

# **Kinematic Analysis of Multi DoF Robotic Arm**

Submitted in partial fulfillment of the requirements  
of the degree of

## **MASTER OF TECHNOLOGY IN MECHATRONICS**

By

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**SCHOOL OF MCHANICAL ENGINEERING  
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**2020**

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# CERTIFICATE

This is to certify that the Research work titled **Kinematic Analysis of Multi DoF Robotics Arm**, that is, being submitted by **Shah Jyoti Gopalbhai**, is in partial fulfillment of the requirements for the award of Master of Technology, is a record of bonafide work done under my guidance. The contents of this research work, in full or in parts, have neither been taken from any other source nor have been submitted to any other Institute or University for award of any degree or diploma.

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**ExternalExaminer**

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This thesis dissertation /project report entitled **Kinematic Analysis of Multi DoF Robotics Arm** by **Shah Jyoti Gopalbhai** is approved for the degree of master of technology in mechanical engineering.

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**Supervisor**

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**Dean**

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Date \_\_\_\_\_

Place \_\_\_\_\_

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I, hereby, declare that this written submission represents my ideas in my own words and where others' ideas or words have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my submission. I understand that any violation of the above will be cause for disciplinary action by the Institute and can also evoke penal action from the sources which have thus not been properly cited or from whom proper permission has not been taken when needed.

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---

Shah Jyoti Gopalbhai

(Department of Mechanical engineering)

# ABSTRACT

In present robotic manipulator are used to complete work like as picking and placing, Drilling Process, Heavy equipment handling, Quality check, Machine work etc. But to complete these task Manipulator face problem to get its desire orientation and position of end effector.

Kinematic Analysis of multi dof robotic arm using robo-analyzer and Mat Lab In this Project we propose to analyze multi degree of analysis. Multi-degree of freedom robots with the help of robo-analyzer and MatLab.

First we determine the D-H parameters and kinematic equations for the given robot and then with help of experimental Set-up further establish different parameters involved.

Finally, we compare our result with compare our result with the previous work done in literature

<b>TABLE OF CONTENT</b>		<b>Page</b>
<b>Certificate</b>		<b>III</b>
<b>Approval sheet</b>		<b>IV</b>
<b>Student declaration</b>		<b>V</b>
<b>ACKNOWLEDGEMENT</b>		<b>VI</b>
<b>Abstract</b>		<b>VII</b>
<b>Table of content</b>		<b>VIII</b>
<b>List of figures</b>		<b>IX</b>
<b>List of abbreviation</b>		<b>X</b>
<b>Chapter 1</b>	<b>Introduction</b>	<b>1</b>
1.1	Project background	1
1.2	Research purpose and meaning	2
1.3	Objective of study	3
<b>Chapter 2</b>	<b>Literature review</b>	<b>4</b>
2.1	Introduction	4
2.2	Reviews	5
<b>Chapter 3</b>	<b>Problem description</b>	<b>13</b>
3.1	Problem description	13
	<b>REFERENCES</b>	<b>36</b>

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## List of abbreviations

- |        |                    |
|--------|--------------------|
| 1. DOF | Degree of freedom  |
| 2. IK  | Inverse kinematic. |
| 3. FK  | Forward kinematic  |
| 4. D-H | Denavit-Hartenberg |



## List of Figure

Figure-1.1: Symbolic representation of robot joints	3
Figure-1.2: Types of Revolute Joint	4
Figure-1.3: Type of Prismatic joints	4
Figure-1.4: Cartesian Robot	6
Figure-1.5: Cylindrical Robot work space	7
Figure-1.6: Work space of Spherical Robot	7
Figure-1.7: (a) Serial and (b) Parallel Robotic Manipulator	8
Figure-3.1: Design base of the robotic manipulator	18
Figure-3.2: Design of upper part of the base	19
Figure-3.3: Design of connecting rod	20
Figure-3.4: Design of link 1	21
Figure-3.5: Design of Link 2	21
Figure-3.6: Design of link3	22
Figure-3.7: Design of End effector	23
Figure-3.8: Initial position of the end effector	24
Figure-3.9: Final position of the end effector	24
Figure-3.10: Maximum height of the manipulator	25
Figure-3.11: Different angle with same position	25
Figure-4.1: Forward Kinematic	29
Figure-4.2: Inverse Kinematic	30
Figure-4.3: Coordinate frames satisfying DH1 and DH2	31
Figure-4.4: coordinate frame for each joint of 5DOF	32
Figure-4.5: The pure translational movement	34
Figure-4.6: rotation of two frame	35
Figure-4.7: coordinate for single joint	35

Figure-4.8: rotation of frame 1 around frame 0	36
Figure-4.9: translational movement of two consecutive frame	37
Figure-4.10: Elbow –in and Elbow-out configuration	41
Figure-4.11: work space of 5 DOF manipulator	43
Figure-5.1: Modelling of 6DOF Robots using CATIA VS PRO	45
Figure-5.2: Finding result using Robo-Analyzer	46
Figure-5.3: Graph between error percentage and number of iteration	46

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# CHAPTER 1

## INTRODUCTION

### 1.1 Introduction

Robots are very important and powerful element of today's industry. Robot are very capable of performing many different task is accurate and do not require common safety.



A robot is formally characterised in the international Standard of Organization as a reprogrammable, multi-functional manipulator produced to move object , parcel , equipment or particular devices by variable programmed motion for the operation of a different tasks.

A robot is a manipulator which when controlled by a computer. robot is always assumed to be computer controlled.it can be regarded as a robot. Industrial robots are designed to work as multi-purpose semi-skilled or unskilled labour. For welding, painting, machining.Special purpose robots are classified as follows:

1. Automatic Guided Vehicle
2. Walking Robots
3. Parallel Robots

The FK evaluation is easy to evaluation of mannequin and calculate the function the usage of the joint angle. But the venture in to analyze the IK answer the use of the position. Complexity of the IK will increase with enlarge the Degree of Freedom.so to analyze Inverse Kkinematics in this Paper use the DH conference and transformation kind solution.

## **1.1 History of Robotic**

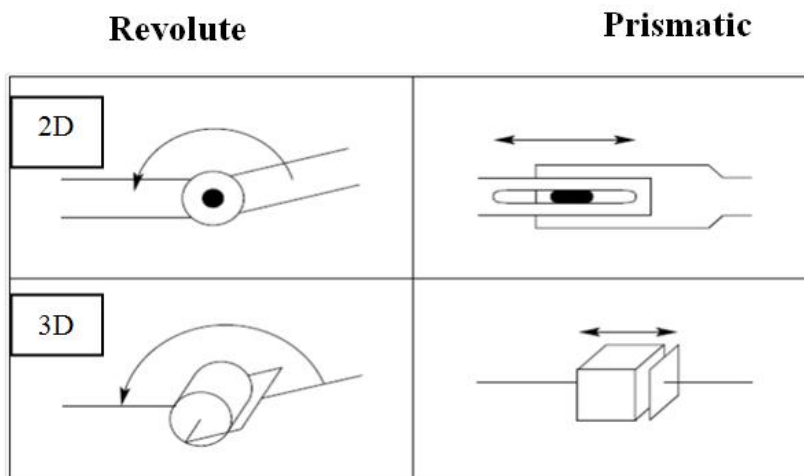
Robot invented through a Czech novelist Capek in 1920. And the 1st industrial robotic UNIMATE designed by means of George Devol in1954 and this was once 1st hooked up at familiar motors plant to work in Die casting machine. The PUMA Robot is developed by way of pioneering robotic Unimation Company in 1978. And used by using NOKIA and NASA, for assembling and for house use.1980s: The robotic enterprise enters a section of fast growth. Many establishments introduce applications and publications in robotics. Robotics publications are unfold throughout engineering, departments. And this is capable to care your toddler in the absence of dad and mom and this additionally discuss to in a similar fashion like as human and appears like as human.

### **1.1.1 Types of joint**

Joint can be categorized in to following

- Revolute joint
- Prismatic or translational joint

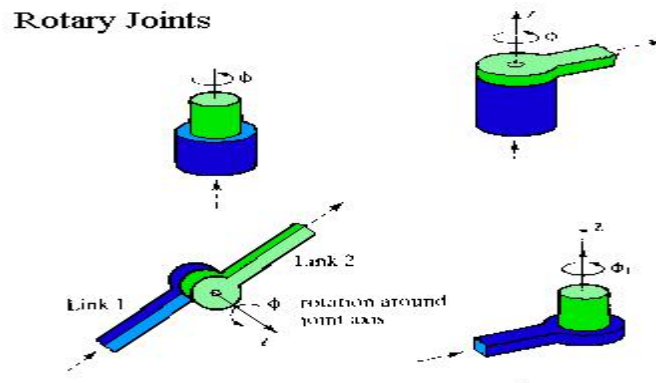
The below figure 1.1 show [29] the 2D and 3D types of joint there are two types of joint one is rotting and one is prismatic or can say that trans lational joint which have linear motion.



**Figure 1.1. Schematic representation of robot joints**

### 1.1.1.1 Revolute joint

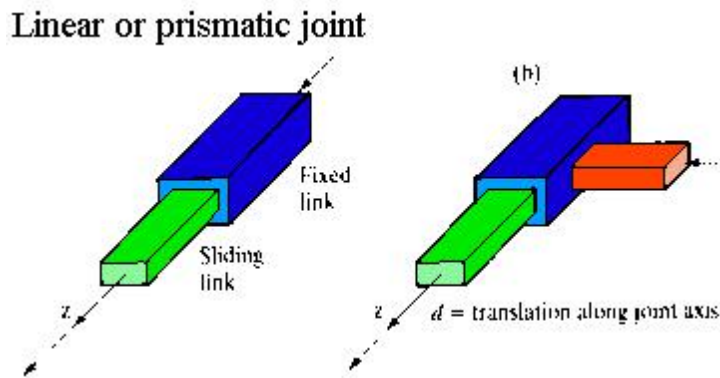
The top two linkages in the figure 4 [29] are generally used as a waist joint. This joint pivots the whole robot. The green rotates about it and the Blue section is fixed. The four joints in this case are only capable of one degree of rotation and the joint variable is the degree of rotation that the joint rotate to. Almost all of rotary joints are not capable to rotate by 360° degrees as they are restrained mechanically.



**Figure 1.2. Types of Revolute Joint**

### 1.1.1.2 Linear or Translational Joint

A prismatic joint is an example of a sliding joint. The usage of it is for linear movement of rigid body. These joints are not as regular as other joints such as the rotary joints however, are useful. Further, The cylindrical robot, having both types of joints allows the link to move around the axis and the linear joints move according to their length.



**Figure 1.3. Type of Prismatic joints**

### 1.1.2 End Effectors:

End effector is an example of gripper tool. It is a special device. The fixture is attached to the Robots arm. It actually do the work. Or can say that this contain the tools to perform the work like as drilling, painting, picking and placing etc.

### 1.1.3 Types of Manipulator

#### a) General manipulator

This is type of manipulator used for general uses like as pick and place type, drilling, material handling in the industries etc.

#### b) Redundant/hyper redundant manipulator

Redundant manipulator have one DOF extra to complete work easily. Or can say that this have one extra link which should not be used in work space.

#### c) Flexible manipulator

Flexible manipulator have maximum length and have more than 10 DOF. This is used tendon type hydraulic system. Flexible manipulator is generally used in space for satellite maintenance.

## 1.2 Classification of industrial robot

Based on numerous factors such as particular task of moving the specific objects to different locations the robots are classified into various ways. This will be discussed in coming sections.

### 1.2.1 Classification based on coordinate frame

The coordinate of the robot is used to planning the motion of the robot and this is generally divided into three types.

#### 1.2.1.1 Cartesian Robots

The robotics arm moves in a rectilinear mode that the direction of x,y, and z coordinates of the rectangular right handed Cartesian coordinate system shown in fig 1.4 . It is called Cartesian or rectangular type. The movement are referred to as travel x-height y and reach z of the arm. A Cartesian robot needs a large volume to operate .

**Advantages:** Easy to see, fixed Structure, Easy mechanical stop, Easy offline programming

**Disadvantages:** Only front and back reaches, it needs big floor space, Axes are fixed to seal .

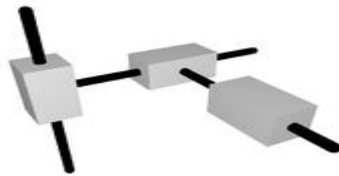


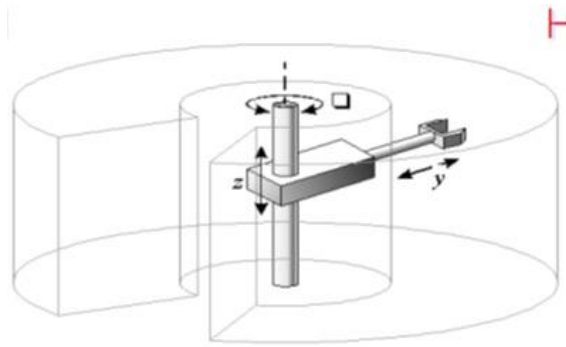
Figure 1.4. Cartesian Robot

#### 1.2.1.2 Cylindrical Robot

The arm of a robot possesses one revolute and two prismatic joint. The prismatic joint Cartesian type show in fig 1.5. A robot with this type of arm angle, height, and radius.

Cylindrical robot one rotation and 2 linear axes. @ base rotation, y height, z reach.

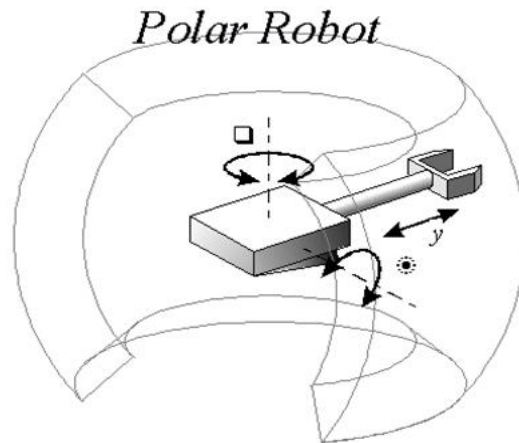
**Advantages:** it reaches to all around , fixed y, z axes .



**Figure 1.5. Cylindrical Robot work space**

### 1.2.1.3 Spherical Robot

A spherical coordinate robotic manipulator can move like as cylindrical but this have one revolute joint than this also cover the area which are not by the cylindrical robot. When we consider the work space of the spherical robotic manipulator this looks like a globe. This is used for placing and picking of the object one place to other. Further, this is widely used in car manufacturing industries on assembly line.

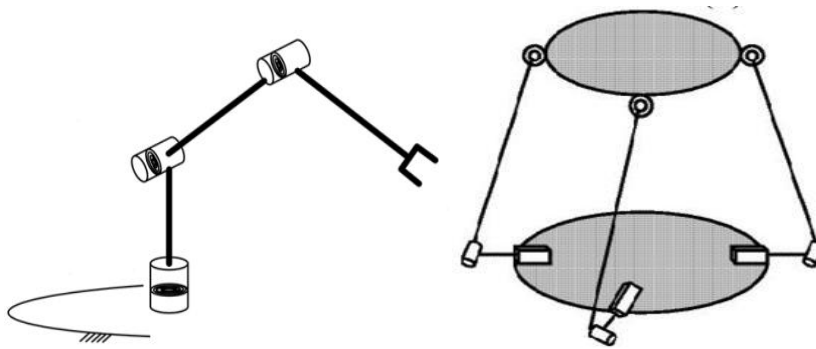


**Figure 1.6. Work space of Spherical Robot**

### 1.2.2 Classification by mechanism

When the robot is classified by their mechanism there are two types one is serial robot where each link is connected to its previous and one end of the manipulator is fixed and one is free to move. And the parallel robot have closed path and can say that both the end joint are fixed and this move only from joints and this type of manipulator are generally used in medical or where high accuracy needed.





**Figure 1.7. (a) Serial and (b) Parallel Robotic Manipulator**

### 1.2.3 Classification by application

Based on the applications manipulators the robots can be categorized into the following types:

- Manipulator use for assembling
- Robotic arm use for Underwater
- Space robot
- Agriculture robot and automatic machine
- Mining manipulators
- Surgical arm and rehabilitation
- Domestic robots
- Educational robots
- Industrial use (welding, drilling, quality check etc.)

### 1.3 Degree of Freedom

The degree of freedom define the first joint movement or in other word this show the number of independent coordinates that define system completely. Simple robot which can move 3 ways down-up ,right- left, forward and inverse this type of robot have 3 DOF.

When talk about human arm which have 7 DOF except hand.

- 1DOF: Shoulder pitch
- 2DOF: Arm Yaw
- 3DOF: Shoulder roll
- 4DOF: Elbow pitch
- 5DOF: Wrist pitch

- 6DOF: wrist yaw
- 7DOF: wrist roll

## **1.4 Motivation**

If we talk about traditionally in industry, manufacturing industries, electronics industries for quality check, die-casting industries for poring of material, material handling etc. A good example of a robotic manipulator is welding or drilling process over a manufacturing line. But main task is controlling of robotic manipulator without overlapping to each other. This has given the cardinal motivation for the research and development of robot manipulator. Controlling of the robotic manipulator is basically done by using microcontroller or any other type of controller but main challenge is to make program according to their position and orientation of the robotic manipulator.

This is the main region to develop a method which takes less time and have less error to get accurate position of end effector. Secondly, there have been technological advances in mechatronics design methods has made it possible to have more compact and integrated robot systems. These new approaches lead to reduced system error and improved performance because position and orientation of the robotic manipulator is main problem.

So the goal of this dissertation is to forward and inverse kinematics solution applicable for the 5DOF manipulator using D-H parameter. As a result appropriate posture and the trajectories of the manipulator could be planned for operation of various work in different fields.

## **1.5 Objective**

The objective of this dissertation is to solve the forward as well as inverse kinematics equations of the 5DOF manipulator. Further, design the manipulator according to the result. It is simple to solve the Forward kinematic analysis but it is very unpredictable in the case of the inverse kinematics equations because of the non-linearity of the equation as well as transcendental function constrain. Because of higher DOF the complexity increases in solving this equation. So the various researchers had used D-H parameter to find the solution for the non-linear and complex equations arise in various

field.

The final objectives of this dissertation can be categorized as:

- Forward and inverse kinematic analysis using analytical method(using D-H parameter)
- Designing of robotic manipulator with 5 degree of freedom according to analytical result
- Comparison of result to getting error

### **1.6 Structure of thesis**

The dissertation is divided into 6 chapters containing the Introduction, literature review, Analysis of robotic manipulator, Designing of robotic manipulator, result and discussion, conclusion and future work as well as references. A concise description of each chapter is given below.

**Chapter 1** is an introduction to the thesis which contains the motivation, and the problem statement of the dissertation. The organization of the dissertation is also included here.

**Chapter 2** presents the literature survey of the dissertation which includes surveys on camera based robot and Also include the sensor based robot strategy that are employed in the recent past.

**Chapter 3** the brief back ground theory for forward and inverse kinematic is provided. A description of fundamental principle and assumption applicable for D-H notation and that give to the formulation and mathematical equation of forward and inverse kinematics of the 5DOF manipulator.

**Chapter 4** the designing of the 5 DOF manipulator is given in this chapter the chapter start with part designing on CATIA and leads to final designing on CATIA software.

**Chapter 5** The result and discussions are carried out.

**Chapter 6** this chapter contain Conclusion and future work.

# **CHAPTER 2**

## **Literature Review**

### **2.1 Overview**

It is really highly essential that, at the onset of a thesis work, a researcher should go through various books, different journals, magazines, and others, to have information on ones research topic. The literature search in today's time is speedier and easier because of data based readily available on line. This chapter will give the background ideas on the issues to be considered in the present work.

### **2.2 Tools used for kinematic solution for survey**

Besides the transformation matrix type solution there are various method to solve or analysis the robotic arm several are mention below.

- Analytical Method
- Iterative Method
- Geometric Method
- Quaternion algebraic Method
- Screws Theory
- Exponential rotational algebra Method
- Lie algebra Method
- Genetic algorithm Method
- Simulated annealing Method
- Bee algorithm
- Particle swarm optimization Method
- Adaptive neuro-fuzzy inference system (ANFIS) using MATLAB
- Fuzzy learning algorithm Method
- Neuro-fuzzy using MATLAB
- Artificial neural network (ANN) using MATLAB

On the basis of these comprehensive survey of literature, the research done in this field are listed below.

**Funda and Paul [1]** in their work A computational evaluation of screw they formulated an approach that was based totally on screw displacement and made them examine end result on the foundation of computational cost. In this task they have decided and characterize rotational and translation of line for universal displacement of inflexible body. They evaluate 4 mathematical answer which impacts the second of inflexible body. They used twin orthogonal matrix, twin unit quaternion, twin distinct unitary matrix and twin Pauli spin matrix. And use PUMA 560 Robotic manipulator for analysis. Drawback of this is time consumable.

**Funda et al. [2]** in their work the arrangement of inverse kinematic of robot controller utilizing quaternion vector pair based technique. This technique is utilized to understand IK of PUMA robot and result is think about based on calculation cost. Also, the mistake can be determined by the tangle lab device for apply autonomy.

**Mitsi et al. [3]** proposed a solution of inverse kinematic solution to avoid the collision of arms. In this work repetitive sort 5DOF mechanical controller are utilized and IK arrangement of 5-dof excess robot controller which depends on ordinary streamlining technique is proposed. In this research punishment work enhancement technique is additionally utilized for the difficult goals. Be that as it may, for forward kinematic arrangement standard scientific technique is utilized

**Xu et al. [4]** in their work repetitive sort 5DOF mechanical controller are utilized and IK arrangement of 5-dof excess robot controller which depends on ordinary streamlining technique is proposed..

**Martn [5]** In this work repetitive sort 5DOF mechanical controller are utilized and IK arrangement of 5-dof excess robot controller which depends on ordinary streamlining

technique is proposed. In this work punishment work enhancement technique is additionally utilized for the difficult goals.

**Wenjun et al. [6]** Gave understanding of numerical study of 5R serial robot. In the work repetitive sort 5DOF mechanical controller are utilized and IK arrangement of 5-dof excess robot controller which depends on ordinary streamlining technique is proposed. Further, punishment work enhancement technique is additionally utilized for the difficult goals.

**Köker [7]** In his research work gave generic algorithm. In this work repetitive sort 5DOF mechanical controller are utilized and IK arrangement of 5-dof excess robot controller which depends on ordinary streamlining technique is proposed. Further, penaltyt work enhancement technique is additionally utilized for the difficult goals.

**De Xu [8]** proposed "Advancement of robot connect movement in reverse kinematic arrangement thinking about crash shirking and joint cutoff." In this work repetitive sort 5DOF mechanical controller are utilized and IK arrangement of 5-dof excess robot controller which depends on ordinary streamlining technique is proposed. In this work punishment work enhancement technique is additionally utilized for the difficult goals. Be that as it may, for forward kinematic arrangement standard scientific technique is utilized ..

**Aspragathos and Dimitros [9]** introduced three strategies for the formula for kinematic equations of robots with inflexible links. The most frequent technique in the subject i.e. robotics neighborhood is dependent totally on matrix transformation of homogeneous type, the 2nd one is dependent on totally on Lie algebra, as well as the 1/3 one on screw idea observed with the aid of twin quaternion algebra. However, the usage of this method has no longer been accompanied in greater Degree of Freedom

**Lai and Menq [10]** Given the two algorithms, iterative and the degenerate axis methods for the motion manipulators with closed-form preferences in the nearby of singularities. These methods analytically assurance a robot's characteristic. The degenerate axis strategy can also moreover now no longer work properly region increments flip out to be finite..

**Pennock and Raghavan [11]** Formulated algorithm to clear up the inverse kinematics of robots primarily dependent on absolutely on integration numerically. Inverse kinematics algorithms chiefly primarily dependent on integration numerically. The methods used in the work has proved exponential stability. Many numerical occasion and a genuine software program are added to its advantages.

**Liang et al. [12]** brought error assessment of SCARA manipulator the utilization of the theory i.e. Screw theory. Authors have added the error obtained by way of way of D-H algorithm as well in distinction the equal with the output totally absolutely in contrast.

**Palacios [14]** proposed a number of strategy for the answer of inverse kinematic of 6DOF robotic manipulators besides thinking about express answer for the chosen manipulator. In this work, sixteen special shape or configurations of the 6DOF manipulators have been introduced and their types on the groundwork of the structure..

**Vivek Deshpande et al [15]** proposed in their paper the goals to mannequin the ahead and inverse kinematics of a 5 Degree of Freedom Robotic Arm for easy pick ans place. A popular D-H illustration of ahead and inverse matrix is produced. A theoretical answer for inverse kinematics of 5 DOF robotic arm given , to asses the motion of robotic It is a reliable and protected robotic machine designed for academic purpose.

**Carlos A. Acosta Calderon et al [16] formulated** an analytical answer for a 5-Degree of freedom manipulator to observe a fixed trajectory whilst preserving the direction of one axis in the frame. The ahead kinematics for a 5-Degree of Freedom manipulator are asseseed systemically.

# CHAPTER 3

## Designing of manipulator

### 4.1 Introduction

To analyze the FoK and IK of 5 DOF manipulator, we must make the robotic manipulator by using CATIA software. This software generally used to design 3D part of mechanical but here we used to design 5 DOF manipulator. Each of the part of manipulator is design according to data is take in this project and check the angle of each arm and also see the orientation by using animation properties. And for comparison the result robo-analyzer software is used.

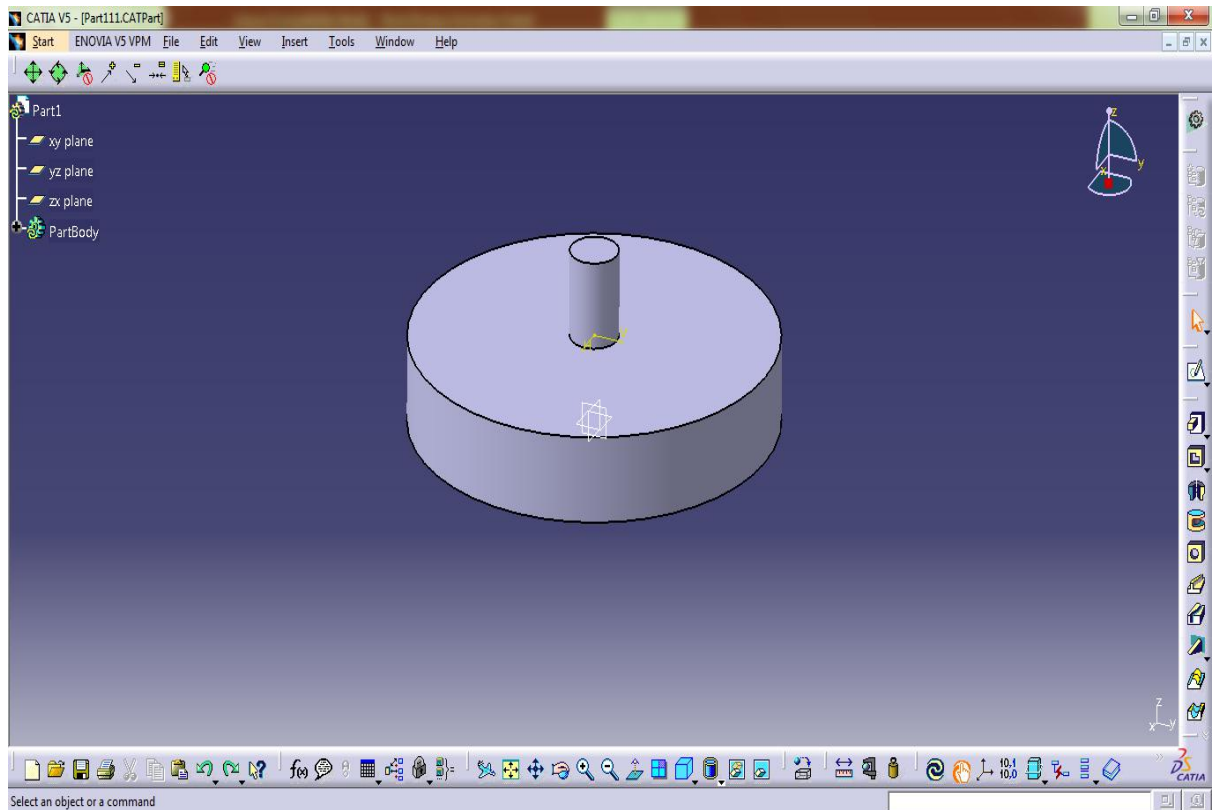
### 4.2 Designing of Part

First design each part of the robotic manipulator and then assemble each part. Specification of each part is given below.

#### 4.2.1 BASE

The base of the manipulator is circular because the manipulator rotate around its base. First make the circle and after that use the extrude command. Similarly make connecting rod for upper part of the robotic base.





**Figure 3.1. Design base of the robotic manipulator**

### **Specification**

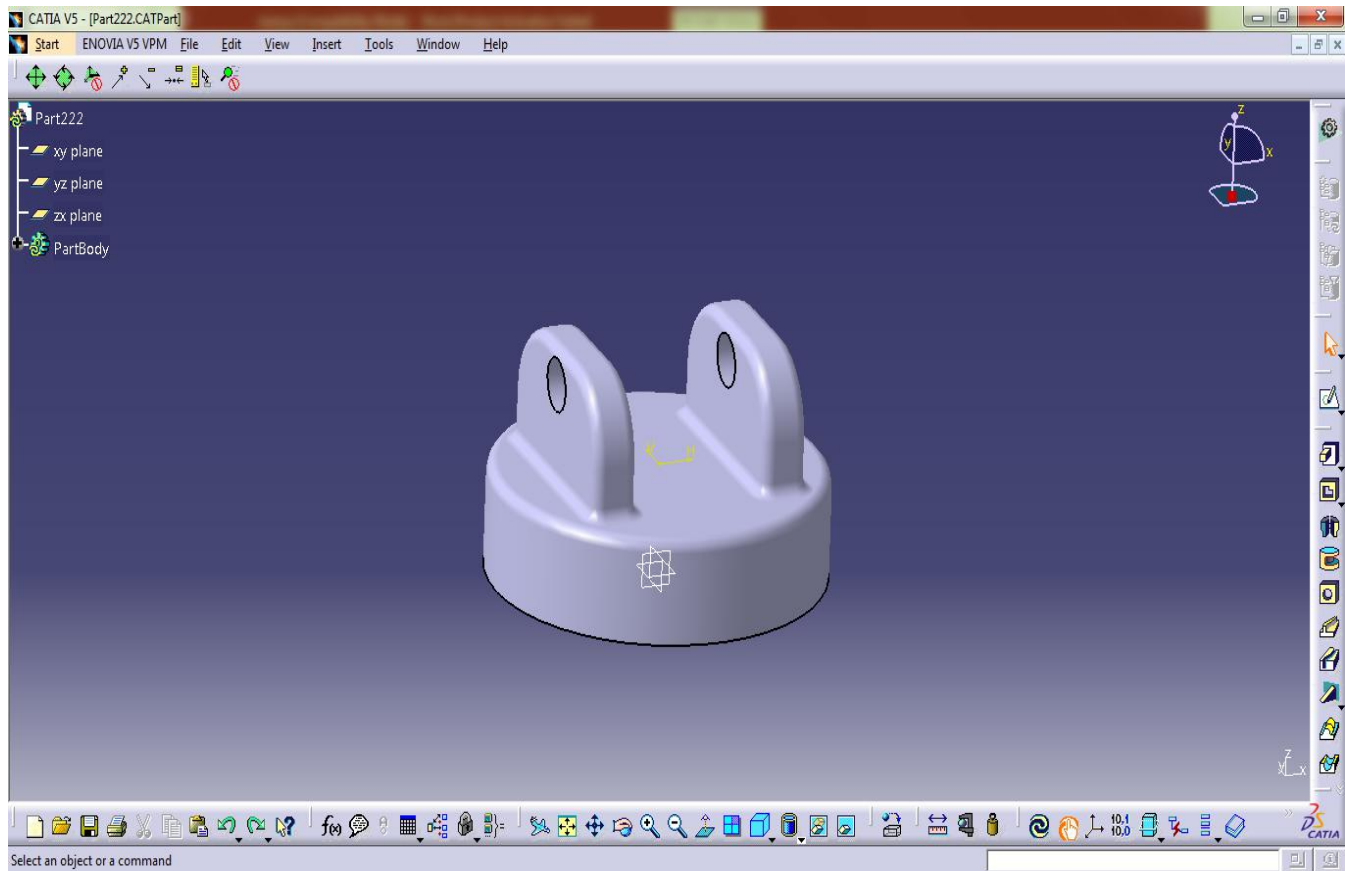
Diameter = 150mm

Height = 200mm

Diameter of connecting rod = 20mm

Height of connecting rod = 20mm

### **Upper part of Base**



**Figure 3.2. Design of upper part of the base**

### **Specification**

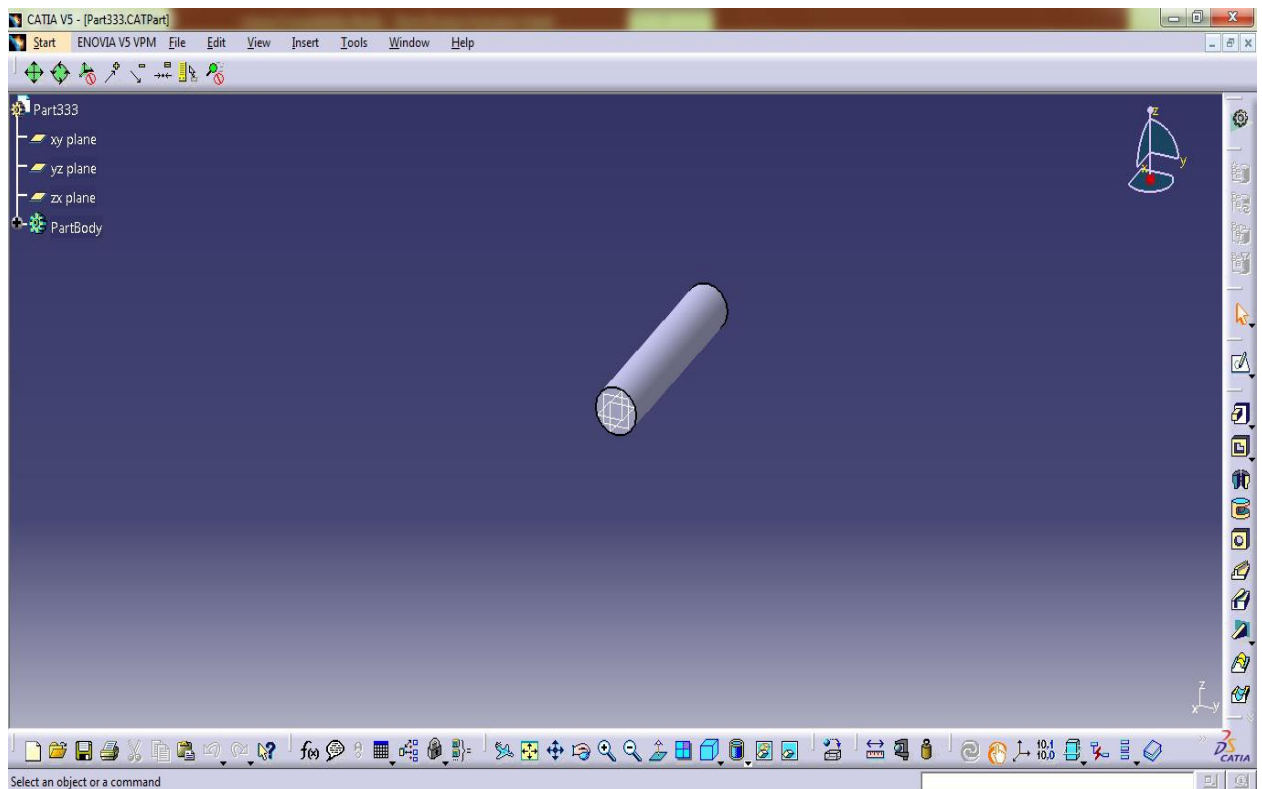
Diameter = 120 mm

Height = 200 mm

Connecting hinge = 60\*40\*20mm

In the above part use extrude, cell and fillet command. Fillet command is used for smoothing the part in catia.

### **4.2.2 Connecting Rod**



**Figure 3.3. Design of connecting rod**

### **Specification**

Diameter = 20mm

Length = 80mm

Quantity = 3

In this simple extrude command is used. First make circle of the 20mm and then extrude it with 80 mm. and this connected rod is save for 3 times use.

### **4.2.3 Designing of Link 1**

#### **Specification**

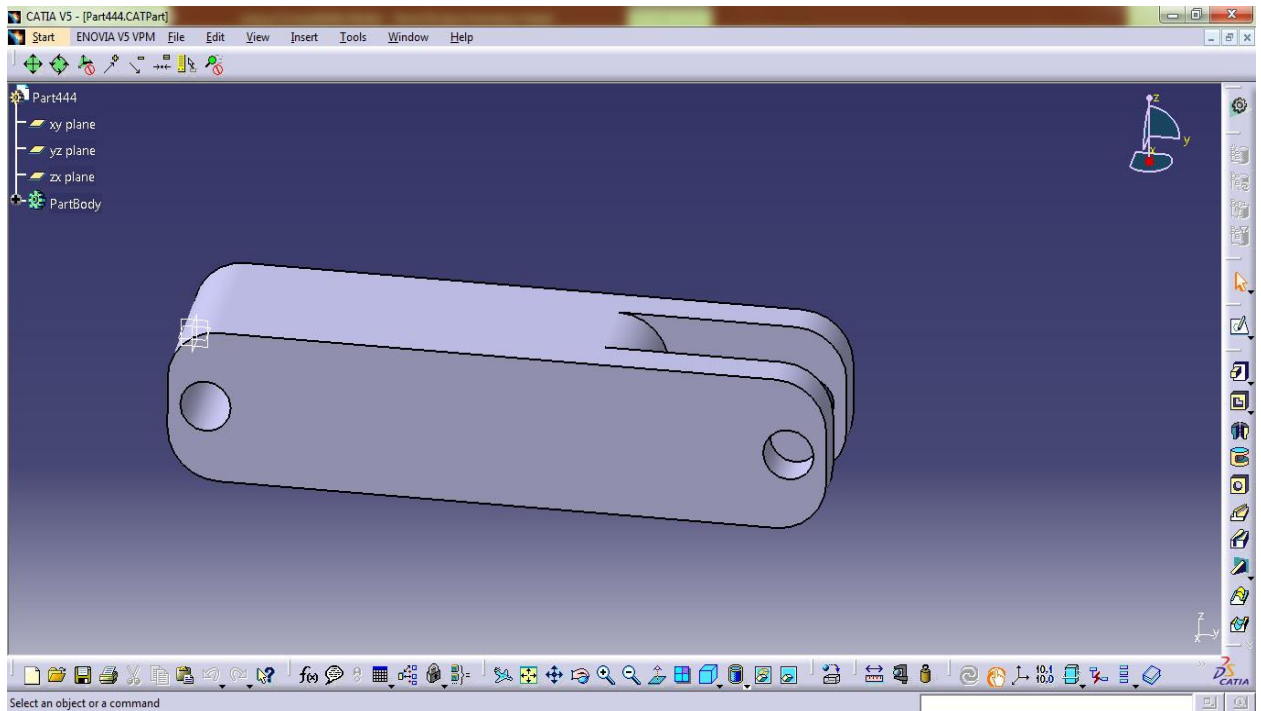
Length = 600mm

Height = 60mm

Hole = 20mm

Trim commend is used for surface smoothing

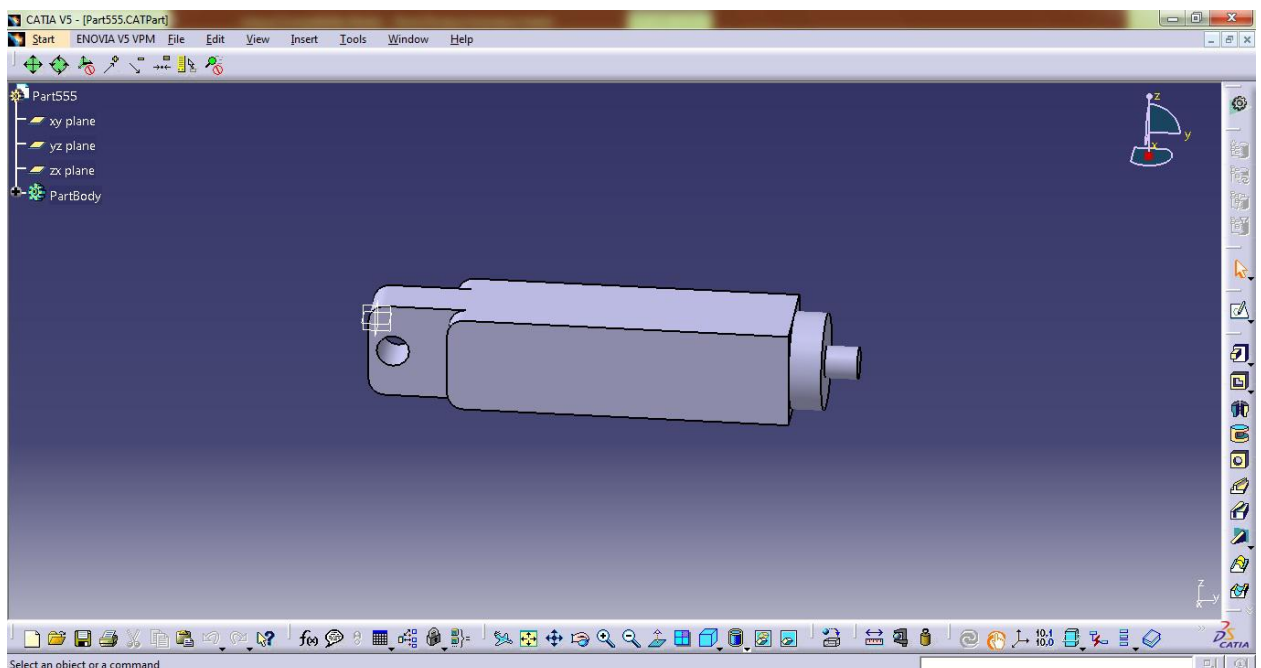
Gap between both of the hinge = 40mm



**Figure 3.4. Design of link 1**

To making above part first make rectangle of 600 mm \* 60mm. then extrude it 60mm. there are two hole of 20mm for connecting other part of the manipulator and make space for next link.

#### 4.2.4 Designing of Link 2



**Figure 3.5. Design of Link 2**

In this part at the end of link circular part is attached to connect next

revolute joint with this link.

### Specification

Diameter of end = 60mm

Length = 400mm

Breadth = 60mm

Hole = 20mm

Front circle diameter = 50mm

### 4.2.5 Design of link 3

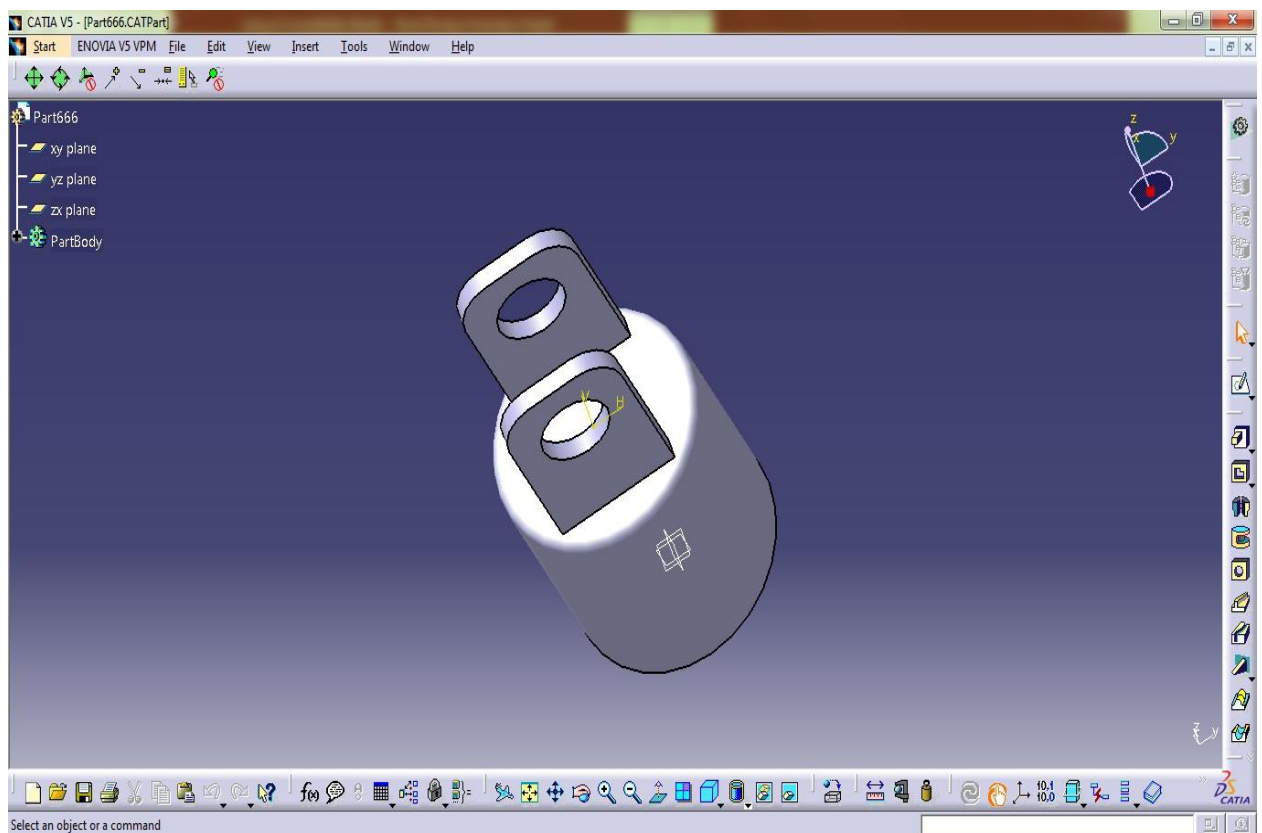


Figure 3.6. Design of link3

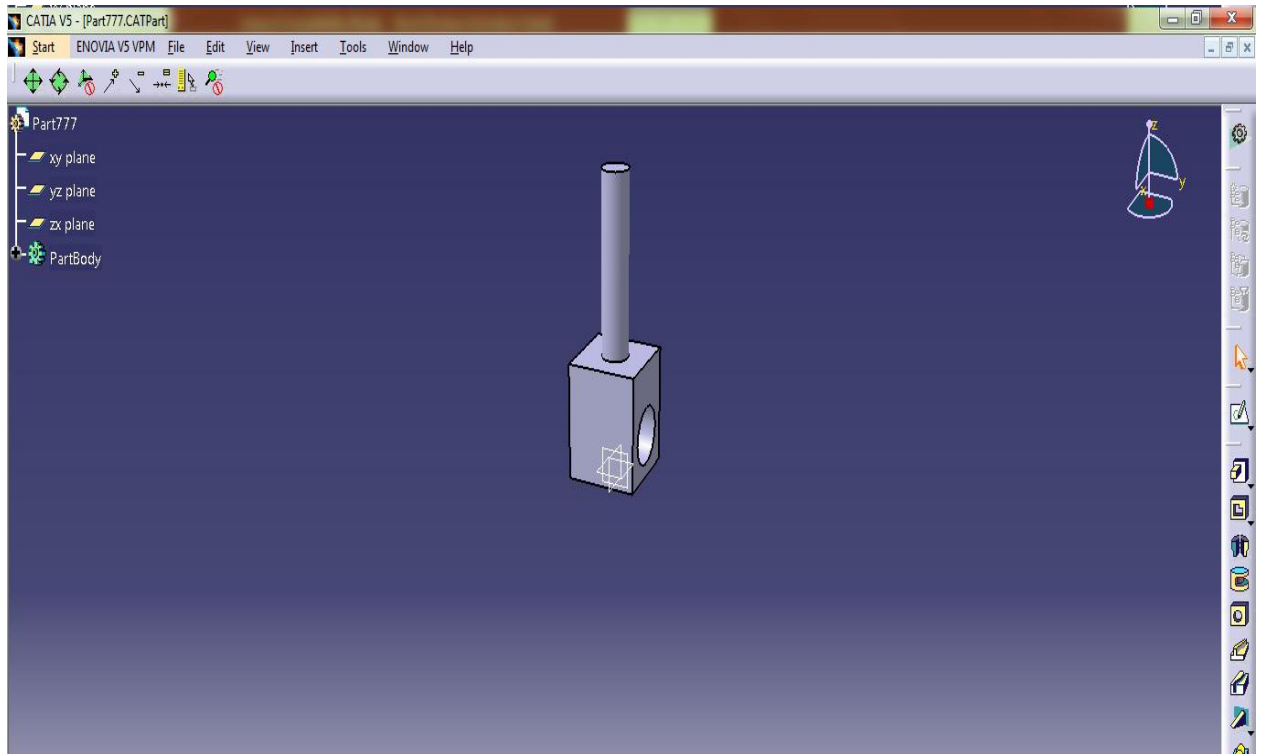
### Specification

Diameter of end = 60mm

Height = 200mm

Hole = 20mm

## 4.2.6 Designing of end effector



**Figure 3.7. Design of End effector**

### Specification

Rectangle = 40 \* 30mm

Length = 70mm

Diameter = 10mm

Diameter of hole = 20mm

## 4.3 Final design of 5 DOF

By using these part we can make 5 DOF manipulator the final design of the manipulator is given below with its different position and angle. These figure show the maximum height of the robotic manipulator. The below figure show the different position of the manipulator with different angles.

### 4.3.1 Initial position of end effector

In the initial position the value of each joint are given below

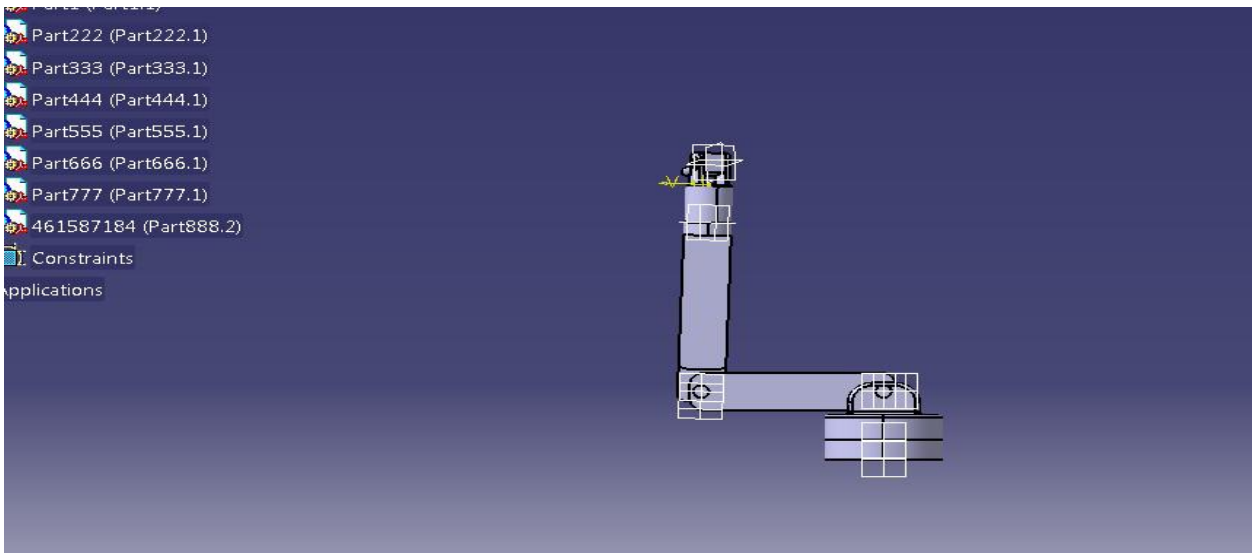
**Table 3 initial value of the end effector**

Joint value 1	Joint value 2	Joint value 3	Joint value 4	Joint value 5
-360	0	-90	-360	-90



**Figure 3.8. Initial position of the end effector**

**4.3.2 Final position of the end effector**



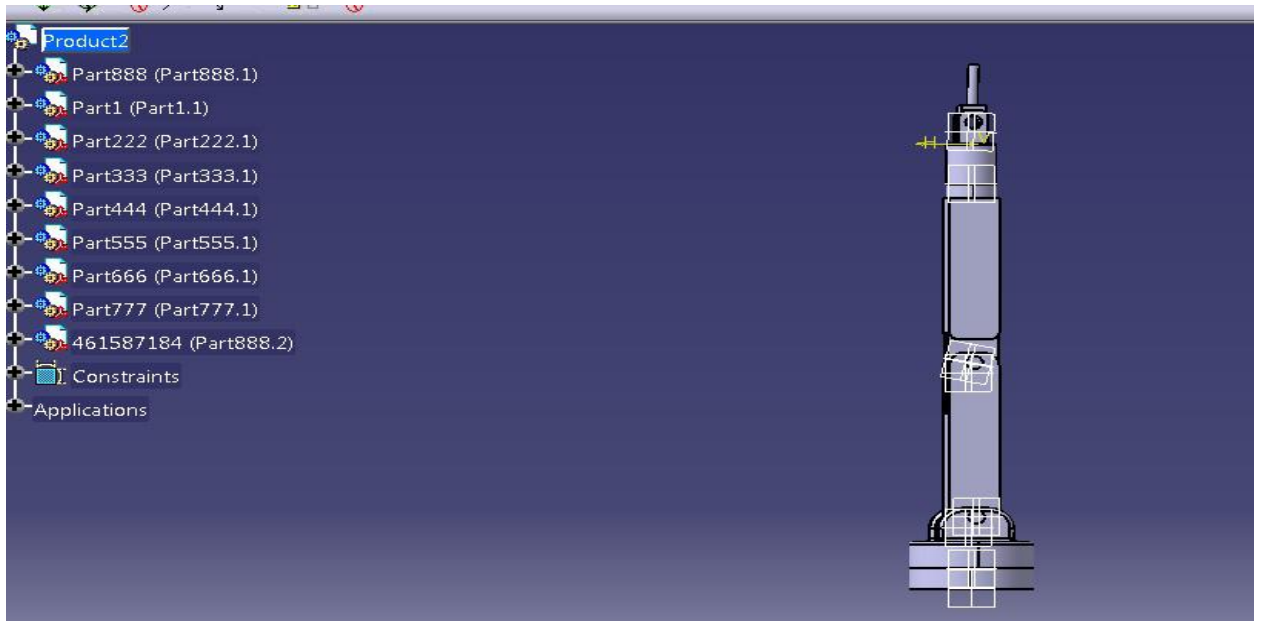
**Figure 3.9. Final Location of the end effector**

In final position the value of each joint are given below

**Table 4 final joint values**

Joint value 1	Joint value 2	Joint value 3	Joint value 4	Joint value 5
360	180	90	360	90

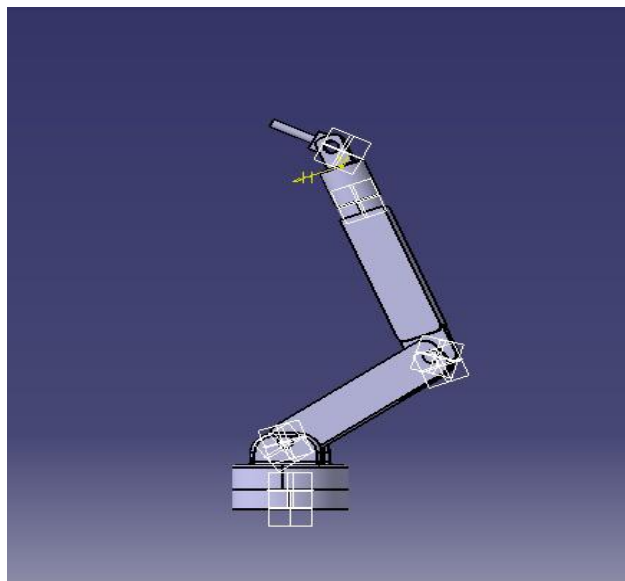
### 4.3.3 Maximum height of the manipulator



**Figure 3.10. Maximum height of the manipulator**

The maximum height of the manipulator is 1620 mm but when we include the height of the end effector tool then this have maximum height of 1740mm.

### 4.3.4 Working position of the manipulator



**Figure 3.11. Different angle with same position**

Figure 3.8 show the initial location of the end effector means all the joint value of end



effector is minimum. When we give command to it then this start working from that position.

Figure 3.9 shows the final location of the EE. All joint value have its maximum value.

Figure 3.10 shows the maximum height of the EE where this can work easily the maximum height of this robotic manipulator is 1740mm.

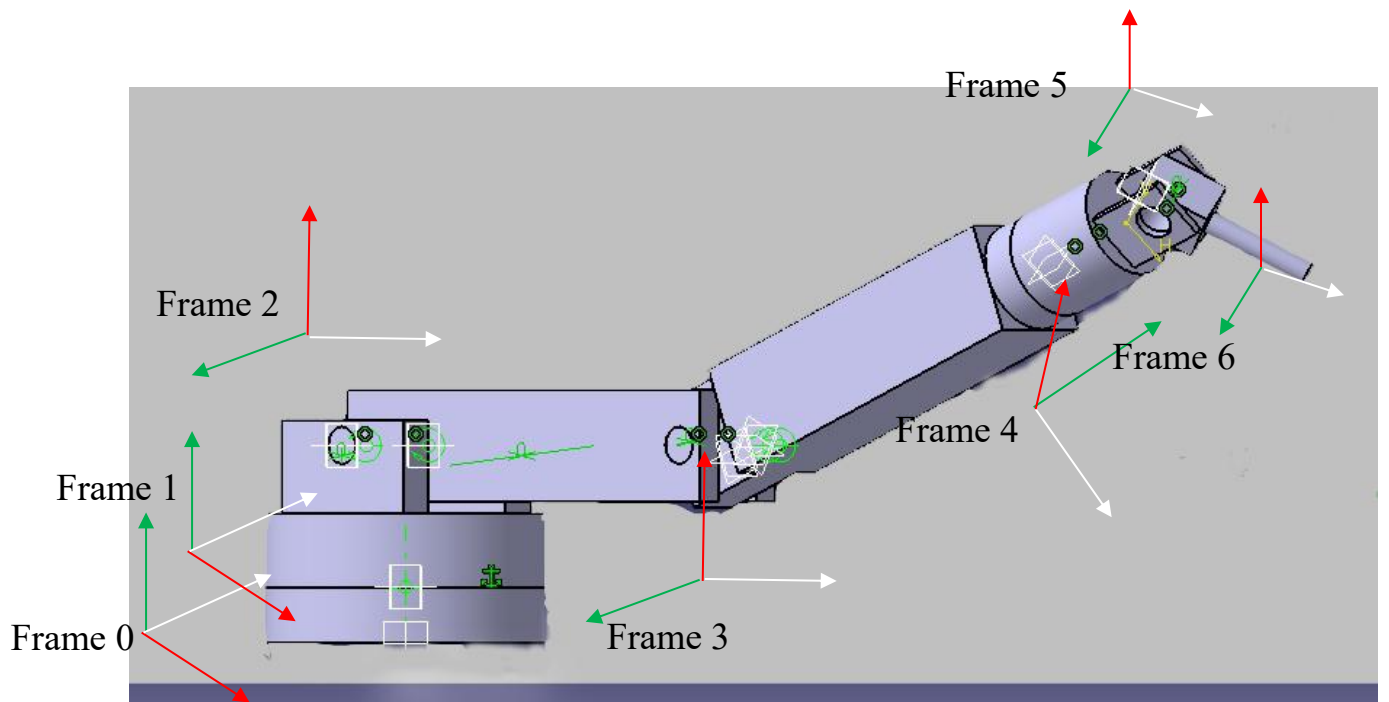
This figure 3.11 shows the different angle of links but the location of the end effector is same and this shows that the IK analysis have more than one solution for a single position.

So that the designing of 5 DOF robotic manipulator is completed with using the software CATIA.

## **CHAPTER 4**

# **Kinematic\_of\_Robotics**

### **4.1 Coordinate frame distribution for 5DOF**



**Figure 4.1 Coordinate frames for each joint of 5DOF**

**Table 3: Joint value or angle of rotation**

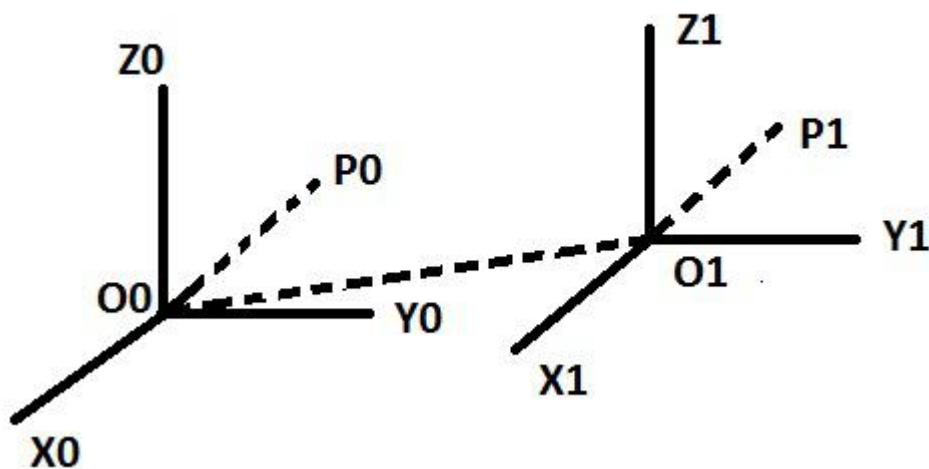
	No. of joints	Joints types	Initial value of orientation	Final value of rotation
Joint1		revolute	-360	360
Joint2		revolute	0	180
Joint3		revolute	-90	90
Joint4		revolute	-360	360
Joint5		revolute	-90	120

This coordinate are used to calculate the homogeneous transformation equation for each frame. The homogeneous equation show that the joint are rotational type or prismatic type according to the frame and this is given below how we calculate the

homogeneous transformation using these frame.

$$T = \begin{bmatrix} \mathbf{R}_{3 \times 3} & \mathbf{P}_{3 \times 1} \\ \boldsymbol{\eta}_{1 \times 3} & \sigma_{1 \times 1} \end{bmatrix}$$

This show the homogeneous matrix which contain 4 parameter one matrix of 3\*3 type is rotational matrix. Here p is the position of end effector and this is 3\*1 type matrix and the eta is 1\*3 matrix and perspective transformation used for computer vision and camera calibration and here consider as 0. Sigma is scale factor in robotics value of sigma is always 1.



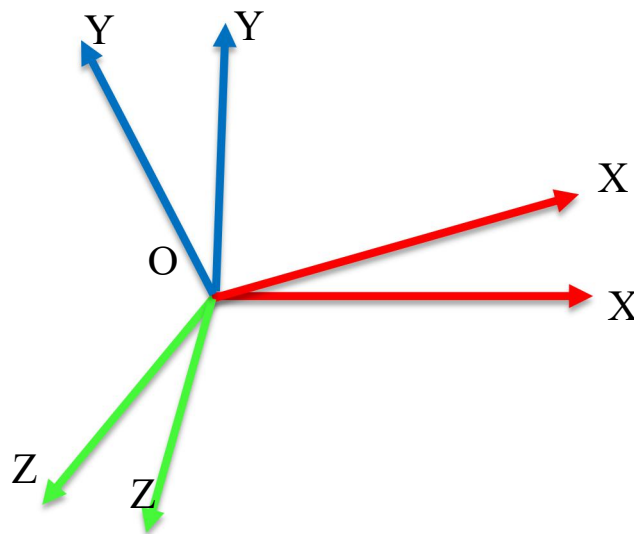
**Figure 4.5. Pure Translational Movement**

Figure 4.5 show the pure translational movement of the coordinate frame of two consecutive joint and the matrix for translational is given below

$$H = \begin{bmatrix} 1 & 0 & 0 & P_x \\ 0 & 1 & 0 & P_y \\ 0 & 0 & 1 & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Here  $P_x$ ,  $P_y$ , and  $P_z$  show the location of the end effector of robotic manipulator.

Now consider two frame with pure rotation or without translational movement. In pure rotation two frame are rotate around there center.



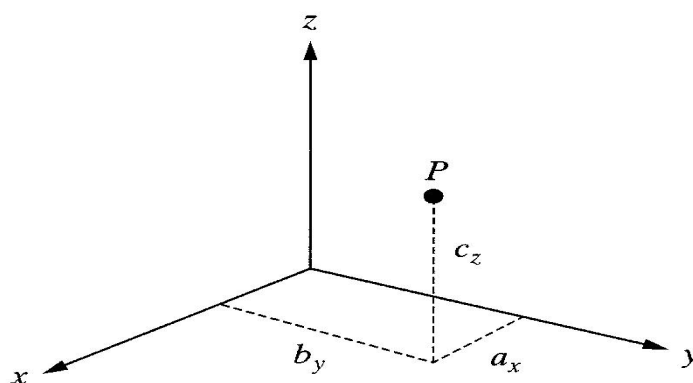
**Figure 4.6. Rotation of two frame**

In the above figure frame is rotate around its center and the matrix for pure rotation is given below

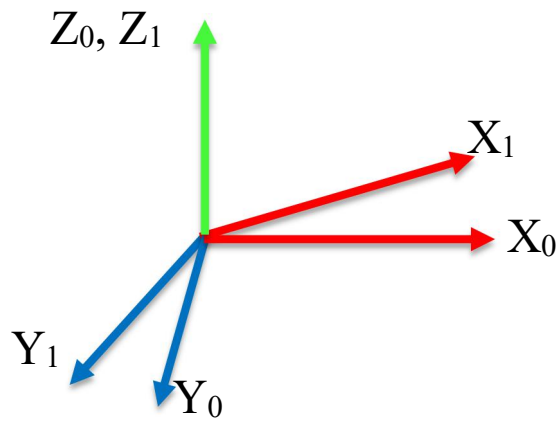
$$H = \begin{bmatrix} \mathbf{n}_x & \mathbf{o}_x & \mathbf{a}_x & \mathbf{0} \\ \mathbf{n}_y & \mathbf{o}_y & \mathbf{a}_y & \mathbf{0} \\ \mathbf{n}_z & \mathbf{o}_z & \mathbf{a}_z & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

This matrix show the direction of the end effector and translational movement so the position matrix is zero.

These matrix is further used to calculate the rotational and translational matrix for each joint.



**Figure 4.7. Coordinate for single joint**



**Figure 4.8. Rotation of frame 1 around frame 0**

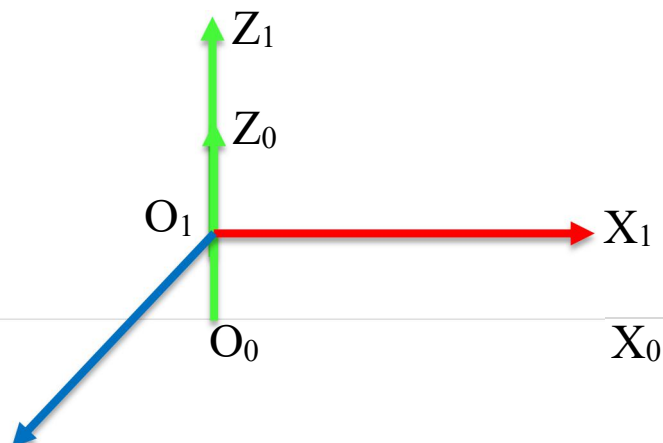
In the figure 4.7 shows that the coordinate assignment for single joint and show the position which is calculate and The above figure 4.8 show the rotation of frame 1 around the Z axis of the Frame 0. This coordinate is used to calculate the basic rotational matrix which are given below

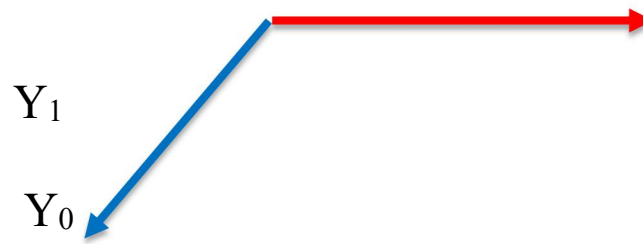
$$\mathbf{R}_z = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2)$$

$$\mathbf{R}_y = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix} \quad (3)$$

$$\mathbf{R}_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix} \quad (4)$$

These three matrix are show the rotation of the frame1 around the z-axis (2), y-axis (3) and x-axis (4).now the translational of the frame 1 and frame0 is take to find the location of the end effector.





**Figure 4.9. Translational movement of two consecutive frame**

This show the parallel translation of distance  $d$  and these two frame are related as

$$\mathbf{P0} = \mathbf{P1} + d \mathbf{1}_0 \quad (5)$$

Here  $P0$  is the location of the end effector from frame 0 and  $P1$  show the distance from frame 1 and the  $d1$  show the movement of frame 1 from frame0.

By using these coordinate frame pure translation matrix are found when the frame is displaced in the direction of  $X, Y$  &  $Z$ .

$$\mathbf{Trans}_{x,a} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

$$\mathbf{Trans}_{y,b} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

$$\text{Trans}_{z,x} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

These matrix show the translational movement of frame1 along the x, y and z-axis.

### 3.6 Analysis of 5 DOF Manipulator Forward Kinematics Analysis

In the forward kinematic analysis the value of the angle is approximate according to design of robotic manipulator.

The Denavit-Hartenberg methodology and notation is used to find the kinematics of the 5-Degree of Freedom manipulator. The frame given and the Denavit-Hartenberg (DH) parameters are shown in Fig 4.4 and listed in Table 4 respectively.

**Table 4. The D-H parameters of the 5-DOF manipulator.**

Joint no.	Joint type	Joint angle $\Theta_i$ (degree)	Joint offset $d_i$ (mm)	Link length $a_i$ (mm)	Twist angle $\alpha_i$ (degree)
1	Revolute	-360 to 360	400mm	0mm	90
2	Revolute	0 to 180	0mm	600mm	180
3	Revolute	-90 to 90	0mm	120mm	-90
4	Revolute	-360 to 360	620mm	0	90
5	Revolute	-90 to 120	0	0	-90

The matrix transformation  $A_i$  and two neighboring frames and is given in equation (1) as,  $O_{i-1}$  and  $O_i$

$$A_i = Rot(Z, \theta_i)trans(Z, d_i)Trans(X, a_i)Rot(X, \alpha_i)$$

Put all the value in above equation from equation 2, 3, 4 and for translational use equation 6, 7, and 8 and by multiplying all these equations the below matrix is find.

The name is given equation 1.

$$= \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i)\cos(\alpha_i) & \sin(\theta_i)\sin(\alpha_i) & a_i\cos(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i)\cos(\alpha_i) & -\cos(\theta_i)\sin(\alpha_i) & a_i\sin(\theta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

$${}^0T_6 = A_1A_2A_3A_4A_5A_6 = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

:

$$\begin{bmatrix} c_1s_{23}c_4c_5 + s_1s_4c_5 & -c_1s_{23}s_4 + s_1c_4 & -c_1s_{23}c_4s_5 - s_1s_4s_5 & -d_6c_1s_{23}c_4s_5 - d_6s_1s_4s_5 + d_6c_1c_{23}c_5 + d_4c_1c_{23} + a_2c_1c_2 + a_1c_1 \\ s_1s_{23}c_4c_5 - c_1s_4c_5 & -s_1s_{23}s_4 - c_1c_4 & -s_1s_{23}c_4s_5 + c_1s_4s_5 & -d_6s_1s_{23}c_4s_5 + d_6c_1s_4s_5 + d_6s_1c_{23}c_5 + d_4s_1c_{23} + a_2s_1c_2 + a_1s_1 \\ c_{23}c_4c_5 - s_{23}s_5 & -c_{23}s_4 & -c_{23}c_4s_5 - s_{23}c_5 & -d_6c_{23}c_4s_5 - d_6s_{23}c_5 - d_4s_{23} - a_2s_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

### 3.7 5 DOF manipulator and inverse kinematic

In the inverse kinematic position of the end effector is known and by using these position joint angle can be obtained. The forward kinematic equation is highly non-linear. And by using the FK equation the calculation of the IK is very difficult. In this project use the transformation matrix for solving the IK equation. By using equation (4) and (13) drive the position equation around the x-axis.

$$P_x - d_6 a_x = C_1 (d_4 C_{23} + a_2 C_2 + a_1) \quad (16)$$

Similarly get other equation by using equation (5) and (14) and get

$$P_y - d_6 a_y = S_1 (d_4 C_{23} + a_2 C_2 + a_1) \quad (17)$$



And by using the above two equation value of  $\Theta_1$  is calculated

$$\Theta_1 = \arctan \left( \frac{2P_y - d_6 a_y}{P_x - d_6 a_x} \right) \quad (18)$$

Now for calculating the value of  $\Theta_2$  and  $\Theta_3$  use the equation (16) and (17) and represented as follow

$$d_4 C_{23} + a_2 C_2 = (P_x - d_6 a_x) / C_1 - a_1 \quad (19)$$

$$d_4 C_{23} + a_2 C_2 = (P_y - d_6 a_y) / S_1 - a_1 \quad (20)$$

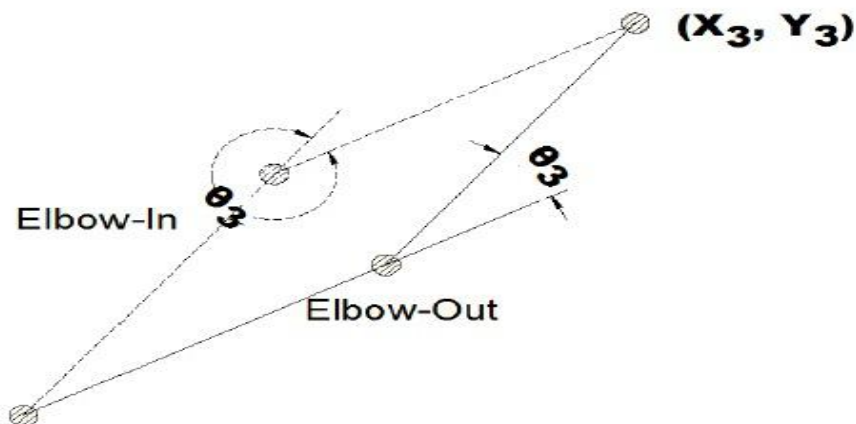
And by using the equation (6) and (15) the following equation can be derived

$$P_z - d_6 a_z = -d_4 S_{23} - a_2 S_2 + d_1 \quad (21)$$

By using these equation the value of the  $\Theta_3$  can be calculated

$$\theta_3 = \pm \arccos \left( \frac{r^2 + r_z^2 - a_2^2 - d_4^2}{2a_2 d_4} \right) \quad (22)$$

$$\theta_3 = \pm \left[ \pi - \arccos \left( \frac{a_2^2 - d_4^2 - r^2 + r_z^2}{2a_2 d_4} \right) \right] \quad (23)$$



### Figure 4.10. Elbow-in and Elbow-out representation

Now calculate the possible solution of  $\Theta_2$  now take equation (21) and written in the form of

$$d_4 s_{23} = B_1 - a_2 s_2 \quad (24)$$

Where  $d_4 a_z - P_z + d_1 = B_1$

Now consider the equation (19) and (20)

$$d_4 c_{23} + a_2 c_2 = \pm \sqrt{(-a_x d_6 + p_x)^2 + (-a_y d_6 + p_y)^2} \quad (25)$$

$$\text{Let } B_2 = \pm \sqrt{(-a_x d_6 + p_x)^2 + (-a_y d_6 + p_y)^2}$$

$$d_4 c_{23} = B_2 - a_2 c_2 \quad (26)$$

Now rearranging equation (23) and (25) and get

$$B_1 = (d_4 c_3 + a_2) s_2 + (d_4 s_3) c_2 \quad (27)$$

$$B_2 = (d_4 c_3 + a_2) c_2 - (d_4 s_3) s_2 \quad (28)$$

Now by using these equation calculate the value of  $\Theta_2$

$$\theta_2 = \text{atan2}(B_1, B_2) - a \cos \frac{(d_4 c_3 + a_2)}{\sqrt{B_1^2 + B_2^2}} + 2m\pi \quad (29)$$

$$\theta_2 = \text{atan2}(B_1, B_2) + a \cos \frac{(d_4 c_3 + a_2)}{\sqrt{B_1^2 + B_2^2}} + 2m\pi \quad (30)$$

Similarly, from the eq (29) and (30), the possible solution of the  $C_4$  is derived as:

$$c_4 = \frac{(o_x - c_1 s_{23} o_z / c_{23})}{s_1} \quad (31)$$

$$c_4 = \frac{-(o_y - s_1 s_{23} o_z / c_{23})}{c_1} \quad (32)$$

Using these equation for small value of  $\Theta_4$  is

$$\theta_4 = a \tan 2 \left( -\frac{o_z}{c_{23}}, \frac{(o_z - c_1 s_{23} o_z / c_{23})}{s_1} \right) \quad (33)$$

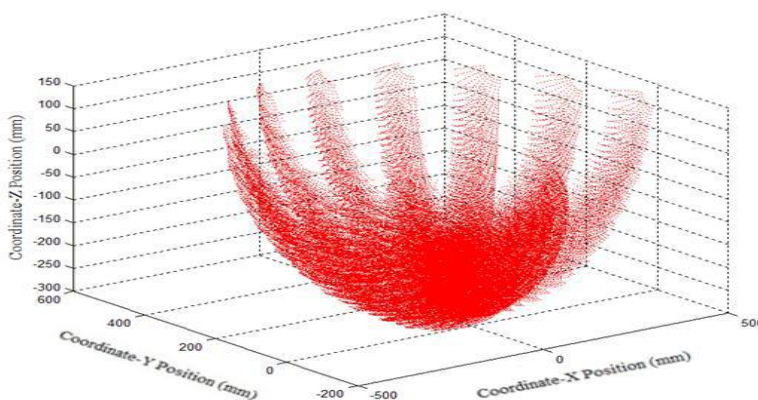
Similarly  $\Theta_5$  can be calculated

$$\theta_5 = a \tan 2 \left\{ -(a_z c_{23} c_4 + s_{23} n_z), (n_z c_{23} c_4 - s_{23} a_z) \right\} \quad (34)$$

The above derivations with different conditions are being taken into account gives a complete theoretical solution to inverse kinematics of 5-Degree of Freedom redundant manipulator. It is to be observed that there consists two viable solutions for all the value of  $\Theta$  depicted in equation (18), (29) or (30), (22) or (23), (33) and (34) respectively. Therefore, to know what solution is good to find the inverse kinematics, all joint angles are found and compared with the help of forward kinematics solution.

### 3.8 Work Space of 5 DOF Manipulator

Figure 4.11 [17] show the maximum work space of 5 DOF manipulator. This show the all possible position and angle of the 5 DOF manipulator.



**Figure 4.11. Work space of 5 DOF manipulator**

## **CHAPTER 5**

### **Result and Discussion**

**Table 1 different parameters for the 6-DoF Robotic manipulator.**

Frame	Joint Offset in m		Length of the manipulator(m)	Twist angle(Degree)	Starting joint angle in degree	The target joint angles in degree
0-1	D1= 774.21	variable	0	-90	0	60
1-2	D2= 101.592	variable	A2=506.628	180	0	60
2-3	D3= -38.1	variable	A3=20	90	0	-60
3-4	D4= 267.696	variable	0	-90	0	60
4-5	0	variable	0	90	0	60
5-EE	D6= 58.42	variable	0	0	-120	120

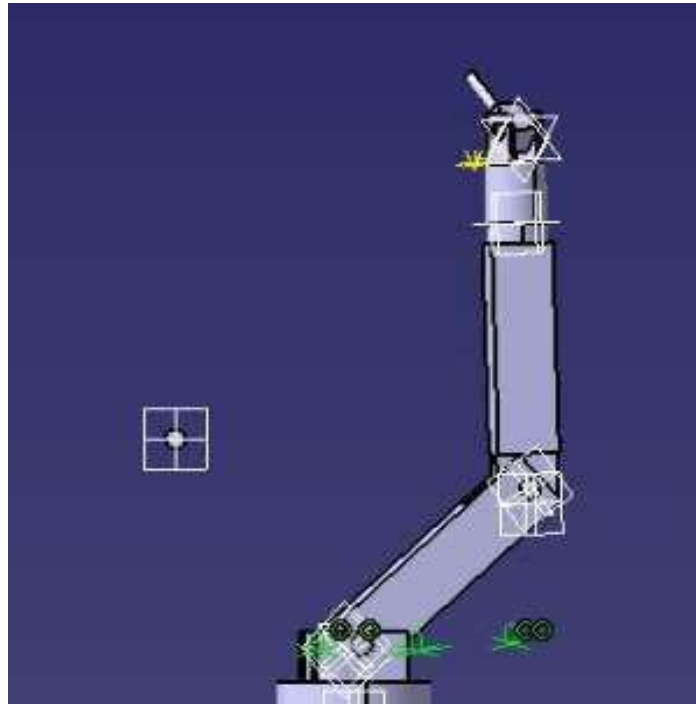


Fig.5.1- Modelling of 6DOF Robots using CATIA VS PRO

Select Robot: 6R Decoupled Mar

Link Length (a) mm	Joint Offset (b) mm	Twist Angle (alpha) deg	End Effector's Position
1: 180	1: 400	1: 90	X (mm): 80
2: 600	2: 135	2: 180	Y (mm): 100
3: 120	3: 135	3: -90	Z (mm): 1200
4: 0	4: 620	4: 90	Orientation Matrix
5: 0	5: 0	5: -90	-0.8086 0 0.5883
6: 0	6: 0	6: 0	0 1 0
			-0.5883 0 -0.8086

IKin Analysis Complete

For FKIn  
 Select Initial Values: Solution 2  
 Select Final Values: Solution 2  
 OK

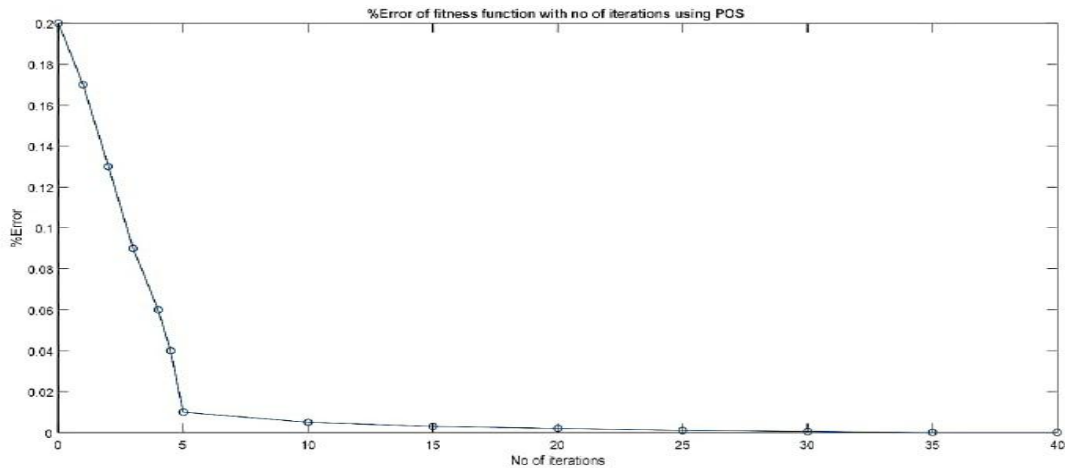
Solution1: Theta(deg)	Solution2: Theta(deg)	Solution5: Theta(deg)	Solution6: Theta(deg)
1: 51.3402	1: 51.3402	1: -128.6598	1: -128.6598
2: 42.598	2: 42.598	2: 158.4767	2: 158.4767
3: -177.8588	3: -177.8588	3: 12.7614	3: 12.7614
4: 241.9236	4: -298.0764	4: 71.7104	4: -108.2896
5: 148.6226	5: -148.6226	5: 151.0633	5: -151.0633
6: 103.31	6: -76.69	6: 114.6166	6: -65.3834

Solution3: Theta(deg)	Solution4: Theta(deg)	Solution7: Theta(deg)	Solution8: Theta(deg)
1: -128.6598	1: -128.6598	1: 51.3402	1: 51.3402
2: 63.6446	2: 63.6446	2: 144.831	2: 144.831
3: -170.8531	3: -170.8531	3: 19.7668	3: 19.7668
4: 152.2108	4: -27.7892	4: 332.2454	4: -207.7546
5: 99.8113	5: -99.8113	5: 99.4236	5: -99.4236
6: -139.8233	6: 40.1767	6: -139.6162	6: 40.3838

Fig. 5.2 : Finding result using Robo-Analyzer

To check the results obtained of the inverse kinematics, an arbitrary point within the working limit of the robot is defined. The inverse kinematics will compute the set of

joint angles. An optimal set of joints is defined when the first arm is positioned perpendicular to the ground. Results can be validated analytically and geometrically. With running the code in the MATLAB[9] we find that it takes only about 30 iteration to converge the solution i.e. error is even less than 0.001%, consequently we stop the simulation run.



**Fig. 5.2 :** Graph between error percentage and number of iteration

In fig.5.2 we have plotted the graph between percentage error and iteration number. From the graph we can find that only it takes 30 iterations to converge the solution with

very less time. That can be very useful for deployment of 6R robots for real time applications. To check the results obtained of the inverse kinematics, an arbitrary point within the working limit of the robot is defined. The inverse kinematics will compute the set of joint angles. An optimal set of joints is defined when the first arm is positioned perpendicular to the ground. Results can be validated analytically and geometrically.

With running the code in the MATLAB[9] we find that it takes only about 30 iteration to converge the solution i.e. error is even less than 0.001%, consequently we stop the simulation run. In fig.3 we have plotted the graph between error percentage and number of iteration. From the graph we can find that only it takes 30 iterations to converge the solution with very less time. That can be very useful for deployment of 6R robots for real time applications.

## Conclusion

In this study, ahead kinematic answer the use of approximate cost and the inverse kinematics answer the usage of D-H parameter for a 5-DOF robotic manipulator is presented. In this task we are introducing a approach that can be used to clear up the ahead and inverse kinematic equation for 5DOF robotic manipulator. And format robotic manipulator in accordance to the result. Because the role and orientation of the manipulator have the vital function to entire any challenge or work in enterprise or any place. The time ingesting in fixing these equation by means of the use of robo analyzer is approx. 20 minute and when use the analytical technique this take two and three hours to remedy FK and IK equation. But robo-analyzer software program have their obstacle of the work and there are some ordinary approach that are normally used to remedy FK and IK equation however these typical technique are extra time consumable and have greater error in fixing of FK and IK equation to discover position, perspective and orientation. So that D-H parameter technique for fixing IK and FK equation is handy and much less time consumable and we see that there are suited error. And most necessary characteristic of this task is particularly correct for discovering the

position, attitude and orientation of the EE of the pick out and location kind robot, welding robot, drilling robotic or can say that the place position, perspective and orientation have essential massive role.

The D-H parameter approach are additionally used for fixing the FK and IK equation for different sorts of Robot like as

- Kuka KR5 robotic arm: this is commonly used for pick out and place.
- Pioneer robotic arm: used for whole drilling and welding tasks.
- Other kinds of robotic which are used in space, below water, manufacturing industry, medical, fabric coping with and so on



# Future Work

The robotics enterprise has reached one plateau with the profitable introduction of robots into automobile manufacturing for spot welding and painting, selecting and placing, manufacturing industries, electronics aspect mounting, first-class checking are areas the place robotic manipulator are nearly used.

A 5 DOF robotic manipulator is used in OPPO cell organisation which are used to vicinity the issue at that place. And quite a few robotic are used for pleasant checking of the phone. So D-H parameter can be used to this robotic for its free positioning and to decide its path. Apart from this, the transformation method can be used in a number area to decide the positions, perspective and orientations. It can be used for:

- Industrial utility ( choosing and placing, excellent checking)
- Material managing robot
- Firefighting, building and agricultural robotic
- Medical utility (operation and surgery)
- For domestic groundwork robotic ( Washing Machine)

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