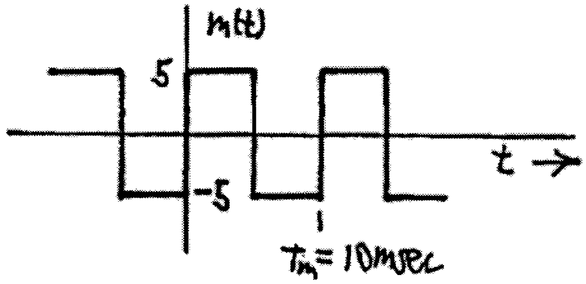
$$S(f) = \frac{1}{2}\left[\sum_{-\infty}^{\infty} c_n\,\delta(f - f_c - nf_m) + \sum_{-\infty}^{\infty} c_n^*\,\delta(f + f_c + nf_m)\right]$$

5-42 Cont'd. $f_m = 1\text{ kHz} = \frac{1}{T_m}$



5-46 NBFM

$$\theta(t) = D_f \int^t m(\lambda)\, d\lambda$$



(a.)

$$\Delta\theta = D_f \int_0^{T_m/2} m(t)\,dt = D_f \int_0^{T_m/2} 5\,dt = D_f\, 5t \Big|_0^{T_m/2} = D_f\, 5\, \frac{T_m}{2} \underset{\text{Set}}{=} (10^\circ)\left(\frac{\pi \text{ rad}}{180^\circ}\right)$$

$$\Rightarrow D_f = \frac{10\pi}{180}\left(\frac{2}{5T_m}\right) = \frac{20\pi}{5(180)10^{-2}} = \underline{\underline{6.98 \frac{\text{rad}}{\text{V}\cdot\text{sec}}}}$$

$$(5\text{-}6) \Rightarrow \Delta F = \frac{D_f V_p}{2\pi} = \frac{6.98(5)}{2\pi} = \underline{\underline{5.55 \text{ Hz}}}$$

(b.) From (5-26) and (5-27)

$$S(f) = \frac{A_c}{2}\Big\{\delta(f-f_c) + \delta(f+f_c) + \frac{D_f}{2\pi}\,\frac{M(f-f_c)}{f-f_c} - \frac{D_f}{2\pi}\,\frac{M(f+f_c)}{f+f_c}$$

$$M(f) = \mathcal{F}[m(t)] = \sum_{-\infty}^{\infty} c_n\, \delta(f - nf_m) \text{ where } f_m = \frac{1}{10 \text{ msec}} = \underline{\underline{100 \text{ Hz}}}$$

$$\underline{\underline{S(f) = \frac{A_c}{2}\Big\{\delta(f-f_c) + \delta(f+f_c) + \frac{D_f}{2\pi}\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty}\left(\frac{c_n}{nf_m}\right)\Big[\delta(f-f_c-nf_m) - \delta(f+f_c-nf_m)\Big]\Big\}}}$$

5-46. (b.) Cont'd. Aside : Evaluate c_n

$$c_n = \frac{1}{T_m}\int_0^{T_m} m(t)\, e^{-jn\omega_m t}\, dt$$

$$= \frac{1}{T_m}\left[\int_0^{T_m/2} 5 e^{-jn\omega_m t}\, dt + \int_{T_m/2}^{T_m} (-5)\, e^{-jn\omega_m t}\, dt\right]$$

$$= \frac{5}{T_m}\left[\left.\frac{e^{-jn\omega_m t}}{-jn\omega_m}\right|_0^{T_m/2} - \left.\frac{e^{-jn\omega_m t}}{-jn\omega_m}\right|_{T_m/2}^{T_m}\right]$$

$$= \frac{5}{T_m}\left[\frac{e^{-jn\omega_m \frac{T_m}{2}} - e^{-j0} - e^{-jn\omega_m T_m} + e^{-jn\omega_m \frac{T_m}{2}}}{-jn\omega_m}\right]$$

$$= \frac{5}{T_m}\left[\frac{2 e^{-jn\pi} - 1 - \overset{1}{e^{-jn2\pi}}}{-jn\omega_m}\right]$$

$$= 10\left[\frac{1 - e^{-jn\pi}}{jn\omega_m T_m}\right] = 10\, e^{-j\frac{n\pi}{2}}\left[\frac{e^{j\frac{n\pi}{2}} - e^{-jn\frac{\pi}{2}}}{j2\pi n}\right]$$

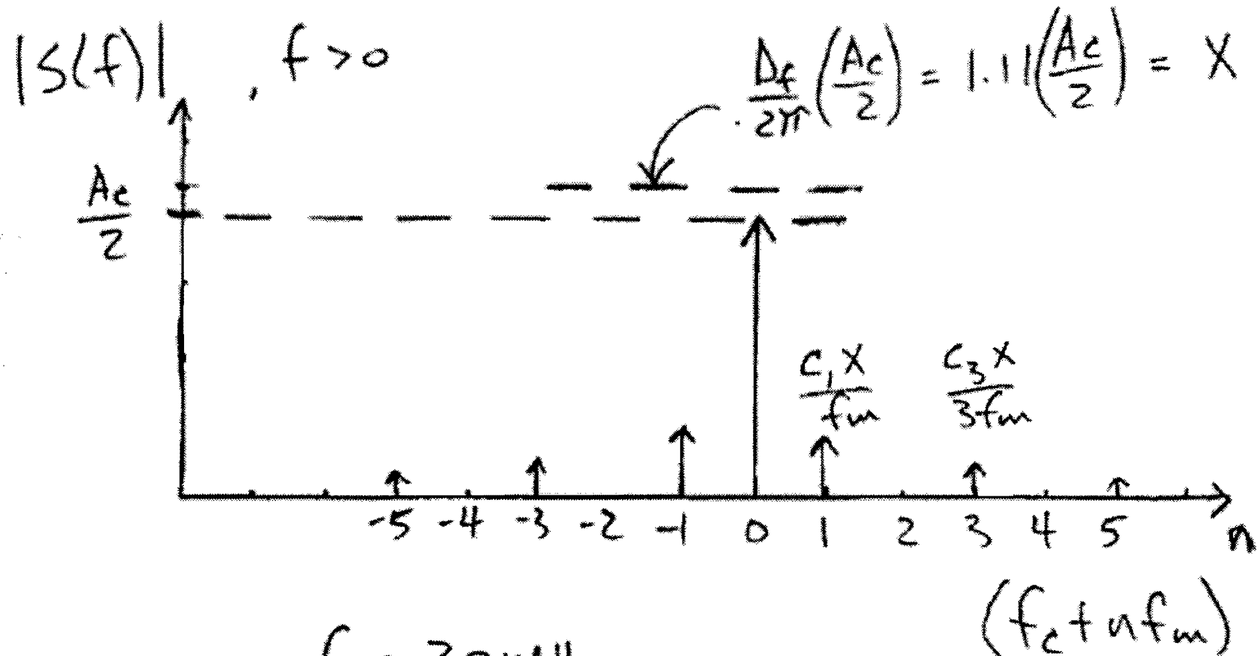
$$= \frac{10}{2}\, e^{-j\frac{n\pi}{2}}\left[\frac{\sin\left(\frac{n\pi}{2}\right)}{n\pi/2}\right]$$

$$\underline{\underline{c_n = 5\,(-j)^n\left[\frac{\sin n\pi/2}{n\pi/2}\right],\; n \neq 0\; ;\; c_0 = 0}}$$

5-46. (b.) Cont'd

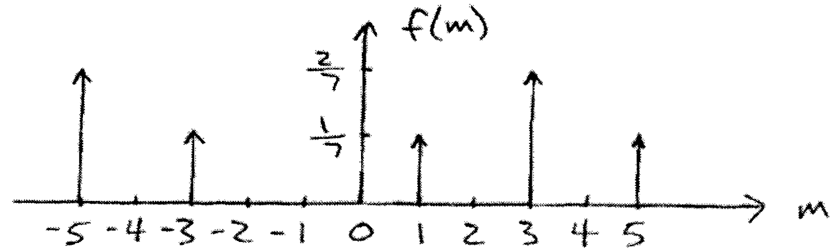
$\underline{\underline{c_n = 0 \quad , \quad n = \text{even}}}$

$|S(f)|$, $f > 0$



$f_c = 30\text{MHz}$

5-49 $A_c = 5$, $f_c = 2\text{ GHz}$, $D_f = 10^5$



$$(4\text{-}131) \Rightarrow \mathcal{P}(f) = \frac{\pi A_c^2}{2 D_f}\left[f\left(\frac{2\pi}{D_f}(f-f_c)\right) + f\left(\frac{2\pi}{D_f}(-f-f_c)\right)\right]$$

$$\delta(ax) = \frac{1}{|a|}\delta(x), \quad a = \frac{2\pi}{D_f}; \quad \delta(-x) = \delta(x)$$

$$= \frac{A_c^2}{4}\Big\{\frac{2}{7}\delta(f-f_c+5f_0) + \frac{1}{7}\delta(f-f_c+3f_0)$$

$$+ \frac{1}{7}\delta(f-f_c-f_0) + \frac{2}{7}\delta(f-f_c-3f_0)$$

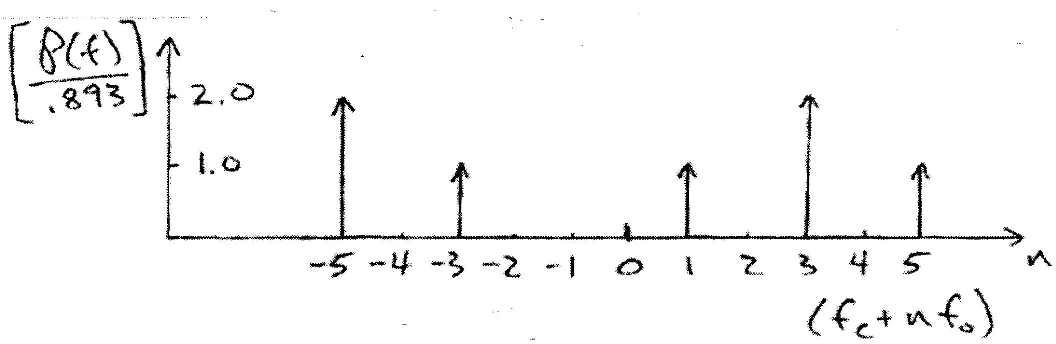
$$+ \frac{1}{7}\delta(f-f_c-5f_0) + \frac{2}{7}\delta(f+f_c-5f_0)$$

$$+ \frac{1}{7}\delta(f+f_c-3f_0) + \frac{1}{7}\delta(f+f_c+f_0)$$

$$+ \frac{2}{7}\delta(f+f_c+3f_0) + \frac{1}{7}\delta(f+f_c+5f_0)\Big\}$$

where $f_0 = \frac{D_f}{2\pi} = 15.9\text{ kHz}$

$$\frac{A_c^2}{4(7)} = \frac{25}{28} = 0.893$$



5-52 (a.) Assuming the BW's are absolute BW's:

The BW of s(t) is $2B_2 = \underline{\underline{20\ \text{KHz}}}$

$$s(t) = m_1(t)\cos\omega_c t + m_2(t)\sin\omega_c t$$

$$= Re\left\{\left[m_1(t) - j\,m_2(t)\right] e^{j\omega_c t}\right\}$$

$$g(t) = m_1(t) - j\,m_2(t)$$

$$G(f) = M_1(f) - j\,M_2(f)$$

$$S(f) = \frac{1}{2}\left[G(f-f_c) + G^*(-f-f_c)\right]$$

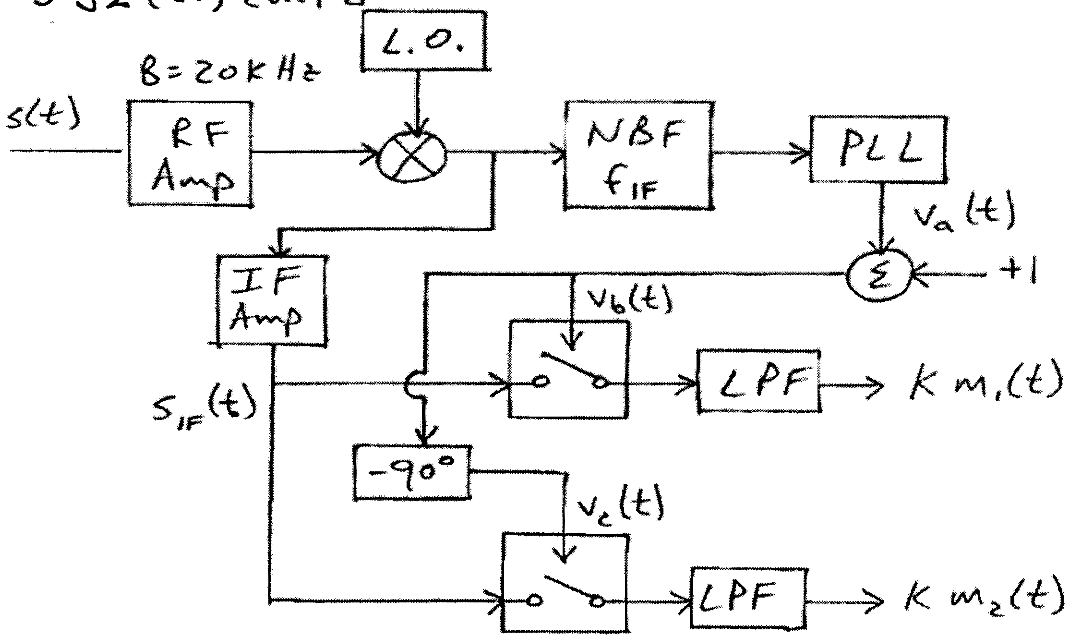
$$= \frac{1}{2}\left[M_1(f-f_c) + M_1^*(-f-f_c) - j\,M_2(f-f_c) + j\,M_2^*(-f-f_c)\right]$$

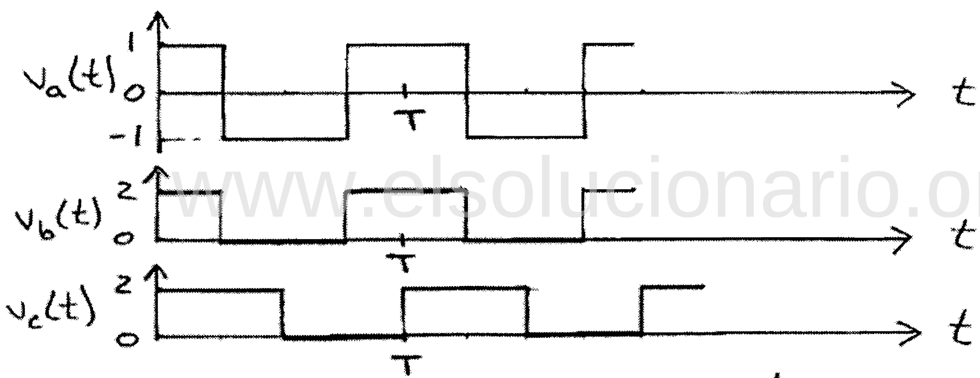
for m(t) real: $M^*(-f) = M(f)$

$$= \frac{1}{2}\left[M_1(f-f_c) + M_1(f+f_c)\right] - \frac{j}{2}\left[M_2(f-f_c) - M_2(f+f_c)\right]$$

(c.) Assume $m_1(t)$ has a constant D.C. value, therefore providing a discrete carrier term.

5-52 (c.) cont'd





$$T = \frac{1}{f_{IF}}$$

$s_{IF}(t)$ is sampled once every T seconds by each switch, therefore

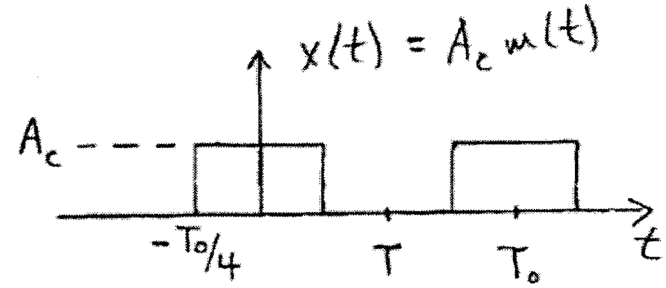
$$f_s = \frac{2}{T} = 2f_{IF}$$

$v_a(t)$ could be obtained by passing the output of the VCO (of the PLL) through a hard limiter.

5-53 (a.) $s(t) = x(t)\cos\omega_c t$, where

$$T = \frac{1}{R} = \frac{1}{24000}$$

$$T_0 = 2T$$



OOK :

$$S(f) = \frac{1}{2}\left[X(f-f_c) + X^*(-f-f_c)\right]$$

$$X(f) = \sum_{-\infty}^{\infty} c_n \delta(f-nf_0) \; ; \; x(t) = \sum_{-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$c_n = \frac{1}{T_0}\int_{-T_0/4}^{T_0/4} A_c e^{-jn\omega_0 t}\,dt = \frac{A_c}{T_0}\left.\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right|_{-T_0/4}^{T_0/4}$$

$$= \frac{A_c}{T_0}\,\frac{e^{-jn\pi/2} - e^{jn\pi/2}}{-jn2\pi/T_0} = \frac{A_c}{2}\,\frac{\sin(n\pi/2)}{n\pi/2}$$

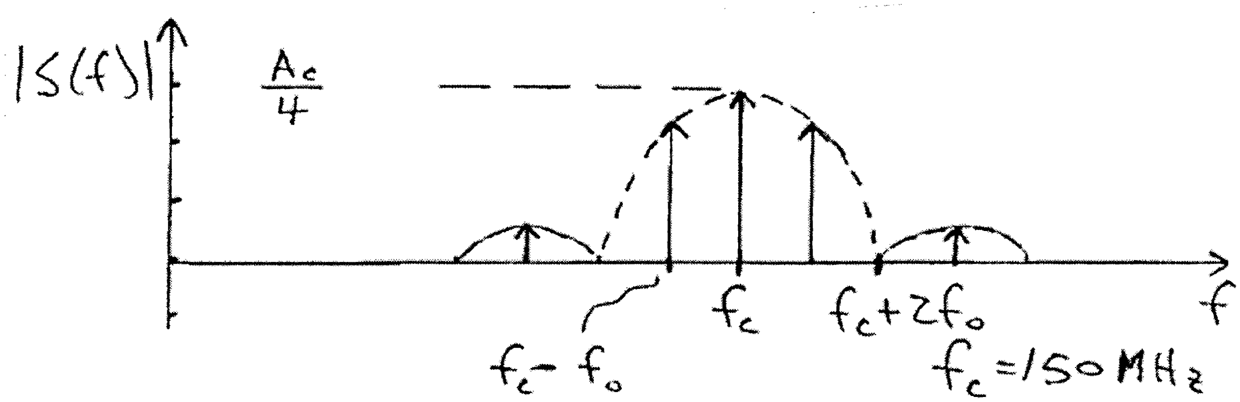
$$\underline{\underline{X(f) = \frac{A_c}{2}\sum_{-\infty}^{\infty}\left[\frac{\sin(n\pi/2)}{n\pi/2}\right]\delta(f-nf_0)}}$$

$$f_0 = \frac{1}{T_0} = \frac{1}{2T} = \frac{R}{2}$$

$$\underline{\underline{S(f) = \frac{1}{2}\left[X(f-f_c) + X^*(-f-f_c)\right]}}$$

5-53 Cont'd.

(b)



$f_c = 150\,MHz$

$f_o = 12\,KHz$

First zero-crossing at $n = 2$

$BW_T = 2(2f_o) = 2R = \underline{\underline{48\,KHz}}$

5-57

$k := 0 \,..\, 7$ $\quad T := 0.0025 \quad t := 0, 0.00008 \,..\, 8\,T$

$a_k :=$ $\quad \omega_c := 1000\,\pi$

a_k
1
-1
-1
1
-1
1
1
-1

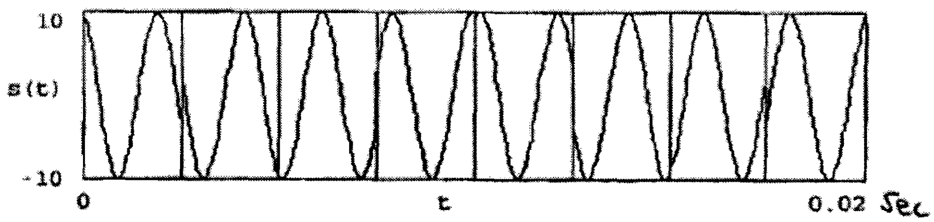
$$\mathrm{rect}(t) := \mathrm{if}\left[\left|t - \frac{T}{2}\right| \le \frac{T}{2}, 1, 0\right]$$

$$m(t) := \sum_k \left[a_k \,\mathrm{rect}(t - k\,T)\right]$$

SET INDEX $\quad N := 64 \quad n := 1 \,..\, N - 1 \quad j := 0 \,..\, \frac{N}{2}$

(a) $h := 0.2 \quad D_p := \frac{h\,\pi}{2} \quad Ts := \frac{0.02}{N} \quad t1_n := n\,Ts$

$$s(t) := 10\cos\left[\omega_c\, t + D_p\, m(t)\right]$$



$M := 6 \quad N := 2^M \quad N = 64 \quad k := 0 \,..\, N - 1 \quad T0 := 8\,T$

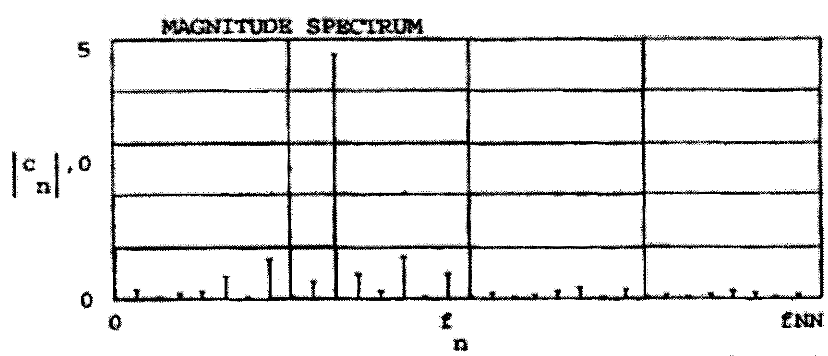
$dt := \frac{T0}{N} \quad dt = 0 \quad t_k := k\,dt$

$ss_k := s(k \cdot dt) \quad n := 0 \,..\, N - 1 \quad NN := 2^{M-1}$

Assume that the signal is periodic with period T0=8T. The spectrum can be obtained from the complex Fourier series coefficients. Furthermore, the complex Fourier series coefficients may be calculated using the FFT by substituting (2-178) into (2-186). Thus,

$c := \left[\frac{1}{\sqrt{N}}\right] \mathrm{icfft}(ss) \quad fs := \frac{1}{dt} \quad fs = 3200$ Hz

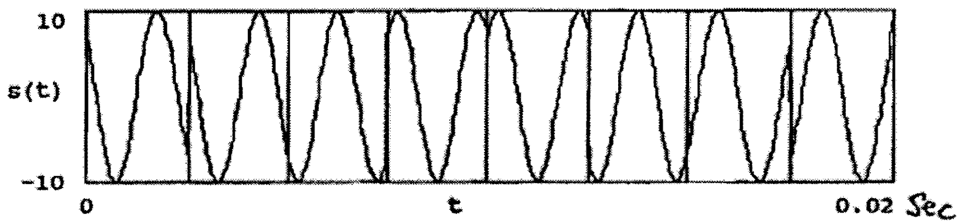
$f_n := \frac{n}{T0} \quad f_1 = 50 \quad fNN := NN\, f_1 \quad fNN = 1600$

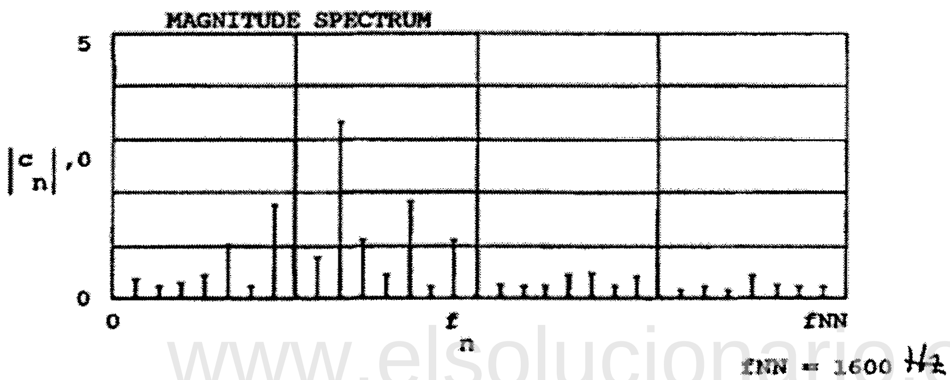


fNN = 1600 Hz

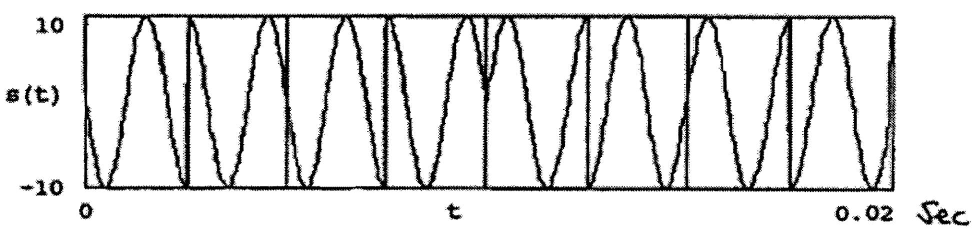
5-57 Cont'd

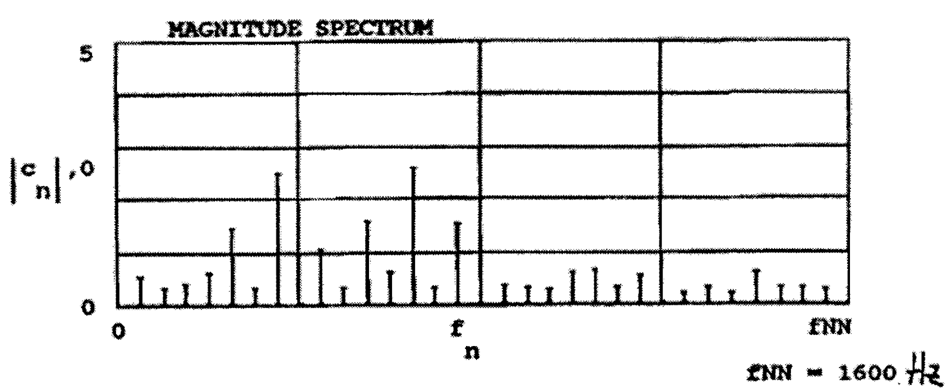
(b.) $h = 0.5$





(c.) $h = 1.0$





5-62 $\Delta = R = \dfrac{2B}{1+r}$

(a.) OOK

(5-2) $B_T = 2B = (1+r)R = (1.5)64k$

$= \underline{\underline{96 \text{ kHz}}}$

(b.) FSK

(5-7) $B_T = 2\Delta F + 2B$

$= 5 \text{ kHz} + 96 \text{ kHz} = \underline{\underline{101 \text{ kHz}}}$

5-67 Use (5-106) $B_T = (1+r)\dfrac{R}{\ell}$ where $\ell = 2$ for QPSK

(a.) $\Rightarrow 24 = (1+r)\dfrac{30}{2} \Rightarrow (1+r) = \dfrac{2(24)}{30} = 1.6$

or $\underline{\underline{r = 0.6}}$

(b.) Max R allowed is when $r = 0$

$\Rightarrow R_{max} = \dfrac{2B_T}{1} = 2(24) = 48$ Mb/sec

$\Rightarrow$ $\underline{\underline{\text{No}}}$. A rolloff factor, r, could $\underline{\text{not}}$ be found support 50Mb/s QPSK signaling

5-68

(a) Using the same procedure that leads to (5-102) where the FT of the square-root raised-cosine-rolloff pulse is obtained from (3-69)

$$F(f) = \sqrt{H_e(f)}$$

Then, using (6-70d) with $m_c = 0$ and $\sigma_c^2 = C$, the PSD of the complex envelope is

$$\mathcal{P}_g(f) = K|F(f)|^2 = K H_e(f)$$

$$\therefore \quad \mathcal{P}_g(f) = K \begin{cases} 1, & |f| < f_1 \\ \frac{1}{2}\left[1 + \cos\left[\frac{\pi(|f| - f_1)}{2f_\Delta}\right]\right], & f_1 < |f| < B \\ 0, & |f| > B \end{cases}$$

where $f_0 = \frac{1}{2\ell T_b} = \frac{1}{2}\left(\frac{R}{\ell}\right)$ and $\ell = 2$ for QPSK

$R = \frac{1}{T_b}$

5-68 Continued

$$f_1 = f_0 - f_\Delta = f_0 - rf_0 = f_0(1-r) = \frac{1}{2}(1-r)\left(\frac{R}{\ell}\right)$$

$$f_\Delta = rf_0 = \frac{1}{2}r\left(\frac{R}{\ell}\right)$$

and

$$B = f_0 + f_\Delta = f_0(1+r) = \frac{1}{2}(1+r)\left(\frac{R}{\ell}\right)$$ ← Absolute BW of complex envelope

$$\mathcal{P}_v(f) = \frac{1}{4}\left[\mathcal{P}_g(f-f_c) + \mathcal{P}_g(-f-f_c)\right]$$

$\mathcal{P}_g(f) = \mathcal{P}_g(-f)$ for QPSK → $\mathcal{P}_g(f+f_c)$

The absolute bandpass bandwidth is

$$B_T = 2B = (1+r)\left(\frac{R}{\ell}\right) \text{ where } \ell = 2 \text{ for QPSK}$$

(b)

$r := 0.35$ ←—— **Enter value of the rolloff factor, *r***

$R := 1$ $\quad fo := 0.5\cdot\left(\frac{R}{2}\right)$ $\quad N := 100$ $\quad n := 0, 1 .. N-1$

$df := 2\cdot\frac{fo}{(N-1)}$ $\quad f_n := n\cdot df$

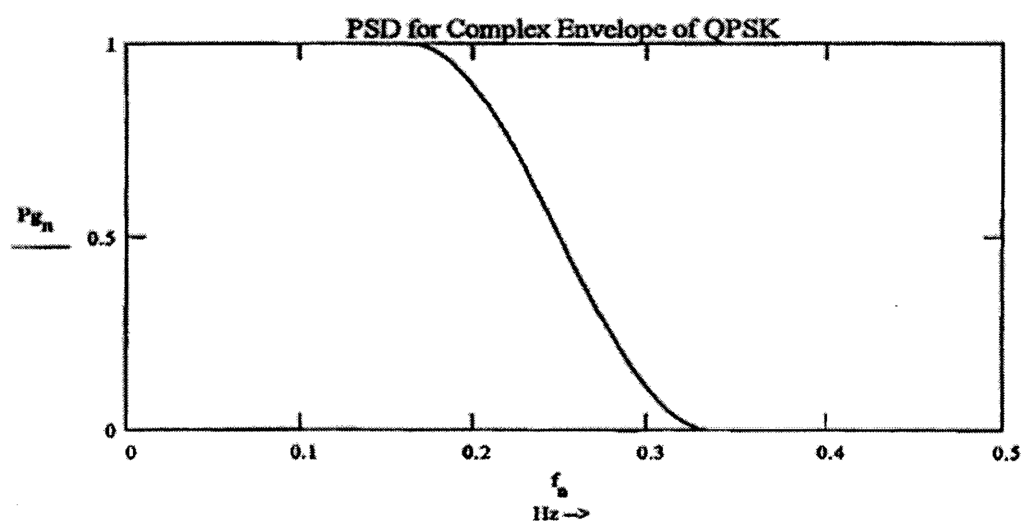
$fdel := r\cdot fo$ $\quad f1 := fo - fdel$ $\quad B := fo + fdel$

Construct the Raised Cosine frequency response using the Mathcad "if" function.

$$\cos1(f) := 0.5\left[1+\cos\left[0.5\cdot\pi\cdot\frac{(f-f1)}{fdel}\right]\right]$$

$Pg1(f) := \mathrm{if}(f > f1, \cos1(f), 1)$ $\quad Pg(f) := \mathrm{if}(f > B, 0, Pg1(f))$

$Pg_n := Pg(n\cdot df)$



5-77

From the description of $\pi/4$ QPSK in Sec. 5-10, use the table shown at the right

Input Bits	$\Delta\theta$
11	$+45°$
01	$+135°$
00	$-135°$
10	$-45°$

Data	10	11	01	00	10	10	10
$\Delta\theta$	$-45°$	$+45°$	$+135°$	$-135°$	$-45°$	$-45°$	$-45°$

5-82

$i := 0 .. 14$ $\quad n := 0 .. 14$ $\quad t := -5, -4.99 .. 5$ $\quad T := 1$ $\quad a_6 := -1$

$a_1 := -1$ $\quad a_2 := -1$ $\quad a_3 := 1$ $\quad a_4 := 1$ $\quad a_5 := 1$ $\quad a0 := -1$

$a_7 := -1$ $\quad a_8 := -1$ $\quad a_9 := 1$ $\quad a_{10} := -1$ $\quad a_{11} := -1$ $\quad a_{12} := 1$

$a_{13} := -1$ $\quad a_{14} := -1$ $\quad tt := 0, 0.2 .. 15$

$h(t) := \Phi(t) - \Phi(t-1)$ $\quad n1 := 0, 2 .. 14$ $\quad n2 := 1, 3 .. 13$

5-82 Cont'd.

$$hx(t) := \Phi(t) - \Phi(t-2) \qquad m(tt) := \sum_{n} a_n \cdot h(tt - n\,T)$$

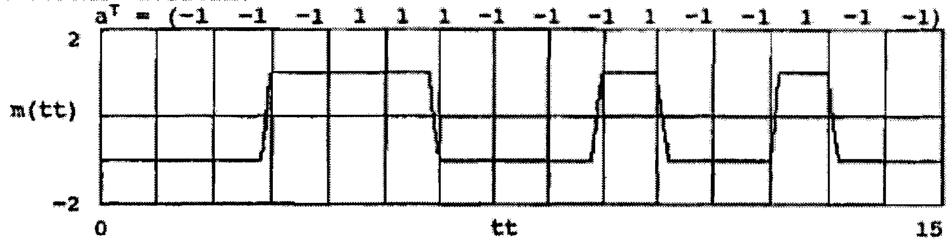
$$x(tt) := \sum_{n1} a_{n1} \cdot hx(tt - n1\,T) \qquad y(tt) := \sum_{n2} a_{n2}\, hx(tt - n2 \cdot T)$$

$$yy(tt) := \sum_{n2} a_{n2}\, hx(tt - n2 \cdot T)\, \sin\left[\pi \frac{tt - n2 \cdot T}{2}\right]$$

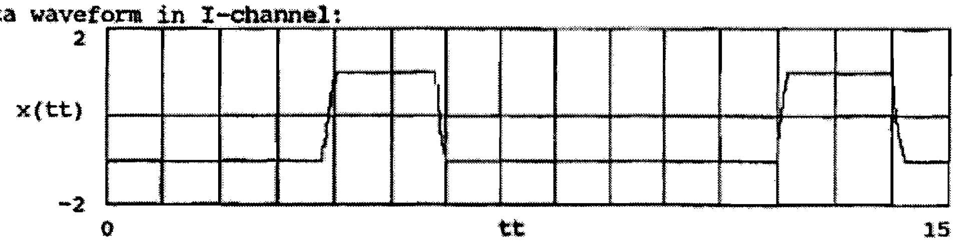
$$xx(tt) := \sum_{n1} a_{n1}\, hx(tt - n1\,T) \cdot \cos\left[\pi \frac{tt - n1\,T - 1}{2}\right]$$

Data stream waveform:

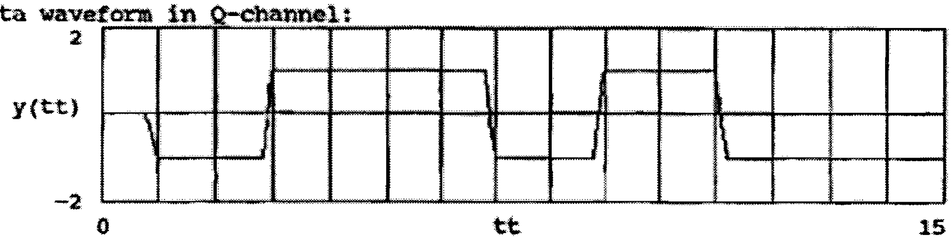
$a^T = (-1 \;\; -1 \;\; -1 \;\; 1 \;\; 1 \;\; 1 \;\; -1 \;\; -1 \;\; -1 \;\; 1 \;\; -1 \;\; -1 \;\; 1 \;\; -1 \;\; -1)$



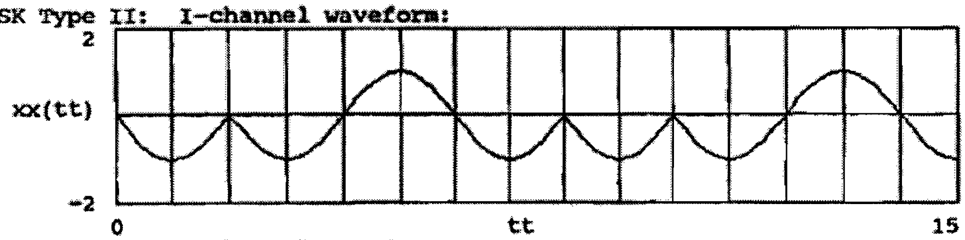
Data waveform in I-channel:



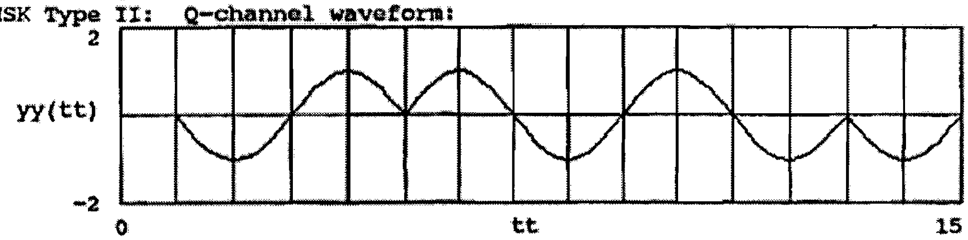
Data waveform in Q-channel:



MSK Type II: I-channel waveform:



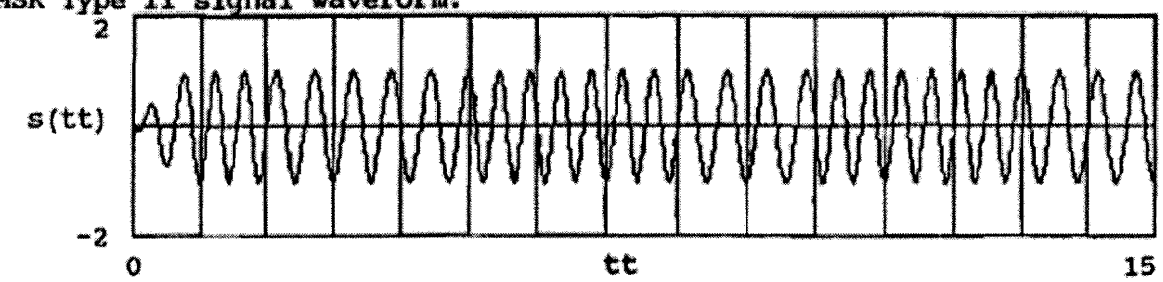
MSK Type II: Q-channel waveform:



5-82 Cont'd.

tt := 0,0.01 ..15
s(tt) := xx(tt) cos(2 π·2 tt) − yy(tt)·sin(2 π·2 tt)

MSK Type II signal waveform:



5-84 (a) $H(f) = e^{-\left(\frac{f}{B}\right)^2\left(\frac{\ln 2}{2}\right)} = \left[e^{-\pi\left(f\sqrt{\frac{\ln 2}{2}}\frac{1}{B}\frac{1}{\sqrt{\pi}}\right)^2}\right] \Rightarrow h(t) = B\sqrt{\frac{2\pi}{\ln 2}}\, e^{-\pi\left(tB\sqrt{\frac{2\pi}{\ln 2}}\right)^2}$

Table 2-2

$$\Pi\left(\frac{t}{T}\right) \longrightarrow \boxed{h(t)} \longrightarrow h_s(t)$$

$$h_s(t) = \Pi\left(\frac{t}{T}\right) * h(t) = \int h(\lambda)\,\Pi\left(\frac{t-\lambda}{T}\right)d\lambda \overset{\Pi(-t)=\Pi(t)}{=} \int h(\lambda)\,\Pi\left(\frac{\lambda-t}{T}\right)d\lambda$$

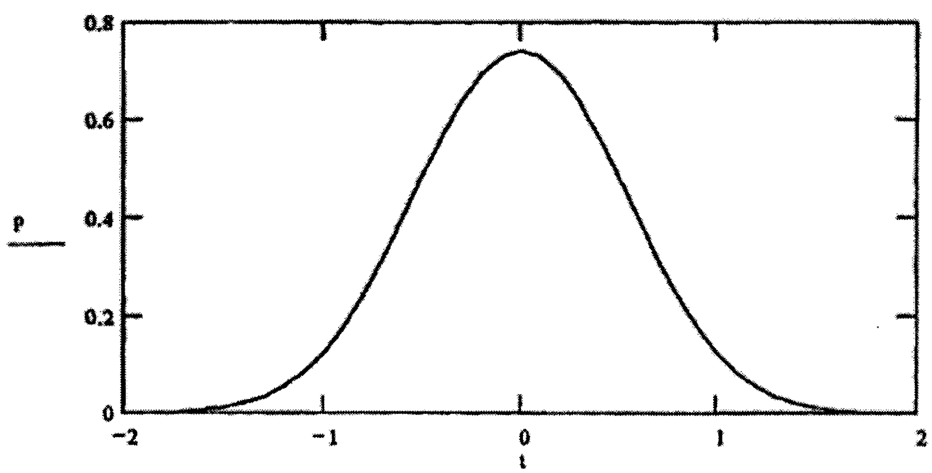
$$= \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} \left(\sqrt{\frac{2\pi}{\ln 2}}\right) B\, e^{-\frac{2\pi^2}{\ln 2}\lambda^2 B^2}\, 1\, d\lambda$$

$$\Rightarrow h_s(t) = \left(\sqrt{\frac{2\pi}{\ln 2}}\right)(BT) \int_{\frac{t}{T}-\frac{1}{2}}^{\frac{t}{T}+\frac{1}{2}} e^{-\frac{2\pi^2}{\ln 2}(BT)^2 x^2}\, dx = p(t)$$

Let $\lambda = Tx$
$d\lambda = T\,dx$
$x = \frac{1}{T}\lambda$

(b) BTb := 0.3 Tb := 1 N := 20 $dt := \frac{2\cdot Tb}{N}$ n := 0, 1 .. 2·N $t_n := (n - N)\cdot dt$

$$p(t) := \left[\sqrt{2\cdot\frac{\pi}{\ln(2)}}\cdot BTb\cdot\int_{\frac{t}{Tb}-0.5}^{\left(\frac{t}{Tb}\right)+0.5} e^{-\left[(2)\cdot\frac{(\pi^2)\cdot(BTb^2)\cdot x^2}{\ln(2)}\right]}dx\right] \qquad p_n := p(t_n)$$



5-90

(a) Referring to Fig 5-42a, the FSK signal is

$$v_1(t) = \cos[\omega_c t + \theta(t)] \quad \text{where } \theta(t) = D_f \int^t m(\lambda)\,d\lambda$$

The output of the FH spreader is

$$v_2(t) = A_c \cos[\omega_c t + \theta(t)] \cos(\omega_i t)$$

$$= \frac{A_c}{2} \cos[(\omega_c - \omega_i)t + \theta(t)] + \frac{A_c}{2} \cos[(\omega_c + \omega_i)t + \theta(t)]$$

The output of the BPF is the sum frequency part of $v_2(t)$,

$$\Rightarrow s(t) = \frac{A_c}{2} \cos[(\omega_c + \omega_i)t + \theta(t)]$$

(b) Referring to Fig 5-42b the signal out of the FH spreader

is $v_5(t) = s(t)\, 2\cos(\omega_i t) = A_c \cos[(\omega_c + \omega_i)t + \theta(t)] \cos(\omega_i t)$

$$= \underbrace{\frac{A_c}{2} \cos[\omega_c t + \theta(t)]}_{\text{diff term}} + \underbrace{\frac{A_c}{2} \cos[(\omega_c + 2\omega_i)t + \theta(t)]}_{\text{sum term}}$$

$\Rightarrow$ The output of the BPF is $v_6(t) = \frac{A_c}{2} \cos[\omega_c t + \theta(t)]$ which is FSK.

Chapter 6

6-2 (a.)

$$\overline{x(t)} = \int_0^{\pi/2} \frac{2}{\pi} \cdot A \cos(\omega_0 t + \theta)\, d\theta$$

$$= \frac{2A}{\pi} \sin(\omega_0 t + \theta) \Big|_0^{\pi/2}$$

$$= \frac{2A}{\pi} \left[\sin(\omega_0 t + \pi/2) - \sin \omega_0 t \right]$$

$$= \frac{2A}{\pi} \left[\cos \omega_0 t - \sin \omega_0 t \right]$$

$$= \frac{2\sqrt{2}A}{\pi} \left[\cos\left(\frac{\pi}{4}\right) \cos \omega_0 t - \sin\left(\frac{\pi}{4}\right) \sin \omega_0 t \right]$$

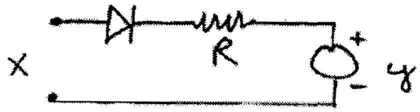
$$\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} = \sin \frac{\pi}{4}$$

$$= \frac{2\sqrt{2}A}{\pi} \cos\left(\omega_0 t + \frac{\pi}{4}\right)$$

(b.) $\overline{x(t)}$ is a function of time

$\therefore$ $x(t)$ is not stationary.

6-5



Let y be the current thru the meter produced by the voltage x.

For $x = V_p \cos \omega_0 t$:

$$\langle y \rangle = \frac{1}{T_0} \int_{-T_0/4}^{T_0/4} \frac{V_p}{R} \cos \omega_0 t \, dt = \frac{2V_p}{T_0 R} \int_0^{T_0/4} \cos \omega_0 t \, dt$$

$$= \frac{2V_p}{R T_0} \left. \frac{\sin \omega_0 t}{\omega_0} \right|_0^{T_0/4} = \frac{V_p}{R\pi} = \frac{\sqrt{2}}{R\pi} V_{rms}$$

$\omega_0 = \frac{2\pi}{T_0}$

$$V_{meter} = V_{rms} = \frac{R\pi}{\sqrt{2}} \langle y \rangle$$

Let x be a zero-mean, ergodic Gaussian noise voltage.

$$\langle y \rangle = \frac{1}{R} \int_0^{\infty} x \, p(x) \, dx = \frac{1}{R\sqrt{2\pi}\,\sigma} \int_0^{\infty} x \, e^{-x^2/2\sigma^2} dx$$

where σ = rms value of x

Let $z = \frac{x^2}{2\sigma^2}$; $dz = \frac{x}{\sigma^2} dx$

$$\langle y \rangle = \frac{\sigma}{R\sqrt{2\pi}} \int_0^{\infty} e^{-x^2/2\sigma^2} \left(\frac{x}{\sigma^2} \right) dx$$

$$= \frac{\sigma}{R\sqrt{2\pi}} \int_0^{\infty} e^{-z} dz = \left. \frac{-\sigma e^{-z}}{R\sqrt{2\pi}} \right|_0^{\infty}$$

$$= \frac{-\sigma}{R\sqrt{2\pi}} \left[e^{-\infty} - e^{-0} \right] = \frac{\sigma}{R\sqrt{2\pi}} = \langle y \rangle$$

$$V_{meter} = \frac{R\pi}{\sqrt{2}} \langle y \rangle = \frac{R\pi}{\sqrt{2}} \left[\frac{\sigma}{R\sqrt{2\pi}} \right] = \frac{\sqrt{\pi}}{2} \sigma$$

$$\therefore \quad \sigma = V_{meter} \left[\frac{2}{\sqrt{\pi}} \right]$$

6-9

Ergodicity $\Rightarrow \langle [\ ] \rangle = \overline{[\ ]}$

(a.) $P = \overline{n^2(t)} = \overline{[n_1(t) + n_2(t)]^2}$

$= \overline{n_1^2(t)} + 2\,\overline{n_1(t)n_2(t)} + \overline{n_2^2(t)}$

$\overline{n_1(t)n_2(t)} \rightarrow 0$ orthogonal

$= \overline{n_1^2(t)} + \overline{n_2^2(t)}$

$= 5 + 10 = \underline{\underline{15 \text{ Watts}}}$

(b.) $P = \overline{n^2(t)} = \overline{n_1^2(t)} + 2\,\overline{n_1(t)n_2(t)} + \overline{n_2^2(t)}$

uncorrelated $\quad 2\,\overline{n_1(t)}\;\overline{n_2(t)}$

$P = 5 + 2(-2)(1) + 10 = \underline{\underline{11 \text{ Watts}}}$

(c.) $P = \overline{n^2(t)} = \overline{[n_1(t) + n_2(t)]^2}$

$= \overline{n_1^2(t)} + 2\,\overline{n_1(t)n_2(t)} + \overline{n_2^2(t)}$

$2R_{n_1n_2}(0)$

$= 5 + 2(2) + 10 = \underline{\underline{19 \text{ Watts}}}$

6-11

(a.) $\sin \omega_0 \tau$ ① $R(\tau) \neq R(-\tau)$ ×

NO

(b.) $\left(\dfrac{\sin \omega_0 \tau}{\omega_0 \tau}\right)$

① $R(\tau) = R(-\tau)$

② $R(0) \geq |R(\tau)|$

YES

③ $\mathcal{F}\left[\dfrac{\sin \omega_0 \tau}{\omega_0 \tau}\right] =$ Non-negative rectangle

(c.) $\cos \omega_0 \tau + \delta(\tau)$ ① $R(\tau) = R(-\tau)$

YES

② $R(0) \geq |R(\tau)|$

③ $\mathcal{F}\left[\cos \omega_0 \tau + \delta(\tau)\right] = \frac{1}{2}\delta(f-f_0) + \frac{1}{2}\delta(f+f_0) + 1$

$$\geq 0$$

(d.) $e^{-a|\tau|}$ where $a<0$ ① $R(\tau) = R(-\tau)$

NO

② $R(0) \ngeq |R(\tau)|$ ×

6-13 $R_x(\tau) = 4e^{-\tau^2}+3$

(a.) $\mathcal{P}_x(f) = \mathcal{F}[R_x(\tau)] = \mathcal{F}[4e^{-\tau^2}] + \mathcal{F}[3] = \mathcal{F}\left[4e^{-\pi\left(\frac{\tau}{\sqrt{\pi}}\right)^2}\right] + \mathcal{F}[3]$

$\Rightarrow \mathcal{P}_x(f) \underset{\uparrow}{=} 4\sqrt{\pi}\, e^{-\pi(f\sqrt{\pi})^2} + 3\delta(f) = \underline{\underline{4\sqrt{\pi}\, e^{-(\pi f)^2} + 3\delta(f)}}$

(Using Table 2-2)

(b.) This is a low-pass spectrum. ⇒ Use (6-97) and (6-98).

$$\overline{f^2} = \frac{\int_{-\infty}^{\infty} f^2 \mathcal{P}_x(f)\,df}{\int_{-\infty}^{\infty} \mathcal{P}_x(\lambda)\,d\lambda} = \frac{4\int_{-\infty}^{\infty} f^2 \frac{1}{\sqrt{2\pi}\left(\frac{1}{\sqrt{2}\pi}\right)} e^{-f^2/2\left(\frac{1}{\sqrt{2}\pi}\right)^2} df + \overbrace{3\int_{-\infty}^{\infty} f^2\delta(f)\,df}^{0}}{4\Bigg(\underbrace{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\left(\frac{1}{\sqrt{2}\pi}\right)} e^{-f^2/2\left(\frac{1}{\sqrt{2}\pi}\right)^2} df}_{1}\Bigg) + 3\underbrace{\int_{-\infty}^{\infty} \delta(f)\,df}_{1}}$$

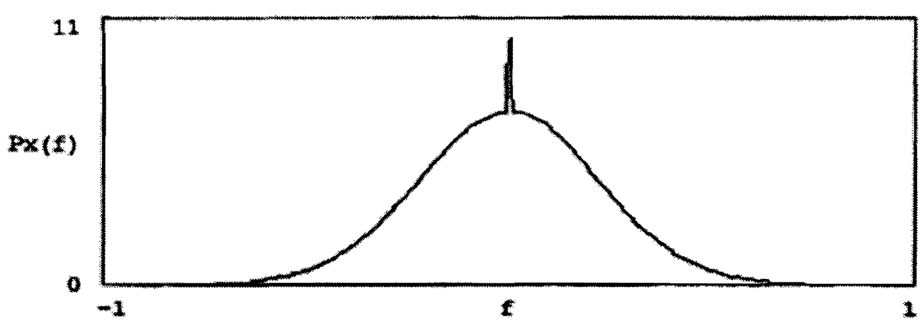
$$\Rightarrow \overline{f^2} = \frac{4\left(\frac{1}{\sqrt{2}\pi}\right)^2}{4+3} = \frac{\overset{2}{\cancel{4}}}{7}\,\frac{1}{\cancel{2}\pi^2} = \frac{2}{7\pi^2}$$

Thus, $B_{rms} = \sqrt{\overline{f^2}} = \sqrt{\frac{2}{7\pi^2}} = \sqrt{\frac{2}{7}}\,\frac{1}{\pi} = \underline{\underline{0.170\ \text{Hz}}}$

δ(0) cannot be plotted since it is infinity. Consequently, following the usual convention in EE, plot the WEIGHT of δ(f) instead at f=0.

f := −1,−0.99 ..1 δ(f) := if(f ≈ 0,1,0)

$Px(f) := 4\sqrt{\pi}\cdot e^{-\left[(\pi\cdot f)^2\right]} + 3\cdot\delta(f)$ $Px(0) = 10.09$



$Brms := \sqrt{\frac{2}{7}}\cdot\left[\frac{1}{\pi}\right]$ Brms = 0.17

6-18

(a.) $x_{rms} = \sqrt{\overline{x^2(t)}}$

$$\overline{x^2(t)} = R_x(0) = \int_{-\infty}^{\infty} \mathcal{P}_x(f)\,df$$

$$= \int_{-B}^{0} \frac{1}{B}(B+f)\,df + \int_{0}^{B} \frac{1}{B}(B-f)\,df$$

$$= \frac{1}{B}\left[\left(Bf+\frac{f^2}{2}\right)\Big|_{-B}^{0} + \left(Bf-\frac{f^2}{2}\right)\Big|_{0}^{B}\right]$$

$$= \frac{1}{B}\left[-\left(-B^2+\frac{B^2}{2}\right) + \left(B^2-\frac{B^2}{2}\right)\right] = \frac{1}{B}\left[2B^2-B^2\right]$$

$$= B \Rightarrow \underline{\underline{x_{rms} = \sqrt{B}}}$$

(b.) $\mathcal{P}_x(f) = \frac{1}{\sqrt{B}}\Pi\left(\frac{f}{B}\right) * \frac{1}{\sqrt{B}}\Pi\left(\frac{f}{B}\right)$

$$R_x(\tau) = \mathcal{F}^{-1}[\mathcal{P}_x(f)] = \frac{1}{B}\left\{\mathcal{F}^{-1}\left[\Pi\left(\frac{f}{B}\right)\right]\right\}^2$$

Table 2-1 · Multiplication property of $\mathcal{F}[\,]$

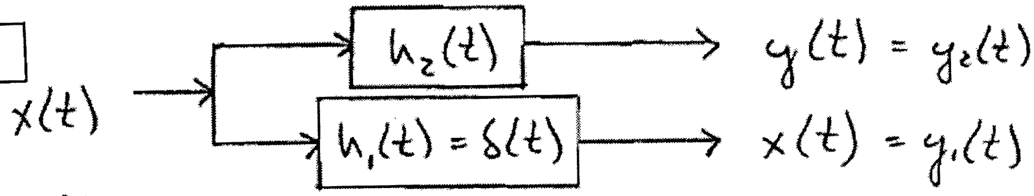
$$R_x(\tau) = \frac{1}{B}\left\{\mathcal{F}^{-1}\left[\Pi\left(\frac{f}{B}\right)\right]\right\}^2$$

Table 2-2

$$= \frac{1}{B}\left\{B\,\frac{\sin(\pi B\tau)}{\pi B\tau}\right\}^2$$

$$\underline{\underline{R_x(\tau) = B\left[\frac{\sin(\pi B\tau)}{\pi B\tau}\right]^2}}$$

6-22



$$\mathcal{P}_x(f) = \frac{N_0}{2} \Rightarrow R_x(\tau) = \frac{N_0}{2}\delta(\tau)$$

(a.) From eqn. (6-86b.)

$$R_{y_1 y_2}(\tau) = h_1(-\tau) * h_2(\tau) * R_{x_1 x_2}(\tau)$$

but $h_1(-\tau) = \delta(-\tau) = \delta(\tau)$

$$\Rightarrow R_{xy}(\tau) = \delta(\tau) * h_2(\tau) * R_x(\tau)$$

$$= \delta(\tau) * \left[h_2(\tau) * \frac{N_0}{2}\delta(\tau)\right]$$

$$= \delta(\tau) * \frac{N_0}{2} h_2(\tau) = \frac{N_0}{2} h_2(\tau)$$

$$\Rightarrow \underline{\underline{h_2(\tau) = R_{xy}(\tau)\left[\frac{2}{N_0}\right]}}$$

(b.) From (a.): $R_{xy}(\tau) = \frac{N_0}{2} h_2(\tau)$

$$\mathcal{P}_{xy}(f) = \mathcal{F}\left[R_{xy}(\tau)\right] = \mathcal{F}\left[\frac{N_0}{2} h_2(\tau)\right] = \frac{N_0}{2} H_2(f)$$

$$\Rightarrow \underline{\underline{H_2(f) = \left[\frac{2}{N_0}\right]\mathcal{P}_{xy}(f)}}$$

6-25 (a.) $\mathcal{P}_y(f) = |H(f)|^2 \mathcal{P}_n(f) = \left|\frac{K}{j2\pi f}\right|^2 \frac{N_0}{2}$

$$\underline{\underline{\mathcal{P}_y(f) = \frac{N_0 K^2}{8\pi^2 f^2}}}$$

(b.) $y_{rms}^2 = R_y(0) = \int_{-\infty}^{\infty} \mathcal{P}_y(f)\, df$

$$= \int_{-\infty}^{\infty} \frac{N_0 K^2}{8\pi^2 f^2} = 2\int_0^{\infty} \left[\frac{N_0 K^2}{8\pi^2}\right] \frac{1}{f^2}\, df$$

$$= \frac{N_0 K^2}{4\pi^2}\left[\frac{-1}{f}\right]\Bigg|_0^{\infty} = \frac{N_0 K^2}{4\pi^2}\left[\frac{-1}{\infty} + \frac{1}{0}\right] = \underline{\underline{\infty}}$$

A practical integrator will have a large (i.e. finite) output.

6-29 From (6-95): $\left(\frac{S}{N}\right)_{out} = \frac{2A_0^2 RC}{N_0[1+(2\pi f_0 RC)^2]}$

Let $RC = \tau$, $2\pi f_0 = \omega_0$

$$\Rightarrow \left(\frac{S}{N}\right)_{out} = \frac{2A_0^2 \tau}{N_0[1+(\omega_0\tau)^2]}$$

For $\max\left[\left(\frac{S}{N}\right)_{out}\right]$, set $\frac{d\left[\left(\frac{S}{N}\right)_{out}\right]}{d\tau} = 0$

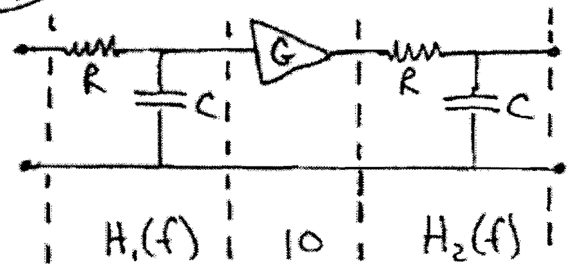
$$\Rightarrow \frac{d\left[\left(\frac{S}{N}\right)_{out}\right]}{d\tau} = \frac{[1+(\omega_0\tau)^2]2A_0^2 - 2A_0^2\tau(2)(\omega_0\tau)\omega_0}{N_0[1+(\omega_0\tau)^2]^2}$$

Set numerator $= 0 \Rightarrow 2A_0^2[1+\omega_0^2\tau^2 - 2\tau\omega_0^2\tau] = 0$

$$\Rightarrow [1-\omega_0^2\tau^2] = 0 \Rightarrow \omega_0^2\tau^2 = 1 \Rightarrow \tau^2 = \frac{1}{\omega_0^2} \Rightarrow \tau = \frac{1}{\omega_0}$$

Thus, $\underline{\underline{RC = \frac{1}{2\pi f_0}}}$ for max $\left(\frac{S}{N}\right)_{out}$

eqn. (6-102) $\doteq f_o$



6-34 (a.)

$$H_1(f) = H_2(f) = \frac{1}{1 + j\, f/f_o}$$

where $f_o = \frac{1}{2\pi RC}$, and $G = 10$

$$H(f) = H_1(f)\{10\} H_2(f) = \underline{\underline{\frac{10}{[1 + j\, f/f_o]^2}}}$$

$$|H(f)| = \frac{10}{\left|1 + j\,2f/f_o - (f/f_o)^2\right|} = \frac{10}{\sqrt{[1 - (f/f_o)^2]^2 + (2f/f_o)^2}}$$

Using a programmable calculator, find the value of $f = f_c$, such that:

$$|H(f_c)| = \frac{10}{\sqrt{2}} \implies \underline{\underline{f_c = 0.690\, f_o \quad ; \quad f_o = \frac{1}{2\pi RC}}}$$

6-37 (a.) x_1 + y_2 uncorrelated $\overset{\text{prop. 3}}{\Longrightarrow}$ Independent

when $R_{xy}(\tau) = \overline{x(t_1)\, y(t_2)} = 10 \sin(2\pi\tau) = \underline{0} = \overset{0}{\cancel{\bar{x}_1}}\, \overset{0}{\cancel{\bar{y}_2}}$

$\Longrightarrow \quad 2\pi\tau = \pm n\pi \Rightarrow$ These r.v.'s are independent only when $t_2 - t_1 = \tau = \pm \frac{n}{2}$; $n = 0, 1, \ldots$

6-37 Cont'd (b.)

$$10\sin(2\pi\tau) = 10\sin[2\pi(t_2-t_1)] = \overline{x(t_1)y(t_2)}$$

This cannot be expressed as $\overline{x(t_1)}\;\overline{y(t_2)}$

$\therefore$ $x(t)$ and $y(t)$ are <u>not indep.</u>

6-39 Evaluate $\overline{x^2(t)} = R_x(0)$

$$\overline{x^2(t)} = \overline{A_0^2\cos^2(\omega_0 t+\theta)} = \frac{A_0^2}{2}\left[1+\overline{\cos(2\omega_0 t+2\theta)}\right]$$

$$= \frac{A_0^2}{2} + \frac{A_0^2}{2}\int_0^{\pi/2}\cos(2\omega_0 t+2\theta)\frac{2}{\pi}\,d\theta$$

$$= \frac{A_0^2}{2} + \frac{A_0^2}{2}\left.\frac{\sin(2\omega_0 t+2\theta)}{2}\left(\frac{2}{\pi}\right)\right|_0^{\pi/2}$$

$$= \frac{A_0^2}{2} + \frac{A_0^2}{2\pi}\left[\sin(2\omega_0 t+\pi) - \sin(2\omega_0 t)\right]$$

$$= \frac{A_0^2}{2} + \frac{A_0^2}{2\pi}\left[-2\sin(2\omega_0 t)\right]$$

Using Sec. A-1

This is a function of t $\therefore$ $x(t)$ <u>not W.S.S.</u>

6-40

Eqn. (6-42) $\mathcal{P}_x(f) = \lim_{T\to\infty}\left[\frac{|X_T(f)|^2}{T}\right]$

where $X_T(f) = \int_{-T/2}^{T/2} x(t)\, e^{-j\omega t}\, dt$

$$= \int_{-T/2}^{T/2} A_o \cos(\omega_o t + \theta)\, e^{-j\omega t}\, dt$$

$$= A_o \int_{-T/2}^{T/2} \frac{e^{j(\omega_o t+\theta)} + e^{-j(\omega_o t+\theta)}}{2}\, e^{-j\omega t}\, dt$$

$$= \frac{A_o}{2} e^{j\theta} \int_{-T/2}^{T/2} e^{j(\omega_o - \omega)t}\, dt + \frac{A_o}{2} e^{-j\theta} \int_{-T/2}^{T/2} e^{-j(\omega_o+\omega)t}\, dt$$

$$= \frac{A_o}{2} e^{j\theta} \left.\frac{e^{j(\omega_o-\omega)t}}{j(\omega_o-\omega)}\right|_{-T/2}^{T/2} + \frac{A_o}{2} e^{-j\theta} \left.\frac{e^{-j(\omega_o+\omega)t}}{-j(\omega_o+\omega)}\right|_{-T/2}^{T/2}$$

$$= A_o\left[e^{j\theta}\, \frac{e^{j(\omega_o-\omega)T/2} - e^{-j(\omega_o-\omega)T/2}}{2j\,(\omega_o-\omega)} + e^{-j\theta}\, \frac{e^{j(\omega_o+\omega)T/2} - e^{-j(\omega_o+\omega)T/2}}{2j\,(\omega_o+\omega)}\right]$$

$$= A_o e^{j\theta} \frac{\sin(\omega_o-\omega)T/2}{(\omega_o-\omega)} + A_o e^{-j\theta} \frac{\sin(\omega_o+\omega)T/2}{(\omega_o+\omega)}$$

Let $x_1 = (\omega_o - \omega)T/2$ and $x_2 = (\omega_o+\omega)T/2$

$$= \frac{A_o T}{2}\left[e^{j\theta} \frac{\sin x_1}{x_1} + e^{-j\theta} \frac{\sin x_2}{x_2}\right]$$

$$\frac{|X_T(f)|^2}{T} = \frac{X_T(f)\, X_T^*(f)}{T} =$$

$$\frac{\left(\frac{A_o T}{2}\right)^2}{T}\left[e^{j\theta}\frac{\sin x_1}{x_1} + e^{-j\theta}\frac{\sin x_2}{x_2}\right]\left[e^{-j\theta}\frac{\sin x_1}{x_1} + e^{j\theta}\frac{\sin x_2}{x_2}\right]$$

6-40 Cont'd

$$\overline{\frac{|X_T(f)|^2}{T}} = \frac{A_0^2 T}{4}\left[\left(\frac{\sin x_1}{x_1}\right)^2 + \overline{e^{j2\theta}}\left(\frac{\sin x_1}{x_1}\right)\left(\frac{\sin x_2}{x_2}\right) + \overline{e^{-j2\theta}}\left(\frac{\sin x_1}{x_1}\right)\left(\frac{\sin x_2}{x_2}\right) + \left(\frac{\sin x_2}{x_2}\right)^2\right]$$

Aside:

$$\overline{e^{j2\theta}} = \int_0^{\pi/2} e^{j2\theta}\cdot\frac{2}{\pi}\,d\theta = j\,2/\pi$$

$$\overline{e^{-j2\theta}} = \int_0^{\pi/2} e^{-j2\theta}\cdot\frac{2}{\pi}\,d\theta = -j\,2/\pi$$

$$\overline{\frac{|X_T(f)|^2}{T}} = \frac{A_0^2}{4}\left[\frac{\overbrace{T\pi}^{a}}{\pi}\left(\frac{\sin \pi T\overbrace{(f-f_0)}^{x}}{\pi T(f-f_0)}\right)^2 + \frac{T\pi}{\pi}\left(\frac{\sin \pi T(f+f_0)}{\pi T\underbrace{(f+f_0)}_{x'}}\right)^2\right]$$

From Sec. A-8

$$\delta(x) = \lim_{a\to\infty}\left[\frac{a}{\pi}\left(\frac{\sin ax}{ax}\right)^2\right]$$

$$\Longrightarrow \quad \underline{\underline{\mathcal{P}_x(f) = \frac{A_0^2}{4}\left[\delta(f-f_0) + \delta(f+f_0)\right]}}$$

6-41 $\overline{x(t)} = A_0\,\overline{\cos(\omega_0 t+\theta)} = A_0\cdot 0 = \underline{0}$

$$R_x(\tau) = \overline{x(t)\,x(t+\tau)}$$

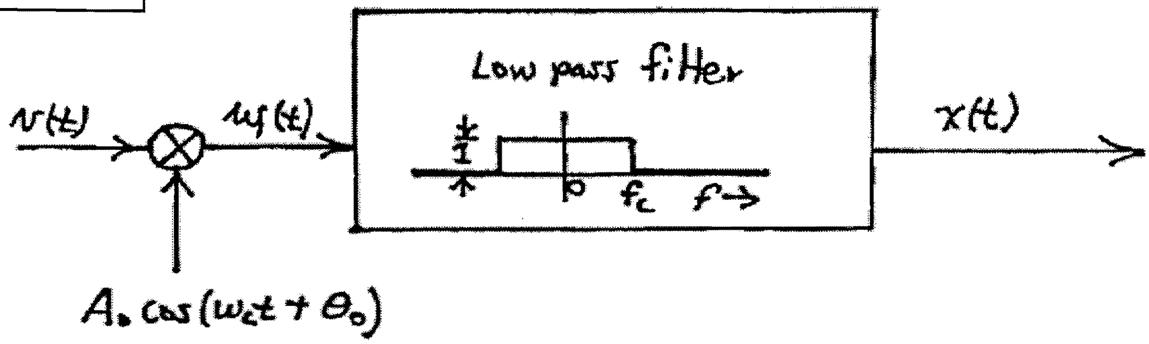
$$= A_0^2\,\overline{\cos(\omega_0 t+\theta)\cos(\omega_0 t+\omega_0\tau+\theta)}$$

$$= \frac{A_0^2}{2}\cos\omega_0\tau + \frac{A_0^2}{2}\underbrace{\overline{\cos(2\omega_0 t+\omega_0\tau+2\theta)}}_{0}$$

$$= \frac{A_0^2}{2}\cos\omega_0\tau \quad ; \text{ not a function of } t$$

$\therefore$ $\underline{\underline{x(t) \text{ is w.s.s.}}}$

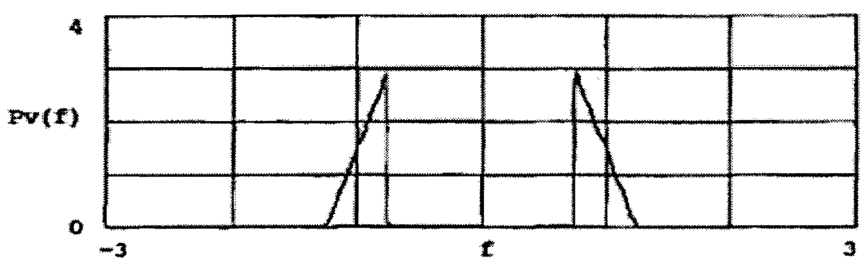
6-43



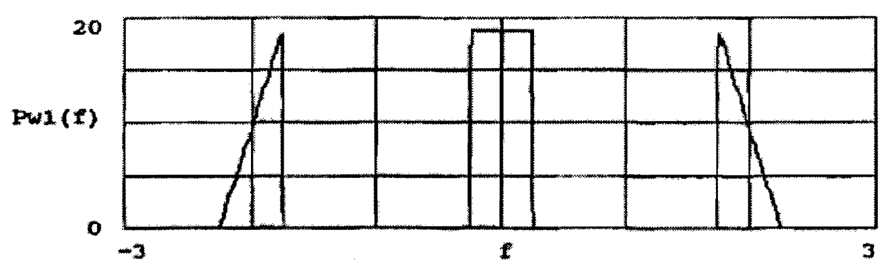
Formulas used in the solution for this problem are derived (essentially) in the textbook. See (6-141) and formulas that follow for Pw1(f) and Px(f) where the 2 in (6-141) is replaced by $(Ao)^2/2$.

f := −3,−2.98 ..3 Ao := 5 fc := 1

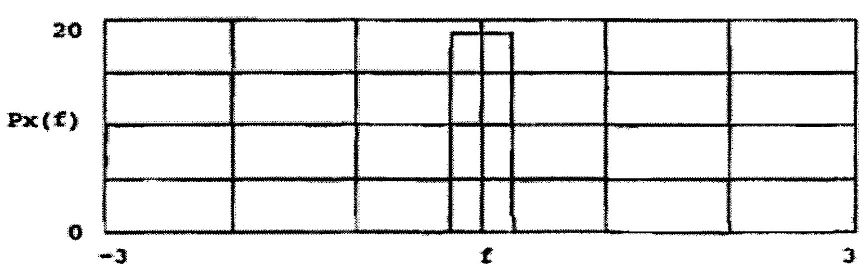
$$Pv(f) := \left[f+\frac{5}{4}\right]\cdot 6\cdot\left[\Phi\left[f+\frac{5}{4}\right]-\Phi\left[f+\frac{3}{4}\right]\right]-\left[f-\frac{5}{4}\right]\cdot 6\left[\Phi\left[f-\frac{3}{4}\right]-\Phi\left[f-\frac{5}{4}\right]\right]$$



$$Pw1(f) := \frac{Ao^2}{4}\cdot(Pv(f-fc)+Pv(f+fc))$$



Px(f) := if(|f| < fc,Pw1(f),0)



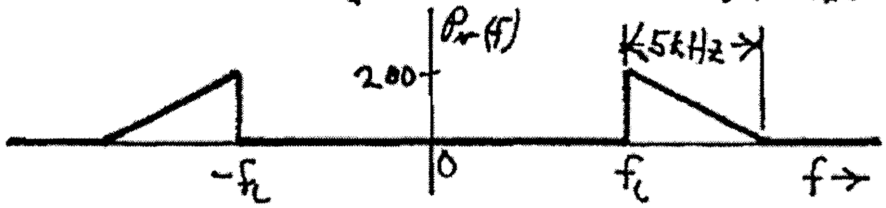
6-45 (a.) $g(t) = 10[x(t) + j\hat{x}(t)]$

$$\Rightarrow G(f) = 10\left\{X(f) + j\begin{bmatrix} -jX(f), & f>0 \\ jX(f), & f<0 \end{bmatrix}\right\} = \begin{Bmatrix} 20X(f), & f>0 \\ 0, & f<0 \end{Bmatrix}$$

$$\Rightarrow \mathcal{P}_v(f) \underset{(6\text{-}133d)}{=} \frac{1}{4}[\mathcal{P}_g(f-f_c) + \mathcal{P}_g(-f-f_c)] = \frac{1}{4}\begin{Bmatrix} 20^2[\mathcal{P}_x(f-f_c) + \mathcal{P}_x(f+f_c)], & |f|>f_c \\ 0, & f \text{ elsewhere} \end{Bmatrix}$$

Thus,

$$\mathcal{P}_v(f) = 100\begin{Bmatrix} [\mathcal{P}_x(f-f_c) + \mathcal{P}_x(f+f_c)], & |f|>f_c \\ 0, & f \text{ elsewhere} \end{Bmatrix}$$



(b.) The normalized average power is

$$P = \int_{-\infty}^{\infty} \mathcal{P}_v(f)\,df \underset{\text{Area under } \mathcal{P}_v(f)}{=} 2\left[\tfrac{1}{2}(200)(5.0k)\right] = 1{,}000k \text{ Watts}$$

$P = 1{,}000k$ Watts (normalized)

6-49 $s(t) = x(t)\cos(\omega_c t + \theta_c)$

$\qquad - y(t)\sin(\omega_c t + \theta_c)$

$\qquad = s_{uSSB}(t) + s_{\ell SSB}(t)$

where

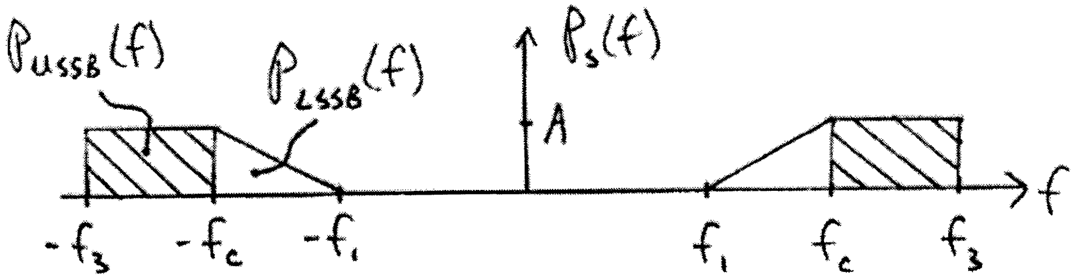
$s_{uSSB}(t) = m_1(t)\cos(\omega_c t + \theta_c) - \hat{m}_1(t)\sin(\omega_c t + \theta_c)$

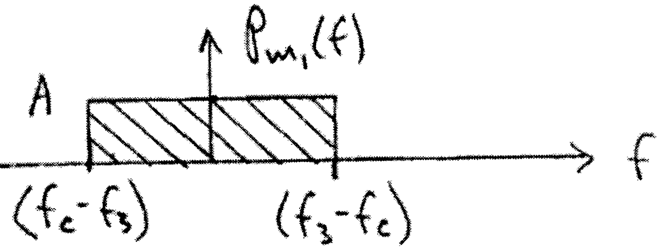
$s_{\ell SSB}(t) = m_2(t)\cos(\omega_c t + \theta_c) + \hat{m}_2(t)\sin(\omega_c t + \theta_c)$

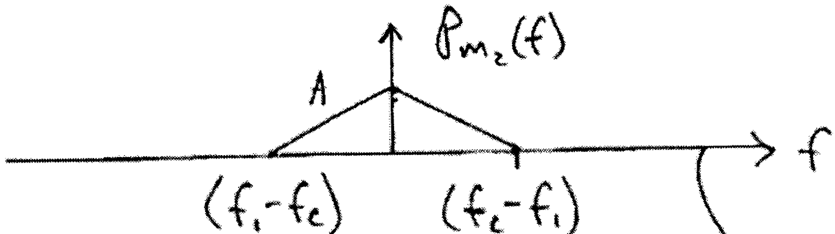
6-49 Cont'd

$$\Rightarrow s(t) = [m_1(t) + m_2(t)]\cos(\omega_c t + \theta_c) - [\hat{m}_1(t) - \hat{m}_2(t)]\sin(\omega_c t + \theta_c)$$

$$\Rightarrow \underline{\underline{x(t) = m_1(t) + m_2(t)}} \;;\; \underline{\underline{y(t) = \hat{m}_1(t) - \hat{m}_2(t)}}$$







$$\underline{\underline{\mathcal{P}_{m_1}(f) = A\,\Pi\left(\frac{f}{2(f_3 - f_c)}\right)}}$$

$$\underline{\underline{\mathcal{P}_{m_2}(f) = A\,\Lambda\left(\frac{f}{f_c - f_1}\right)}}$$

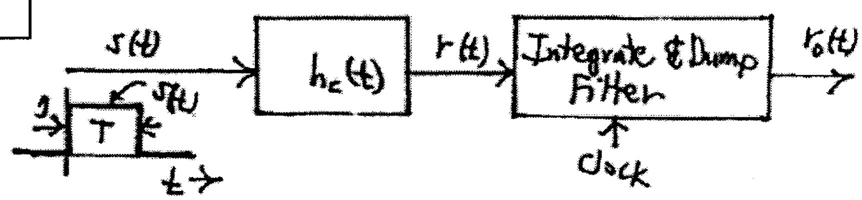
6-52 From example 6-12:

$$\mathcal{P}_v(f) = \frac{1}{4}\left[\mathcal{P}_x(f-f_c) + \mathcal{P}_x(-f-f_c)\right]$$

Where, from solution to problem 6-19:

$$\mathcal{P}_x(f) = T_b\left[\frac{1-\cos(\pi f T_b)}{\pi f T_b}\right]^2 = \mathcal{P}_x(-f)$$

6-56



$$H_c(f) = \frac{B}{B+jf} = \frac{1}{1+j\left(\frac{f}{B}\right)}$$

Let $a = 2\pi B$

Using Table 2-2

$$\Rightarrow h_c(t) = \begin{cases} 2\pi B e^{-2\pi B t}, & t>0 \\ 0, & t<0 \end{cases} = \begin{cases} a e^{-at}, & t>0 \\ 0, & t<0 \end{cases}$$

$$r(t) = s(t) * h_c(t) = \int_0^t s(\lambda) h_c(t-\lambda)\, d\lambda = \begin{cases} \int_0^t a e^{-a(t-\lambda)}\, d\lambda, & 0<t<T \\ \int_0^T a e^{-a(t-\lambda)}\, d\lambda, & t>T \end{cases}$$

$$\Rightarrow r(t) = \begin{cases} 1-e^{-at}, & 0<t<T \\ e^{-at}\left[e^{aT}-1\right], & t>T \end{cases}$$

$$r_0(t) = \begin{cases} \int_0^t r(\lambda)\, d\lambda, & 0<t<T \\ \int_T^t r(\lambda)\, d\lambda, & T<t<2T \end{cases} = \begin{cases} \int_0^t \left[1-e^{-a\lambda}\right] d\lambda, & 0<t<T \\ (e^{aT}-1)\int_T^t e^{-a\lambda}\, d\lambda, & T<t<2T \end{cases}$$

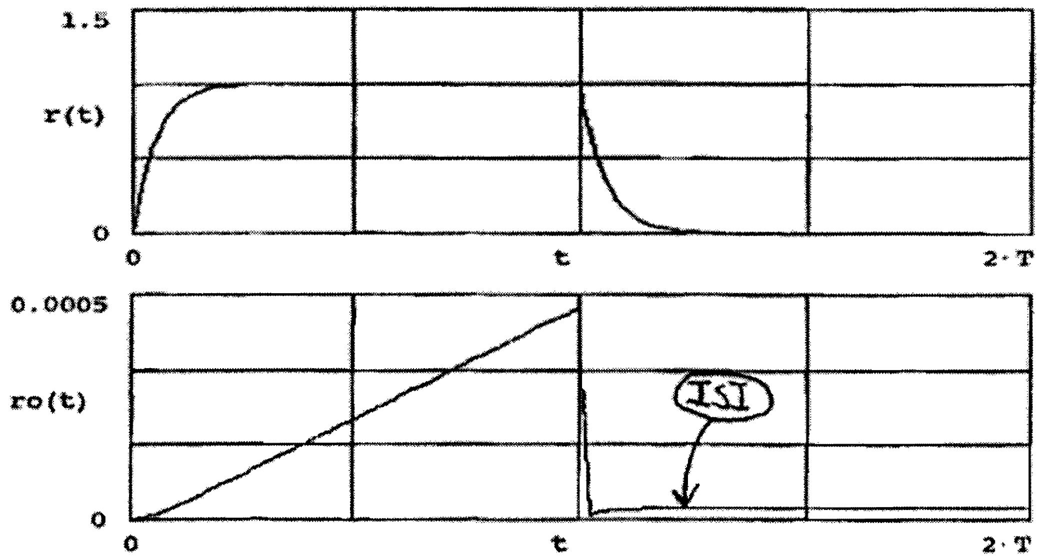
$$\Rightarrow r_0(t) = \begin{cases} t + \frac{1}{a}(e^{-at}-1), & 0<t<T \\ \frac{1}{a}(e^{aT}-1)(e^{-aT}-e^{-at}), & T<t<2T \end{cases}$$

6-56 Cont'd (a)

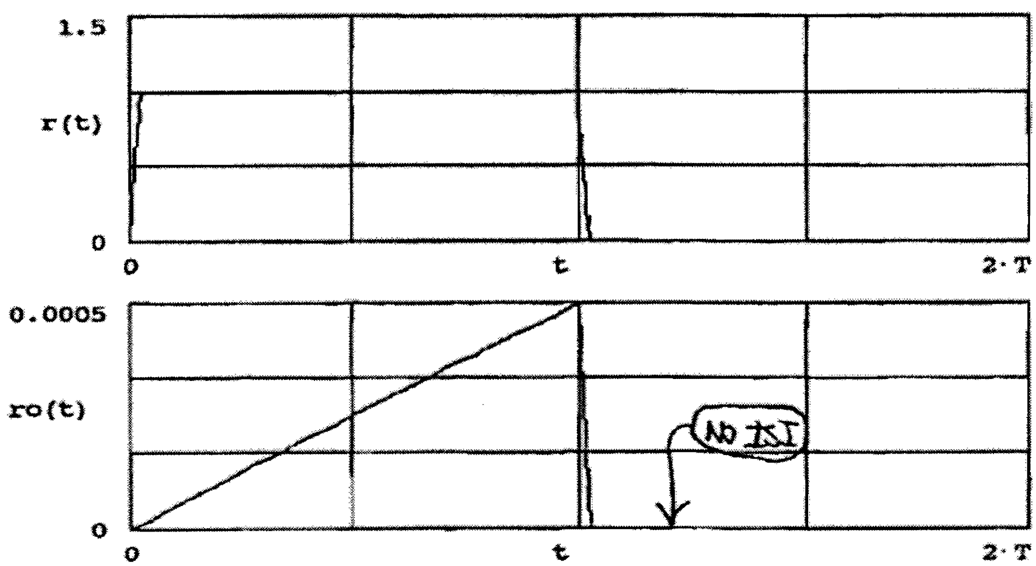
$$R := 2000 \qquad T := \frac{1}{R} \qquad B := 6000 \qquad a := 2\cdot\pi\cdot B \qquad t := 0, \frac{T}{40} .. 2\,T$$

$$r(t) := \text{if}\left[t \le T, 1 - e^{-a\,t}, \left[e^{a\,T} - 1\right]\cdot e^{-a\cdot t}\right]$$

$$ro(t) := \text{if}\left[t \le T, t + \frac{e^{-a\cdot t} - 1}{a}, \frac{e^{a\cdot T} - 1}{a}\cdot\left[e^{-a\,T} - e^{-a\cdot t}\right]\right]$$



(b.) For all pass channel ⇒ Let $B \to \infty$. ⇒ Use $B = 100{,}000$.
Then, results are as follows.

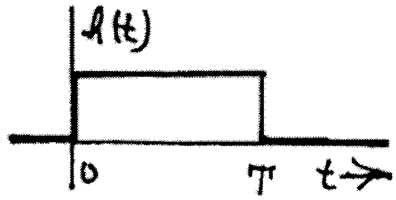


6-58 (a.) $y(t) = \int_{-\infty}^{t} [x(t_1) - x(t_1 - T)]\, dt_1$

For the impulse response let $x(t) = \delta(t)$.

$\Rightarrow y(t) = h(t) = \int_{-\infty}^{t} [\delta(t_1) - \delta(t_1 - T)]\, dt_1$

$$h(t) = \begin{cases} \frac{1}{2}, & t = 0 \\ 1, & 0 < t < T \\ \frac{1}{2}, & t = T \\ 0, & t \text{ elsewhere} \end{cases}$$



(b.) $s(t) = h(t_0 - t)$ where $t_0 = T$ $\Rightarrow s(t) = h(t)$ above.

6-59 (a.) $s(t) = s_1(t) = A\cos\omega_1 t$

$w_1(t) = s_1(t)\cos(\omega_1 t) = A\cos^2(\omega_1 t) = \frac{1}{2}A[1 + \cos(2\omega_1 t)]$

$v_1(t) = \int_0^t w_1(\lambda)\,d\lambda = \frac{1}{2}A\int_0^t [1 + \cos(2\omega_1\lambda)]\,d\lambda = \frac{1}{2}A\left[\lambda + \frac{\sin(2\omega_1\lambda)}{2\omega_1}\right]_0^t$

$$\Rightarrow \underline{\underline{v_1(t) = \frac{1}{2}A\left[t + \frac{\sin(2\omega_1 t)}{2\omega_1}\right]}}$$

$w_2(t) = s_1(t)\cos(\omega_2 t) = A\cos(\omega_1 t)\cos(\omega_2 t)$

$= \frac{1}{2}A[\cos(\omega_1 - \omega_2)t + \cos(\omega_1 + \omega_2)t]$

or $w_2(t) = \frac{1}{2}A[\cos(2\Delta\omega t) + \cos((\omega_1 + \omega_2)t)]$ where $\Delta\omega = 2\pi\Delta F$

$v_2(t) = \int_0^t w_2(\lambda)\,d\lambda = \frac{1}{2}A\int_0^t [\cos(2\Delta\omega\lambda) + \cos(\omega_1 + \omega_2)\lambda]\,d\lambda$

$$\Rightarrow \underline{\underline{v_2(t) = \frac{1}{2}A\left[\frac{\sin(2\Delta\omega t)}{2\Delta\omega} + \frac{\sin(\omega_1 + \omega_2)t}{\omega_1 + \omega_2}\right]}}$$

$$\underline{\underline{r_0(t) = v_1(t) - v_2(t)}}$$

(b.) $r(t) = s(t) = A\cos\omega_1 t$, $0 < t < T$ $\Rightarrow h(t) = A\cos[\omega_1(T - t)]$

Thus, $r_0(t) = r(t) * h(t) = \int_0^t r(\lambda)h(t - \lambda)$, $0 < t < T$

6-59 (b.) Cont'd.

$$r_o(t) = A^2 \int_0^t \cos(\omega_1\lambda)\cos[\omega_1(T-(t-\lambda))]\,d\lambda$$

$$= \tfrac{1}{2}A^2 \int_0^t [\cos(\omega_1 T - \omega_1 t) + \cos(2\omega_1\lambda + \omega_1 T - \omega_1 t)]\,d\lambda$$

$$= \tfrac{1}{2}A^2 \left[\lambda\cos(\omega_1 T - \omega_1 t) + \frac{\sin(2\omega_1\lambda + \omega_1 T - \omega_1 t)}{2\omega_1}\right]\Bigg|_0^t$$

$$= \tfrac{1}{2}A^2 \left[t\cos(\omega_1 T - \omega_1 t) + \frac{\sin(2\omega_1 t + \omega_1 T - \omega_1 t) - \sin(\omega_1 T - \omega_1 t)}{2\omega_1}\right]$$

Thus,

$$\underline{\underline{r_o(t) = \tfrac{1}{2}A^2\left[t\cos\omega_1(T-t) + \frac{\sin\omega_1(T+t) - \sin\omega_1(T-t)}{2\omega_1}\right]}}$$

$f_c := 1000 \qquad \delta f := 50 \qquad \omega 1 := 2\cdot\pi\cdot[f_c - \delta f] \qquad \omega 2 := 2\cdot\pi\cdot[f_c + \delta f]$

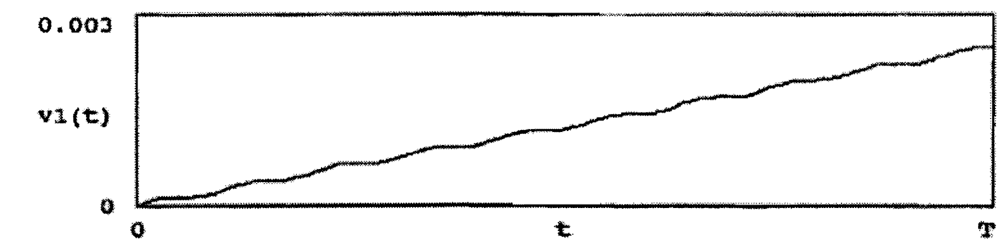
$A := 1 \qquad T := \dfrac{1}{4\,\delta f} \qquad t := 0, \dfrac{T}{100} .. T \qquad \delta\omega := 2\cdot\pi\cdot\delta f$

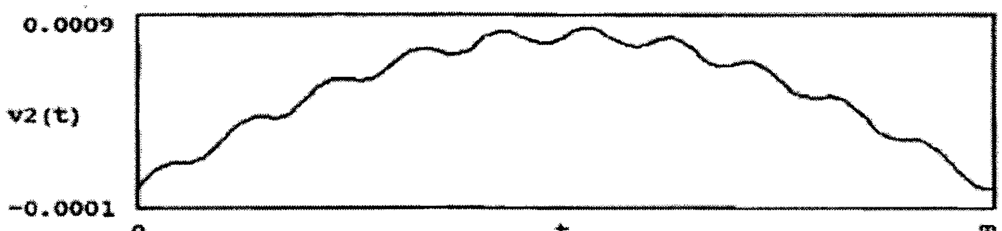
(a.)

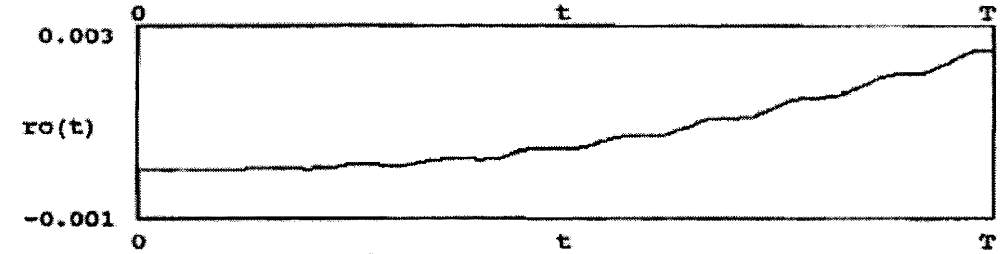
$$v1(t) := \frac{A}{2}\cdot\left[t + \frac{\sin(2\cdot\omega 1\cdot t)}{2\cdot\omega 1}\right]$$

$$v2(t) := \frac{A}{2}\cdot\left[\frac{\sin((\omega 1 + \omega 2)\cdot t)}{\omega 1 + \omega 2} + \frac{\sin(2\cdot\delta\omega\cdot t)}{2\cdot\delta\omega}\right]$$

$ro(t) := v1(t) - v2(t) \qquad T = 0.005$





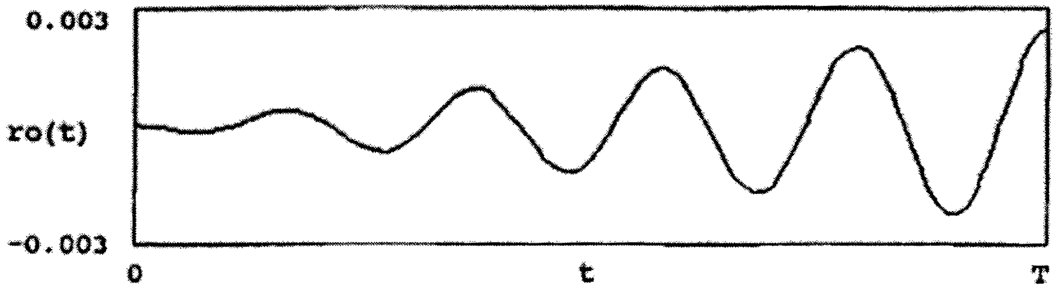


$ro(0.5\ T) = 4.123\cdot 10^{-4} \qquad \underline{ro(T) = 0.003}$

6-59 Cont'd.

(b.)

$$ro(t) := \frac{A^2}{2}\cdot\left[t\cdot\cos(\omega 1\ (T - t)) + \frac{\sin(\omega 1\ (T + t)) - \sin(\omega 1\cdot(T - t))}{2\ \omega 1}\right]$$



$ro(0.5\cdot T) = -8.839\cdot 10^{-4}$ $\qquad$ $\underline{\underline{ro(T) = 0.003}}$

(c.) The results for parts (a.) and (b.) are different for $0 < t < T$. However, at the sampling time, $t = T$, the results are identical. That is, MathCAD computes $r_0(T)$ to be 0.003 for both cases.

Chapter 7

7-1 (a.) $r_0 = \begin{Bmatrix} A + n_0 \ , \ s_1 \text{ sent} \\ -A + n_0 \ , \ s_2 \text{ sent} \end{Bmatrix}$

$$\Rightarrow f(r_0|s_1) = \frac{1}{\sqrt{2}\Delta_0} e^{-\frac{\sqrt{2}|r_0 - A|}{\Delta_0}}$$

$$f(r_0|s_2) = \frac{1}{\sqrt{2}\Delta_0} e^{-\frac{\sqrt{2}|r_0 + A|}{\Delta_0}}$$

Using (7-8)

$$P_e = \frac{1}{2}\int_{-\infty}^{V_T} \frac{1}{\sqrt{2}\Delta_0} e^{-\frac{\sqrt{2}|r_0 - A|}{\Delta_0}} dr_0 + \frac{1}{2}\int_{V_T}^{\infty} \frac{1}{\sqrt{2}\Delta_0} e^{-\frac{\sqrt{2}|r_0 + A|}{\Delta_0}} dr_0$$

$m_{r_{01}} = A$, $m_{r_{02}} = -A$, the source probabilities are equally likely, and the conditional probabilities have symmetrical shapes about $\pm A$.

Thus $\boxed{V_T = 0}$.

$$\therefore P_e = \frac{1}{2\sqrt{2}\Delta_0}\left[\int_{-\infty}^{0} e^{-\frac{\sqrt{2}|r_0 - A|}{\Delta_0}} dr_0 + \int_0^{\infty} e^{-\frac{\sqrt{2}|r_0 + A|}{\Delta_0}} dr_0\right]$$

$$= \frac{1}{2\sqrt{2}\Delta_0}\left[\int_{-\infty}^{0} e^{\frac{\sqrt{2}(r_0 - A)}{\Delta_0}} dr_0 + \int_0^{\infty} e^{-\frac{\sqrt{2}(r_0 + A)}{\Delta_0}} dr_0\right]$$

Let $x_1 = \frac{\sqrt{2}(r_0 - A)}{\Delta_0}$ and $x_2 = \frac{-\sqrt{2}(r_0 + A)}{\Delta_0}$

$dx_1 = \frac{\sqrt{2}}{\Delta_0} dr_0 \qquad dx_2 = \frac{-\sqrt{2}}{\Delta_0} dr_0$

$$= \frac{1}{2\sqrt{2}\Delta_0}\left[\int_{-\infty}^{-\sqrt{2}A/\Delta_0} e^{x_1}\left(\frac{\Delta_0}{\sqrt{2}} dx_1\right) + \int_{-\sqrt{2}A/\Delta_0}^{-\infty} e^{x_2}\left(\frac{-\Delta_0}{\sqrt{2}} dx_2\right)\right]$$

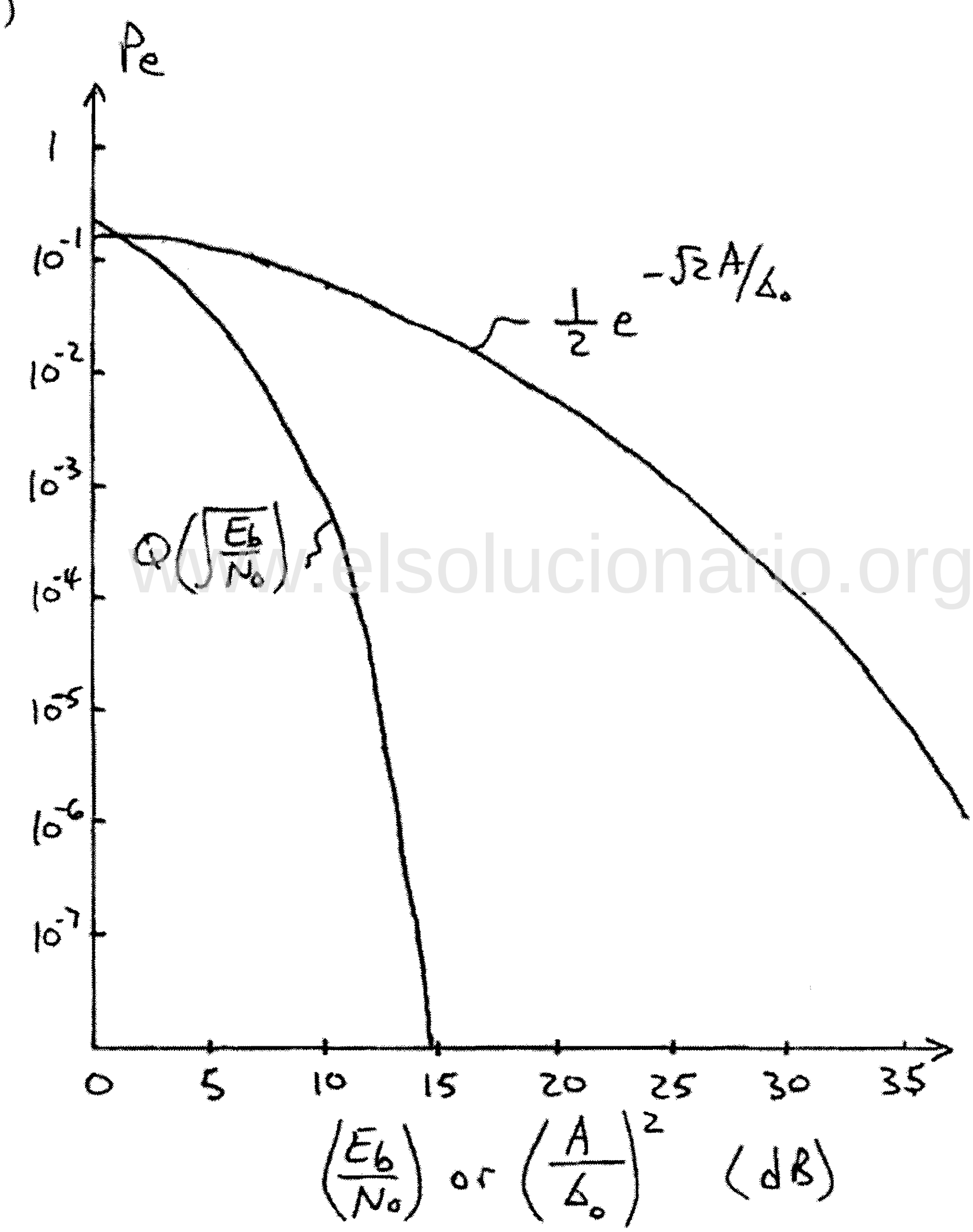
$$= \frac{1}{4}\left[\int_{-\infty}^{-\sqrt{2}A/\Delta_0} e^{x_1} dx_1 + \int_{-\infty}^{-\sqrt{2}A/\Delta_0} e^{x_2} dx_2\right]$$

7-1. (a.) Cont'd

$$P_e = \frac{1}{2}\left[\int_{-\infty}^{-\sqrt{2}A/\Delta_0} e^{x}\,dx\right] = \frac{1}{2}\, e^{x}\Big|_{-\infty}^{-\sqrt{2}A/\Delta_0}$$

$$= \frac{1}{2}\left\{e^{-\sqrt{2}A/\Delta_0} - e^{-\infty}\right\} = \underline{\underline{\frac{1}{2}\, e^{-\sqrt{2}A/\Delta_0}}} = P_e$$

(b.)



P_e <u>much</u> larger for Laplacian Noise.

7-6

$$\left(\frac{S}{N}\right)_{in} = \frac{\frac{E_b}{T_b}}{\left(\frac{N_0}{2}\right)(2B_{eq})} = \frac{E_b R}{N_0 B_{eq}} \Rightarrow \frac{E_b}{N_0} = \frac{B_{eq}}{R}\left(\frac{S}{N}\right)_{in}$$

Aside:

$$B_{eq} = \frac{\int_{-\infty}^{\infty}|H(f)|^2 df}{2|H(0)|^2} \overset{\uparrow}{=} \frac{\left(\frac{2K}{N_0}\right)^2\int_{-\infty}^{\infty}|S(f)|^2 df}{2\left(\frac{2K}{N_0}\right)^2|S(0)|^2} = \frac{\int_{-\infty}^{\infty}|S(f)|^2 df}{2|S(0)|^2}$$

Using (6-155) for MF: $H(f) = \frac{K S^*(f) e^{-j\omega t_0}}{N_0/2}$

$$\Rightarrow B_{eq} = \frac{T_b^2\int_{-\infty}^{\infty}\left(\frac{\sin(\pi T_b f)}{\pi T_b f}\right)^2 df}{2 T_b^2 \, 1^2} = \frac{1}{2\pi T_b}\int_{-\infty}^{\infty}\left(\frac{\sin x}{x}\right)^2 dx = \frac{\pi}{2\pi T_b} = \frac{1}{2}R$$

$s(t) = \Pi\left(\frac{t}{T_b}\right) \leftrightarrow S(f) = T_b\frac{\sin(\pi T_b f)}{\pi T_b f}$

Let $x = \pi T_b f$
$dx = \pi T_b \, df$

$$\Rightarrow \frac{E_b}{N_0} = \frac{\frac{1}{2}R}{R}\left(\frac{S}{N}\right)_{in} = \underline{\frac{1}{2}\left(\frac{S}{N}\right)_{in} = \frac{E_b}{N_0}}$$

Using (7-24b):

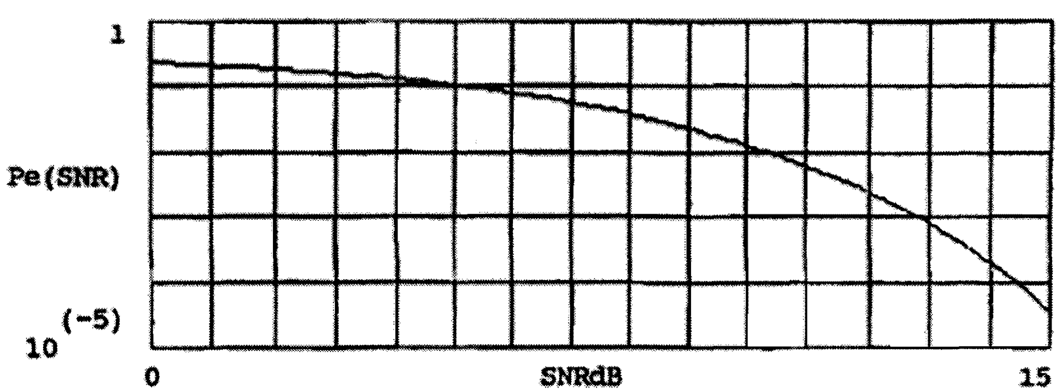
$$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right) = Q\left(\sqrt{\frac{1}{2}\left(\frac{S}{N}\right)_{in}}\right)$$

SNRdB := 0,0.1 ..15

Q(x) := 1 − cnorm(x)

$$SNR(SNRdB) := 10^{\frac{SNRdB}{10}}$$

$$Pe(SNR) := Q\left[\sqrt{0.5 \cdot SNR(SNRdB)}\right]$$



7-9 (a.) For derivation of P_e, follow the same procedure as used in the solution for Prob. 7-8.

$$B_{eq} = \int_0^\infty \frac{|H(f)|^2}{|H(0)|^2}\,df = \int_0^\infty \frac{1}{1+\left(\frac{f}{f_0}\right)^4}\,df = f_0 \int_0^\infty \frac{1}{1+x^4}\,dx = \frac{f_0 \pi}{2\sqrt{2}}$$

Let $x = f/f_0$ Using Sec. A-3

$$s_{01} = \int_0^T s_{01}(T-\lambda)\,h(\lambda)\,d\lambda = \int_0^T A\left[\sqrt{2}\,\omega_0 e^{-(\omega_0/\sqrt{2})\lambda} \sin\left(\frac{\omega_0}{\sqrt{2}}\lambda\right)\right] d\lambda$$

or

$$s_{01} = \int_0^T 2A\,e^{-x} \sin(x)\,dx = A\left[1 - e^{-\sqrt{2}\pi}\left(\sin(\sqrt{2}\pi) + \cos(\sqrt{2}\pi)\right)\right]$$

Let $x = \frac{\omega_0}{\sqrt{2}}\lambda$ Using Sec. A-5

$$\Rightarrow \underline{s_{01} = 1.01447\,A}$$

$f_0 = \frac{1}{T}$

$$\sigma_0^2 = \frac{N_0}{2}\,2B_{eq} = N_0 B_{eq} = \frac{N_0 \pi f_0}{2\sqrt{2}} = \frac{N_0 \pi}{2\sqrt{2}\,T}$$

Using (7-17) where $s_{01} = -s_{02}$,

$$P_e = Q\left(\sqrt{\frac{s_{01}^2}{\sigma_0^2}}\right) = Q\left(\sqrt{\frac{(1.01447)^2 A^2}{\frac{N_0 \pi}{2\sqrt{2}\,T}}}\right) = Q\left(\sqrt{\frac{2\sqrt{2}(1.01447)^2 A^2 T}{\pi N_0}}\right)$$

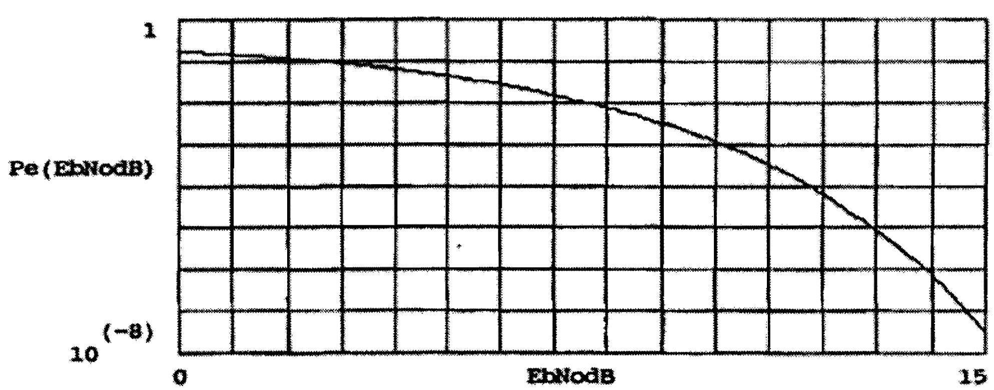
$$\Rightarrow P_e = Q\left(\sqrt{\frac{2\sqrt{2}(1.01447)^2}{\pi}\left(\frac{E_b}{N_0}\right)}\right) = \underline{\underline{Q\left(\sqrt{0.92656\left(\frac{E_b}{N_0}\right)}\right)}}$$

(b.) EbNodB := 0,0.1 ..15

Q(x) := 1 - cnorm(x)

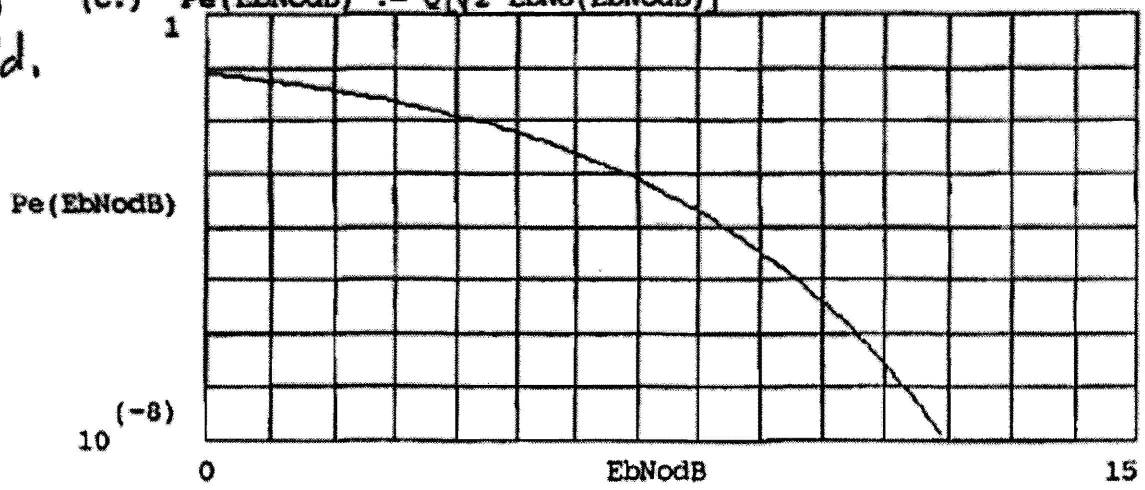
$$\text{EbNo(EbNodB)} := 10^{\frac{\text{EbNodB}}{10}}$$

Pe(EbNodB) := $Q\left[\sqrt{0.92656 \cdot \text{EbNo(EbNodB)}}\right]$



7-9 Cont'd.

(c.) $Pe(EbNodB) := Q\left[\sqrt{2\ EbNo(EbNodB)}\right]$



7-12

(a.) Referring to the solution for SA 7-3,

$$P_e = Q\left(\sqrt{\frac{A^2}{4N_0B}}\right) \qquad (7\text{-}24a)$$

where $B = \frac{2}{T} = 2R$ and $E_b = \left(\frac{A^2}{2}\right)T = \frac{A^2}{2R}$

Thus,

$$P_e = Q\left(\sqrt{\frac{A^2}{8N_0R}}\right) = Q\left(\sqrt{\frac{A^2}{4N_0 2R}}\right) = Q\left(\sqrt{\frac{1}{4}\left(\frac{E_b}{N_0}\right)}\right)$$

7-12 Cont'd

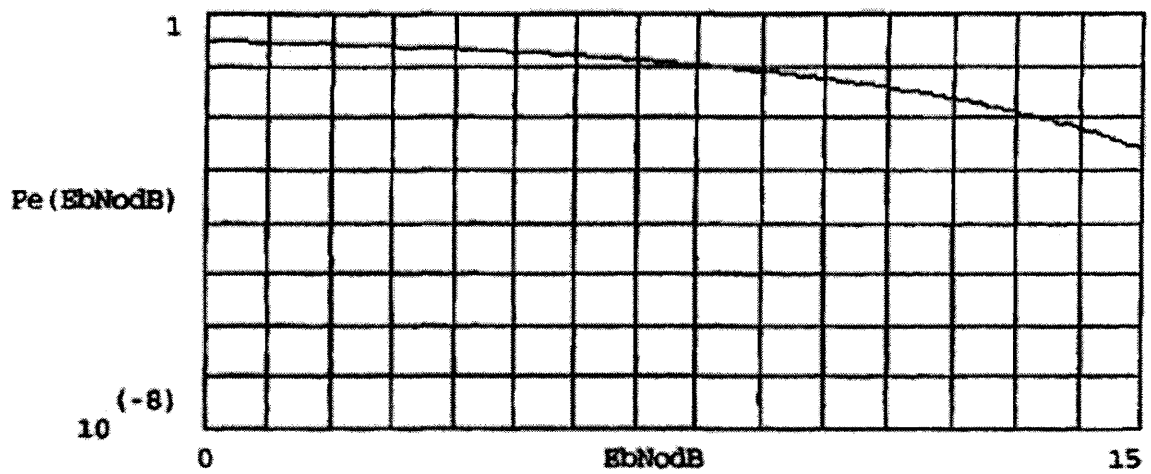
$EbNodB := 0,0.1..15$

$Q(x) := 1 - cnorm(x)$

$$EbNo(EbNodB) := 10^{\frac{EbNodB}{10}}$$

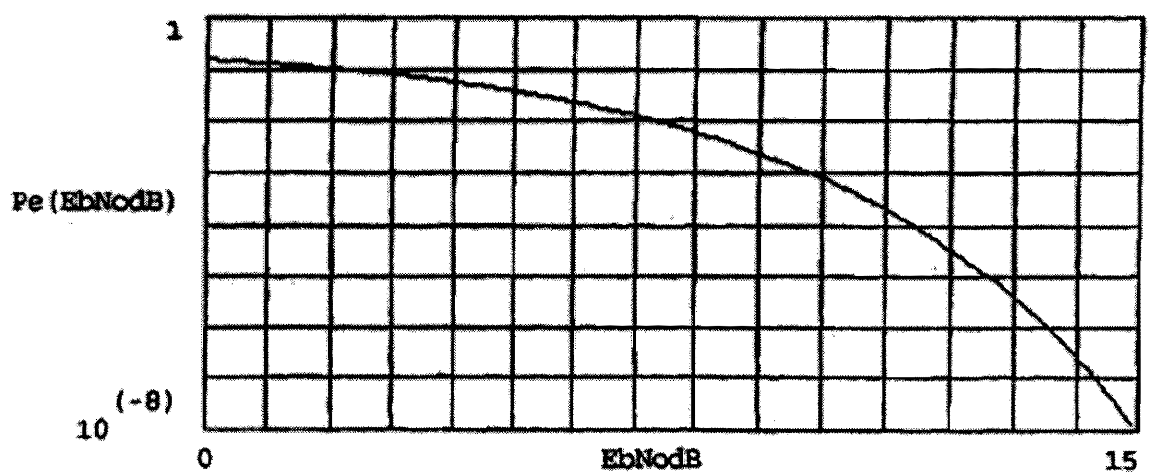
(a.)

$Pe(EbNodB) := Q\left[\sqrt{0.25 \cdot EbNo(EbNodB)}\right]$ <---- LPF with ISI Result



(b.)

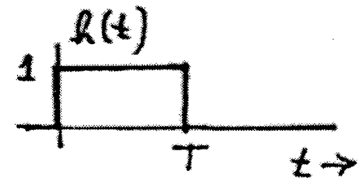
$Pe(EbNodB) := Q\left[\sqrt{EbNo(EbNodB)}\right]$ Matched Filter Result <-- (7-24b)



7-15

(a) The impulse response is:

$$h(t) = s_{01}(T-t) = \underline{\underline{s_{01}(t)}}$$



$$H(f) = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt = \int_0^T e^{-j\omega t}\, dt = \left.\frac{e^{-j\omega t}}{-j\omega}\right|_0^T = \frac{e^{-j\omega T} - e^{-j(0)}}{-j\omega}$$

$$\Rightarrow H(f) = e^{-j\omega T/2}\left[\frac{e^{j\omega T/2} - e^{-j\omega T/2}}{j 2\pi f}\right] = \underline{\underline{T e^{-j\pi f T}\left[\frac{\sin(\pi f T)}{\pi f T}\right]}}$$

See Fig. 6-17

(b)

$$B_{eq} = \frac{\int_0^{\infty} |H(f)|^2\, df}{|H(0)|^2} = \frac{T^2 \int_0^{\infty}\left[\frac{\sin(\pi f T)}{\pi f T}\right]^2 df}{T^2}$$

$$= \int_0^{\infty}\left(\frac{\sin x}{x}\right)^2\left(\frac{1}{\pi T}\, dx\right) = \frac{1}{\pi T}\left(\frac{\pi}{2}\right) = \underline{\underline{\frac{1}{2T} = B_{eq}}}$$

Let $x = \pi f T$; $dx = \pi T\, df$

Using Sec. A-5

7-17 From (7-8)

$$P_e = P(1)\int_{-\infty}^{V_T} f(r_0|s_1)\, dr_0 + P(0)\int_{V_T}^{\infty} f(r_0|s_2)\, dr_0$$

where

$$r_0 = \begin{cases} A + n_0, & \text{for a binary 1 sent} \\ -A + n_0, & \text{" 0 "} \end{cases}$$

Thus

$$P_e = P(1)\int_{-\infty}^{V_T} \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{(r_0 - A)^2}{2\sigma^2}}\, dr_0 + P(0)\int_{V_T}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{(r_0 + A)^2}{2\sigma^2}}\, dr_0$$

Let $\lambda_1 = -(r_0 - A)/\sigma$; $\lambda_2 = (r_0 + A)/\sigma$

$d\lambda_1 = -\frac{1}{\sigma}\, dr_0$ $\quad d\lambda_2 = \frac{1}{\sigma}\, dr_0$

7-17 Cont'd

$$\Rightarrow P_e = P(1)\int_{(-V_T+A)/\sigma}^{\infty} \frac{1}{\sqrt{2\pi}}\, e^{-\lambda_1^2/2}\, d\lambda_1 + P(0)\int_{(V_T+A)/\sigma}^{\infty} \frac{1}{\sqrt{2\pi}}\, e^{-\lambda_2^2/2}\, d\lambda_2$$

$$\text{or}\quad P_e = P(1)\, Q\left[\frac{-V_T+A}{\sigma}\right] + P(0)\, Q\left[\frac{V_T+A}{\sigma}\right]$$

$$= P(1)\, Q\left[\sqrt{\frac{(-V_T+A)^2\, 2T}{N_0}}\right] + P(0)\, Q\left[\sqrt{\frac{(V_T+A)^2\, 2T}{N_0}}\right]$$

$\sigma^2 = \frac{N_0}{2T}$

To Check: Let $P(1) = P(0) = \frac{1}{2}$; $V_T = 0$

$$P_e = \frac{1}{2}\, Q\left[\sqrt{\frac{A^2\, 2T}{N_0}}\right] + \frac{1}{2}\, Q\left[\sqrt{\frac{A^2\, 2T}{N_0}}\right]$$

$$= Q\left[\sqrt{\frac{2A^2 T}{N_0}}\right]$$; This checks with eqn. (7-26b)

7-21 Referring to Fig. 7-7

(a.) Let $s_1(t) = A\cos w_c t$, $s_2(t) = -A\cos w_c t$

$n(t) = x(t)\cos w_c t - y(t)\sin w_c t$

Coherent reference $= 2\cos(w_c t + \theta_e)$

$$\Rightarrow r_0(t) \underset{(A\text{-}11)}{=} \pm A\cos\theta_e + n_0(t) = s_{0_{1\,\text{or}\,2}}(t) + n_0(t)$$

where $n_0(t) = x(t)\cos\theta_e + y(t)\sin\theta_e$

$$\overline{n_0^2(t)} = \overline{x^2(t)}\cos^2\theta_e + 2\,\underbrace{\overline{x(t)\,y(t)}}_{0\ (6\text{-}133k)}\cos\theta_e\sin\theta_e + \overline{y^2(t)}\sin^2\theta_e$$

$$\Rightarrow \overline{n_0^2(t)} \underset{\overline{x^2} = \overline{y^2} = 2N_0B}{=} 2N_0B(\cos^2\theta_e + \sin^2\theta_e) = 2N_0B$$

Using (7-17): $H(f) =$ LPF

Corresponds to (7-36)

$$P_e = Q\left[\pm\sqrt{\frac{(s_{01}-s_{02})^2}{4\sigma_0^2}}\right] = Q\left[\pm\sqrt{\frac{4A^2\cos^2\theta_e}{8N_0B}}\right] = Q\left[\pm\sqrt{\frac{A^2\cos^2\theta_e}{2N_0B}}\right]$$

where $\left\{\begin{array}{l} + \text{ is used when } s_{01} > s_{02} \Rightarrow |\theta_e| < \pi/2 \\ - \text{ is used when } s_{02} > s_{01} \Rightarrow \frac{\pi}{2} < |\theta_e| < \pi \end{array}\right\}$

7-21 Cont'd

(b.) If $H(f)$ is matched to the output of the multiplier, then

$$P_e = Q\left(\sqrt{\frac{E_d}{2N_0'}}\right) \quad \text{using (7-20)}$$

Where $N_0' = 2N_0$ and

$$E_d = \int_0^T [s_{01}(t) - s_{02}(t)]^2 dt = \int_0^T 4A^2 \cos^2\theta_e \, dt$$

or

$$E_d = 4A^2T\cos^2\theta_e = 8E_b\cos^2\theta_e$$

$E_b = \frac{1}{2}E_{s_1} + \frac{1}{2}E_{s_2} = \frac{1}{2}\left(\frac{1}{2}A^2T\right) + \frac{1}{2}\left(\frac{1}{2}A^2T\right) = \frac{1}{2}A^2T$

$$\Rightarrow \underline{\underline{P_e = Q\left[\sqrt{2\left(\frac{E_b}{N_0}\right)\cos^2\theta_e}\right]}}$$

Corresponds to (7-38)

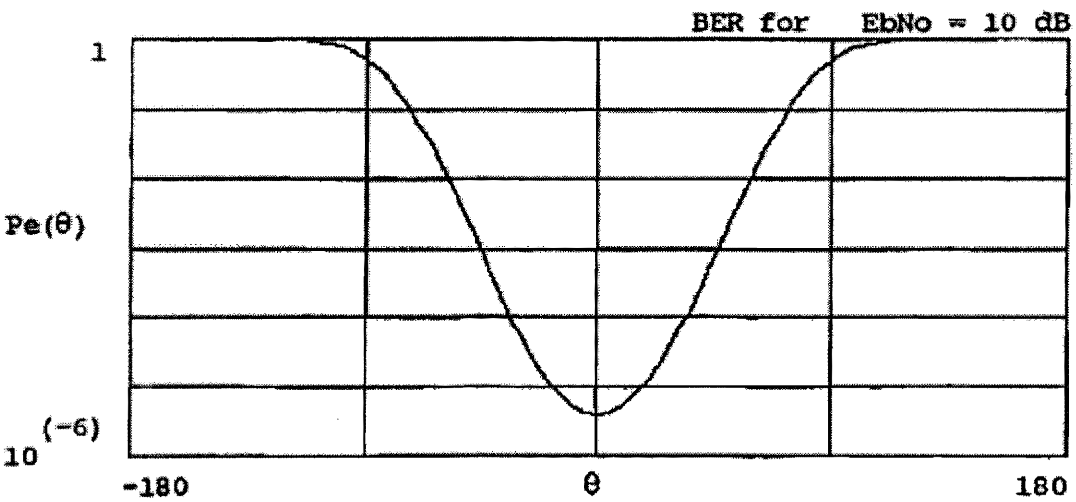
(c.) EbNodB := 10 θ := −180, −175 ..180

$EbNo := 10^{\frac{EbNodB}{10}}$

Q(x) := 1 − cnorm(x)

$$z(\theta) := \sqrt{2\ EbNo \cdot \left[\cos\left[\pi \frac{\theta}{180}\right]\right]^2}$$

Pe(θ) := if(|θ| ≤ 90, Q(z(θ)), Q(−z(θ)))



7-22 (a.) Overall $P_e = 10\,(P_e)_i = \underline{\underline{5 \times 10^{-7}}}$

(b.) When repeaters were used, the E_b/N_0 at the input to each was described by:

$$P_e = Q\left[\sqrt{2\left(\frac{E_b}{N_0}\right)}\right] = 5 \times 10^{-8} \underset{\text{Sec. A-10}}{\approx} \frac{1}{\sqrt{2\pi\left(\frac{2E_b}{N_0}\right)}}\, e^{-E_b/N_0}$$

$$\Rightarrow \frac{E_b}{N_0} = \underline{14.2}$$

Now with 10 amplifiers, the Rx input consists of the BPSK signal with E_b energy/bit plus a noise level 10 times that present before (since the line from one amp to the next contributes a PSD of $N_0/2$, and there are 10 such lines). Thus $\left(\frac{E_b}{N_0}\right)' = \frac{14.2}{10} = \underline{1.42}$

$$\therefore\ P_e' = Q\left(\sqrt{2(1.42)}\right) = Q(1.69) \approx Q(1.7) = \underline{\underline{4.4 \times 10^{-2}}}$$

7-23 (a.) $B_T = 2700 - 300 = 2400\ \text{Hz}$ (Band pass System)

The largest bit rate that can be accomodated w/o I.S.I. is (Table 7-1.)

$$B = \underline{\underline{R = 2400}}\ \text{bits/sec}$$

(b.) From Table 7.1 for BPSK:

$$P_e = Q\left(\sqrt{2\left(\frac{E_b}{N_0}\right)}\right) \underset{\text{Sec. A-10}}{\approx} \frac{1}{\sqrt{4\pi\left(\frac{E_b}{N_0}\right)}}\, e^{-E_b/N_0}$$

, for each repeater

$$\text{But}\ \left(\frac{S}{N}\right) = \frac{P_s}{N_0 B_T} \underset{B_T = R}{=} \frac{P_s}{N_0 R} \underset{R = \frac{1}{T}}{=} \frac{P_s T}{N_0} = \frac{E_b}{N_0} = \frac{S}{N} = 15\,\text{dB}$$

7-23 Cont'd

$$\Rightarrow \frac{E_b}{N_o} = 15\,dB = 31.6$$

$$\therefore P_e = \frac{1}{\sqrt{4\pi(31.6)}}\, e^{-31.6} = 9.26 \times 10^{-16}/\text{repeater}$$

There are $n = \dfrac{600 \text{ mi.}}{50 \text{ mi/rep}} = 12$ repeaters (including R_x)

$$\therefore \text{Overall } (P_e) \approx n P_e = 12(9.26\times 10^{-16}) = \underline{\underline{1.11\times 10^{-14}}}$$

Note: If there are n repeaters, there is an error at the end of the line only if there are an odd number of errors along the line (for the bit in question).

$$\Rightarrow P(\underline{K}\text{ errors}) = \binom{n}{K} P_e^{K}(1-P_e)^{n-K} \; ; \; \binom{n}{K} = \frac{n!}{(n-K)!\,K!}$$

$$\Rightarrow \text{Overall } P_e = \sum_{\substack{K=1 \\ K\text{ odd}}}^{n} P(K \text{ errors}) \approx n P_e \quad \text{If } nP_e \ll 1$$

7-29 From Table 7-1 for FSK w/ noncoherent detection:

$$P_e = \frac{1}{2} e^{-\frac{1}{2}(E_b/N_{total})}$$

$$N_{total} = k(T_o + T_{eff}) = k(T_o + (F-1)T_o) = kFT_o$$

$$T_{eff} = (F-1)T_o \qquad = (1.38\times 10^{-23})(10^{6/10})\,290$$

$$\frac{E_b}{N_{total}} = \frac{P_s T}{N_{tot}} = \frac{P_s}{N_{tot} R} = \frac{V_s^2/R_A}{kFT_o R} = 28.53$$

$$R = 1/T$$

$$P_e = \frac{1}{2} e^{-\frac{1}{2}(28.53)} = \underline{\underline{3.2\times 10^{-7}}}$$

7-33 See Table 7-1.

(a.) For QPSK the largest R and min P_e are obtained.

$$R = 2B = 2(2700-300) = \underline{\underline{4800 \text{ b/s}}}$$

$$\frac{S}{N} = \frac{P_s}{\frac{N_0}{2}(2B)} \underset{\uparrow}{=} \frac{2P_s}{N_0 R} \underset{\uparrow}{=} \frac{2P_sT}{N_0} = \frac{2E_b}{N_0}$$

$B = \frac{1}{2}R$ $\quad$ $R = \frac{1}{T}$

$$\Rightarrow \frac{E_b}{N_0} = \frac{1}{2}\left(\frac{S}{N}\right) = \frac{1}{2}\left(10^{2.5}\right) = 158.1 \Rightarrow 22 \text{ dB}$$

$$P_e = Q\left(\sqrt{2\left(\frac{E_b}{N_0}\right)}\right) \ll 10^{-5} \text{ for } \frac{E_b}{N_0} = 22 \text{ dB}$$

Figure 7-14.

$$C = B\log_2\left(1+\frac{S}{N}\right) = 2400\log_2\left(1+10^{2.5}\right)$$

$$= \frac{2400}{\ln 2}\ln(1+316.23) = 1.99\times10^4 = \underline{\underline{19{,}900 \text{ b/s}}}$$

7-34 (a.) $R = \left(8K \frac{\text{samples}}{\text{sec}}\right)\left(8 \frac{\text{bits}}{\text{sample}}\right) = 64\ K\ b/s$

$$\frac{S}{N} = \frac{P_s}{N_o B} \underset{\uparrow}{=} \frac{P_s T_b}{N_o} = \frac{E_b}{N_o} = 10^{.8} = 6.3$$

$B = R = \frac{1}{T_b}$ ← Table 7-1. for DPSK :

$$P_e = \frac{1}{2} e^{-\left(\frac{E_b}{N_o}\right)} = \frac{1}{2} e^{-6.3} = \underline{\underline{9.18 \times 10^{-4}}}$$

(b.) Using (7-70) with $M = 2^8 = 256$:

$$\left(\frac{S}{N}\right)_{OUT} = \frac{3M^2}{1+4(M^2-1)P_e} = \frac{3(256)^2}{1+4[(256)^2-1]9.18\times10^{-4}}$$

$$= 813.6 \Longrightarrow \underline{\underline{29.1\ dB}}$$

7-36 Using (7-89),

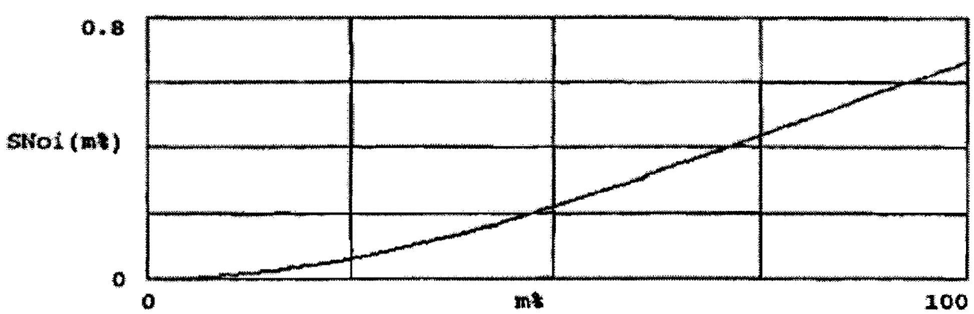
$$SN_{oi} \triangleq \frac{(S/N)_{out}}{(S/N)_{in}} = \frac{2\overline{m^2}}{1+\overline{m^2}}$$

Let $m(t) = \left(\frac{m\%}{100}\right)\cos\omega_m t \Rightarrow \overline{m^2} = \frac{1}{2}\left(\frac{m\%}{100}\right)^2$

$$\Rightarrow SN_{oi} = \frac{\left(\frac{m\%}{100}\right)^2}{1+\frac{1}{2}\left(\frac{m\%}{100}\right)^2}$$

m% := 0,5 ..100

$$SNoi(m\%) := \frac{\left[\frac{m\%}{100}\right]^2}{1 + 0.5\left[\frac{m\%}{100}\right]^2}$$



7-37 $m(t) = 0.4 \sin \omega_a t \Rightarrow \overline{m^2} = \frac{(0.4)^2}{2} = 0.08$

For AM, with product detector, use (7-90).

$$\frac{(S/N)_{out}}{(S/N)_{base}} = \frac{\overline{m^2}}{1+\overline{m^2}} = \frac{0.08}{1.08} = \underline{\underline{0.0741 \Rightarrow -11.3\,dB}}$$

Also get same result for env. det when (S/N) is large.

For DSB-SC, use (7-98).

$$\frac{(S/N)_{out}}{(S/N)_{base}} = \underline{\underline{1 \Rightarrow 0\,dB}} \Rightarrow \underline{\underline{\text{The AM system is inferior by } 11.3\,dB}}$$

7-44 Using Carson's Rule:

$$B_{IF} = 2(\beta_f + 1)B \Rightarrow 25\,kHz = 2(\beta_f+1)\,5\,kHz \Rightarrow \beta_f = 1.5$$

$$f_1 = 2.1\,kHz \; ; \; \frac{B}{f_1} = \frac{5}{2.1} \not\gg 1 \quad \therefore \text{(7-139) is not valid}$$

Equ. (7-124a.) $$s_o(t) = \frac{K D_f}{2\pi} m(t) = \frac{K B \beta_f}{V_p} m(t)$$

$$\overline{s_o^2(t)} = K^2 B^2 \beta_f^2 \overline{\left(\frac{m}{V_p}\right)^2} \; ; \; \overline{\left(\frac{m}{V_p}\right)^2} = \frac{1}{2} \text{ for sinusoid}$$

Equ. (7-136) $$\overline{[\tilde{n}_o(t)]^2} = 2\left(\frac{K}{A_c}\right)^2 N_o f_1^3 \left[\frac{B}{f_1} - \tan^{-1}\left(\frac{B}{f_1}\right)\right]$$

$$\therefore \left(\frac{S}{N}\right)_o = \frac{K^2 B^2 \beta_f^2 \overline{\left(\frac{m}{V_p}\right)^2}}{2\left(\frac{K}{A_c}\right)^2 N_o f_1^3 \left[\frac{B}{f_1} - \tan^{-1}\left(\frac{B}{f_1}\right)\right]}$$

7-44 Cont'd

Eqn. (7-128) $\left(\frac{S}{N}\right)_{in} = \frac{A_c^2}{4N_0(\beta_f+1)B}$

$$\frac{(S/N)_o}{(S/N)_{in}} = \frac{2\left(\frac{B}{f_1}\right)^3 \beta_f^2(\beta_f+1)\overline{\left(\frac{m}{V_p}\right)^2}}{\left[\frac{B}{f_1} - \tan^{-1}\left(\frac{B}{f_1}\right)\right]} = \frac{2(13.5)(2.25)(2.5)\frac{1}{2}}{1.21} = 62.9$$

$N_{total} = kFT_0$; $F = 10^{1.2} = 15.8$

$$\left(\frac{S}{N}\right)_{in} = \frac{P_s}{\frac{N_{tot}}{2}(2B_{IF})} = \frac{P_s}{kFT_0B_{IF}} = \frac{P_s}{1.57\times10^{-15}}$$

$$\Rightarrow P_s = \left(\frac{S}{N}\right)_o\left(\frac{1}{62.9}\right)(1.57\times10^{-15})$$

$$= 10^{3.5}\left(\frac{1}{62.9}\right)(1.57\times10^{-15}) = 7.89\times10^{-14}\text{ W}$$

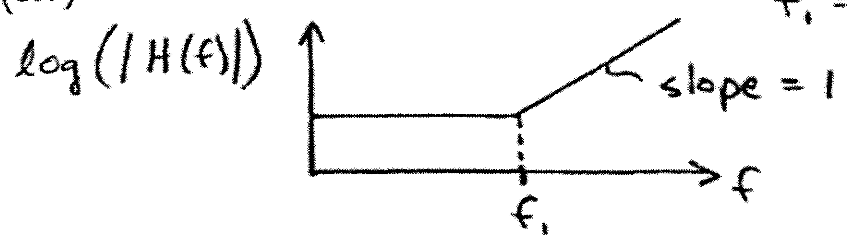
$$10\log_{10}\left(\frac{7.89\times10^{-14}}{10^{-3}}\right) = \underline{\underline{-101\text{ dBm}}} = P_{s\,min}$$

7-49

$H(f) = [1 + jf/f_1]$ for preemphasis

$f_1 = 2.1$ kHz

(a.)



At $f = 15$ kHz the gain is:

$$|H(f)| = \left|1 + j\left(\frac{15}{2.1}\right)\right| = \sqrt{1+\left(\frac{15}{2.1}\right)^2} = 7.21$$

At $f = 1$ kHz:

$$|H(f)| = \left|1 + j\left(\frac{1}{2.1}\right)\right| = \sqrt{1+\left(\frac{1}{2.1}\right)^2} = 1.10$$

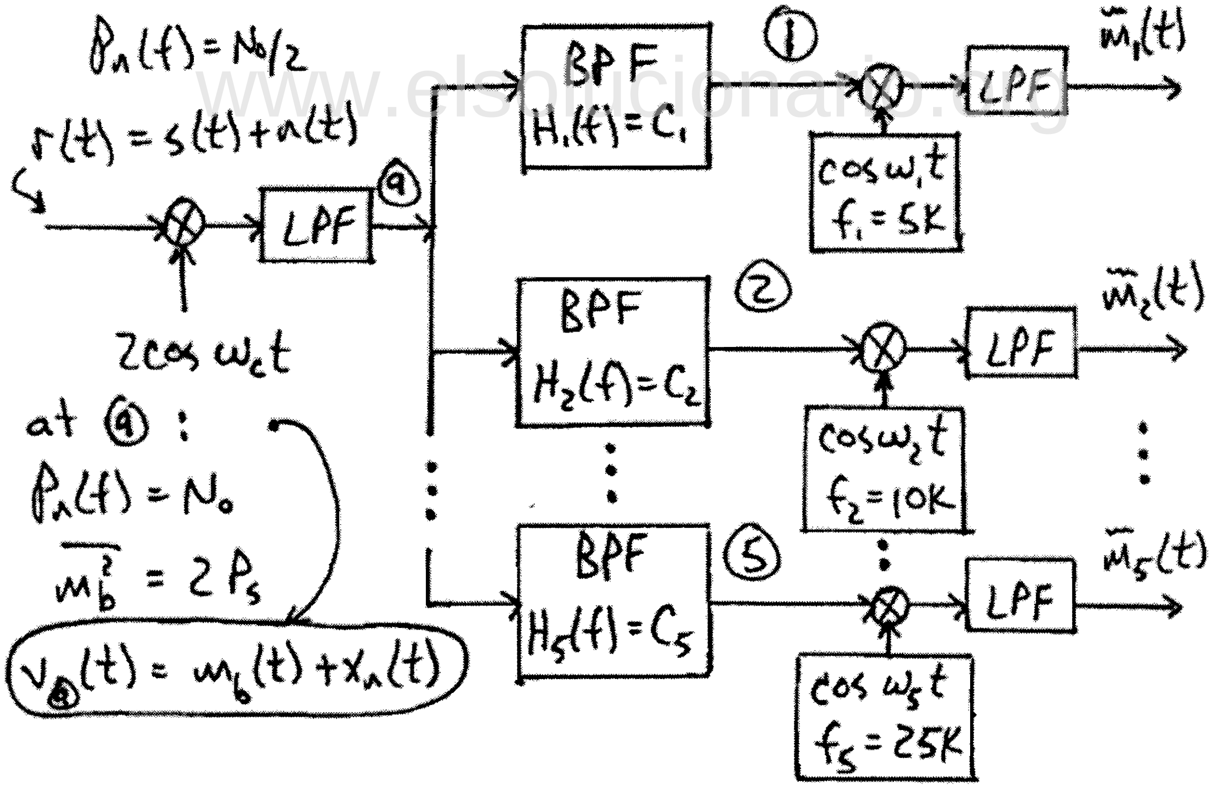
$$\Delta F = 75\text{ kHz}\left(\frac{7.21}{1.10}\right) = \underline{\underline{488\text{ kHz}}}$$

$$\%\text{ mod} = \frac{488}{75}(100) = \underline{\underline{651\%\text{ mod}}}$$

7-49 Cont'd

(b.) The amplitudes of the high frequency audio components are much smaller than those of the low frequency components. For example, if the components at 15 KHz are more than $20 \log_{10}\left(\frac{7.21}{1.1}\right) = 16.3$ dB below the 1 KHz components, there is no problem.

7-51 (a.) $s(t) = m_b(t) \cos \omega_c t$; $P_s = \frac{1}{2} \overline{m_b^2}$



at (a):

$\mathcal{P}_n(f) = N_0$

$\overline{m_b^2} = 2P_s$

$v_a(t) = m_b(t) + x_n(t)$

7-51 Cont'd (b.) Using Fig. P7-51, it is seen that the signal power at point ① is 1/5 the signal power at ⓐ.

$$S_1 = \overline{m_1^2} = \frac{1}{5}\overline{m_b^2} = \frac{2}{5}P_s$$

The noise power at point ① is:

$$N_1 = N_0[2(4\text{KHz})] = (8\times10^3)N_0$$

$$\Rightarrow \left(\frac{S}{N}\right)_1 = \frac{\frac{2}{5}P_s}{(8K)N_0} = \frac{P_s}{(20\times10^3)N_0} = (5\times10^{-5})\frac{P_s}{N_0}$$

Also for SSB $(S/N)_{out} = (S/N)_{in}$

$$\therefore \left(\frac{S}{N}\right)_{01} = (5\times10^{-5})P_s/N_0$$

; the same result is obtained for the other four channels.

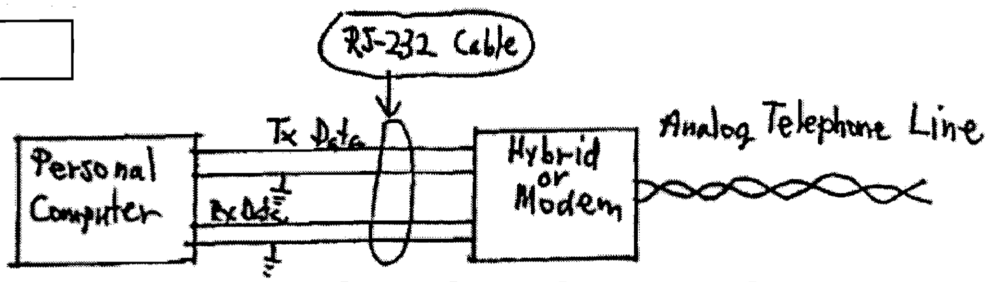
Chapter 8

8-1

Assuming that the 150 G.Lite subscribers have VF service over the DSL lines, as well as VF service to the 300 VF subscriber lines, we get

$$300 \times 64\,\text{kb/s} + 150 \times 64\,\text{kb/s} + 150 \times 1{,}500\,\text{kb/s}$$
$$= 253{,}800\,\text{kb/s} = \underline{253.8\,\text{Mb/s}}$$

8-5



Suppose the serial data port on a PC is interfaced to the VF telephone line without a modem. The transmit (Tx) and receive (Rx) serial data from the PC are each represented by a Polar NRZ line code as described by the RS-232 serial port standard in Appendix C. A hybrid (4 wire to 2 wire) circuit could be used to couple the Tx and Rx line-code signals to a single twisted-pair VF telephone line. However, the spectrum of the polar line-code is not compatible with the frequency response of the VF telephone line. That is, as shown in Fig. 3-16b, the dominant frequencies in the Polar NRZ line code are near zero (i.e. baseband) but the telephone line is a bandpass channel with a frequency response from 300 to 2700 Hz. Thus, the Polar NRZ waveform would be distorted by the telephone line so that the data could not be detected at the receiving end. Thus, a modem is needed to provide a bandpass signal that is generated by some sort of modulation technique, such as QAM. That is, the baseband line-code waveform is modulated onto a carrier. This produces a bandpass signal with a spectrum that falls within the bandpass of the VF telephone line.

8-9 $P_{Tx} = 0.1\,W$, $f_c = 2\,GHz$, $3.28\,ft/meter$

(a.) $G_A = \frac{7A}{\lambda^2} \approx \frac{7.0\pi\left(\frac{2}{3.28}\right)^2}{\left(\frac{3\times10^8}{2\times10^9}\right)^2} = \underline{\underline{363.14}} = \underline{\underline{25.6\,dB}}$

$A = \pi r^2$, $\lambda = c/f$

$10\log(363.4) = 25.6$

(b.) $P_{EIRP} = P_{Tx}\,G_{AT} = 0.1\,(363.4) = \underline{\underline{36.3\,W}}$

(c.) $P_{Rx} = P_{Tx}\,G_{AT}\,G_{AR}\left(\frac{\lambda}{4\pi d}\right)^2 = (0.1)(363.4)^2\left[\frac{(0.15)(3.28)}{4\pi(15)(5280)}\right]$

$\Rightarrow P_{Rx} = 3.23\times10^{-9}\,W$

or $(P_{Rx})_{dBm} = 10\log\left(\frac{3.23\times10^{-9}}{10^{-3}}\right) = \underline{\underline{-54.9\,dBm}}$

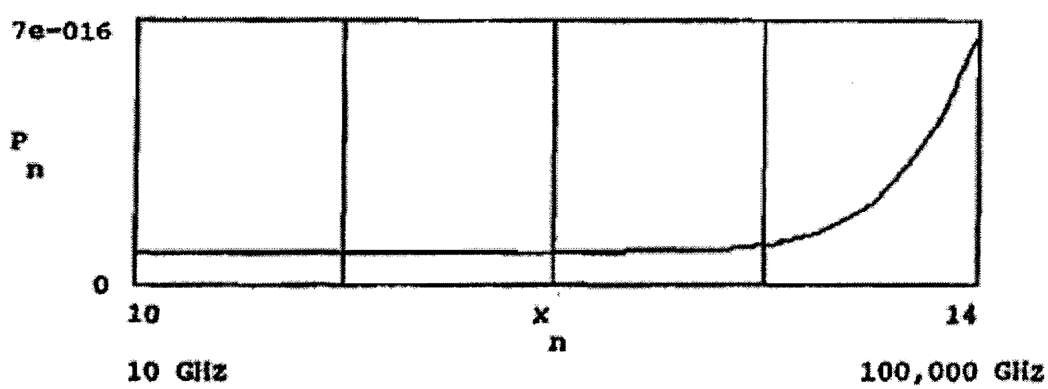
8-14

$n := 100..140$ $\quad h := 6.6252\cdot10^{-34}$ $\quad k := 1.381\cdot10^{-23}$

$T := 300$

$R := 10000$ $\quad x_n := \frac{n}{10}$ $\quad f_n := 10^{x_n}$

$$P_n := 2\cdot R\left[\frac{h\,f_n}{2} + \frac{h\cdot f_n}{e^{\frac{h\cdot f_n}{k\cdot T}} - 1}\right]$$



8-17

$$(P_a)_{in} = R_s\left(\frac{E_s}{2R_s}\right)^2 = \frac{E_s^2}{4R_s}, \quad (P_a)_{out} = \frac{V_{out}^2}{R_{out}} \text{ where } R_L = R$$

$$V_{out} = (h_{fe}\, i)\frac{R}{2} = \frac{h_{fe}}{2}\left(\frac{E_s}{R_s + h_{ie}}\right)R$$

$i = E_s/(R_s + h_{ie})$

$R = 1/h_{oe}$

$$\Rightarrow (P_a)_{out} = \frac{\left(\frac{h_{fe}}{2}\right)^2\left(\frac{E_s}{R_s + h_{ie}}\right)^2 R^2}{R} = \left(\frac{h_{fe}}{2}\right)^2\left(\frac{E_s}{R_s + h_{ie}}\right)^2\left(\frac{1}{h_{oe}}\right)$$

$$\Rightarrow G_a = \frac{(P_a)_{out}}{(P_a)_{in}} = \frac{\frac{1}{4} h_{fe}^2 \frac{1}{h_{oe}}\left(\frac{1}{R_s + h_{ie}}\right)^2 E_s^2}{\frac{1}{4} E_s^2/R_s}$$

$$\Rightarrow \underline{\underline{G_a = \frac{h_{fe}^2 R_s}{h_{oe}(R_s + h_{ie})^2}}}$$

8-21 (a.)

$$T_{eff} = T_0(F-1)$$
$$= 290(10^{.16} - 1) = \underline{\underline{129^\circ K}}$$

(b.)

$$P_{a_{out}} = K\, T_{IN}\, B\, G_a$$
$$= (1.38 \times 10^{-23})(30^\circ + 129^\circ)(10 \times 10^6)(10^3)$$
$$= \underline{\underline{2.2 \times 10^{-11}\ W}}$$
$$= 10 \log_{10}\left(\frac{2.2 \times 10^{-11}}{10^{-3}}\right) = \underline{\underline{-76.6\ dBm}}$$

8-23

From Table 7-1 for FSK w/ noncoherent detection:

$$P_e = \frac{1}{2} e^{-\frac{1}{2}(E_b/N_{total})}$$

$$N_{total} = k(T_0 + T_{eff}) = k(T_0 + (F-1)T_0) = kFT_0$$

$T_{eff} = (F-1)T_0$

$$= (1.38 \times 10^{-23})(10^{6/10})(290)$$

$$\frac{E_b}{N_{total}} = \frac{P_s T}{N_{total}} = \frac{P_s}{N_{tot} R} = \frac{V_s^2/R_{in}}{kFT_0 R} = 28.53$$

$$P_e = \frac{1}{2} e^{-\frac{1}{2}(28.53)} = \underline{\underline{3.2 \times 10^{-7}}}$$

8-26 (a.) $F = F_1 + \dfrac{F_2 - 1}{G_1}$

$$F_1 = 125\,ft\left(\frac{3\,dB}{100\,ft}\right) = 3.75\,dB = 10^{.375} = 2.37$$

$$F = 2.37 + \frac{10^{1.6} - 1}{1/2.37} = \underline{\underline{94.4}} \Longrightarrow \underline{\underline{19.8\,dB}}$$

8-26 Cont'd.

(b)

$$\rightarrow \boxed{\begin{matrix} G_1 = -3.75\,dB \\ F_1 = 3.75\,dB \end{matrix}} \rightarrow \boxed{\begin{matrix} G_2 = 20\,dB \\ F_2 = 3\,dB \end{matrix}} \rightarrow \boxed{F_3 = 16\,dB}$$

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2}$$

$$= 2.37 + \frac{(10^{.3} - 1)}{1/2.37} + \frac{(10^{1.6} - 1)}{10^2/2.37}$$

$$= 2.37 + 2.36 + 0.92 = \underline{\underline{5.65}} \Rightarrow \underline{\underline{7.52\,dB}}$$

$$\rightarrow \boxed{\begin{matrix} G_1 = 20\,dB \\ F_1 = 3\,dB \end{matrix}} \rightarrow \boxed{\begin{matrix} G_2 = -3.75\,dB \\ F_2 = 3.75\,dB \end{matrix}} \rightarrow \boxed{F_3 = 16\,dB}$$

$$F = 2 + \frac{2.37 - 1}{100} + \frac{10^{1.6} - 1}{\frac{100}{2.37}}$$

$$\Rightarrow F = 2 + 0.0137 + 0.92 = \underline{\underline{2.93}} = \underline{\underline{4.67\,dB}}$$

8-30

$$T_A = 160K \quad T_{sys} \rightarrow \overbrace{\boxed{\begin{matrix} L_1 = 2\,dB \\ F_1 = 2\,dB \end{matrix}} \rightarrow \boxed{\begin{matrix} Te_2 = 800K \\ B = 1MHz \end{matrix}}}^{G = 120\,dB} \rightarrow \text{IF Output} \quad P_{no} = k\,T_{sys}BG$$

$$Te_1 = T_0(F_1 - 1) = T_0(L_1 - 1)$$

$$T_{sys} = T_A + Te = T_A + \left(Te_1 + \frac{Te_2}{G_1}\right) = T_A + (Te_1 + Te_2 L_1)$$

$$\Rightarrow T_{sys} = T_A + \underbrace{\left[T_0(L_1 - 1) + Te_2 L_1\right]}_{Te}. \quad \text{Also } Te = T_0(F - 1) \Rightarrow F = \frac{Te}{T_0} + 1$$

8-30 Cont'd.

(a) Compute the system noise temperature Ts evaluated at the antenna input of the waveguide:

Ta := 160 K   To := 290 K   Te2 := 800 K

$L1 := 10^{0.2}$ $G := 10^{12}$ $B := 10^{6}$ Hz

Te := To·(L1 − 1) + Te2 L1   Ts := Ta + Te   Ts = 1597.534 K

(b) Noise figure F :

$F := \frac{Te}{To} + 1$ F = 5.957   FdB := 10·log(F)   FdB = 7.75

(c) The available output noise power Pno:

$Pno := 1.38 \cdot 10^{-23} \cdot Ts \cdot B \cdot G$ Pno = 0.022 Watt

8-32 Assume 4 GHz downlink

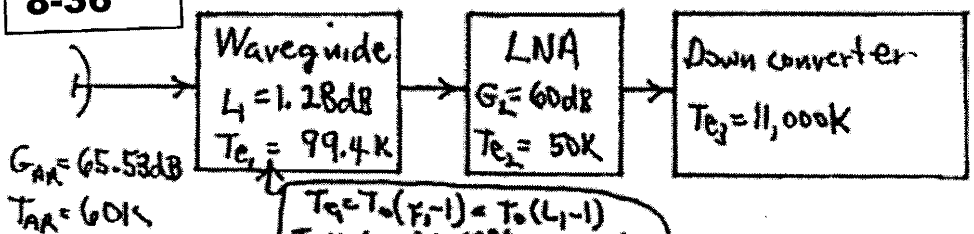
$$\frac{G_{AR}}{T_{sys}} = 10^4 = \frac{4\pi\eta\left[\pi(15)^2\right]}{85\left(3\times10^8/4\times10^9\right)^2} = 1.86\times10^4\eta$$

$\Rightarrow$ $\eta = 54\%$ for 30m antenna

$$10^4 = 1.86\times10^4\eta\left[\frac{(12.5)^2}{(15)^2}\right] = 1.29\times10^4\eta$$

$\Rightarrow$ $\eta = 77.4\%$ for 25m antenna

8-36



$T_e = T_o(F_1 - 1) = T_o(L_1 - 1)$
$T_e = (290)(10^{1.28/10} - 1) = 99.4$

(a.) $T_s = (T_{AR} + T_{e_1})G_1 + \left(T_{e_2} + \frac{T_{e_3}}{G_2}\right) = \frac{T_{AR} + T_{e_1}}{L_1} + \left(T_{e_2} + \frac{T_{e_3}}{G_2}\right)$

$G_s = G_{AR}G_1 = \frac{G_{AR}}{L_1}$

(b.) $T_s = T_{AR} + T_e = T_{AR} + \left(T_{e_1} + \frac{T_{e_2}}{G_1} + \frac{T_{e_3}}{G_1 G_2}\right)$

$\therefore T_s = T_{AR} + \left(T_{e_1} + T_{e_2}L_1 + T_{e_3}\frac{L_1}{G_2}\right)$

$G_s = G_{AR}$

(c.) $T_s = (T_{AR} + T_{e_1})G_1G_2 + T_{e_2}G_2 + T_{e_3} = (T_{AR} + T_{e_1})\frac{G_2}{L_1} + T_{e_2}G_2 + T_{e_3}$

$G_s = G_{AR}G_1G_2 = \frac{G_{AR}G_2}{L_1}$

$Gar := 10^{\frac{65.53}{10}}$ $\quad L1 := 10^{\frac{1.28}{10}}$ $\quad G2 := 10^{\frac{60}{10}}$ $\quad Te3 := 11000$

$Tar := 60$ $\quad Te1 := 290\cdot(L1 - 1)$ $\quad Te2 := 50$

(a.)

$Ts := \frac{Tar + Te1}{L1} + Te2 + \frac{Te3}{G2}$ $\quad \underline{Ts = 168.723}$

$Gs := \frac{Gar}{L1}$ $\quad \underline{Gs = 2.661\cdot 10^{6}}$ $\quad GTsdB := 10\cdot\log\left(\frac{Gs}{Ts}\right)$ $\quad \underline{GTsdB = 41.978}$

(b.)

$Ts := Tar + Te1 + Te2\cdot L1 + Te3\cdot\frac{L1}{G2}$ $\quad \underline{Ts = 226.555}$

$Gs := Gar$ $\quad \underline{Gs = 3.573\cdot 10^{6}}$ $\quad GTsdB := 10\cdot\log\left(\frac{Gs}{Ts}\right)$ $\quad \underline{GTsdB = 41.978}$

(c.)

$Ts := (Tar + Te1)\cdot\frac{G2}{L1} + Te2\cdot G2 + Te3$ $\quad \underline{Ts = 1.687\cdot 10^{8}}$

$Gs := Gar\cdot\frac{G2}{L1}$ $\quad \underline{Gs = 2.661\cdot 10^{12}}$ $\quad GTsdB := 10\cdot\log\left(\frac{Gs}{Ts}\right)$ $\quad \underline{GTsdB = 41.978}$

8-38

$f_c = 2\,GHz \;;\; \lambda = \frac{c}{f} = \frac{3\times10^8}{2\times10^9} = .15\,m$

$$\frac{E_b}{N_0} = \frac{P_{Tx}\,G_{AT}\,G_{FS}\,G_{AR}}{K\,T_{sys}\,R}$$

$$G_{AT} = \frac{E_b}{N_0}\,\frac{k\,T_{sys}\,R}{P_{Tx}\,G_{FS}\,G_{AR}}$$

$$G_{FS} = \left(\frac{\lambda}{4\pi d}\right)^2 \cdot L_{incidental} = \left(\frac{0.15}{4\pi(7.5\times10^{12})}\right)^2 (10^{-0.2}) = 1.6\times10^{-30}$$

$$G_{AR} = 7\pi\left(\frac{r}{\lambda}\right)^2 = 7\pi\left(\frac{32}{0.15}\right)^2 = 1\times10^6$$

$$G_{AT} = \frac{10^{0.988}\,(1.38\times10^{-23})(16)(300)}{10\,(1.6\times10^{-30})(10^6)} = 4.03\times10^4 = \frac{7A}{\lambda^2} = \frac{7A}{(0.15)^2}$$

parabolic

$$\Rightarrow A = 129.44\,m^2 = \pi r^2$$

$$\Rightarrow r = \sqrt{\frac{129.44}{\pi}} = 6.42\,m \Rightarrow \underline{\underline{D = 2r = 12.84\,m}}$$

parabolic ant (rather large)

8-42 Using (8-47) and (8-8)

$$P_{dBm}(d) = P_{dBm}(d_0) - 10n\log\left(\frac{d}{d_0}\right)$$

where $P_{dBm}(d_0) = (P_T)_{dBm} + (G_{TA})_{dBm} - 20\log\left(\frac{4\pi d_0}{\lambda}\right) + (G_{AR})_{dBm}$

$$= 40 + 18 - \underbrace{20\log\left(\frac{4\pi(0.25\text{ miles})(5280\text{ ft/mile})\left(\frac{1\text{ m}}{3.28\text{ ft}}\right)}{3\times10^8/1.8\times10^9}\right)}_{\to\ 89.64} + 0$$

$\Rightarrow P_{dBm}(d_0) = -31.64$ dBm

	Power Received (dBm)				
Distance (miles)	0.25	1	2	5	10
n=2 (free space)	-31.6	-43.7	-49.7	-57.7	-63.7
n=3	-31.6	-49.7	-58.7	-70.6	-79.7
n=4	-31.6	-55.7	-67.8	-83.7	-95.7

8-46 First determine the value of K by using the average power reading:

$s(t) = m(t)\cos\omega_c t$; $m(t)$ shown in figure P8-46

$$S_{rms}^2 = \frac{1}{2}\langle m^2(t)\rangle$$

$$= \frac{1}{2T}\int_0^T m^2(t)\,dt \qquad \left\{\begin{array}{rl} T &= 63.5\,\mu\text{sec} \\ 0.835\,T &= 53\,\mu\text{sec} \\ 0.165\,T &= 10.5\,\mu\text{sec} \\ .075\,T &= 4.76\,\mu\text{sec}\end{array}\right.$$

$$= \frac{1}{2T}\left[(0.75K)^2(0.165T - 0.075T) + K^2(0.075T) + (0.5K)^2(0.835T)\right]$$

8-46 Cont'd.

$$= \frac{K^2}{2}\left[(.75)^2(.09) + 1(.075) + (.5)^2(.835)\right]$$

$$= \frac{K^2}{2}\left[.0506 + .075 + .2087\right] = \frac{K^2}{2}(.3343)$$

$$\Longrightarrow S_{rms}^2 = 0.1672\,K^2$$

$$P_{AV} = 6.9\times10^3 = \frac{.1672K^2}{50} = \frac{S_{rms}^2}{50}$$

$$\Longrightarrow K = \sqrt{\frac{50(6.9\times10^3)}{.1672}} = 1436.6$$

$$P_{PEP} = \frac{V_{max}^2}{2(50)} = \frac{K^2}{100} = \frac{(1436.6)^2}{100} = \underline{\underline{20.64\ \text{kW}}}$$

8-49

$$R_{\text{with Coding}} = (R_{\text{without coding}})\left(\frac{1}{R_{TCM}}\right)\left(\frac{1}{R_{RS}}\right) = 19.39\left(\frac{3}{2}\right)\left(\frac{207}{187}\right)$$

$$\Rightarrow R_{\text{withCoding}} = 32.20\ \text{Mb/s}$$

$$\Rightarrow D_{\text{withcoding}} = \frac{R_{\text{withcoding}}}{\ell} = \frac{32.2}{3} = 10.73\ \text{Mbaud}$$

$\ell = 3$ for 8 levels

The segment sync replaces the payload sync at the beginning of each segment. One segment of training data is added after 312 segments.

Thus,

$$D_{\text{overall}} = (D_{\text{withcoding}})\left(\frac{312+1}{312}\right) = 10.73\left(\frac{312+1}{312}\right) = \underline{\underline{10.76\ \text{Mbaud}}}$$

Appendix B

B-1 $P(1) = \frac{n_1}{n} = \frac{1428}{1428 + 2668} = \underline{\underline{0.3486}}$

B-2 Total # of outcomes $= 6(6) = 36$

(a.) $8 =: (2+6), (3+5), (4+4), (5+3), (6+2)$

$\Rightarrow \underline{\underline{P(8) = 5/36}}$

(b.) $5 =: (1+4), (2+3), (3+2), (4+1) \Rightarrow P(5) = \frac{4}{36}$

$7 =: (1+6), (2+5), (3+4), (4+3), (5+2), (6+1)$

$\Rightarrow P(7) = \frac{6}{36}$

$P(5+7+8) = P(5) + P(7) + P(8)$

Mutually exclusive

$= \frac{1}{36}[4 + 6 + 5] = \underline{\underline{15/36}}$

B-4 $P(1+3+5) = P(1) + P(3) + P(5) = \frac{3}{6} = \underline{\underline{\frac{1}{2}}}$

B-5 $P(4/E) = \frac{P(4 \cdot E)}{P(E)} = \frac{P(4)}{P(E)} = \frac{1/6}{1/2} = \underline{\underline{\frac{1}{3}}}$

B-9

$$P\left(-\frac{A}{4} \le y \le \frac{A}{4}\right) = \int_{-A/4}^{A/4} f(y)\,dy = 2\int_0^{A/4} f(y)\,dy$$

$$= -\frac{2}{A^2}\int_0^{A/4} (y-A)\,dy =$$

Let $x = y - A$; $dx = dy$

$$2\int_0^{A/4} f(y)\,dy = \frac{-2}{A^2}\int_{-A}^{-3A/4} x\,dx = -\frac{x^2}{A^2}\Bigg|_{-A}^{-3A/4}$$

$$= \frac{-A^2}{A^2}\left[\left(-\frac{3}{4}\right)^2 - 1\right] = \underline{\underline{0.4375}}$$

B-12

(a.) $\int_{-\infty}^{\infty} f(x)\,dx = 1 = \int_0^{\infty} k e^{-bx}\,dx$

$$= \frac{ke^{-bx}}{-b}\Bigg|_0^{\infty} = \frac{k}{b}\left[e^{-\infty} - e^0\right] = \frac{k}{b} = 1 \Rightarrow \underline{\underline{k = b}}$$

(b.) $f(x) = b e^{-bx}$

$$m = \bar{x} = \int_{-\infty}^{\infty} x f(x)\,dx = \int_0^{\infty} b x e^{-bx}\,dx$$

$$= b\left[\frac{-xe^{-bx}}{b}\Bigg|_0^{\infty} + \int_0^{\infty} \frac{e^{-bx}}{b}\,dx\right] = b\left[\frac{e^{-bx}}{-b^2}\Bigg|_0^{\infty}\right]$$

Let $u = x$ $\quad dv = e^{-bx}dx$
$du = dx$ $\quad v = -e^{-bx}/b$

$$\Rightarrow m = b\left[-\frac{1}{b}\left(e^{-\infty} - e^0\right)\right] = \underline{\underline{\frac{1}{b} = m}}$$

(c.) $\sigma^2 = \overline{x^2} - (\bar{x})^2$ where $\overline{x^2} = \int_{-\infty}^{\infty} x^2 b e^{-bx}\,dx$

Using Sec. A-5 $\overline{x^2} = be^{-bx}\left[\frac{x^2}{-b} - \frac{2x}{b^2} - \frac{2}{b^3}\right]\Bigg|_0^{\infty} = -be^{-0}\left(-\frac{2}{b^3}\right) = \frac{2}{b^2}$

$$\Rightarrow \sigma^2 = \frac{2}{b^2} - \left(\frac{1}{b}\right)^2 = \underline{\underline{\frac{1}{b^2} = \sigma^2}}$$

B-17 $n := 160 \quad p := 0.1 \quad q := 1 - p \quad \lambda := n \cdot p$

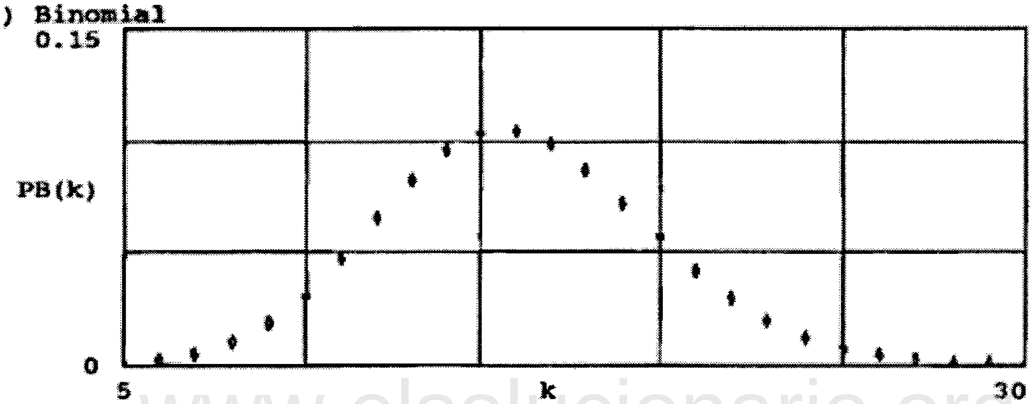
$m := n\,p \quad \sigma := \sqrt{n\,p \cdot q} \quad k := 0\,..2\,\lambda \quad m = 16$

$$PB(k) := \frac{n!}{k!\,(n-k)!}\,p^k\,q^{n-k}$$ Binomial

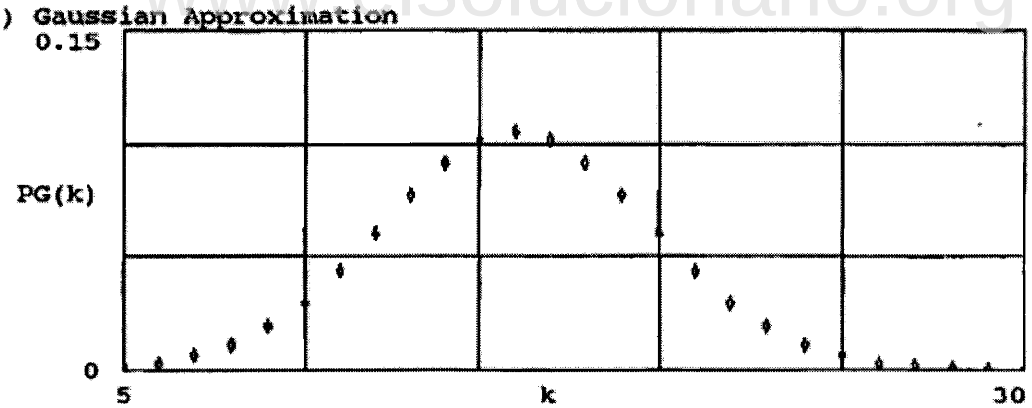
$$PP(k) := e^{-\lambda} \cdot \frac{\lambda^k}{k!}$$ Poisson

$$PG(k) := \frac{1}{\sigma \cdot \sqrt{2 \cdot \pi}}\,e^{\frac{-(k-m)^2}{2 \cdot \sigma^2}}$$

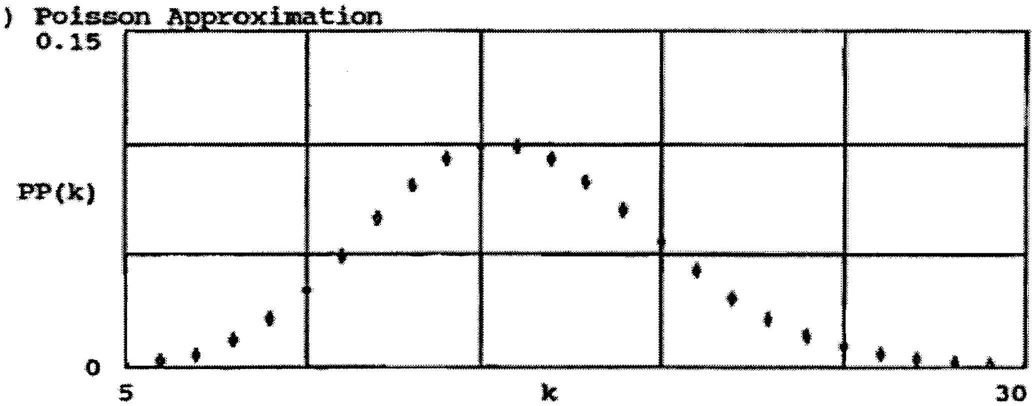
(a.) Binomial



(b.) Gaussian Approximation



(c.) Poisson Approximation



B-23 Let $f(x)$ = pdf of R values where x = R value

$$\Rightarrow \int_{.9\bar{x}}^{1.1\bar{x}} f(x)\,dx \overset{\text{set}}{=} .95$$

$$\Rightarrow \int_{.9\bar{x}}^{1.1\bar{x}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\bar{x})^2}{2\sigma^2}} dx = .95$$

Let $x_1 = x - \bar{x}$; $dx_1 = dx$

$$= \int_{-.1\bar{x}}^{.1\bar{x}} \frac{1}{\sqrt{2\pi}\sigma} e^{-x_1^2/2\sigma^2} dx = .95 = 1 - 2Q\left(\frac{.1\bar{x}}{\sigma}\right)$$

$$\Rightarrow Q\left(\frac{.1\bar{x}}{\sigma}\right) = \frac{1.-.95}{2} = .025 \overset{\text{A-10}}{\Rightarrow} \frac{.1\bar{x}}{\sigma} = 1.96$$

$$\Rightarrow \sigma = \frac{.1\bar{x}}{1.96} = \frac{(.1)(1000)}{1.96} = \underline{\underline{51.0 \text{ ohms} = \sigma}}$$

B-30 (a.) $P(x \le 1) = F(1) = Q\left(\frac{5-1}{.6}\right) = Q(6.66)$

$$= \frac{1}{\sqrt{2\pi}(6.66)} e^{-(6.66)^2/2} = \underline{\underline{1.337 \times 10^{-11} = P(x \le 1)}}$$

(b.) $P(x \le 6) = F(6) = Q\left(\frac{5-6}{.6}\right) = Q(-1.667)$

$$= 1 - Q(1.667) \overset{\text{A-10}}{=} 1 - .04798 = \underline{\underline{.9520 = P(x \le 6)}}$$

B-34

$$f(y) = \left.\frac{f(x)}{\left|\frac{dy}{dx}\right|}\right|_{x_i = h^{-1}(y)} = \left.\frac{f(x_1)}{|2x_1|}\right|_{x_1 = -\sqrt{y}} + \left.\frac{f(x_2)}{|2x_2|}\right|_{x_2 = \sqrt{y}} ; y \geq 0$$

$$y = x^2 \Rightarrow dy = 2x\,dx$$

$$= \frac{f(-\sqrt{y})}{2\sqrt{y}} + \frac{f(+\sqrt{y})}{2\sqrt{y}} = \frac{1}{2\sqrt{y}}\left[\frac{1}{\sqrt{2\pi}\sigma}\left(e^{-(-\sqrt{y}-m)^2/2\sigma^2} + e^{-(\sqrt{y}-m)^2/2\sigma^2}\right)\right]$$

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-m)^2/2\sigma^2}$$

$$\Rightarrow f(y) = \begin{cases} \dfrac{1}{2\sigma\sqrt{2\pi y}}\left[e^{-\frac{(\sqrt{y}+m)^2}{2\sigma^2}} + e^{-\frac{(\sqrt{y}-m)^2}{2\sigma^2}}\right], & y \geq 0 \\ 0, & y < 0 \end{cases}$$

B-38 The input is $x(t) = A \sin \omega_m t$ where $A = 8$. The output consists of a quantized sinusoid similar to that shown in Fig. 3-8b. The PDF of the output, $y(t)$, will consist of δ functions at the quantized values. Thus,

$$f(y) = \sum_{k=1}^{M} P_k\, \delta(y - y_k)$$

where $M = 8$, the step size is $\delta = \frac{2A}{M} = \frac{16}{8} = 2$, and the

B-38 Cont'd

quantized values are: $y_k = \frac{(2k-M-1)}{2}\delta$

$$P_k = \int_{y_k-\delta/2}^{y_k+\delta/2} f_x(x)\,dx = \int_{y_k-\delta/2}^{y_k+\delta/2} \frac{1}{\pi\sqrt{A^2-x^2}}\,dx = \int_{y_k-\delta/2}^{y_k+\delta/2} \frac{\frac{1}{A}}{\pi\sqrt{1-\left(\frac{x}{A}\right)^2}}\,dx$$

Using (B-67)

$$= \frac{1}{\pi}\sin^{-1}\left(\frac{x}{A}\right)\Big|_{y_k-\delta/2}^{y_k+\delta/2} = \frac{1}{\pi}\left[\sin^{-1}\left(\frac{y_k+\delta/2}{A}\right) - \sin^{-1}\left(\frac{y_k-\delta/2}{A}\right)\right]$$

Using (A-29)

or

$$P_k = \frac{1}{\pi}\left[\sin^{-1}\left(\frac{(2k-M-1+1)\delta}{2A}\right) - \sin^{-1}\left(\frac{(2k-M-1-1)\delta}{2A}\right)\right]$$

$$P_k = \frac{1}{\pi}\left[\sin^{-1}\left(\frac{(2k-M)\frac{2A}{M}}{2A}\right) - \sin^{-1}\left(\frac{(2k-M-2)\frac{2A}{M}}{2A}\right)\right] = \frac{1}{\pi}\left[\sin^{-1}\left(\frac{2k-M}{M}\right) - \sin^{-1}\left(\frac{2k-M-2}{M}\right)\right]$$

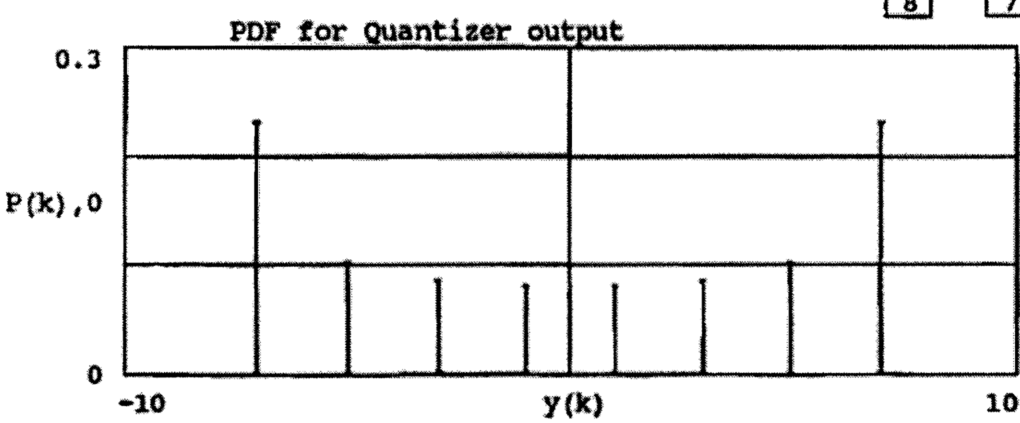
A := 8 M := 8 $\delta := 2\frac{A}{M}$ $\delta = 2$ <--Step size

k := 1 ..M

y(k) := (2 k − M − 1) 0.5·δ

$$P(k) := \frac{1}{\pi}\left[\operatorname{asin}\left[\frac{2\cdot k - M}{M}\right] - \operatorname{asin}\left[\frac{2\,k - M - 2}{M}\right]\right]$$

k	y(k)
1	−7
2	−5
3	−3
4	−1
5	1
6	3
7	5
8	7



B-40

$$\bar{y} = \int_{-\infty}^{\infty} y\, f(y)\, dy = \int_{0}^{\infty} \frac{y}{\sqrt{2\pi}\,\beta\Delta} e^{-y^2/2\beta^2\Delta^2} dy + \cancelto{0}{\frac{1}{2}\int_{-\infty}^{\infty} y\,\delta(y)\, dy}$$

$$= \left(\frac{-\beta\Delta}{\sqrt{2\pi}}\right) \int_{0}^{\infty} e^{-y^2/2\beta^2\Delta^2} \left(\frac{-y}{\beta^2\Delta^2}\right) dy$$

$$= \left(\frac{-\beta\Delta}{\sqrt{2\pi}}\right) \int_{0}^{-\infty} e^{z}\, dz = \left(\frac{-\beta\Delta}{\sqrt{2\pi}}\right) e^{z}\Big|_{0}^{-\infty}$$

$$\boxed{\begin{aligned} \text{Let } z &= \frac{-y^2}{2\beta^2\Delta^2} \\ dz &= \frac{-y}{\beta^2\Delta^2}\, dy \end{aligned}}$$

$$= \frac{\beta\Delta}{\sqrt{2\pi}} = \bar{y}$$

B-42 (a.)

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x_1, x_2)\, dx_1\, dx_2 \overset{\text{set}}{=} 1$$

$$\int_{x_2=0}^{4}\int_{x_1=0}^{1} K\,(x_1 + x_1 x_2)\, dx_1\, dx_2 = \int_{x_2=0}^{4}\left[\int_{x_1=0}^{1} K x_1 (1+x_2)\, dx_1\right] dx_2$$

$$= \int_{x_2=0}^{4}\left[K\,(1+x^2)\,\frac{x_1^2}{2}\Big|_0^1\right] dx_2 = \int_{x_2=0}^{4} K\,(1+x_2)\,\frac{1}{2}\, dx_2$$

$$= \frac{K}{2}\left[x_2 + \frac{x_2^2}{2}\right]\Big|_0^4 = \frac{K}{2}\left[4+8\right] = 6K = 1$$

$$\Longrightarrow K = 1/6$$

B-42 Cont'd

(b.) $$f(x_1) = \int_0^4 f(x_1, x_2)\, dx_2 = \frac{1}{6}\int_0^4 x_1(1+x_2)\, dx_2$$

$$= \frac{1}{6} x_1 \left[x_2 + x_2^2/2\right]\Big|_0^4 = \frac{1}{6} x_1 [4+8] = \underline{\underline{2x_1}}$$

$$f(x_2) = \frac{1}{6}\int_0^1 (1+x_2)\, x_1\, dx_1 = \frac{(1+x_2)}{6}\,\frac{x_1^2}{2}\Big|_0^1$$

$$= \underline{\underline{\left(\frac{1+x_2}{12}\right)}}$$

$$f(x_1) f(x_2) = 2x_1\left(\frac{(1+x_2)}{12}\right) = \frac{1}{6}(x_1 + x_1 x_2)$$

$$= f(x_1, x_2)$$

$\Longrightarrow$ x_1 & x_2 $\underline{\underline{\text{indep.}}}$

$$F_{x_1,x_2}(0.5, 2) = \frac{1}{6}\int_0^{.5} x_1 \int_0^2 (1+x_2)\, dx_2\, dx_1$$

$$= \frac{1}{6}\int_0^{.5} x_1 \left[x_2 + \frac{x_2^2}{2}\right]\Big|_0^2 dx_1$$

$$= \frac{1}{6}\int_0^{.5} x_1 (4)\, dx_1 = \frac{2}{3}\,\frac{x_1^2}{2}\Big|_0^{.5} = \underline{\underline{\frac{1}{12}}}$$

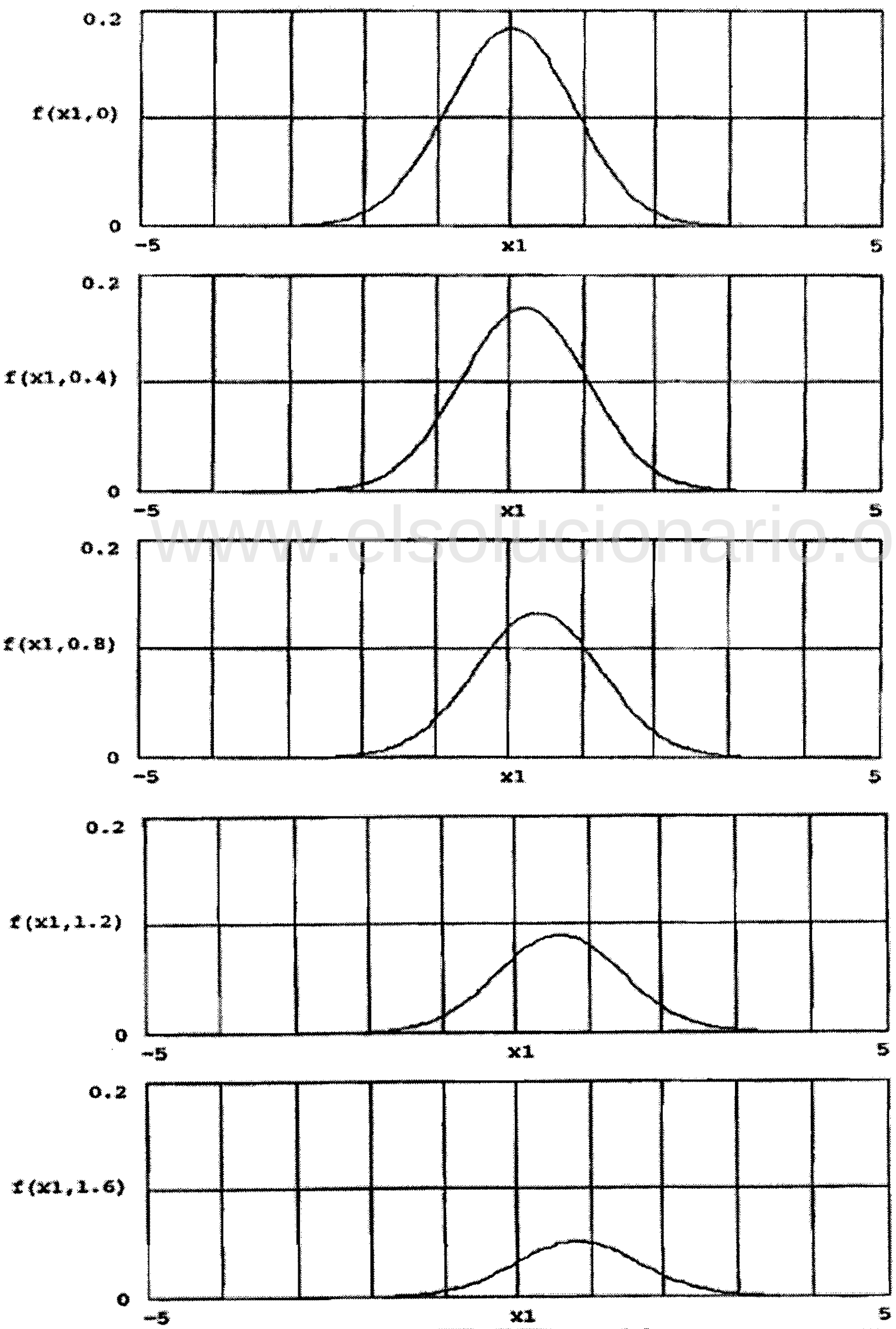
(d.) $$f_{x_2/x_1}(x_2/x_1) = \frac{f(x_2, x_1)}{f(x_1)} = \frac{f(x_2) f(x_1)}{f(x_1)}$$

$$= f(x_2) = \begin{cases} \frac{1}{12}(1+x_2) & ;\ 0 \le x_1 \le 1,\ 0 \le x_2 \le 4 \\ 0 & x_1 \text{ \& } x_2 \text{ elsewhere} \end{cases}$$

B-45

$\sigma := 1 \qquad \rho := 0.5 \qquad x1 := -5, -4.95 \,..\, 5$

$$f(x1,x2) := \frac{1}{2\pi\sigma^2\sqrt{1-\rho^2}}\, e^{-1\cdot\frac{x1^2 - 2\cdot\rho\; x1\; x2 + x2^2}{2\,\sigma^2\cdot\left[1-\rho^2\right]}}$$



B-52

$$mx := \begin{bmatrix} 2 \\ -1 \end{bmatrix} \qquad Cx := \begin{bmatrix} 5 & \frac{-2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & 4 \end{bmatrix} \qquad T := \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix}$$

(a.) Compute the mean vector for y:

$$my := T\, mx \qquad \underline{\underline{my = \begin{bmatrix} 1.5 \\ 0 \end{bmatrix}}}$$

(b.) Compute the covariance matrix, Cy:

$$Cy := T \cdot Cx \cdot T^T \qquad \underline{\underline{Cy = \begin{bmatrix} 5.106 & 3.382 \\ 3.382 & 4.356 \end{bmatrix}}}$$

(c.) Compute the correlation coefficient for y1 and y2:

$$\rho := \frac{Cy_{0,1}}{\left[\sqrt{Cy_{0,0}}\right]\left[\sqrt{Cy_{1,1}}\right]} \qquad \underline{\underline{\rho = 0.717}}$$

B-54 Let $y_1 = A x_1 x_2$; $y_2 = x_2$

$$f(y_1, y_2) = \frac{f(x_1, x_2)}{|J(y/x)|} = \frac{f(y_1/Ax_2, y_2)}{|A y_2|}$$

$$J(y/x) = \det \begin{bmatrix} Ax_2 & Ax_1 \\ 0 & 1 \end{bmatrix} = Ax_2 = Ay_2$$

$$\Rightarrow f(y_1) = \int_{-\infty}^{\infty} \frac{f(y_1/Ay_2, y_2)}{|Ay_2|}\, dy_2$$

(a.) $$\underline{\underline{f(y) = \int_{-\infty}^{\infty} \frac{f(y/Ax_2, x_2)}{|Ax_2|}\, dx_2}}$$

(b.) $$\underline{\underline{f(y) = \int_{-\infty}^{\infty} \frac{f_{x_1}(y/Ax_2)\, f_{x_2}(x_2)}{|Ax_2|}\, dx_2}}$$

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