FIRST PRINCIPLES

OF

SYMMETRICAL BEAUTY.

BY

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FIRST PRINCIPLES

OF

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ERRATA.

Page 7, line 7 from top, for "harmony. Such," read *harmony, such.*

Page 30, column 3, lines 5, 6, and 7 from bottom, for "10^0 15^0 16^0," read 10 15 16.

simply to convey as much instruction regarding the nature of symmetrical beauty and its application to art, as the humblest work on English Grammar conveys regarding the primary elements of written language and their application to literature, consequently it has no more to do with high art than the spelling-book has to do with poetry.

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SYMMETRICAL BEAUTY.

INTRODUCTION.

It is requisite at the outset, to caution the reader against an error that many seem to have fallen into regarding my former works, namely, the supposition, that in attempting to define the laws of symmetry, upon which the primary beauty of form depends, and which is the governing principle in ornamental design, I pretend to give rules for that kind of beauty which genius alone can produce in works of high art. But I make no such attempt—as well might it be said of the author of an elementary school-book, that in attempting to instruct his young readers in the elements of their mother-tongue, he was pretending to teach them rules for producing poetical conceptions and other creations of the imagination—as that I, in laying before my readers the first principles of symmetry, am giving rules for the exercise of genius in the arts of design. My object has been, and still is, of a less aspiring, and more practical nature, and all that is attempted in this Treatise, is simply to convey as much instruction regarding the nature of symmetrical beauty and its application to art, as the humblest work on English Grammar conveys regarding the primary elements of written language and their application to literature, consequently it has no more to do with high art than the spelling-book has to do with poetry.

Students in the art of ornamental design, are generally recommended to copy from nature; and, doubtless, all beauty is to be found in her organic forms; but in these the first principles of symmetrical beauty are so blended with the picturesque, and operate in a manner so exquisitely refined and subtile, that mankind have as yet been unable to systematize them, and therefore we cannot, from individual objects in nature, deduce an intelligible system of such practical rules, as may form the basis of a general mode of instruction in the first principles of ornamental art. It is never assumed, that because nature affords the poet some of the finest themes for the exercise of his genius, that it also supplies the knowledge of language which enables the generality of mankind to read and understand his poetry. Neither are the beauties of nature to be transferred to works of ornamental art by means of mere imitation, more than their mere description constitutes poetry; the hand that traces the semblance of her organic forms, must be guided by a mind so constituted as to possess a keen and quick perception of the most subtile developements of the principles of beauty, and deeply imbued with that faculty which reciprocates at once to these developements. Such a mind constitutes genius—a quality which cannot be inculcated by any process of tuition, and therefore none but men who possess it intuitively, are capable of imitating properly the beauties of nature, either in her general aspect or individual forms. When the attempt is made by ordinary men, whose works should only be such as are subject to rules that can be taught, caricature is the result, and instead of exciting the sympathies and admiration of well-constituted minds, they only occasion regret, and excite ridicule. To study the beauties of nature is, doubtless, one of the most delightful as well as one of the most beneficial employments of the perceptive and reflective powers of the mind; but to attempt to imitate them in their picturesque beauty, without the qualification of genius, is a waste of labour, and the adaptation of those defective imitations indiscriminately, to ornamental purposes, has done more to degrade art than any other species of barbarism. The account handed down to us of the origin of the capital of the Corinthian column, is so appropriate in this place, and points out so clearly the mode in which natural objects may be made available in some of the arts of ornamental design, that familiar as it may be to many of my readers, I shall here repeat it. "A Corinthian virgin of marriageable years, fell a victim to a violent disorder; after her interment, her nurse, collecting in a basket those articles to which she had shown a partiality when alive, carried them to her tomb, and placed a tile on the basket for the longer preservation of its contents. The basket was accidentally placed on the root of an acanthus plant, which, pressed by the weight, shot forth towards spring its stems and large foliage, and in the course of its growth reached the angles of the tile, and thus formed volutes at the extremities. Callimachus, who for his great ingenuity and taste was called Catatechnos by the Athenians, happening at this time to pass by the tomb, observed the basket, and the delicacy of the foliage that surrounded it. Pleased with the form and novelty of the combination, he constructed, from the hint thus afforded, columns with capitals of this species about Corinth, and arranged their proportions, determining their proper measures by perfect rules."

We all feel that a certain degree of order, harmony, or proportion of parts, is a necessary constituent of elegance in every thing; but it ought always to be apparent and simple in works of an ornamental nature. From our earliest recollections we can trace a love of order and uniformity; and although in works of ornamental design we may thus adopt the forms of natural objects, they must be symmetrized by being arranged with some degree of regularity. There is a truth in the picturesque beauties of nature which can be given in works of imitative art by men of genius only, and it is this truth that touches our feelings, and excites our sympathies. Hence, in the compositions of high art, where picturesque effect is combined with accurate imitations of nature in her general aspect, as well as in her more particular details, the principles of symmetrical beauty are so subtilely imparted as not to show themselves. But in the art of ornamental design, which is not an imitative art, the parts ought to be systematically regular, and no subject of this kind is injured by being, like the Corinthian capital, so decidedly uniform as to be obviously artificial. Imitations of the human figure especially, ought to be confined to works of high art; for the higher the position which an object holds in creation, the more intolerable are defective imitations of it in art. Merely conventional figures, in which there is no attempt at imitation, such as the kings, queens, and knaves in a pack of cards, are much more endurable than most of those pictorial imitations of the human figure, to be found in the works of the Birmingham japanner, in such mechanical statuary as occupies the site of the modern Parthenon of Modern Athens, and in many other of the artistical productions of this country. The sculptured decorations of the Parthenon, and the painted decorations of the Vatican, are often referred to as examples of the successful employment of the human figure as an element in ornamental design; but it must be recollected, that the one was the work of a Phidias and the other of a Rafaelle.

In the imitative arts all beauty is of a relative description, and owes its excellence to the conformity or degree of unity that exists between it and the original from which it is taken, and which it ought not only to imitate, but to represent in its effect upon the mind. The original may either be a natural object or an established idea, and the imitation may still be beautiful as a work of art, whether or not there be *absolute* beauty in the object or the idea depicted. An imitation of a healthy, youthful, and well proportioned human figure, may make a more pleasing picture than that of old age and deformity; yet although the subject of the one may be more beautiful than that of the other, both may stand equally high as works of art; and their beauty as such, estimated by the success with which nature has been thus imitated to the eye and represented to the mind. But original or absolute beauty does not depend upon a resemblance to any other original. This beauty is relative only to the human mind by which it is perceived, for here there seems implanted a faculty that reciprocates in some degree or other to certain modes of combination in the visible elements of creation. The first element of this species of beauty is uniformity, and the second, variety. When we look upon the outline of a circle, we are satisfied that it exhibits perfect uniformity in the parts of which it is composed, without reference to any other object whatever, and if we inscribe within it, either an equilateral triangle or a square, we are also satisfied of the agreement that exists between either of these rectilinear figures and the circle. Variety is imparted to combinations of this kind, by a change in one or other of the parts. For instance, when we inscribe within a circle an isosceles triangle, we find the first species of variety, because the circumference of the circle is divided into three arcs, two of which are equal to each other, and one unequal, being either larger or smaller than either of the other two. Thus the impression made upon the mind through the eye, meets an immediate response, and this is the foundation of that species of beauty which *can* be subjected to rules, and which we can apply with precision and certainty, not only in the arts of ornamental design, but in such of the ordinary requirements of mankind as admit of symmetry, proportion, or harmony of parts. When the laws by which this species of beauty is governed are obeyed, in the production of any work, however humble it may be, there can be nothing to offend the eye of taste, while imperfect imitations of natural objects are always offensive.

For instance, let us suppose that a block of white marble was to be applied to the decoration of a flower-garden. If a sculptor of real genius were employed to perform the work, he might convert the shapeless mass into a Venus or a Hercules, thus combining a perfect imitation of the human figure with that of the established idea either of beauty or of strength; and according to the powers of his genius would be the perfection of his work. But suppose that the proprietor of the block of marble could not afford to employ an artist of the highest order, but still wished to have the highest class of subject, he would have in all probability a very imperfect imitation of a female or male figure—without the requisite embodiment of the idea of beauty, or of strength. Let it now be supposed that the proprietor of the block of marble foresaw this failure, and that, as he could not employ real genius upon the work, instead of any attempt at imitative art, he only required an ornamental design where the beauty of form might be applied in the abstract; and supposing that he could get an ingenious mechanic who had been taught the first principles of symmetrical beauty, he would then have every probability of seeing his marble converted into some merely ornamental design, which, although not calculated, like a work of high art, to excite our sympathies, and form a theme for the imagination to dwell upon, would still be æsthetically pleasing; and could never excite in the mind that species of dislike which arises from an imperfect imitation of nature.

The mechanic, in the execution of such a design, can get no direct assistance from the beauties of nature, unless they be applied systematically through the medium of the laws of symmetry; for in the humblest of her organic forms, the laws which constitute their beauty generally lie so concealed as only to be ascertained by the careful investigation of the naturalist, or felt by the keen perception of the man of genius. Therefore no mere imitation of the various natural beauties that were to surround his work, would conduce half as much to its perfection as an application of the most elementary laws of geometric harmony. Such as Callimachus is said to have applied in the production of the Corinthian capital.

It is the same in architecture as in sculpture with regard to the employment of genius. When a public building or palace is to be erected, the established idea of grandeur can only be imparted by an architect of true genius. But the street elevation of a tradesman's house, in its division into doors and windows, may be rendered much more beautiful by an application of the elementary laws of symmetry, than by the usual mode of attempting to produce grandeur by the misapplication of the architectural decorations of a Grecian temple. In like manner, the potter in fashioning his clay, and the decorator in drawing his ornamental design, may have in those laws a guide to as much beauty as the nature of their respective works will admit of, without the danger of falling into the errors of defective imitation either of the works of others or of natural objects.

In opposition to the laying down of any kind of rules in art, it is often asserted that the greatest works of genius generally exhibit a departure from the known laws of harmony. But I strongly suspect that this supposition is a popular error, and that instead of such works exhibiting a departure from those laws, a little investigation would prove that their excellence depended upon a more refined and subtle application of them, emanating from intuitive feeling, and thereby more closely resembling the works of nature. Such apparent deviations from rule are appropriately called flights of genius; but they are flights in the right direction, and such as none but men of genius can successfully take.

The fact that we stand in need of a more general diffusion of a knowledge of, and a love for the beautiful, is often reiterated. But perhaps it has nowhere been more eloquently insisted upon than in a paper in the Athenæum, for the introduction of a portion of which, in this place, its own intrinsic merit will be a sufficient apology. The writer of this paper asks:—"Is it true then, that 'we need little knowledge but that which clothes and feeds us,' that which works our steam engines, spins our cotton, grows our corn, sails our ships, rules our colonies, makes our laws and executes them? Is the useful and the sensible the only real in nature? Is gold the great good, and wealth the chief happiness?—or is it, on the other hand, true that the human mind is to itself the most real of existences—that moral right and moral wrong are powers more mighty than wealth or want—that the acquisition of truth, the attainment of worth, and the appreciation of the beautiful, are purposes of life and of death, incontrovertible and momentous?

"The study of *the true*, *the good*, and *the beautiful*, has formed an important occupation of life in all highly civilized nations, and has been inculcated by the truest patriots and the highest philanthropists. Science, virtue, and beauty, form the noblest elements of creation and of the human soul—they form the first objects of our national institutions, the highest elements of a national character, and the best themes of a national literature. To quote a distinguished modern moralist, 'a people should honour and cultivate that literature which expresses and communicates energy of thought, fruitfulness of invention, force of moral purpose, a thirst for the true, and a delight in the beautiful.'

"It would be a great step in the advancement of our national civilization, were the love of the beautiful, and the power of appreciating the value of its manifestations, more intimately mixed up with the associations and habits of our countrymen. That we have artists of high powers—architects of consummate skill—is matter of national congratulation; but does little to prove the existence of a high standard of national taste. The habit of enjoying the beautiful, and the power of appreciating it, should pervade the national character—should determine its national institutions—and be diffused among the peasantry of our streets and hamlets. 'The farmer and the mechanic (we quote Channing once more) should cultivate the *perception of beauty*.'—'Every man should aim to impart this perfection to his labours.' Were every man a judge and appreciator of beauty, then indeed might we expect forms of loveliness and grace to pervade the regions of domestic and every-day life, to replace in our streets the expensive ugliness of our street decoration—in our homes the vulgarities of ornamental deformity—and in our churches the distortions and anomalies of meretricious decoration. It is in the cottage and the church that national taste must receive its best lessons, and until the love of beauty and the intelligence of its principles pervade the national character, it can never sustain a high standard of national art. To educate the community in the beautiful, is the first condition of a high state of art. High art, to be encouraged, must be widely appreciated and highly enjoyed, and it is most true of works of taste and genius, as of profound investigations of philosophy, that they 'can only be estimated and enjoyed through a culture and power corresponding to that from which themselves have sprung.'"*

The question here naturally arises,—what is the best method of thus advancing our national civilization, and by what means

* *Athenæum,* No. 815, page 541.

can an appreciation and consequent love of the beautiful be more intimately mixed up with our associations and habits? It would appear that we have hitherto been making efforts to reach the top of the ladder, without using the first steps, and that we have consequently failed to make any ascent. We have academies and schools of design, in which copying of the highest works of art, both ancient and modern, is practised, with the understanding that, if after some years of this mode of study, any of the pupils that do not show sufficient genius to become professors of high art will, at all events, be capable of producing ornamental designs. But this is a great mistake, for it has been proved to me, by thirty years' experience, that youths of ordinary capacity who have never had a previous lesson, are more easily instructed in the precise rules of ornamental art than those who have had years of such practice as our academies of art and schools of design afford. Hazlitt says truly, that "a constant reference to the best models of art necessarily tends to enervate the mind, to intercept our view of nature, and to distract the attention by a variety of unattainable excellence. An intimate acquaintance with the works of the celebrated masters, may indeed add to the indolent refinements of taste, but will never produce one work of original genius. * * * The thoughtless imitator, in his attempt to grasp all, loses his hold of what was placed within his reach; and from aspiring at universal excellence, sinks into uniform mediocrity. There is a certain pedantry, a given division of labour, an almost exclusive attention to some one object which is necessary in art, as in all the works of man. Without this the unavoidable consequence is a gradual dissipation and prostitution of intellect, which leaves the mind without energy to devote to any pursuit the pains necessary to excel in it, and suspends every purpose in irritable imbecility."

But instruction in the first principles or elementary laws of beauty in form and colour, can have no such hurtful tendency, because they are capable of being systematized, and consequently of being taught, to youths of ordinary capacity, unconnectedly with high art, and may thus enable them generally to understand the nature of absolute beauty. Those who follow such of the useful arts as depend for a portion of their excellence upon beauty of form and colour, would be enabled to apply them practically in their works, while all would be capable, in some degree or other, of understanding and appreciating such applications. If, therefore, some means could be adopted of inculcating to the rising generation, along with the other elementary branches of education, the simple and teachable laws of visible beauty, then might we look forward with some degree of hope to the time when an appreciation and love of the beautiful will form a feature in our national character.

Let us now inquire into the means adopted for disseminating that appreciation and love of literature that distinguishes our country, as also into the method by which its principles have been inculcated generally, so as to elevate our national character as an intelligent people.

Humble as the occupation of the Parish Schoolmaster is considered, while employed in teaching his young pupils the first simple modes of combining the letters of the alphabet in the formation of words, and those of combining the numerals in expressing numbers, yet none can deny the greatness of the effects thus produced in forwarding our national intelligence and prosperity. What are the classics in such humble schools of literature compared to the spelling-book? What purpose could the highest literary and scientific productions of the human mind serve at this early stage? None, but to bewilder and confuse; for they could not be understood.

In arithmetic the children learn the most simple rules of addition, multiplication, and division. The teacher does not lay before them the calculations by which the greatest of our natural philosophers have immortalized their names, and tell them that as these are the greatest works of the kind ever produced, they will, by transcribing them, attain a perfect knowledge of figures. Neither does he tell them, that the instructions he gives them are merely intended as a pedestal upon which genius is to exhibit the highest order of literary effort or algebraic calculation. No, the master is content to convey, and his pupils to receive, the elementary principles of grammar and calculation, for the purposes of every-day life, leaving out of the question, in the meantime, the higher orders of literature and mathematics. By such primary means of inculcating the principles of literature, have we arrived at our present greatness as an intelligent people.

But as to our general refinement—our knowledge and appreciation of the beautiful, as connected with the useful arts, we are confessedly far behind. We have certainly a wide diffusion of a pretended knowledge or connoisseurship in the fine arts, but then how few can give a reason for the opinions they form upon works of art, and how very few, indeed, can tell why the form of one utensil upon his table is more beautiful than that of another, or why one kind of proportion in an apartment is agreeable to the eye, while another is not. I cannot help attributing the failure of all our attempts to diffuse a knowledge of the beautiful, to an improper method having been adopted,—a method the very opposite to that by which the other useful branches of education are disseminated.

The first principles of art which give beauty to the productions of the mechanic in his humble workshop, and to those of the professor of the fine arts in his studio, are identical. But so are the first principles of that literature which enables the mechanic to correspond with his employer, and the poet to delight mankind by the emanations of his genius. There is also an identity between the first principles of calculation by which the mechanic ascertains the quantities of his work, and renders his accounts, and those by which the natural philosopher calculates the motions of the heavenly bodies. But the letter of the mechanic is not, necessarily, a work of literary genius, neither are the calculations of his account to be compared to the solution of a mathematical problem. Hence the division of the front of a plain street house, the formation of a soup-tureen or tea-pot, the diaper pattern which enriches a window-curtain, a carpet, or the walls of a room, although subject to the first principles of art, are no more necessarily works of artistical genius than the letter of the mechanic is necessarily a work of literary genius, or his account necessarily the work of a mathematician, although the first principles of literature and calculation are in each case the same.

In our schools of design I believe it is a regulation, that no pupils are admitted but such as have shown decided indications of a genius for the graphic art, consequently the instruction which these schools profess to afford is limited to a very small portion of the community, and therefore cannot diffuse a general knowledge and love of the beautiful. How, for instance, could we have become an educated people, had our schools of literature been so constituted, that no pupils would have been admitted but such as had previously given indications of a genius for literary pursuits? We might probably have had, under such circumstances, as many authors as we have at present, but there would have been very few readers to understand and appreciate their works; such, however, is the present state of the arts of design in this country. We have producers of beauty, and always will, while intuitive genius occasionally appears amongst us, but for want of a mode of general instruction in the most elementary principles of art, there are few who understand or appreciate the productions of genius, or even the application of the most simple rules of harmony.

A knowledge of the principles of symmetrical beauty cannot be inculcated by setting boys down to copy drawings or prints even of a high class, any more than instruction in the principles of language could be conveyed to them by making them repeat sentences of classic literature before they had a knowledge of the principles of grammar by which the words were put together.

Collections of works of high art, in painting and sculpture,—casts from the finest statues, and ornamental works of the ancients, and engravings of the highest class, are, doubtless, fit objects for contemplation and study, and are the proper materials to form a museum for the advancement of æsthetic culture. So, likewise, are the works of the ancient poets and philosophers, and such other productions of genius, the fit and proper constituents of a public library for the intellectual and moral improvement of such of the community as have leisure and inclination to study them. But the former are no more fitted as a medium for the purpose of teaching the first principles of symmetrical beauty, as applicable to the requirements of ordinary life, than the latter are for the first lessons in the parish school.

The writer in the Athenæum already quoted, not content with so eloquently enlarging upon the necessity of the general diffusion of a knowledge and appreciation of the beautiful—indicates a system in the following words:—" Beauty in nature (argued the Platonists) is the manifestation of the one great and good mind of the great form-worker of the Universe. No reasons can possibly be assigned why this one great, one good mind should make any thing otherwise than good, and the best possible. The manifestation of the goodness and excellence of this supreme mind in the material universe is its beauty, and excellence, and perfection. Let us inquire how a good and great mind, versant about the forms of matter, must necessarily proceed. The operations of such a mind must be guided by certain principles; these principles must be the principles of beauty and perfection.

"To discover the laws of material beauty is, therefore, first of all to determine the laws of mind: for the laws of the divine mind we must examine our own. The only type of intellect and goodness we possess is that furnished us by human nature. 'God'—says an eminent Christian philosopher—'God is another name for human intelligence raised above all error and imperfection, and extended to all possible truth.'—'We discover the impress of God's attributes in the universe (continues the same author) by accordance of nature, and enjoy them through sympathy.' This we conceive to be the true theory of the enjoyment of nature; we see the developement there of a high, and good, and glorious, and loveable mind—of a mind resembling all that is best in our own, refined and purified above all error and imperfection; and in our enjoyment of the works of the Divine Artist, sympathy is a principal element.

"The Platonists tell us what some of the conditions of matter are which are not good, which are not beautiful, which do not manifest the working of mind, and out of which has come the beautiful of universal nature. To want definite and intelligible figure—to fluctuate confusedly—to manifest no design, intention, or proportion—to be without method—to have no resemblances or relations of one to another—to be in this condition, was a state of being which excited the vehement longing of the great and good God, and out of which He brought the design of the universe, endued with definite and intelligible form; manifesting design, method, definite proportion, symmetry. So also Moses. The heavens and the earth were without form, and void, and in darkness of chaos, and they were afterwards bounded, defined, proportioned, symmetrized, made all very good. Intelligible form, defined, proportioned, manifesting method, order, symmetry, purpose, is beautiful and accomplished in the highest degree—is the best and most beautiful.

"Let us begin with the most elementary conceptions of form, and the relations of which these are capable.

"Form consists in the assemblage of separate parts of matter into one whole. This assemblage is the first condition of order. The relation between a whole and its parts, is the first great conception of the intellect versant about matter: it is essential to our conception of form. Let us call it *the law of Unity*. To the construction of any work of art, the first condition is that it possess unity, and manifest in all its parts a definite relation to a whole. Let, then, there be a mass before us of unformed matter; let it be the clay of the sculptor, or the stone of the architect, (or 'the clay in the hand of the potter;') its parts must all assume some intelligible relation to some unit or whole, as a necessary condition of beauty, or the manifestation of mind. That the figure be a triangle, a globe, a pyramid, that its parts assume such relations as to belong to one intelligible whole, is to conform to the first great law of beauty, unity—the subordination of all the parts to the completeness of the whole.

"The second element of beauty, after the relation of the parts to the whole, is—let us call it the *law of Symmetry*—the relation of these parts to each other. That individual parts may belong to the same whole, they must have a definite proportion to each other: they must be homogeneous, or by some relation homologated to each other in the relation of parts of that whole. Now, if the whole be made up of parts, we may have regard to the number of those parts, or their nature. The composite (or whole) may consist of parts which are identical each with the other; that is to say, each may form the precise half of the whole; and this identity of parts constitutes the closest degree of relationship—the nearest bond of connexion. Hence beautiful objects in nature and art are composed of identical halves—the human body, the human face, a Greek temple, a Gothic church, being compositions of two identical halves around a given mesial line or axis. This is the first degree of symmetry or proportion—the most perfect and intimate and simple of relations. Let us call it the binary proportion or relation—the *symmetry of the first degree*—the first great relation of proportion.

"But we may add to this closest relation of identity of two halves with each other, another relation essentially distinct from, although of the same genus with the former. From the binary composition, the mind passes forward to a ternary composition. A ternary symmetry is also a conceivable idea; a whole composition, or figure, may be such as to arise out of the perfect unison of three identical and equal parts, and these may be united in a whole as perfectly as the two identical halves. A proportion of three to one is thus recognized as a condition of symmetry; and compositions of three figures, or parts, or elements, each identical with each, is a relation of recognized and universal beauty. Let us call it *symmetry of the second degree*, or the ternary proportion or relation.

"Thus flow directly and necessarily from the great laws of unity of composition, the proportions of one, two, and three—the symmetry of parts in the first and second degrees; and hence we see that the elementary numbers are necessarily introduced as elements of beauty in form.

"But these parts, besides resemblance of form, must be connected with each other, either in fact or in idea, in order to conspire and form a unit; that is, they must be united to form a whole. Three things, or forms, may be perfectly identical, and yet not form a whole. To unite them some bond of union is necessary, some cement, some manifest relation, not only of each to each, but of each to each and to the whole. Three identical circles, three identical balls, three identical lines, three identical figures, form not necessarily a whole, a composition, a group. There is necessary a bond of union. To be beautiful, this bond must exist and be manifest. The same bond which unites one part to the whole, must unite each and every part to the whole. Let us call the identification of each part with the other and the whole, *the law of Continuity.*

"Unity, Symmetry, Continuity, are the three ruling principles of composition in design—the great relations of form ; the perfect manifestations of which are at least essential conditions of beauty. They are necessary elements of the human mind in regarding the forms of matter."*

Such is the excellent idea given by this able writer of the principles which are necessary conditions of symmetrical beauty, and it shall now be my task to take up the subject where he has left it off, and to attempt to explain, systematize, and carry it out in such a way as to come within the simple nature and object of this Treatise.

* Athenæum, No. 817, p. 584.

FIRST PRINCIPLES

OF

SYMMETRICAL BEAUTY.

PART I.

THE science which derives the first principles in all the arts of design from those combinations, which excite in us the idea of beauty by the harmony of their parts, is called Æsthetics.

In this science the human mind is the subject, and external nature the object. Each individual mind, as I have elsewhere observed, may be considered as a homogeneous existence—a unity in creation—a world within itself. These two separate existences, the individual mind, and the world at large, have a relation to each other: the subject is affected by the object, and the media of communication by which this is performed, are the sensorium, and its inlets the organs of sense—the former being in direct contact with the subject, and the latter with the object. The organs of sense are acted upon in various ways, and by various modifications of the elements of the external world, but æsthetically the mind is affected in two ways only. These affections are either pleasing or displeasing—harmonious or discordant—the absence of the one quality constituting the presence of the other. When these qualities are equally balanced, no more effect is produced on the mind, than two opposite colours produce on the organ of vision when they destroy each other in neutrality. This effect upon the subject results from a homogeneous principle existing in external nature, to the operations of which the internal sense responds. This response is called perception, and the science of æsthetics is devoted to the investigation of the modes in which external objects—natural and artificial—affect this power of the mind. Although the organs of sense are various, yet the mode in which they act appears to be uniform, and of a mathematical nature, so that the effects of the object upon the subject are either harmonious or discordant, according to the degree in which this principle is exhibited and responded to.

Harmony is not a simple quality, but, as Aristotle defines it, "the union of contrary principles having a ratio to each other." Harmony thus operates in the production of all that is beautiful in nature, whether in the combinations, in the motions, or in the affinities of the elements of matter. In form, harmony constitutes proportion, which, as Vitruvius expresses it, "is the commensuration of the various constituent parts with the whole; in the existence of which symmetry is found to consist." (Note A.)

The contrary principles to which Aristotle alludes, are those of uniformity and variety, which give rise to two distinct kinds of beauty, according to the predominance of the one or the other of these principles in the object. The one kind may be called symmetrical beauty, and the other picturesque beauty—the first allied to the principle of uniformity in being based upon precise laws that may be taught to men of ordinary capacity, so as to enable them to produce it in their works—the second allied to the principle of variety to so great a degree that no precise laws can be laid down for its production. The generality of mankind may be capable of perceiving this latter kind of beauty, and of feeling its effects, but men of genius only of producing it in works of art. Natural objects have in general a preponderance of picturesque beauty, but the highest degree of perfection is the result of an equal balance of both kinds. Of this the human figure is an example; because, when it is of those proportions generally reckoned the most perfect, its symmetrical beauty bears to its picturesque beauty an apparently equal ratio. For instance, its lateral halves are perfectly uniform to the eye, and its principal divisions relate to each other as follows:—From the crown of the head to the *os pubis,* is one half of the whole length, from the thigh-joint to the knee-joint, from the knee-joint to the heel, and from the elbow-joint to the end of the longest finger, are each one fourth of the whole length. From the crown of the head to the end of the chin is one eighth, and from the elbow-joint to the shoulder-joint, is one fifth of the whole length. These are the most apparent divisions, and are those which principally operate in the motions and attitudes of the human frame.

The symmetrical beauty of the facial surface, as depicted on the retina, or referred to a plane, when viewed in full front, consists in a similar kind of proportion. The form is an oval, upon the conjugate diameter of which the eyes are horizontally placed, thus dividing it into two equal parts. From the eyes to the end of the nose, from the end of the nose to that of the chin, and from the top to the bottom of the ear, are each one fourth of the whole length. The mouth is placed upon one third of the length, between the nose and end of the chin. The mouth and each of the eyes are, horizontally, one fifth of the conjugate diameter. These are only a few of the various proportions that constitute the symmetrical beauty of the human figure; but the whole are easily defined, and are thus subject to rules which may be taught. (See Note A.) It is very different with the picturesque beauty of the human figure. This consists, in the first place, in light, shade, and colour; in the second place, in the ever varying undulations of the forms of the external muscles, in the changes of these forms by the innumerable positions and motions of the members to which the muscles belong; and in the third place, it consists in the expression of the countenance, depending upon the operations of the mind, and the correspondence between this expression and that of the attitude or motion of the whole figure. These are the picturesque beauties with which genius only can deal—for they are subject to no rules that can be taught, and belong exclusively to the imitative arts.

There are objects in nature, however, which have no symmetrical beauty, but are, nevertheless, beautiful. An ancient oak, for example, is one of the most picturesque objects in nature, and its beauty is enhanced by want of apparent symmetry. Thus, the more fantastically crooked its branches, and the greater the dissimilarity and variety it exhibits in its masses of foliage—the more beautiful it appears to the artist and the amateur. And, as in the human figure, any attempt to produce variety in the proportions of its lateral halves would be destructive of its symmetrical beauty—so in the oak tree, any attempt to produce palpable similarity between its opposite sides would equally deteriorate its picturesque beauty. As in nature, there are objects which are beautiful without apparent symmetry—so in art, there are others which are beautiful without that degree of variety which produces the picturesque. Such objects depend, principally, upon symmetry for their beauty, and are thus calculated to please the perception æsthetically, without exciting the higher emotions of the mind. It is to such objects that the rules I am about to point out may be most profitably applied.

As symmetry consists in the commensurability of the various constituent parts with the whole, and as the two contrary principles of uniformity and variety, or similarity and dissimilarity are in operation in every harmonious combination of the elements of form, we must first have recourse to numbers in the abstract, in order to understand the nature of the harmonic ratios when applied to numerical quantity, and thus be able to apply them in the production of symmetry.

These contrary principles arise, in abstract numbers, from the various modes in which the unit may be combined. Its first and most uniform combination is with another unit by which its first multiple 2 is produced, which number is a submultiple of 4, 6, 8, progressively as 2, 3, 4. It is the first even number, that is, it consists of two equal and similar portions, and these are its only aliquot parts. It therefore may be termed the first principle of uniformity. The second combination of the unit is 3, which is the first odd number, and is a submultiple of the numbers, 6, 9, 12, these being progressive multiples of 3, by 2, 3, 4. This number being also the combination of 1 and 2, may be called the first principle of inequality or variety. The third mode of union is the combination of the two contrary principles of 2 and 3 in the number 5. This number is the third simple multiple of the unit, because it has no other aliquot parts, and combines the binary and ternary principles of union amongst units. It is a submultiple of the numbers 10, 15, 20, progressively as 2, 3, 4. The numbers 2, 3, and 5, are, therefore, the first three multiples of 1 that have no other aliquot parts, and it will be shown, that by the union in proper proportions of the contrary principles which they exhibit, the proportion, symmetry, and beauty of abstract form, may be educed. (Note B.)

In form, as exhibited by plane figures, these principles may be applied through the medium of the circle, because by the division of its arc into parts called degrees, minutes, &c., the units are formed, by which the harmonic ratios of numerical quantity may be applied to that primary species of figure upon which the beauty of form especially depends, namely the triangle.

The circle has, from time immemorial, been divided into 360 degrees. It is not known by whom this division was originally made, nor for what particular purpose; but so far as I have been able to investigate its properties, I think I may with safety say, that no other number combines in the same harmonious manner, the numbers to which I have just alluded, namely, 2, 3, and 5. Of this I shall give the following example, by which it will be seen that, in the first place, it is divisible by 2 three times, by 3 two times, and by 5 once. The quotients arising from these modes of division, commencing with the greatest, are in like manner divisible by 2 three times, by 3 two times, and by 5 once. Lastly, the quotients of this second mode of division are divisible in the same manner, thus:—

A	B	C
2)360	3)360	5)360
2)180	3)120	72
2)90	40	
45		
C	A	B
2)72	3)45	5)40
2)36	3)15	8
2)18	5	
9		
B	C	A
2)8	3)9	5)5
2)4	3)3	1
2)2	1	
1		

The division of the circle by its diameter into equal parts, gives the base line from which all angles are calculated by the degrees contained in the arc which stands upon it, and these are in number 180° divisible into minutes, &c., as just stated. The division of the semicircle by two gives the perpendicular line to the base which divides the arc into quadrants, each of which contains 90°, and the angle thus formed with the base line, is called the right angle, because it is the most perfect of its kind, all angles having more degrees being obtuse, and those having fewer being acute. This angle may therefore be taken as the fundamental angle, to which all others, in the first instance, should relate as to a natural key, and it is by this means that the principle of harmonic ratio may be applied to forms, see Plate I.

A rectilinear triangle is the simplest component figure of any other rectilinear plane figure; because it has the smallest number of sides and angles. It must, therefore, be to this figure that we are to look for the most simple operation of those first principles which give symmetry to forms.

Of symmetry there appears to be three primary orders, namely, that which relates to the number 2; that which relates to the number 3, and that which relates to the number 5, and of these there are various degrees, gradually receding from the principle of simplicity of ratio.

Symmetry of the first order is most appropriately represented by the perfect square, because it is composed of two right-angled isosceles triangles, which have the peculiar quality of being divisible and subdivisible into halves, which are uniformly triangles of precisely the same description and relative proportions amongst their parts. The proportions of all triangles are regulated by the numbers of degrees in the arc of the semicircle which their angles contain, and this peculiar triangle, which may be termed the primary isosceles triangle, having two angles of 45°, and one of 90°, exhibits the ratio of 1 to 2, see Plate II.

Symmetry of the second order is, in like manner, most appropriately represented by the equilateral triangle, for, by drawing a line from any angle perpendicular to the opposite side, it may be divided into two scalene triangles, the angles of which being of 30°, 60°, and 90°, exhibit in the ratios of 1 to 3 and 2 to 3, the first principle of variety, see Plate III.

Symmetry of the third order is most appropriately represented by the acute angled isosceles triangle which has one angle of 36° and two of 72°; for a perpendicular line drawn from its vertex to its base, divides it into two scalene triangles, each of which have angles of 18°, 72°, and 90°, and these are in the numerical ratio to the right angle of 1 to 5 and 4 to 5, see Plate IV.

Thus we find in those three orders of symmetry—*first*, the ratios of 1 to 1 and 1 to 2; *second*, those of 1 to 2, 1 to 3, and 2 to 3; and *third*, those of 1 to 4, 1 to 5, and 4 to 5, exhibited in the primary isosceles triangle, in the scalene of the equilateral triangle, and in the scalene of what may be termed the primary acute angled isosceles triangle, thus:—

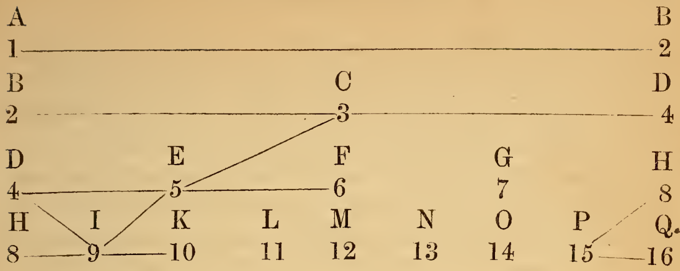
II.		III.		V.	
90°		90°		90°	
I.	I.	I.	II.	I.	IV.
45°	45°	30°	60°	18°	72°

Each of these harmonic ratios is compounded of others of a less simple kind. Thus the simple ratio of 1 to 2 between 45° and 90°, is compounded of the two less simple ratios of 2 to 3 and 3 to 4, in 45° to 67° 30′ and 67° 30′ to 90°; and these two ratios being multiplied together, the lesser term of the one by the lesser term of the other, and the greater term of the one by the greater term of the other, produce 6 and 12, which have the same ratio to each other as 1 to 2. Again, the ratio of 2 to 3 between 60° and 90°, may be divided into the less simple ratios of 4 to 5 and 5 to 6, in 60° to 75° and 75° to 90°, and these ratios being multiplied as before will produce 20 and 30, which numbers have the same ratio to each other as 2 to 3. Lastly, the ratio of 4 to 5 between 72° and 90°, is divisible into the two less simple ratios of 8 to 9 and 9 to 10, in 72° to 81° and 81° to 90°, and these ratios being multiplied as before, give 72 and 90, which numbers are to each other in the ratio of 4 to 5.

But the two following series of numbers and relative series of degrees of the quadrant, will show more clearly than any description, the manner in which the harmonic ratios arise progressively in numbers, and how they may be extended and applied in the production of symmetry amongst angles.

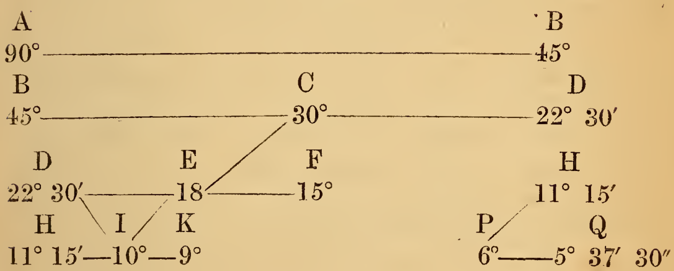
In the first series, the number C is compounded of A and B, E is compounded of B and C, I is compounded of D and E, and P is compounded of G and H. D is a multiple of 2 by 2, F a multiple of 3 by 2, and K a multiple of 5 by 2. F is likewise a multiple of 2 by 3, I a multiple of 3 by 3, and P a multiple of 5 by 3.

In the second series, the degrees at C are to those at A as 1 to 3, and to those at B as 2 to 3. The degrees at E are to those at D as 4 to 5, and to those at F as 6 to 5. The degrees at I are to those at D as 4 to 9, and to E as 5 to 9.



$3)\underline{9}$ $\quad$ $3)\underline{6}$ $\quad$ $3)\underline{15}$

3 $\quad$ 2 $\quad$ 5



$2)\underline{10°}$ $\quad$ $5)\underline{15°}$ $\quad$ $3)\underline{6°}$

5 $\quad$ 3 $\quad$ 2

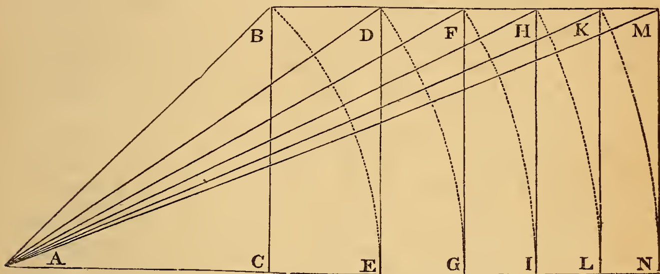
It will be observed, that in the first table the numbers *ascend* in arithmetical progression from 1 to 16, but that in the second, the numbers of degrees which give the angles, *descend* by the same number of intervals progressively from 90° to 5° 37′ 30″, which are to 90° as 1 to 16. It will also be observed that there is no number of degrees to represent seven, or any of those that lie between ten and fifteen. The cause of the contrary mode of progression is, that the numbers in the first scale act as divisors in the second, and the ratios are therefore precisely the same both in reference to the number from which they proceed, and to each other, although their positions are reversed. And the reason that there is no number of degrees put down to represent 7, 11, 13, or 14, is simply that the quadrant of 90° will not divide by any of these numbers, in such a way as to form one of the elements of symmetry, see Note B.

It will also be observed that both scales end with multiples of 2, 3, and 5, and that the following series of harmonic ratios are evolved, as pointed out upon the scales by the straight lines between the numbers.

Ratios in Scale of Numbers.	Degrees.	Ratios in Scale of Degrees.
1 to 2	90° to 45°	2 to 1
2 — 3	45° — 30°	3 — 2
3 — 4	30° — 22° 30′	4 — 3
3 — 5	30° — 18°	5 — 3
4 — 5	22° 30′ — 18°	5 — 4
5 — 6	18° — 15°	6 — 5
4 — 9	22° 30′ — 10°	9 — 4
5 — 9	18° — 10°	9 — 5
8 — 9	11° 15′ — 10°	9 — 8
9 — 10	10° — 9°	10° — 9
8 — 15	11° 15′ — 6°	15° — 8
15 — 16	6° — 5° 37′ 30″	16° — 15

I shall now proceed to show that a series of peculiarly symmetrical rectangles arise naturally from the elementary figure of the first order of symmetry, namely, the right angled isosceles triangle, which is half of the square.

Construct the right angled isosceles triangle ABC, as under, lengthen AC indefinitely, and through B draw BM parallel to AN. Make AE equal to AB, through E draw ED at right angles to AE, and join AD. Next make AG equal to AD, draw GF at right angles to AG, and join AF. In like manner, the triangle AHI is derived from AFG, by making AI equal to AF; and so also AKL and AMN are obtained. (Note C.)



The hypotenuses of the scalene triangles, so constructed, form with the base line AN, angles of 35° 16′, 30, 26° 34′, 24° 5′, 22° 12′, or nearly.

By uniting these scalene triangles in pairs by their hypotenuses, we have a series of six rectangles having diagonals of 45°, 35° 16′, 30°, 26° 34′, 24° 5′, 22° 12′, or nearly, when horizontally placed, and diagonals of 45°, 54° 44′, 60°, 63° 26′, 65° 55′, 67° 48′, or nearly, when vertically placed. These rectangles have peculiarities which distinguish them in a remarkable manner from all other figures of the same species.

The first is the square, which, in common with all other rectangles, may be divided by lines drawn across its area parallel to its four sides, into four of its own kind and relative proportions, or into any other number produced by the power of four. But the other five rectangles are oblongs or parallelograms, possessing the following peculiarities :—

The first is divisible into two of its own kind and relative proportions, by a line drawn across its area parallel to its shortest sides ; the second is, in like manner, divisible into three ; the third into four ; the fourth into five ; and the fifth into six of its own kind and relative proportions. These oblong rectangles are given in Plates VI., VII., VIII., IX., and X. The first is thus a multiple of 2, 4, 8, &c. The second, a multiple of 3, 9, 27, &c. The fourth, a multiple of 5, 25, 125, &c. ; and the fifth, a multiple of 6, 36, 216, &c. of figures of its own kind and proportions. This is evident, for each parallelogram being divisible into a certain number of parallelograms similar to itself, each of these are again divisible into the same number of similar parallelograms, and so on to infinity. Thus the second parallelogram, Plate VI., is divisible into two figures similar to itself, each of these is again divisible into two similar to itself,—these again are each divisible into two similar to itself, and thus the whole figure is successively divided into 2, 4, 8, 16, 32, &c., parallelograms, all similar to the original figure. (Note D.)

In order to form the scalene triangles, arising from the hypotenuses by the process just described, into such a series as correspond to the harmonic ratios, it is only requisite to modify the second, fourth, fifth, and sixth to 36°, 27°, 24°, and 22°, 30′. The series will then be as follows :—45°, 36°, 30°, 27°, 24°, 22° 30′, as shown on Plate V.

These numbers have the following ratios to each other, which I shall give in the order of their consonancy or simplicity, thus:— (Note E.)

22° 30′	have the ratio	to 45°	of	1	to	2
30°	„ „	to 45°	„	2	„	3
27°	„ „	to 36°	„	3	„	4
27°	„ „	to 45°	„	3	„	5
36°	„ „	to 45°	„	4	„	5
30°	„ „	to 36°	„	5	„	6
22° 30′	„ „	to 36°	„	5	„	8
24°	„ „	to 27°	„	8	„	9
27°	„ „	to 30°	„	9	„	10
24°	„ „	to 45°	„	8	„	15
22° 30′	„ „	to 24°	„	15	„	16

Every rectangle has two positions—a horizontal and a vertical—and its diagonal, or the hypotenuse of the scalene of which it is composed, forms, in the first case, an angle with its longest side or base having a smaller number of degrees than 45; and in the second, an angle with its shortest side or base, having a greater number of degrees than 45. The ratios which the series just given have to the diagonal of the square—to their own vertical position, and to the right angle taken in the order of their simplicity, are as follows:—

			Ratio.
Sixth rectangle,	Horizontal position,	22° 30′	1
	Square, . . .	45°	2
	Vertical position,	67° 30′	3
	Right angle, .	90°	4

			Ratio.
Third rectangle,	Horizontal position,	30°	2
	Square, . . .	45°	3
	Vertical position,	60°	4
	Right angle, . .	90°	6
Second rectangle,	Horizontal position,	36°	4
	Square, . . .	45°	5
	Vertical position,	54°	6
	Right angle, . .	90°	10
Fourth rectangle,	Horizontal position,	27°	3
	Square, . . .	45°	5
	Vertical position,	63°	7
	Right angle, . .	90°	10
Fifth rectangle,	Horizontal position,	24°	8
	Square, . . .	45°	15
	Vertical position,	66°	22
	Right angle, . .	90°	30

These oblong rectangles are given in both their positions, along with the scalene triangles, which produce them in Plates XI, XII, XIII, XIV, and XV. From this scale there proceeds an extended series of rectangles, embracing every variety of which the harmony of form is susceptible. Such an extended series I have given in a former work.*

Before leaving this part of the subject, I may observe, that the proportioning of rectangles to one another, is the primary element of symmetry in every architectural work. The elevation of a Greek temple, and that of the plainest

* "Proportion, or the Geometric Principle of Beauty Analyzed." Blackwoods: Edinburgh and London.

street house, are alike reducible to an assemblage of scalene triangles composing rectangles, in the requisite solids and vacuities of which such buildings are constituted—the first by its division into columns, intercolumniations, entablature, and pediment; and the second, by its simple division into doors and windows.

If these elementary parts of the façade of any building—whether they be few or many—have diagonals that bear harmonic ratios to one another, in the numbers of the degrees contained in the angles which they form with their bases, the first and most obvious species of symmetrical beauty has been produced.

All ornamental additions ought to take their proportions from these elements, and relate to them subordinately; and where a deviation from the strict laws of harmonic proportion has necessarily occurred, such additions should be employed to assist in remedying the defect, instead of being considered, as they too often are, the primary elements of beauty in architectural works. I have often heard architects of acknowledged genius, express the difficulty they experienced in pleasing themselves with the arrangement of rectangles in cases where none of the rules relating to the orders of architecture were applicable, and for which they could find no precedent to guide them. But in general little or no trouble is taken in such matters, and the idea seems prevalent, that no species of architectural beauty can be produced, unless it be by elaborate decoration, or by a servile imitation of some great work of antiquity. Hence we find, that in the ordinary practice of this art, there is as little attention paid to the general proportions of the interior apartments, or to the relations of the various parts into which they are necessarily subdivided, as to those of the exteriors of our ordinary dwelling-houses, and they consequently often exhibit the most incongruous mixtures of disproportionate figures. It is in such cases that the introduction of symmetrical beauty is most desirable, and where it is most likely to advance our national appreciation of the truth in matters of taste. Having given examples in some of my former works, of exterior proportion agreeably to the harmony of diagonals,* and as this treatise is intended more to elucidate principles, than exhibit their application in particular cases, I shall here confine myself to one or two examples.

I shall take for the first example, the general proportion of the interior of an apartment. Every ordinary room is composed of a floor, a ceiling, two sides, and two ends, all of which are rectangles, divisible, as has just been shown, into triangles, the hypotenuses of which form their diagonals. Suppose a drawing-room to be the subject, and that its length was to be 42 feet, a diagonal of 30° will give the most elegant rectangle for the floor, and produce a width of 24 feet. Suppose this adopted as the plan of the floor—the size might be increased or diminished to any extent, but while the diagonal had an inclination to the longest side of 30° the relative proportions would continue the same, and would be always divisible into three of its own kind. If the height were to be equal to the width, the rectangles forming the sides would also have a dia-

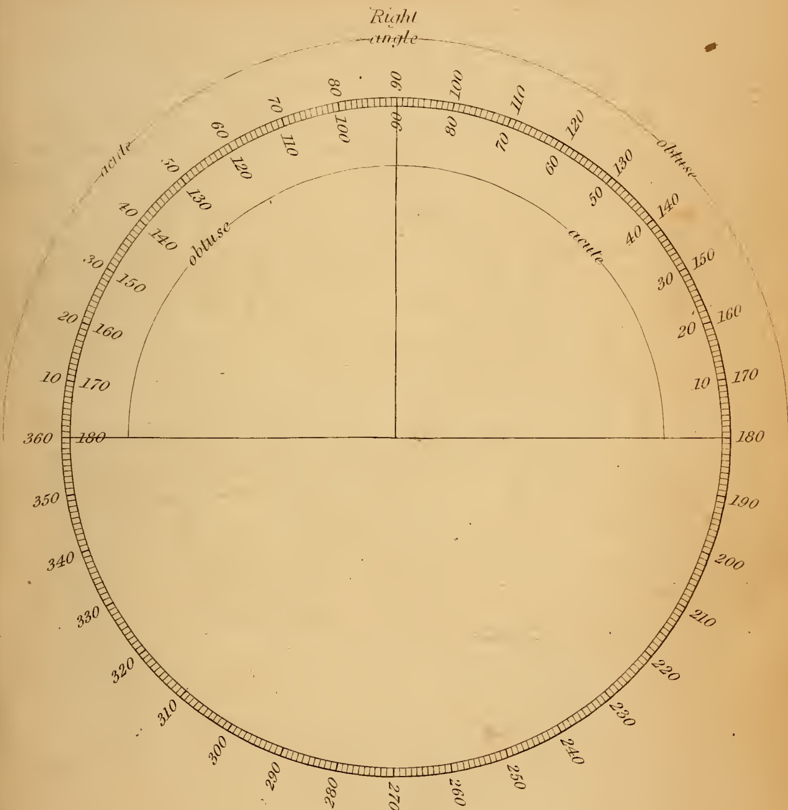
* "Essay on Ornamental Design." D. Bogue, London: J. Menzies, Edinburgh.

gonal of 30°, and the ends would each be square, having a diagonal of 45°. The ends would therefore have a ratio to the floor, ceiling, and sides, of 3 to 2 in the inclination of their diagonals, and the apartment would exhibit one of the simplest kinds of symmetrical elegance. According to the ordinary dimensions of such apartments, however, the height would be considered too great for the width, so that some modification of these simple proportions would become necessary. Let, therefore, the diagonal of the sides be reduced to 27°, and the end to about 40°, and the ratios to the floor would be 10 to 9 and 4 to 3. By this modification the symmetry is reduced, but the variety of contrary principles increased, and although the beauty be thus rendered of a less imposing description, it would, perhaps, be of a more pleasing nature. I give this simple illustration of the mode of proportioning an apartment in Plates XVI. and XVII. The requisite subdivisions of the rectangles which form the sides of an apartment into doors, windows, &c., or their ornamental division into dados, entablatures, or panels, may be rendered symmetrically beautiful by attention to the same laws of proportion. For example, the vertical oblong, the diagonal of which is 67° 30′ when placed on each side of the perfect square, whose diagonal is 45°, will produce a symmetrical arrangement, because their diagonals are in the ratio of 3 to 2, see Plate XVIII.; so will those of 60° and 45°, because their diagonals are in the ratio of 4 to 3, see Plate XIX. In the first, the dissimilarity or variety is greater than in the second, but the symmetrical beauty of both is equal, so that it becomes a matter of fancy to which of the two the preference ought to be given. The space between these rectangles is also determined by the same principle of ratio. The diagonal of this space is 80°, which is to 45° in the ratio of 16 to 9, to 67° 30′ in that of 32 to 27, and to 60° in that of 4 to 3.

When these rectangles are considered, as horizontally placed, the accompaniment becomes the principal, and although their relations be thus altered, their symmetrical proportion remains the same. The horizontal of 67-30 is 22° 30′, which, if placed between two squares, whose diagonals are 45°, the ratio is that of 1 to 2, and symmetrical beauty is still the result. It is the same with the other example when horizontally placed; the centre rectangle has a diagonal of 30°, which are to 45° in the ratio of 2 to 3.

As these ratios become less simple, the figures become less symmetrical, until they degenerate into dissonance. For instance, 45° to 48° is in the ratio of 15 to 16, and the result of such a combination is, that the two vertical rectangles, losing the oblong form which, in the other examples, contrasted so beautifully with the square, become themselves like imperfect squares. See Plate XX.

I



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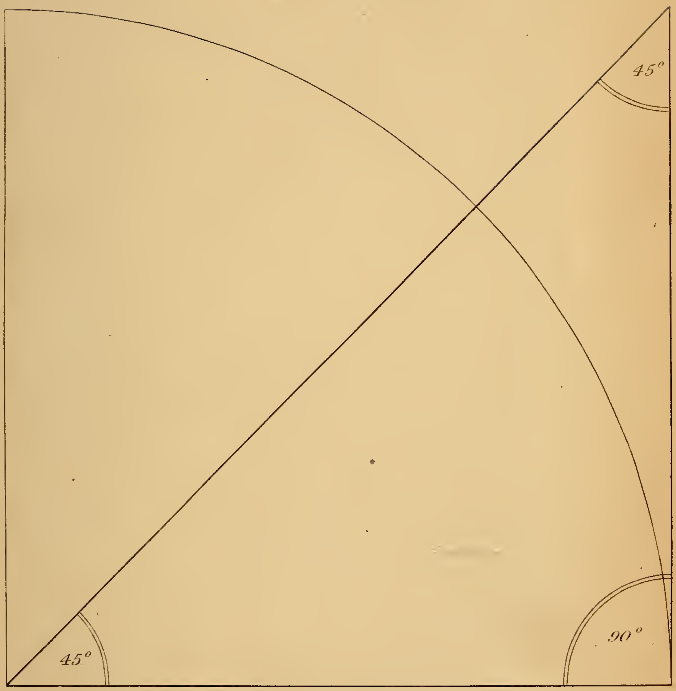
II



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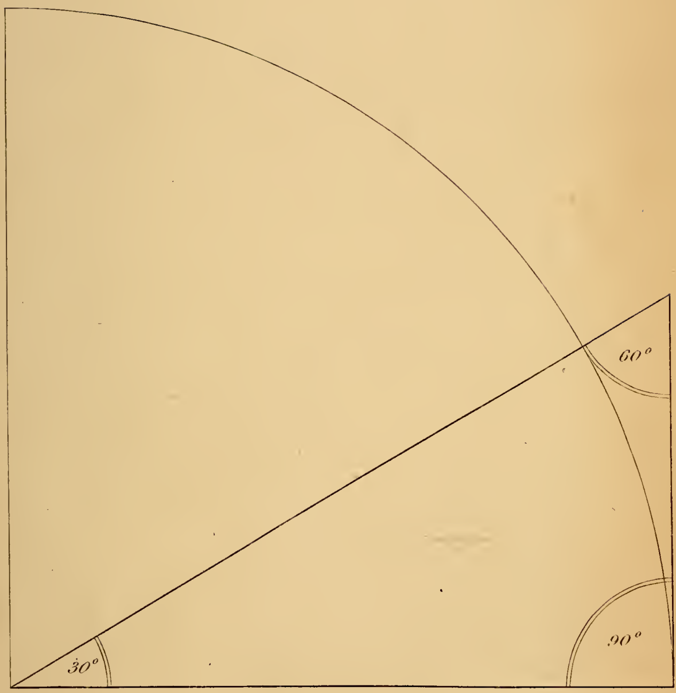

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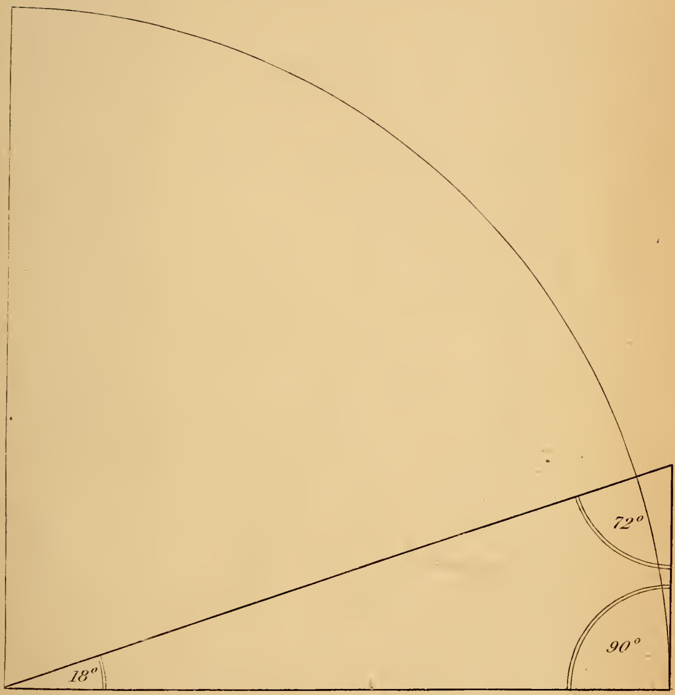

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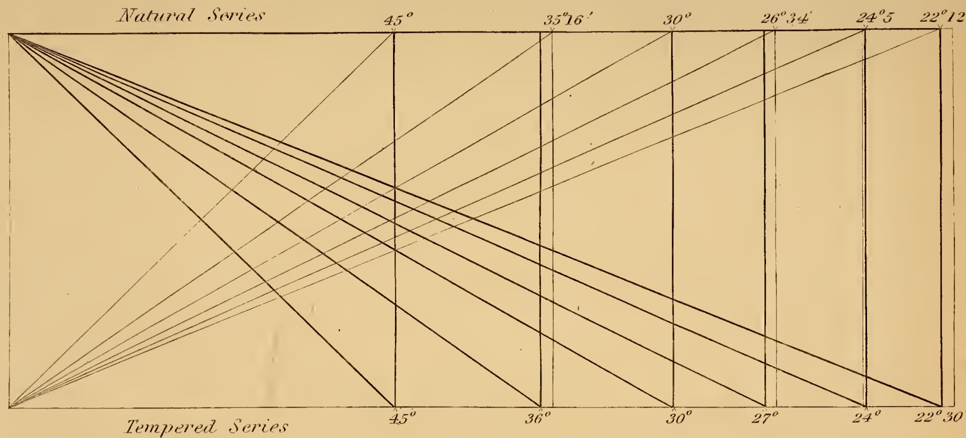

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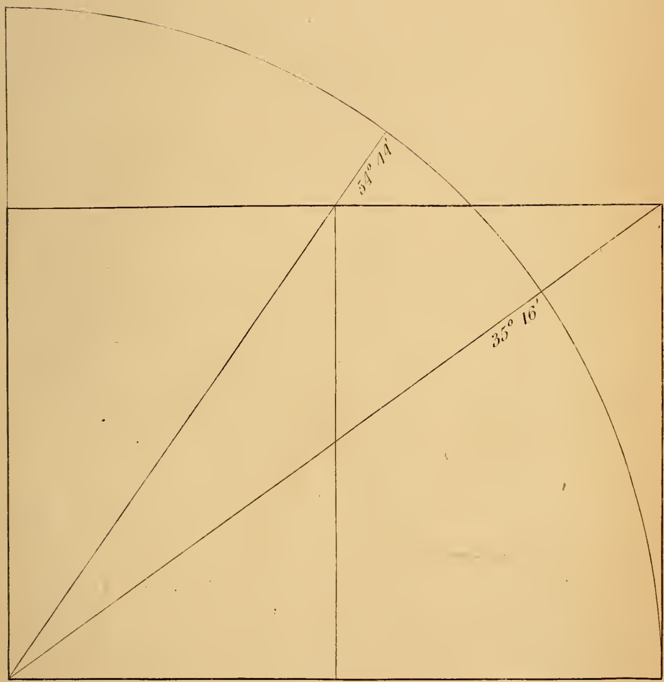

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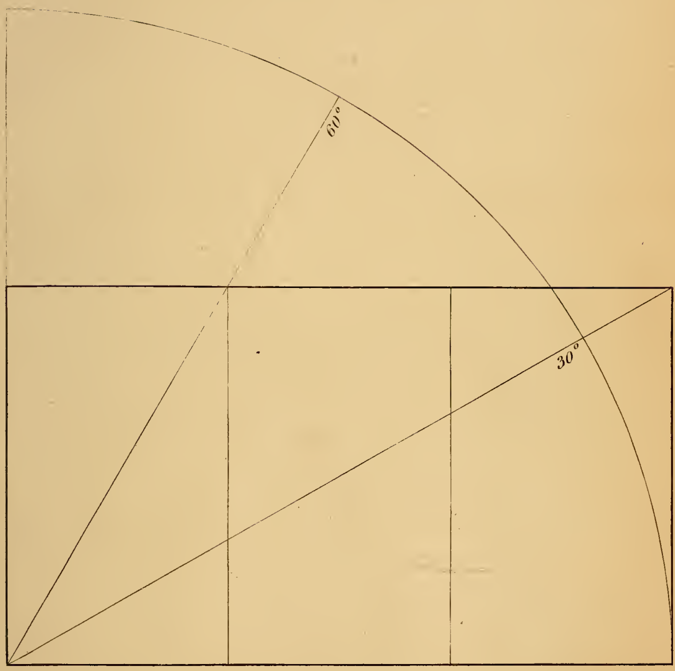
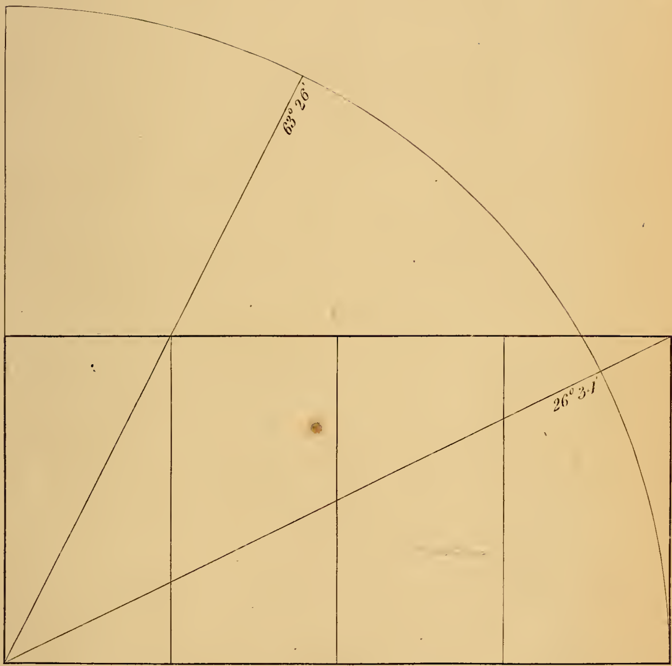
VII



D. R. Hay *G. Aikman*

VIII.



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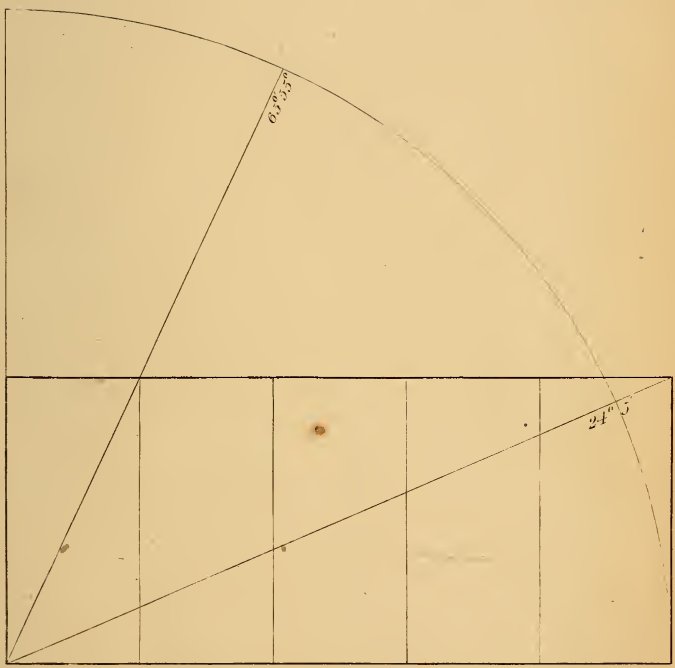
IX



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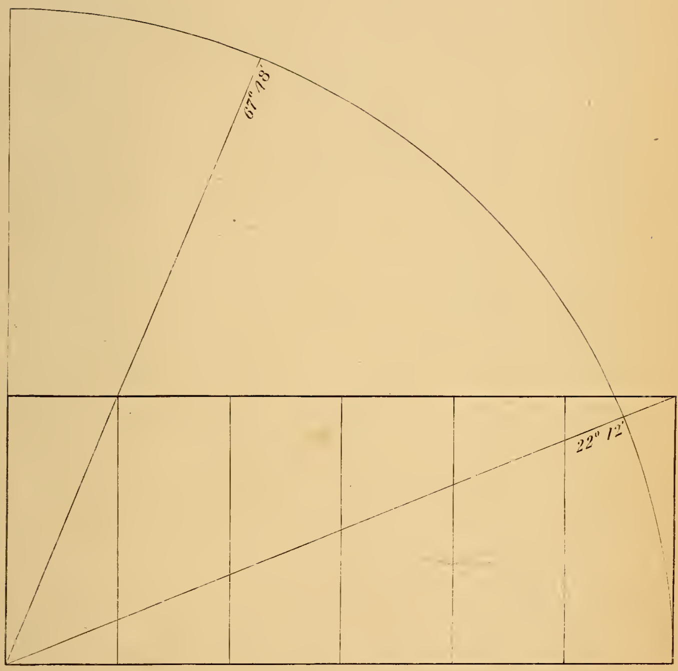

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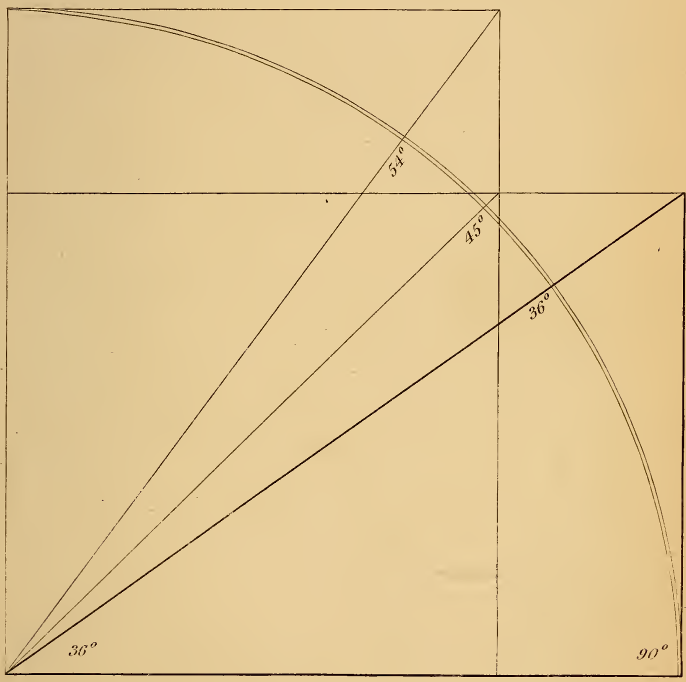

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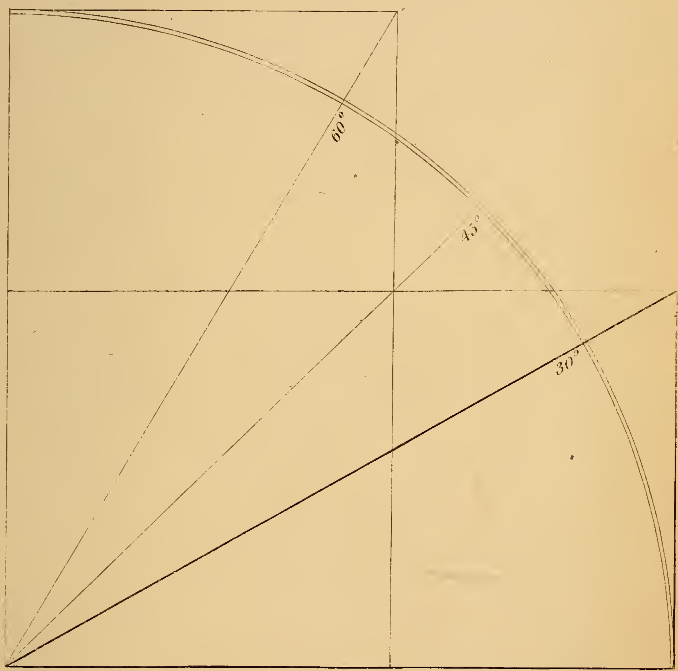

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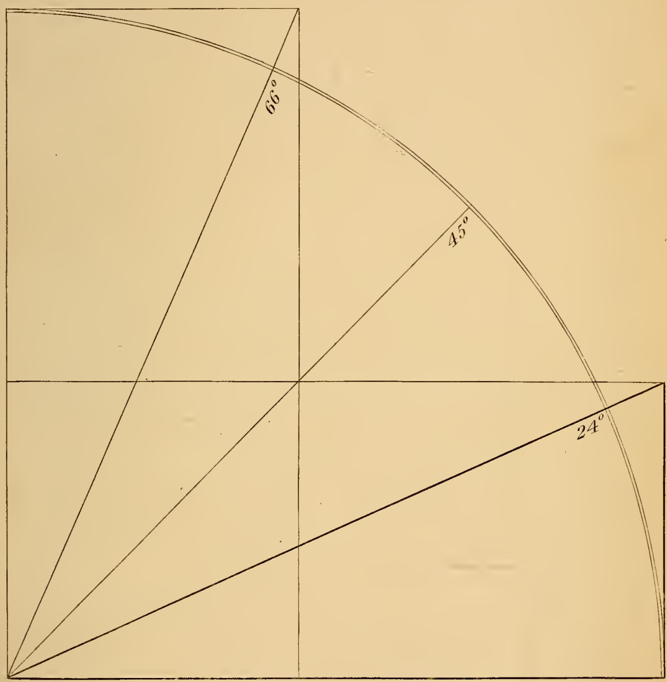

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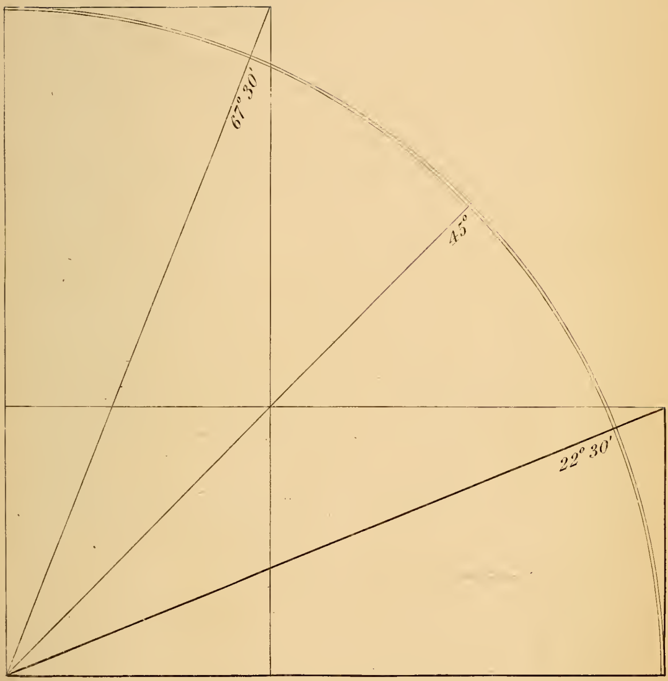

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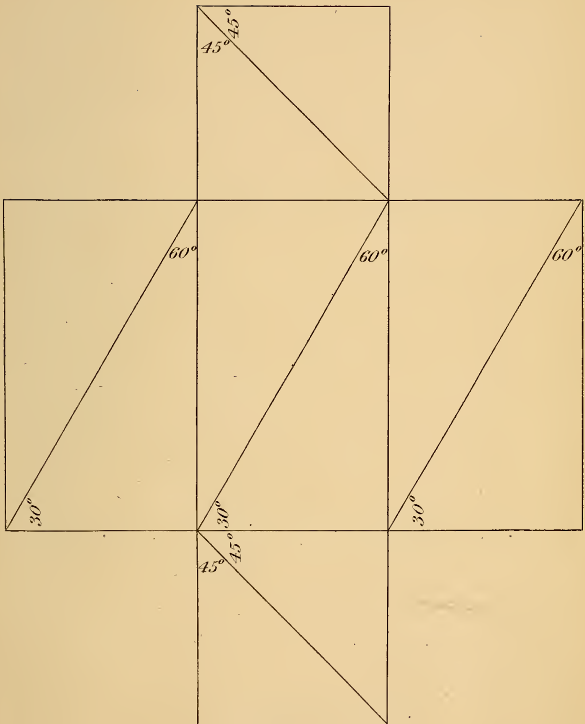

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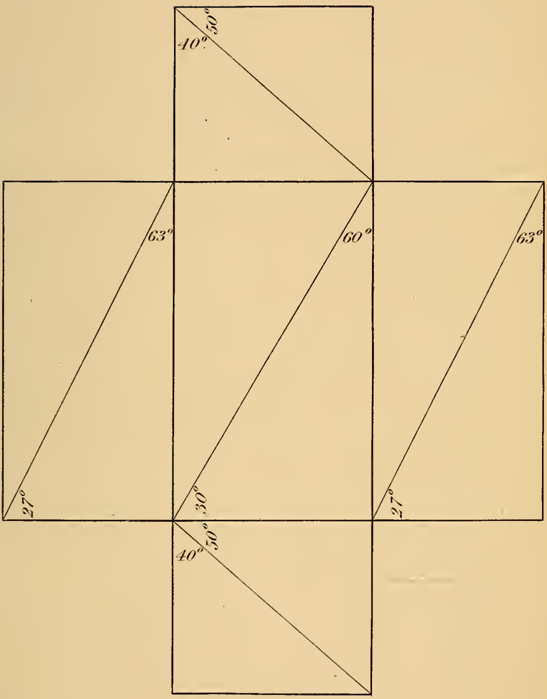

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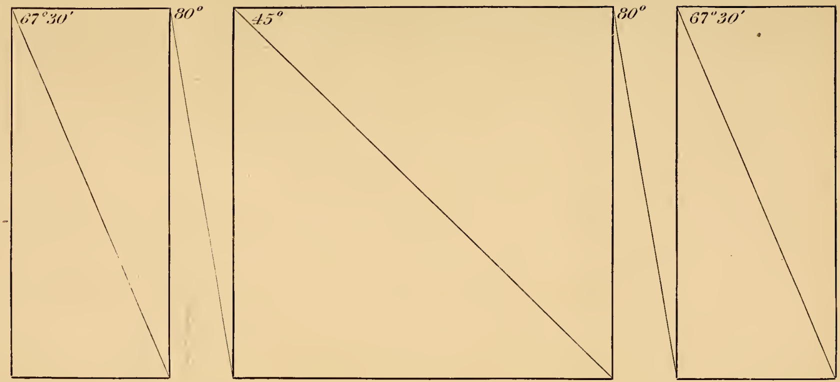

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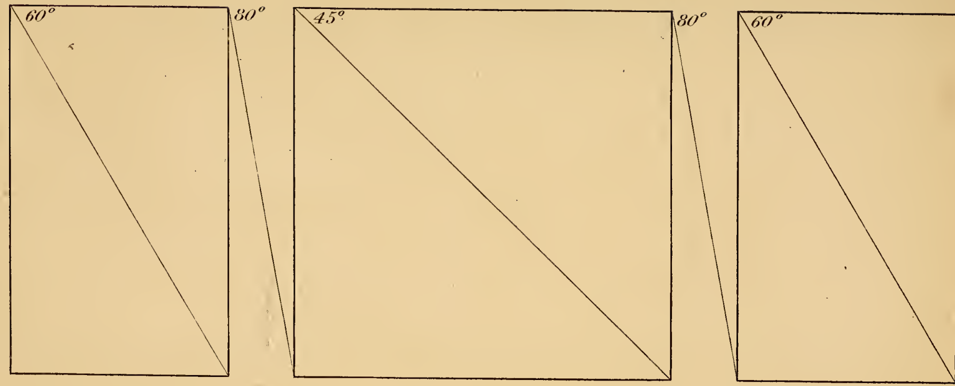

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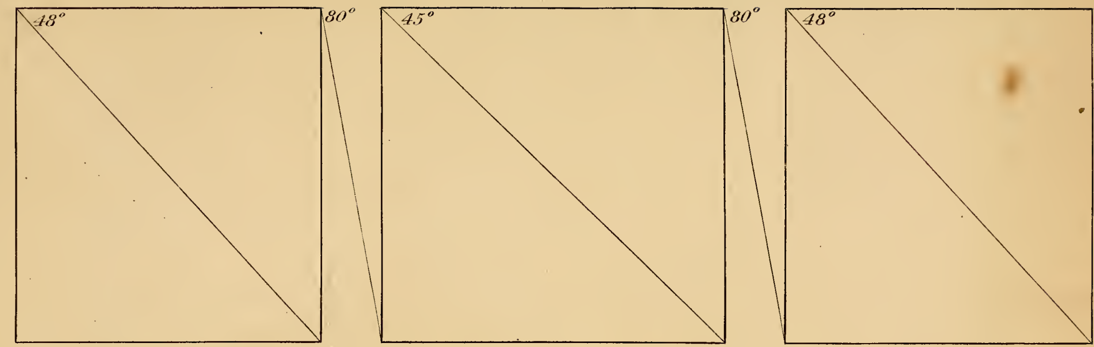

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PART II.

Before proceeding with this part of my subject, I think it requisite to state, that all forms here treated of are understood to be what, in the language of geometry, are termed plane figures, having no other dimensions than those of length and breadth, and the configuration of which may be understood without the assistance of light and shade upon their surface. In this way all figures, whether they be solid bodies or plain surfaces, are depicted upon the retina of the eye, while their other dimensions and modes of configuration, are only guessed at by the light and shade, real or artificial, which appears upon their surface. I say guessed at, for it is not difficult, by artificial means, to make a plane circle appear to the eye to be a sphere, and a plane square to be a cube; but it is impossible, by art, to deceive the eye in respect to the configuration of their external form, as plane figures. Plato has said, "It is necessary that every depth should comprehend the nature of a plane;" which means, that plane figures are the basis of every description of solid body. As such, therefore, they must convey to the bodies of which they form the bases, the same degree of symmetry which they exhibited in their primitive state. I shall, therefore, continue to point out what appears to me to be the laws of symmetrical beauty in forms by means of plane figures, as the most simple mode of treating the subject.

The circle, the square, and the equilateral triangle, being the primary elements of plane figures, must, consequently, exhibit in their configuration, the first principles of that proportion and symmetry which constitute the beauty of all forms employed in ornamental design.

These figures are homogeneous in their parts, and the manner in which they evolve the modes or orders of symmetry by their union is remarkable. The first part of this Treatise having been confined to an attempt to elucidate the operation of the harmonic ratios upon rectilinear figures, and as I am now about to connect these figures with the curvilinear kind, and to endeavour to show, that by their union, the same laws of harmony are as clearly evolved, I shall first explain the nature of the circle and the ellipse, which may be termed the primary and secondary curvilinear figures. The circle has been already referred to, and is a figure so well known, that it is only necessary here to say, that its centre is a point from which every part of its circumference is equally distant, this point is called its focus, and a line from it to any part of the circumference, a radius ; that any line bisecting it is called a diameter, any portion of the circumference, an arc ; and the straight line which cuts it off from the circle, is called a chord; while the area inclosed by the arc and chord of a circle, is called a segment. The centre or focus of a circle, inscribed by a square, is at the tangential point (Plate XXI, A) of two semicircles described within the area, and upon opposite sides of this rectangle, and the radius will, of course, be a line from this point to the centre of either of the semicircles. (Plate XXI, B.) The ellipse is of a different nature, inasmuch as its centre is not a point but a straight line, the ends of which are its foci, and the sum of the distances of the foci from any point in the curve, is constantly equal to a line drawn through the foci and terminated at the ends by the curve. The length of this line is determined by two semicircles, whose diameters are the longer sides of the rectangle, within which the ellipse is to be inscribed ; because the intersection of those semicircles will give the foci (A) of such an ellipse, and lines drawn from these to the focus (B) of either of the semicircles, are its radii. The radii are together always equal in length to the longest diameter of the ellipse, and, consequently, double that of the radius of a circle of the same diameter. Not having given more than two ellipses in any of my former works, and in order that the nature of this figure may be more clearly understood, I have given in Plates XXII, XXIII, XXIV, XXV, XXVI, and XXVII, a series of six ellipses produced in this way, inscribed in as many rectangles having the harmonic diagonals of 48°, 54°, 60°, 30°, 72°, and 80°, which will explain the process better than words. In each of these Plates, AA are the foci of the ellipse, and B the focus of the semicircle, where it will be observed, that the radii of each ellipse are the same. I have said that the square is the most appropriate representative of the first order of symmetry, which order has relation to the number two. From this figure it will now be shown, that the representative of the second order of symmetry, which has relation to the number three, is generated by a natural process—the circle acting as a medium; and again, that the third order, which has relation to the number 5, is, in like manner, generated from the second order—the ellipse acting as a medium. The process is as follows:—Let a square be described about a circle, divide each of its sides into four equal parts, and unite those divisions by parallel lines drawn across its area, thus dividing the perimeter of the square as well as its area, into sixteen parts. It will now be found that those lines have cut the circumference of the circle into twelve equal arcs, the binary mode of division upon the perimeter of the square, producing a ternary mode upon the circumference of the circle, the one number being a composite of two, and the other of three, thus so far evolving the two first orders of symmetry. See Plate XXVIII. From the centre division of the base of the square draw lines to the points of intersection, marked 2, 3, 4, 8, 9, 10, and there will thus be produced the platonic elements of beauty, in the representatives of the two orders of symmetry, namely, upon the centre chord the right-angled isosceles triangle, which is half of the square, upon the upper chord, the equilateral triangle, and upon the under chord, the obtuse-angled isosceles triangle, which is divisible into two scalene triangles of the same proportions as those which form the lateral halves of the equilateral triangle. See Plate XXIX. The ratios of the angles of these figures have already been explained. It has also been explained, that as the square is composed of two right-angled isosceles triangles, so is every other rectangle composed of two scalene triangles, united by their hypotenuses. The two scalene triangles which form the equilateral triangle, thus produce the oblong rectangle, which belongs to the second order of symmetry, as the square does to the first. This rectangle has a curvilinear figure—the ellipse—which it harmonically inscribes, and to which it is in every respect related, as the square is to the circle. Let this rectangle be constructed, and having inscribed within it its own ellipse, by the method already explained, let each of its four sides be divided into four parts. Unite those divisions by parallel lines, and the circumference of the inscribed ellipse will be harmonically divided into twelve arcs; their length decreasing in proportion as the curvature becomes greater, see Plate XXX.

Repeat the same process with the ellipse, thus divided, by which the platonic elements were produced within the circle; that is, draw lines from the centre point of its base to the divisions 2, 3, 4, 8, 9, 10, and there will thus be produced, along with the elements of the first and second orders of symmetry, the representative of the third order also; thus, on the under chord, the right-angled isosceles triangle; on the centre chord, the equilateral triangle; and on the upper chord, the primary acute-angled triangle, the scalene triangles of which relate to the number five, see Plate XXXI. Let two scalene triangles of this latter kind be united by their hypotenuses, and the rectangle of the third order of symmetry will be produced; let its appropriate ellipse be inscribed within it; let its perimeter be divided in the same manner as that of the preceding figure, Plate XXXII. Draw lines from the base and within the area of this ellipse, to the intersections, 2, 3, 4, 8, 9, and 10, and there will be produced upon the under chord, the equilateral triangle, the representative of the second order of symmetry; upon the middle chord the primary acute isosceles triangle, the representative of the third order of symmetry; and upon the upper chord, an acute-angled isosceles triangle, whose smallest angle is 20°, and the scalenes of which have angles of 10°, 80°, 90°, giving the ratio of 1 to 9 and 8 to 9, see Plate XXXIII. This last is a compound order of symmetry, 8 being a composite of 2, and 9 of 3. Others of a more complex kind will evolve themselves by a continuation of the process, similar to what would arise from an extension of the scales of numbers and angles at p. 29.

The harmonic ratios of number are thus proved to be an inherent quality in geometric figures, as they arise from the combination and division of their primary elements—the circle, the square, and the equilateral triangle.

The diagonals of the rectangles which have arisen naturally out of this process, commencing with the perfect square, are vertically, 45°, 60°, and 72°, and they consequently are to the right angle in the harmonic ratios of 1 to 2, 2 to 3, and 4 to 5.

The nature of the triangles has already been explained; but it is worthy of remark in this place that the characteristic angles of the three isosceles triangles produced by this process, having the following numbers of degrees, 90°, 60°, and 36°, are respectively to the semicircle of 180°

in the harmonic ratios of 1 to 2, 1 to 3, and 1 to 5. The first ellipse, which belongs to the second order of symmetry, has also peculiarities that are worthy of notice in this place. I have just shown, that upon the three chords which divide its transverse diameter into four parts, the triangles which represent the three first orders of symmetry are harmonically produced within its area. If the first and third of these chords be united by vertical lines, a perfect square is formed, which the ellipse thus harmonically inscribes, see Plate XXXIV; and if diagonal lines be drawn across its area, uniting the points of intersection 2, 8, 6, 4, 10, 12, 2, two perfect squares are formed, which it also harmonically inscribes. In like manner, by the union of the points 3, 6, 9, 12, 3, a rhombus composed of two equilateral triangles is produced, see Plate XXXVI.

The second ellipse, which belongs to the third order of symmetry, has also its peculiarities. By the union of its first and third chords vertically, the oblong of the second order of symmetry is produced, see Plate XXXVII; and by drawing diagonal lines through its area, uniting the points 2, 8, 6, 4, 10, 12, 2, two rhombs, composed of four equilateral triangles, are produced, harmonically inscribed in its circumference, see Plate XXXVIII. These two ellipses are therefore symmetrically allied, in distinct degrees, to the primary figures.

Every rectangle has thus a curvilinear figure that exclusively belongs to it, and every such curvilinear figure generates, by the process just detailed, two isosceles triangles, one of which belongs to the rectangle which inscribes the curvilinear figure, and the other gives the elements of the next degree of symmetry.

PART III.

It has just been shown that the primary curvilinear figure is the circle, and that it associates harmonically with either of the two primary rectilinear figures—the square and the equilateral triangle. It has likewise been shown that the secondary curvilinear figure is the ellipse, and that it associates harmonically with the two secondary rectilinear figures—the oblong rectangle, and the rhomb, or lozenge, which, as quadrilateral figures, are allied to the primary rectangle, the perfect square. So also, from the primary or equilateral triangle, a series of figures of its own kind proceed. These secondary triangles are called isosceles, and may be right-angled, acute-angled, or obtuse-angled. With this class of rectilinear figures there has, as yet, been no curvilinear figure associated. Such a figure, however, not only exists, but its curve and form prevail in the animal and vegetable kingdoms to a far greater extent than those of any other figure —I mean a figure of which the oval or egg form is the type. (Note F.)

The name of oval not being sufficiently comprehensive for this figure, I shall call it the composite ellipse, because, although it commences with the egg form, from which the term oval is derived, it gradually, in continuing the same process by which that form is produced, passes into a more oblate figure, in the same way that the isosceles triangle passes from the acute-angled to the right-angled, and from the right-angled to the obtuse-angled ; and more particularly, because it is actually composed of various ellipses, either harmonically combined or fluxionally blended into each other. It is distinguished from the other two curvilinear figures by the following peculiarities.

The primary curvilinear figure—the circle—has one focus ; the secondary curvilinear figure—the ellipse—has two foci ; but this tertiary curvilinear figure has three foci, which, being united by straight lines, form an isosceles triangle. Therefore it may be said that the circle has for its centre a point, the ellipse a line, and the composite ellipse a figure.

In the formation of this tertiary figure, another isosceles triangle is required besides that which forms its centre, and these two triangles must have a harmonic relation to each other, otherwise the figure will not be of a graceful form. These triangles are united by one of their sides, thereby forming a four-sided figure, one pair of the subtending angles of which are always equal, and the other pair always unequal, in the number of their degrees. The vertices of the pair of angles that are equal, and one of those, (the more acute of the two,) that are unequal, form the foci ; and the vertex of the remaining angle determines the length of the radii. If, therefore, these four points be harmonically arranged, then the length of the radii will be in symmetrical proportion to the triangle which forms the basis of the composite ellipse. Four pairs of radii operate in the formation of this figure, whatever its relative proportions may be; and the angle formed by the first, or upper pair, with the two equal angles of the isosceles triangle, should not be less than 90°, because a more acute angle would render the curvature of that part of the figure too great. Upon this principle, I have divided the composite ellipse into five classes. The radii of the first class form an angle of 90° with the apices of the equal angles of the isosceles triangle; the second, 108°; the third, 120°; the fourth, 135°; and the fifth, 144°. The numbers of degrees in these angles relate harmonically to the number contained in the semicircle, (180°,) by being to it in the ratios of 1 to 2, 3 to 5, 2 to 3, 3 to 4, and 4 to 5. Of each of the two first of these classes I have only given six examples, as their upper angle, which determines the relative length of the radii, is not sufficiently obtuse to produce the oblate figure. Of each of the three latter I have given ten examples, because their upper angle admits of their producing the oblate, as well as the egg-shaped composite ellipse. The following are the proportions of these examples; with the ratios of the numbers of degrees contained in the subtending unequal angles:—

First Class.

Plate.	Degree.	Lower Angle.	Upper Angle.	Ratio.
XXXIX,	1st,	45° ,, ,,	90° ,, ,,	1 to 2
XL,	2d,	30° ,, ,,	,, ,, ,,	1 — 3

Plate.	Degree.	Lower Angle.	Upper Angle.	Ratio.
XLI,	3d,	22° 30′ „	90° „ „	1 to 4
XLII,	4th,	18° „ „	„ „ „	1 — 5
XLIII,	5th,	15° „ „	„ „ „	1 — 6
XLIV,	6th,	11° 15′ „	„ „ „	1 — 8

Second Class.

Plate.	Degree.	Lower Angle.	Upper Angle.	Ratio.
XLV,	1st,	54° „ „	108° „ „	1 to 2
XLVI,	2d,	36° „ „	„ „ „	1 — 3
XLVII,	3d,	27° „ „	„ „ „	1 — 4
XLVIII,	4th,	21° 36′ „	„ „ „	1 — 5
XLIX,	5th,	18° „ „	„ „ „	1 — 6
L,	6th,	13° 30′ „	„ „ „	1 — 8

Third Class.

Plate.	Degree.	Lower Angle.	Upper Angle.	Ratio.
LI,	1st,	60° „ „	120° „ „	1 to 2
LII,	2d,	40° „ „	„ „ „	1 — 3
LIII,	3d,	30° „ „	„ „ „	1 — 4
LIV,	4th,	24° „ „	„ „ „	1 — 5
LV,	5th,	20° „ „	„ „ „	1 — 6
LVI,	6th,	15° „ „	„ „ „	1 — 8
LVII,	7th,	72° „ „	„ „ „	3 — 5
LVIII,	8th,	80° „ „	„ „ „	2 — 3
LIX,	9th,	90° „ „	„ „ „	3 — 4
LX,	10th,	96° „ „	„ „ „	4 — 5

D

Fourth Class.

Plate.	Degree.	Lower Angle.			Upper Angle.			Ratio.
LXI,	1st,	67°	30′	,,	135°	,,	,,	1 to 2
LXII,	2d,	45°	,,	,,	,,	,,	,,	1 — 3
LXIII,	3d,	33°	45′	,,	,,	,,	,,	1 — 4
LXIV,	4th,	27°	,,	,,	,,	,,	,,	1 — 5
LXV,	5th,	22°	30′	,,	,,	,,	,,	1 — 6
LXVI,	6th,	16°	52′	30″	,,	,,	,,	1 — 8
LXVII,	7th,	71°	,,	,,	,,	,,	,,	3 — 5
LXVIII,	8th,	90°	,,	,,	,,	,,	,,	2 — 3
LXIX,	9th,	101°	15′	,,	,,	,,	,,	3 — 4
LXX,	10th,	108°	,,	,,	,,	,,	,,	4 — 5

Fifth Class.

Plate.	Degree.	Lower Angle.			Upper Angle.			Ratio.
LXXI,	1st,	72°	,,	,,	144°	,,	,,	1 to 2
LXXII,	2d,	48°	,,	,,	,,	,,	,,	1 — 3
LXXIII,	3d,	36°	,,	,,	,,	,,	,,	1 — 4
LXXIV,	4th,	28°	48′	,,	,,	,,	,,	1 — 5
LXXV,	5th,	24°	,,	,,	,,	,,	,,	1 — 6
LXXVI,	6th,	18°	,,	,,	,,	,,	,,	1 — 8
LXXVII,	7th,	86°	24′	,,	,,	,,	,,	3 — 5
LXXVIII,	8th,	96°	,,	,,	,,	,,	,,	2 — 3
LXXIX,	9th,	108°	,,	,,	,,	,,	,,	3 — 4
LXXX,	10th,	115°	12′	,,	,,	,,	,,	4 — 5

The following is the process by which these figures are described :—

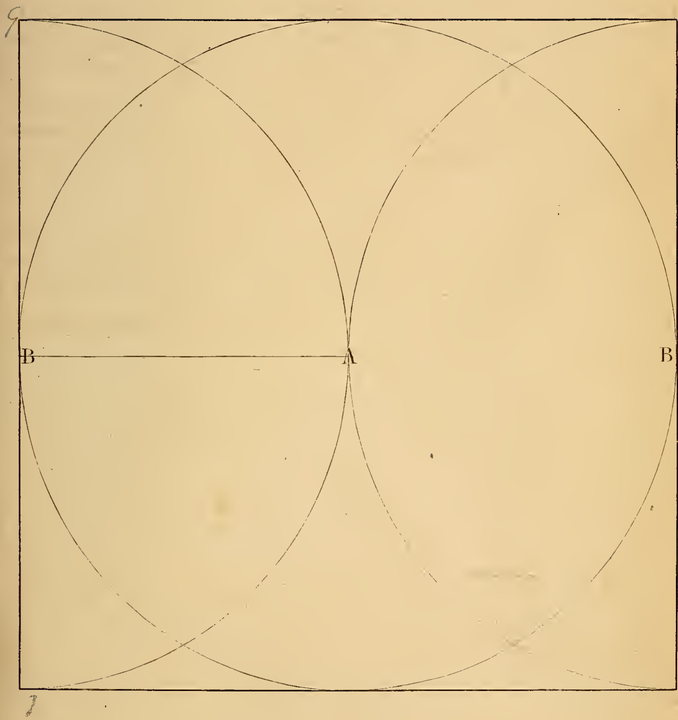
Construct on any line, A B, Plate LXXX, the isosceles triangles, C E D, C F D, (or any other of the pairs of isosceles triangles in the forty-one preceding plates,) and fix pins at the points C E D F. Then, having tied a string tightly round the pins, remove the pin from the point E, and the string will now lie loosely about the remaining pins at C D and F. If a pencil be now introduced within the string so as to restore it to its original tension, and be carried round so as to keep it always equally stretched, it will trace the composite ellipse. The strength of the pins and of the string will, of course, depend upon the size of the figure to be traced, and to be suitable for the purpose, the string should be flexible without being elastic, and tied with a knot that will not slip during the process of tracing the figure.

In the composite ellipse it will be found that relatively to its axis, every part of the curvature of each of its lateral halves varies ; its two sides are therefore, in the parts of which they are composed, perfectly dissimilar in their curvature as relates to its axis.

The circle has only one diameter, which always divides its circumference into equal and similar parts, and its curve is therefore homogeneous in its nature. The ellipse has two diameters—the transverse and the conjugate —which divide the circumference of that figure into four equal and similar parts, and its variety is consequently to its uniformity in the ratio of 1 to 4. But, as just shown, the variety in the curve of the composite ellipse is to its uniformity in the simple ratio of 1 to 2 ; that is, it cannot be divided in any other way into similar parts than by one of its diameters. When the radius of a circle is increased in length, no alteration whatever takes place in the nature or relative proportions of the figure which it produces, but simply an increase of magnitude. When the radii of an ellipse are increased in length, while its foci remain at the same distance from each other, the proportions of the figure they produce are altered, and its size increased in the ratio of the increased length of radii; the difference between its two diameters decreasing in a like ratio. The figure, however, is still a perfect ellipse.

But with respect to the composite ellipse, the four pair of radii must be in symmetrical proportion, otherwise deformity would be the result—therefore the ellipses of which this figure is composed must always unite in harmonious combination with each other.

XXI



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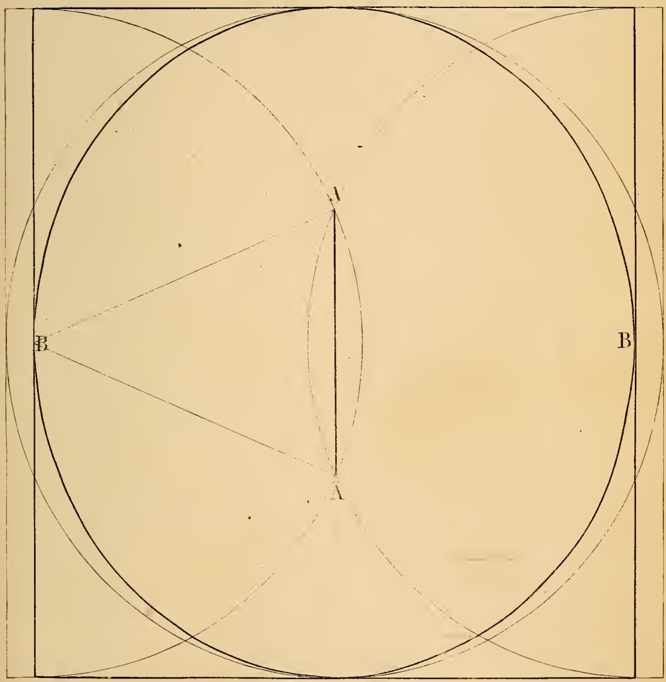

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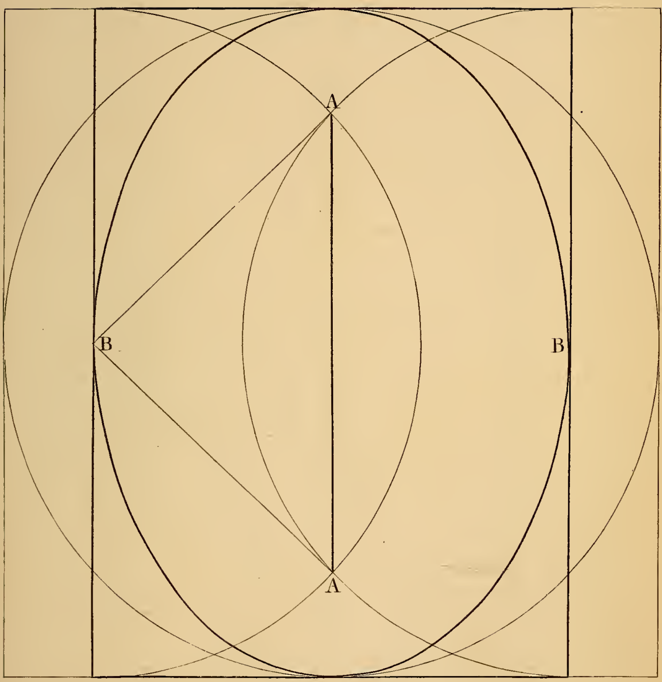

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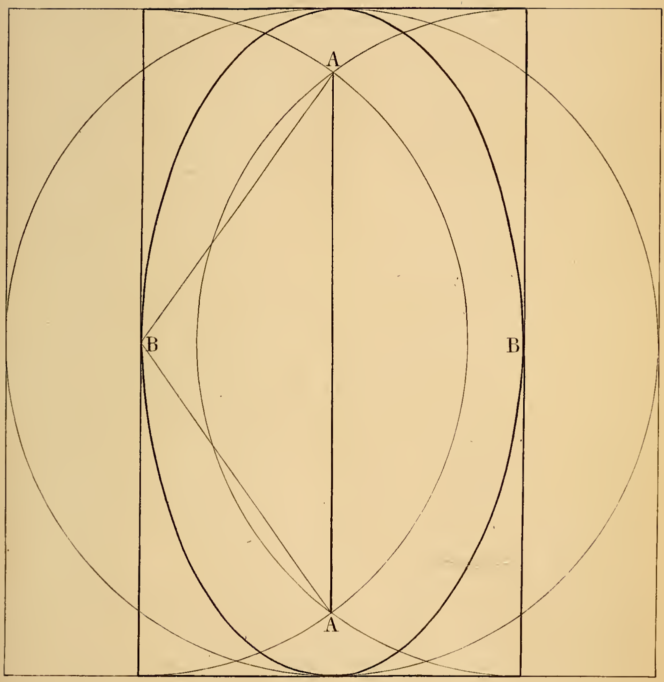

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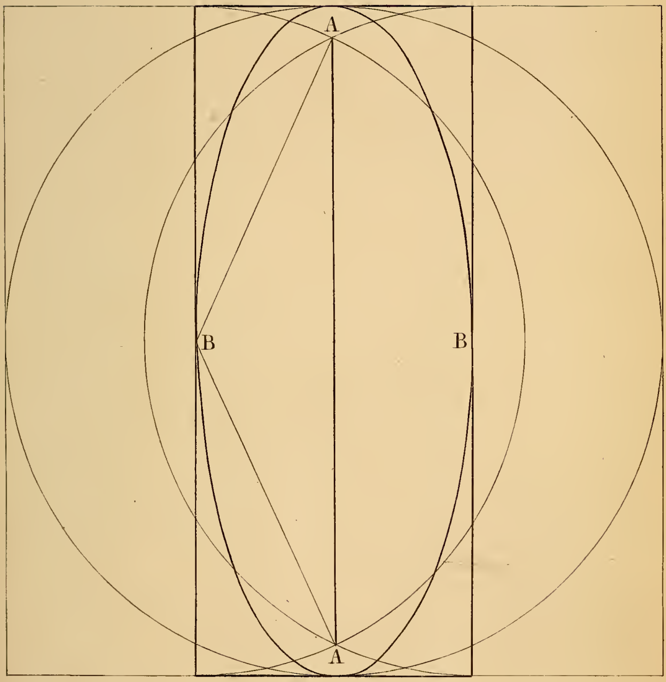

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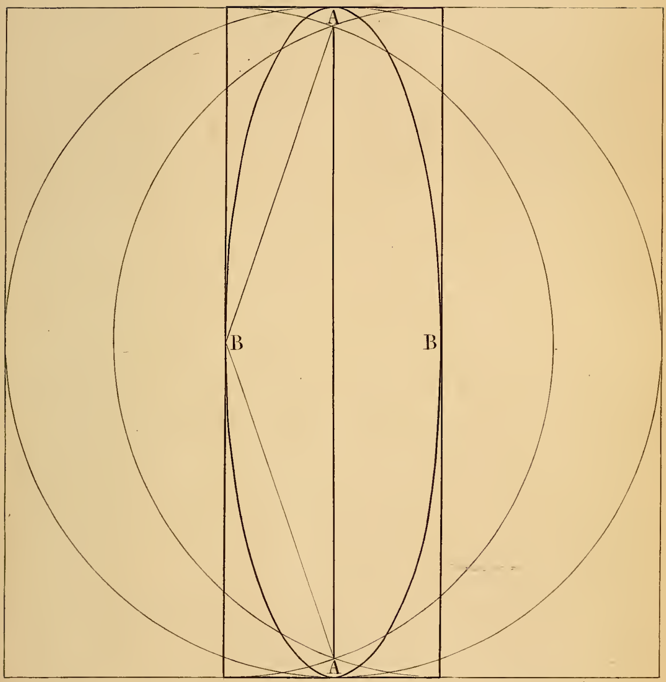

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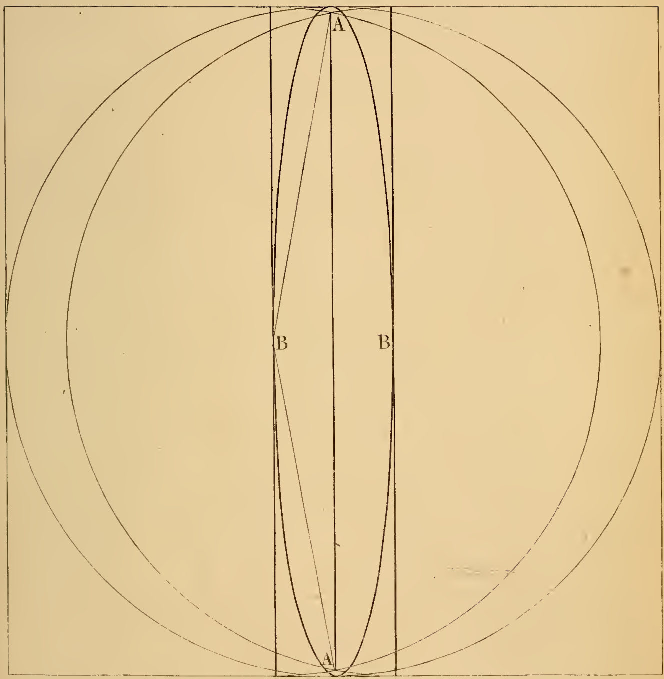
XXVII



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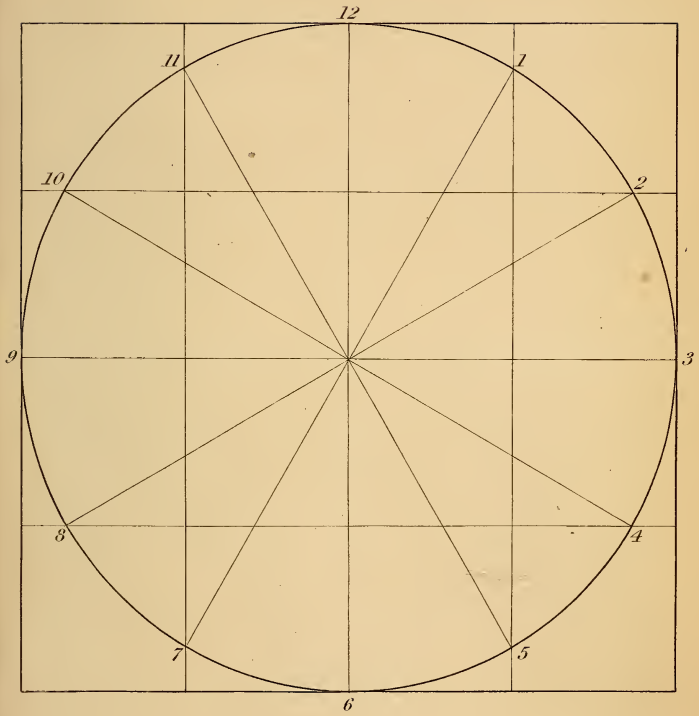

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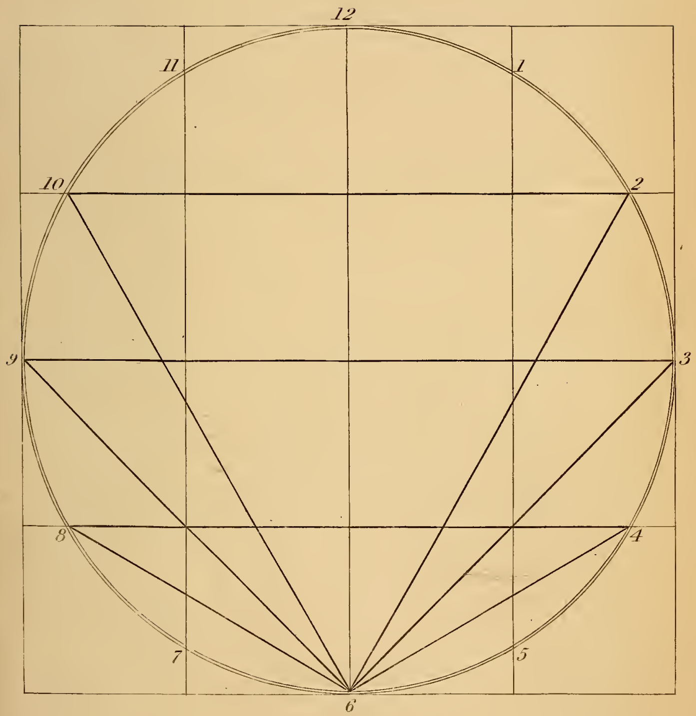

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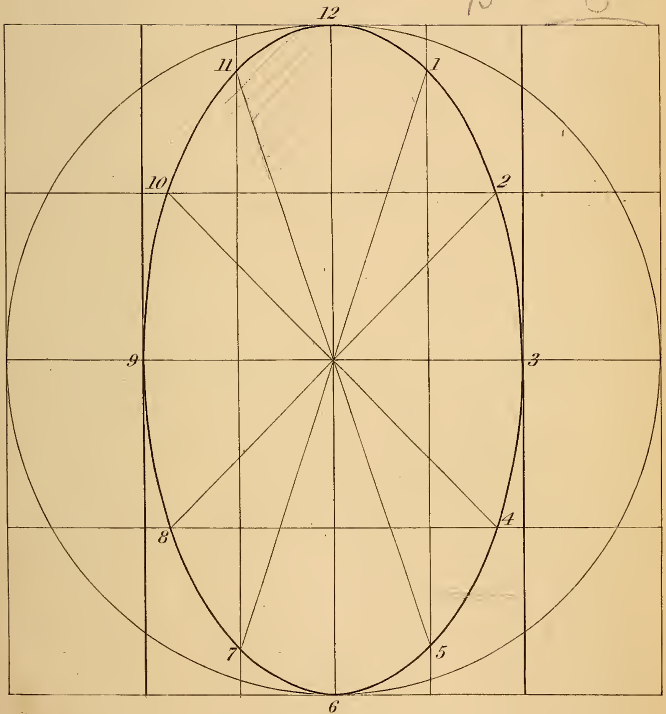

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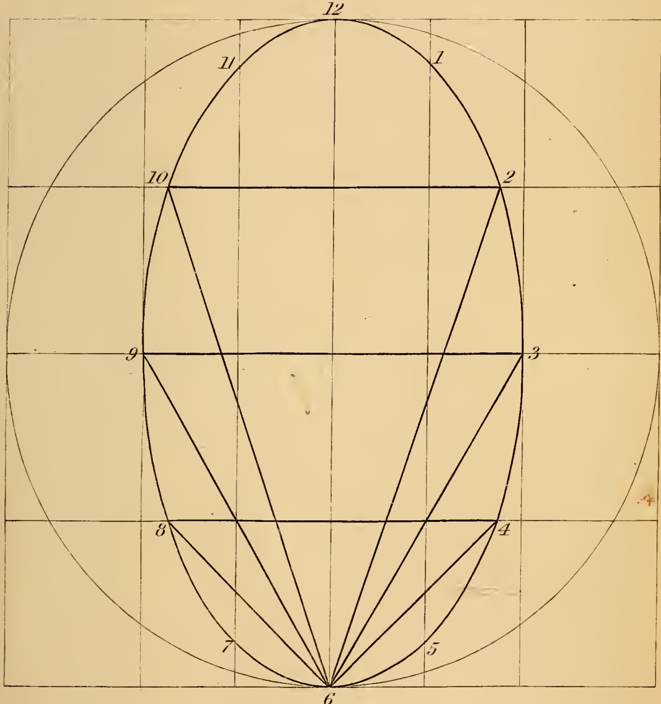
XXXI



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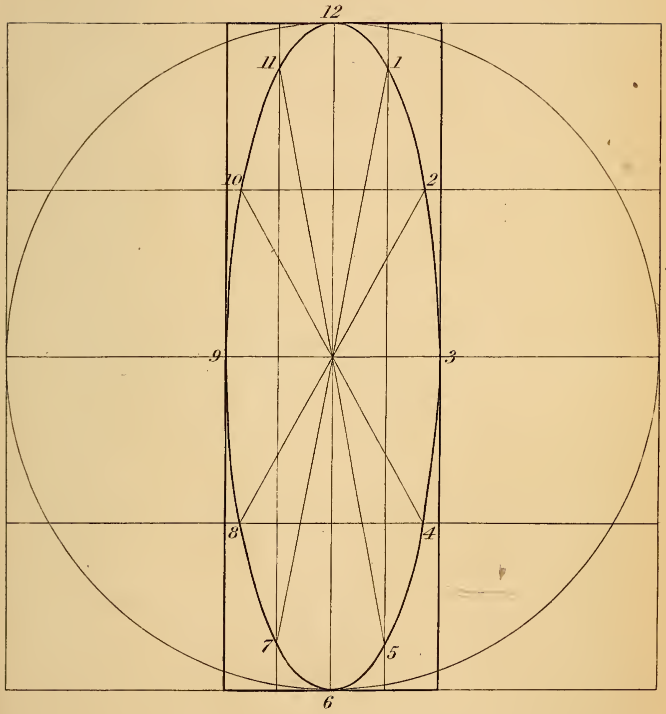
XXXII



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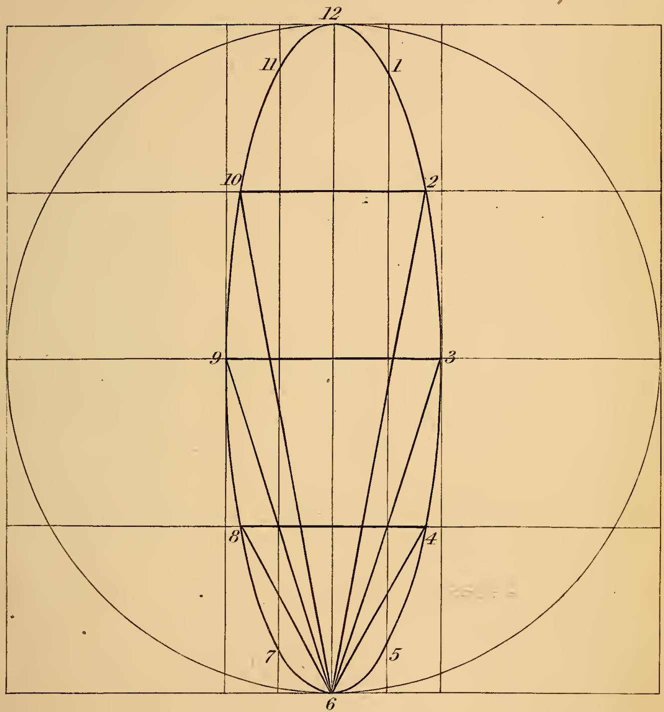
XXXIII



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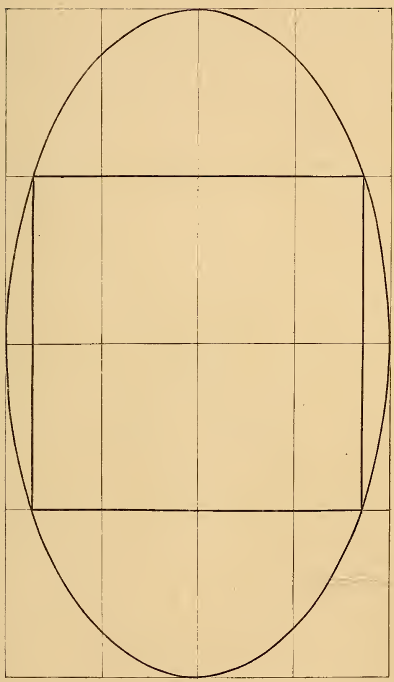

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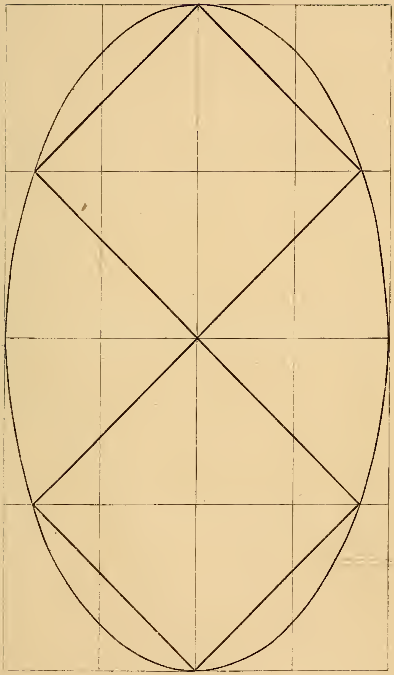

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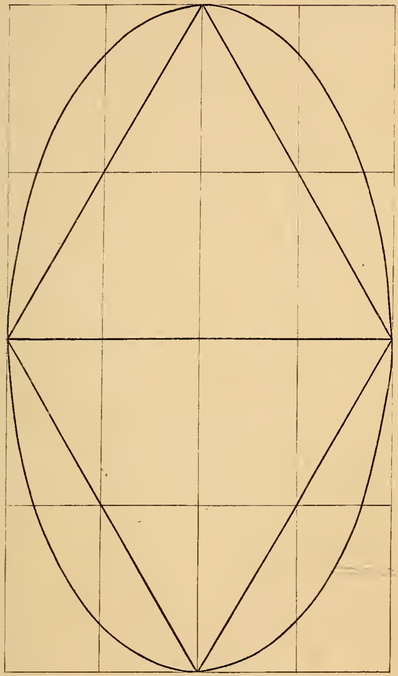

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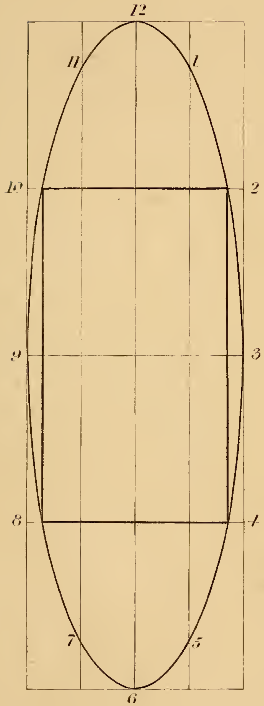

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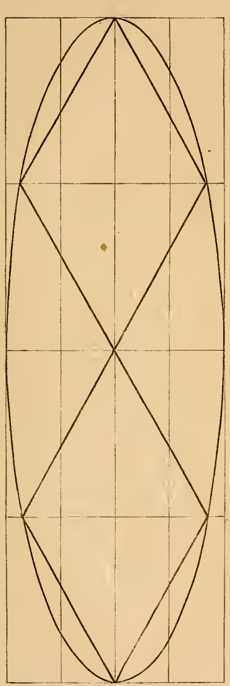

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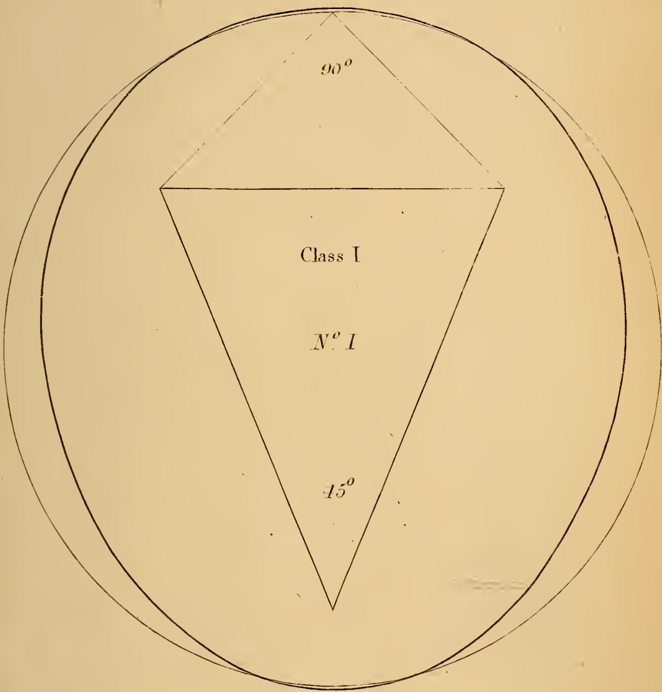

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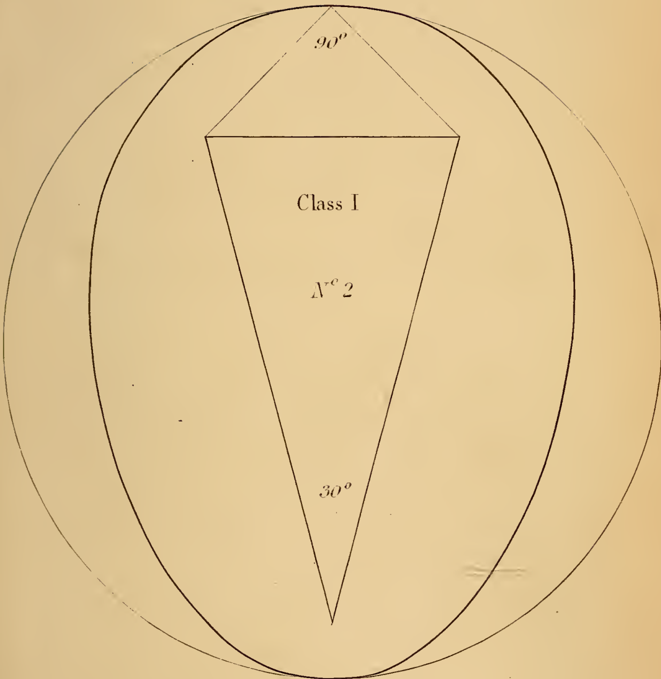

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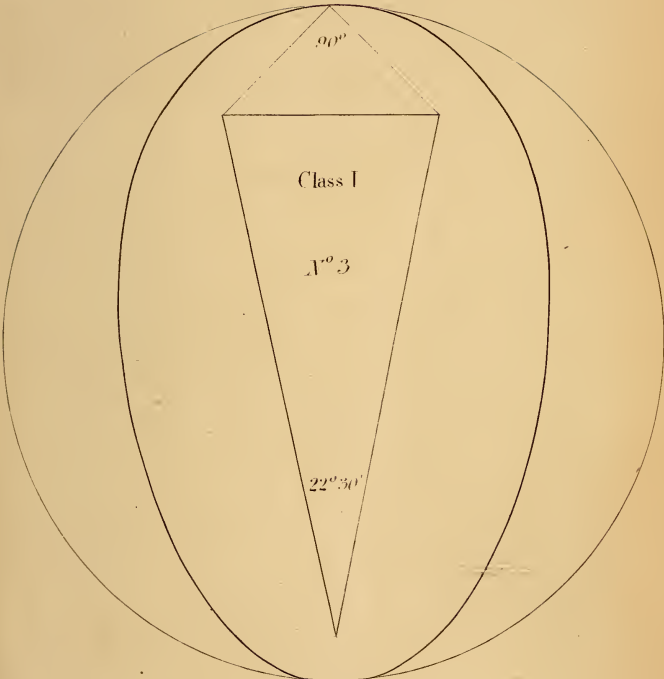

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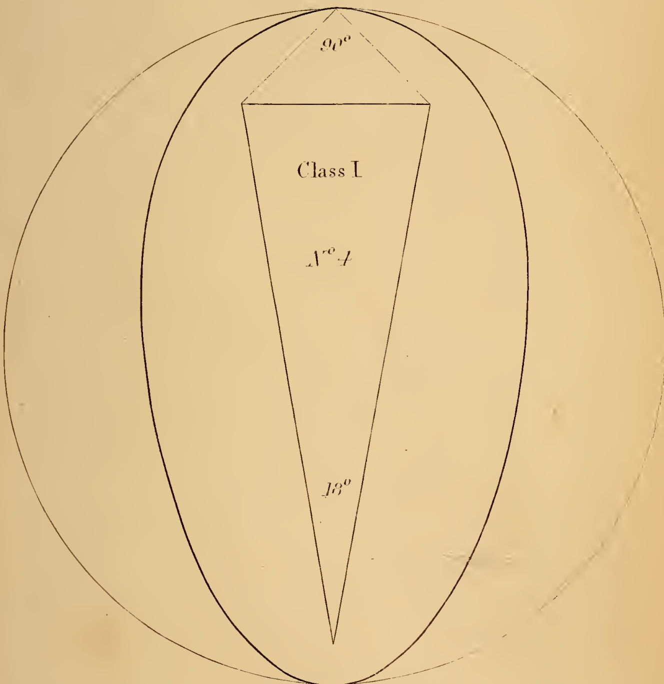

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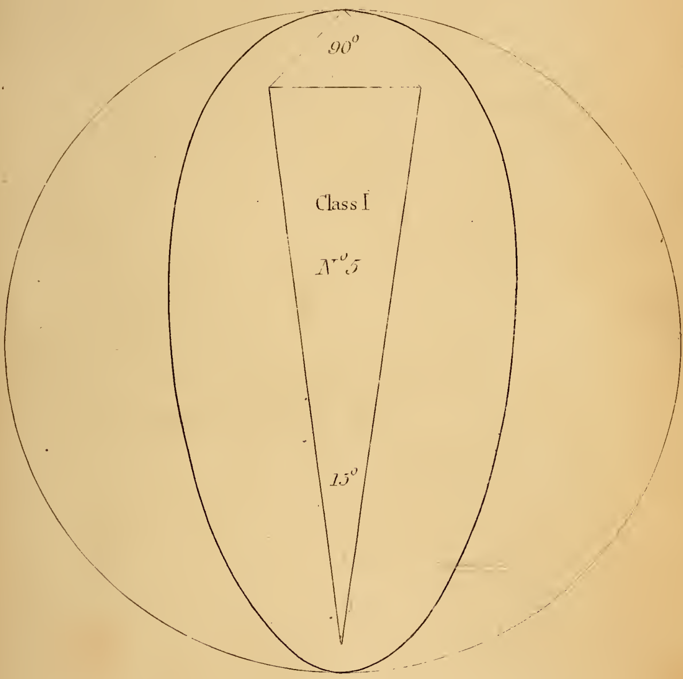

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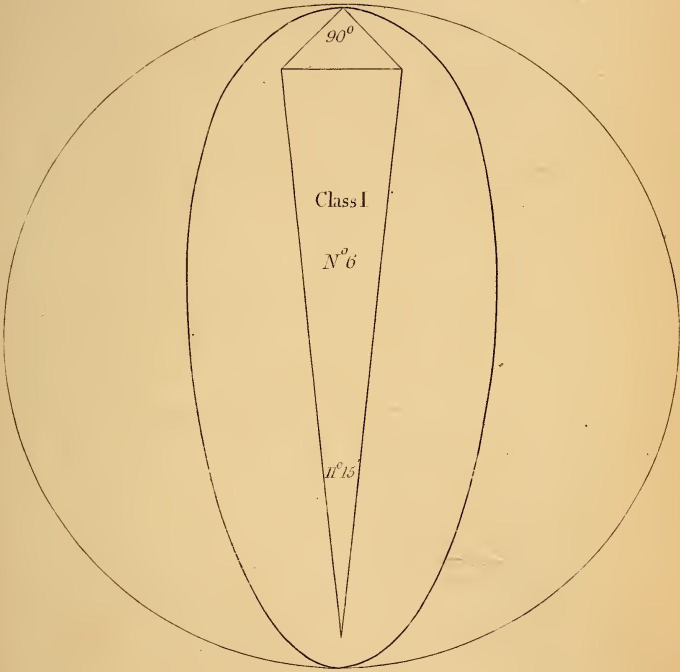

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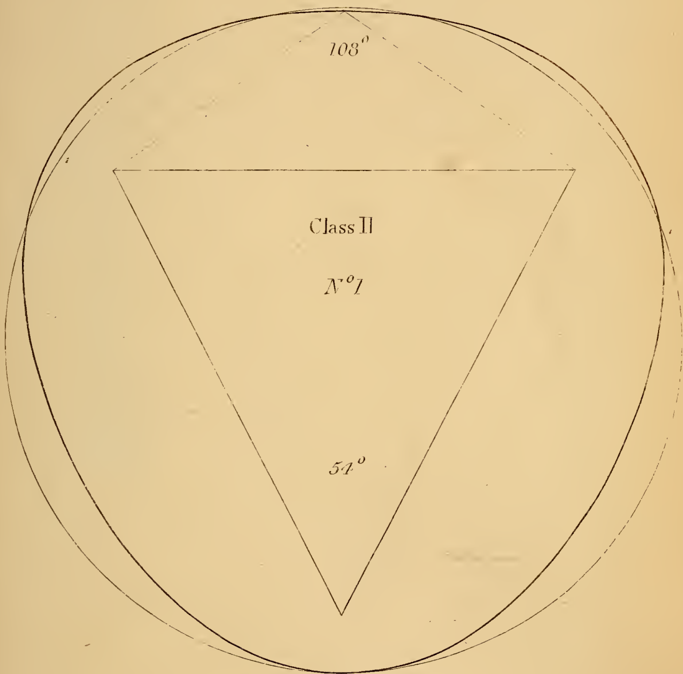

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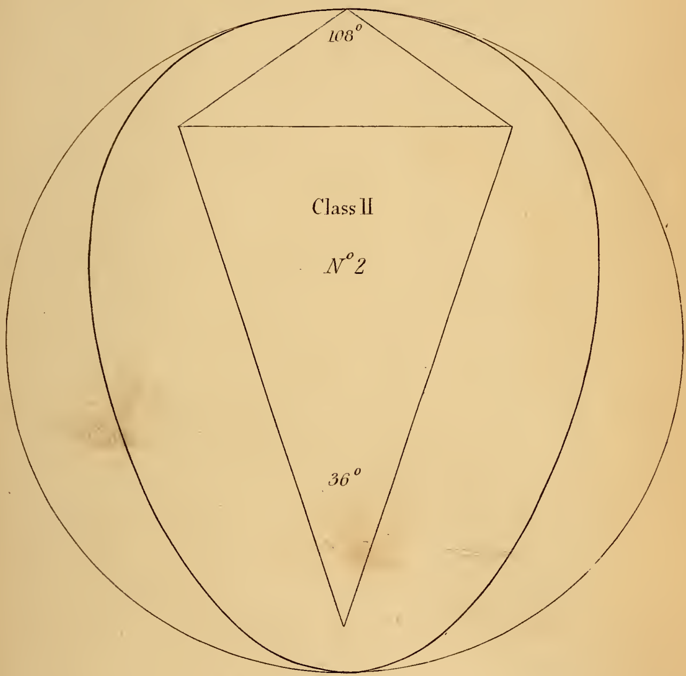

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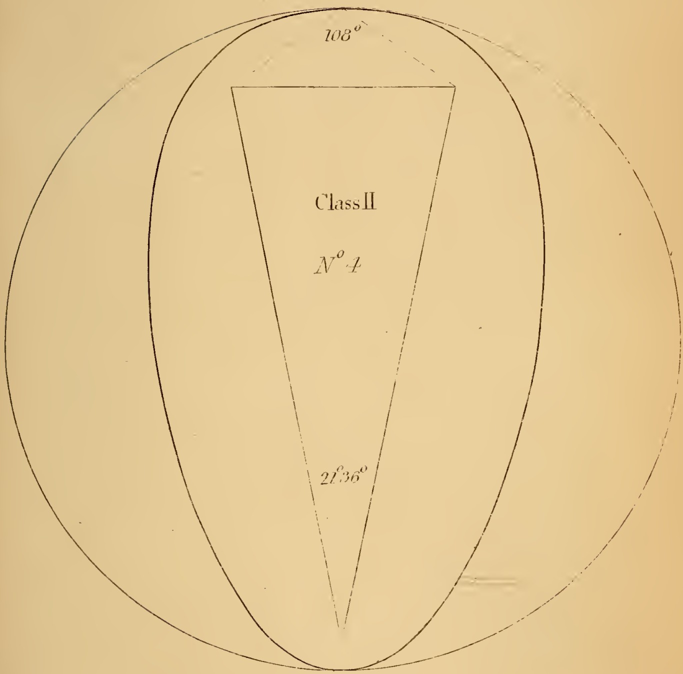

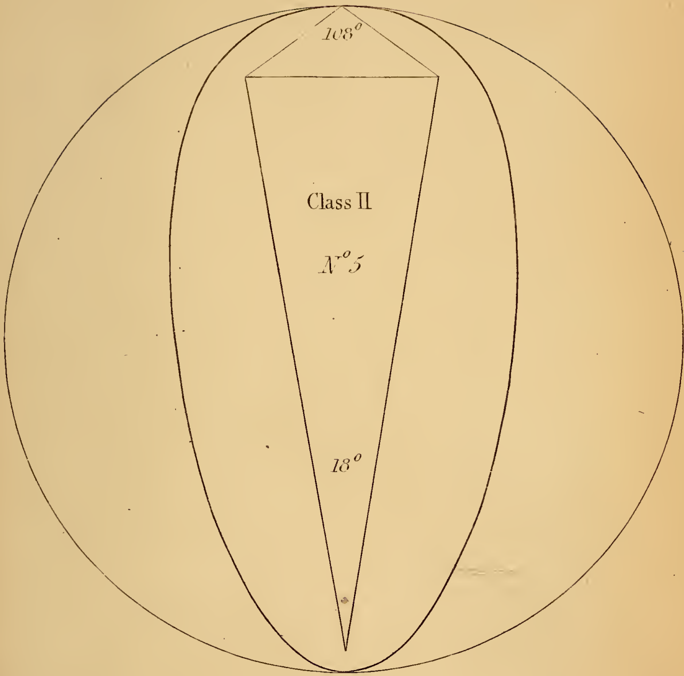

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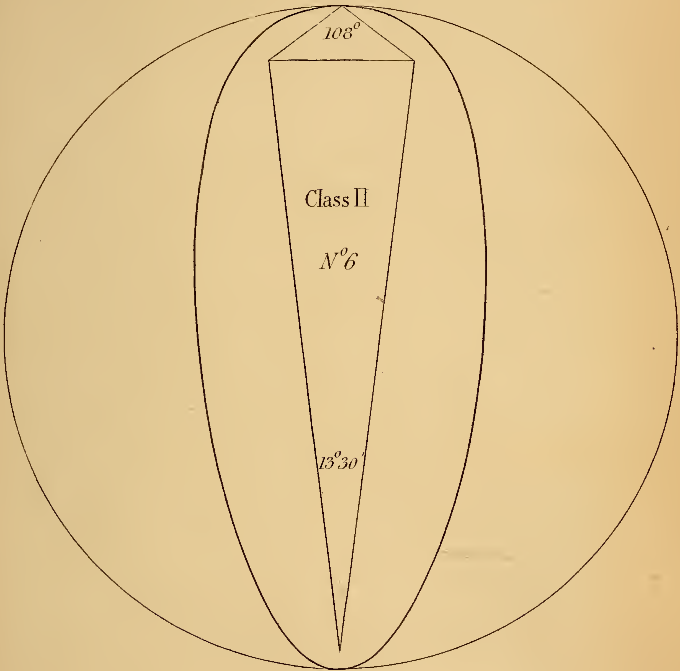

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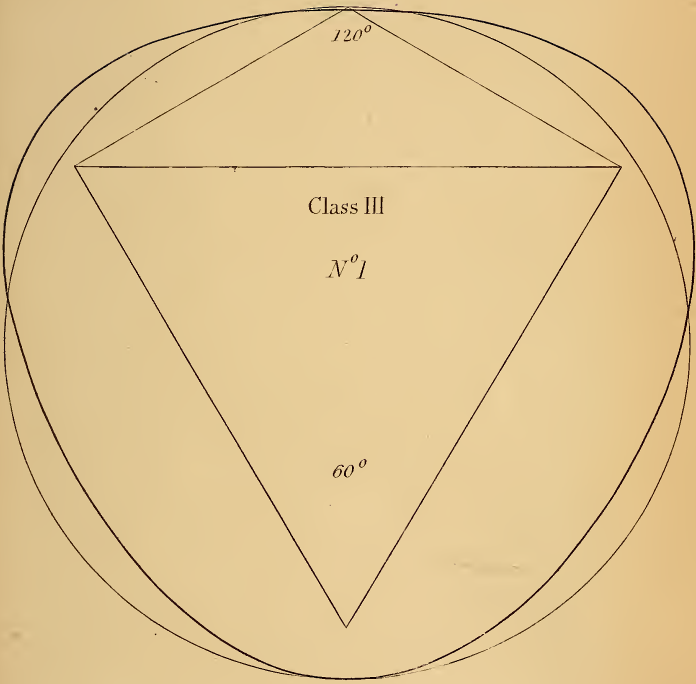

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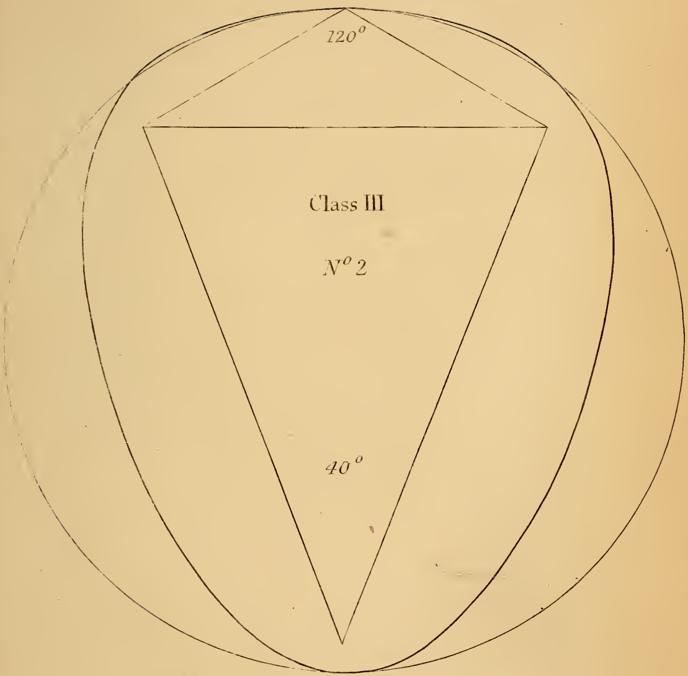

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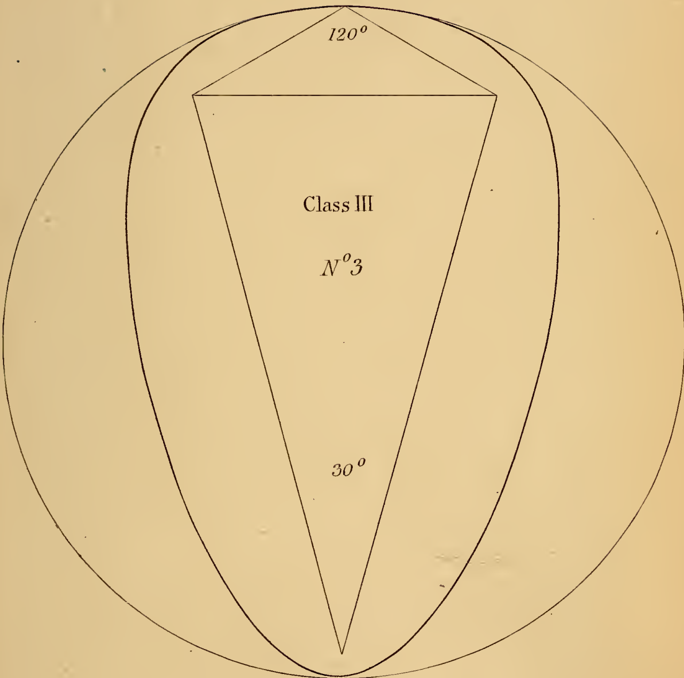

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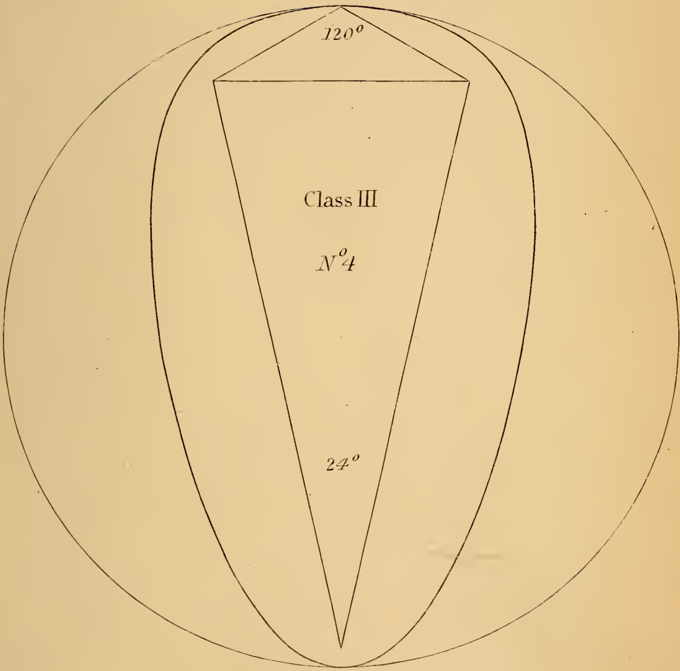

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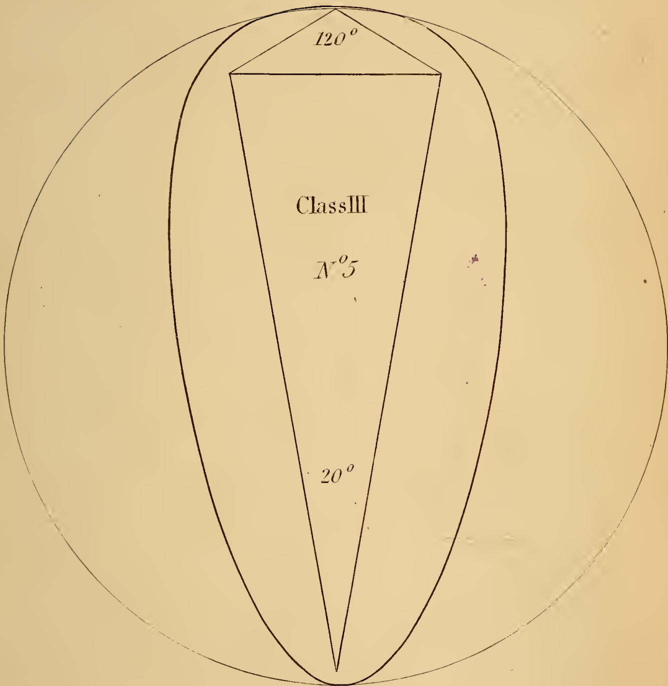

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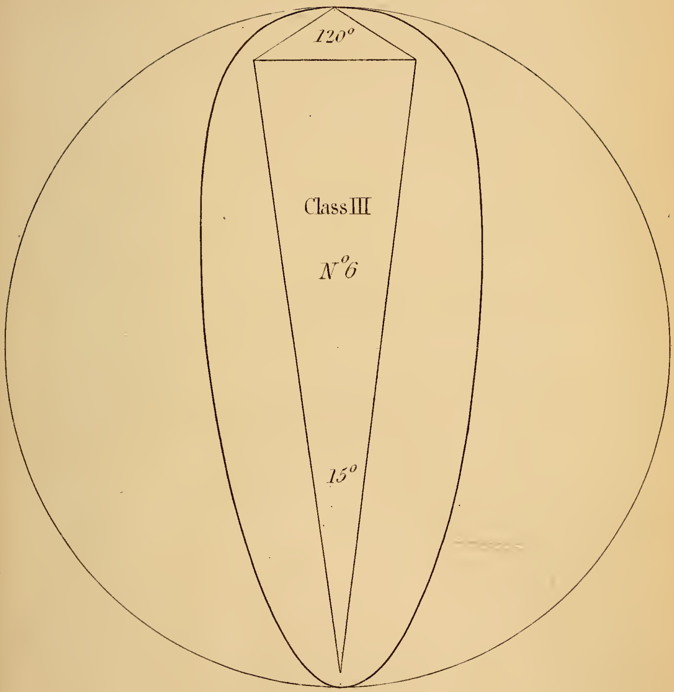

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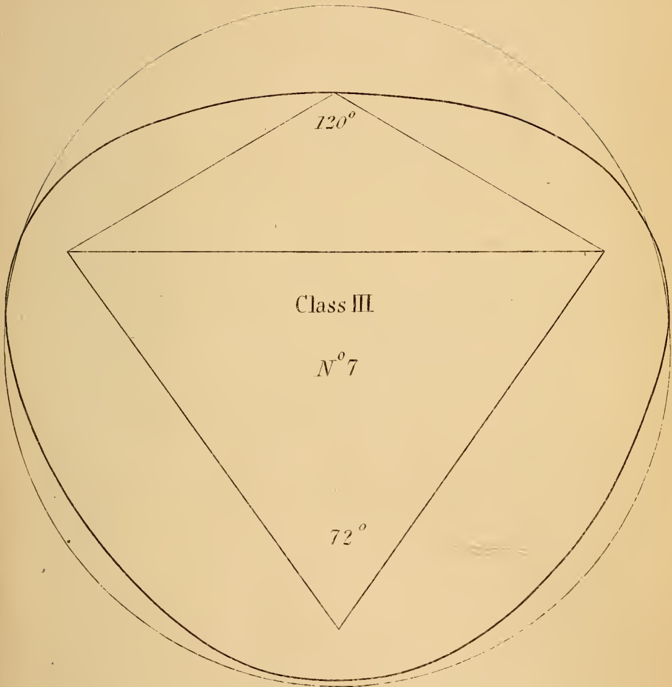

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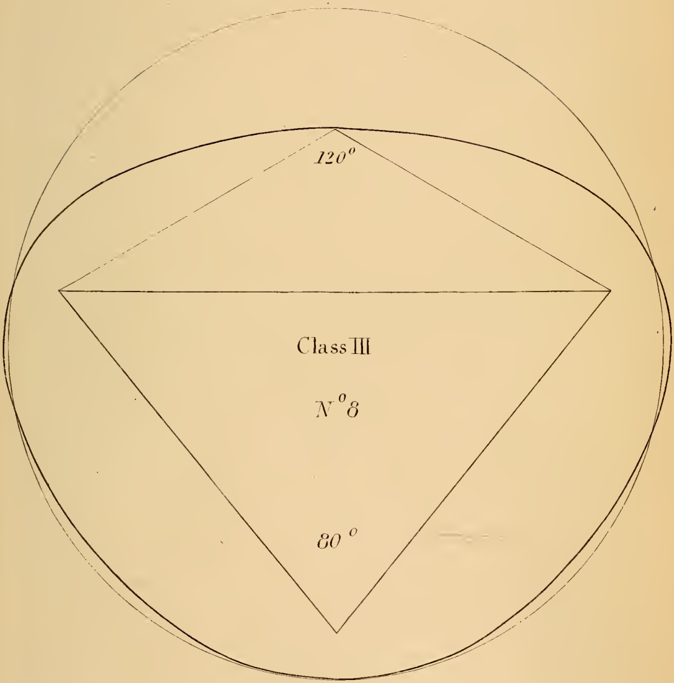

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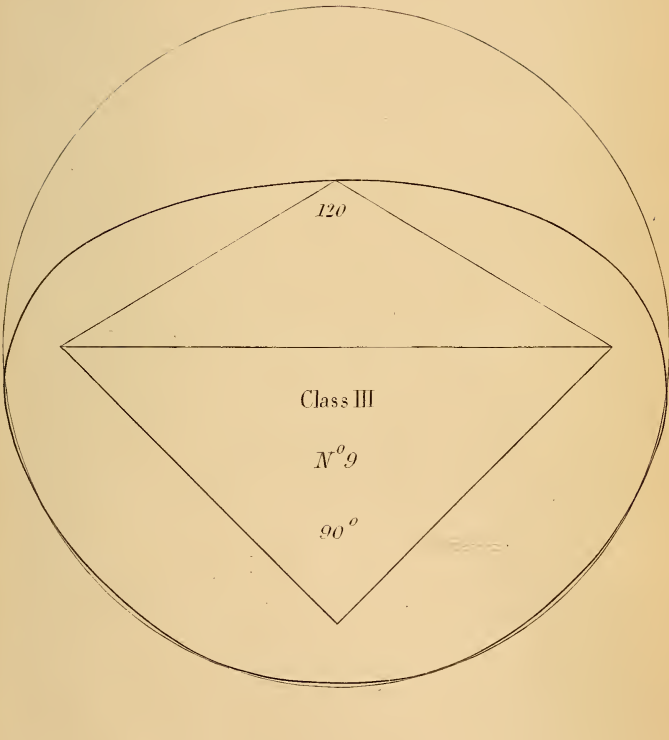

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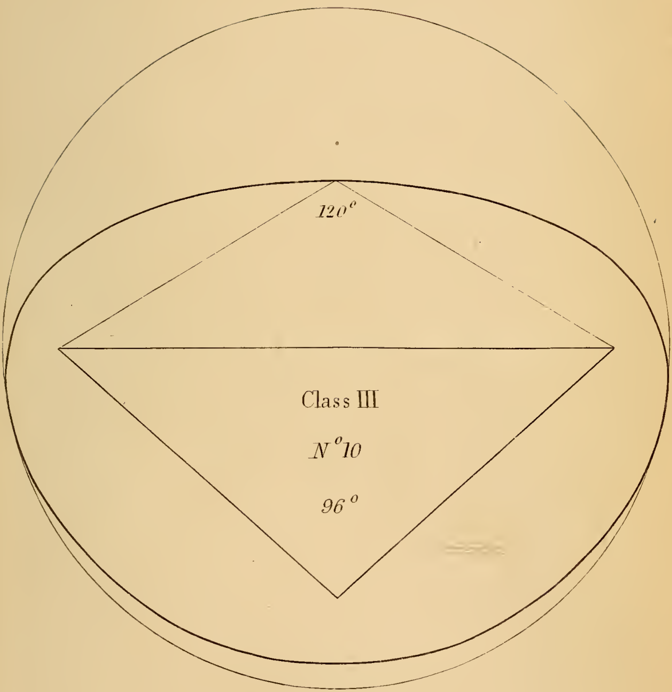

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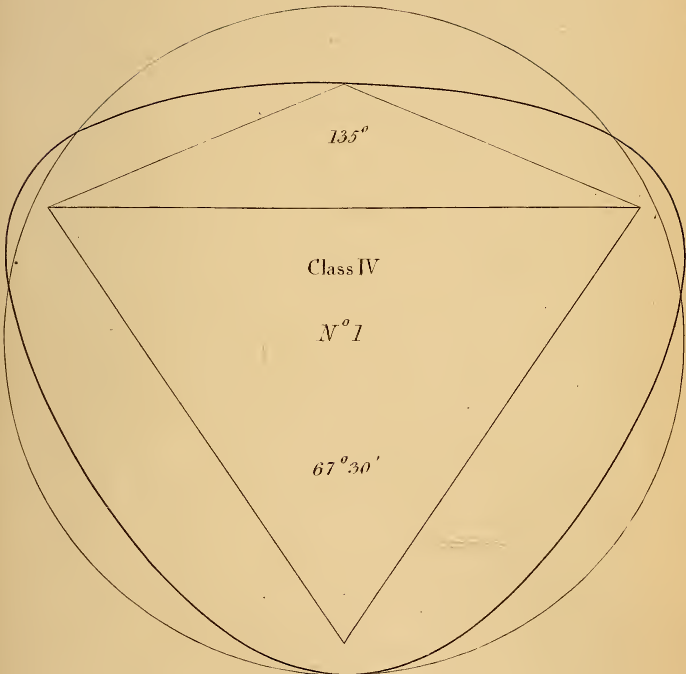

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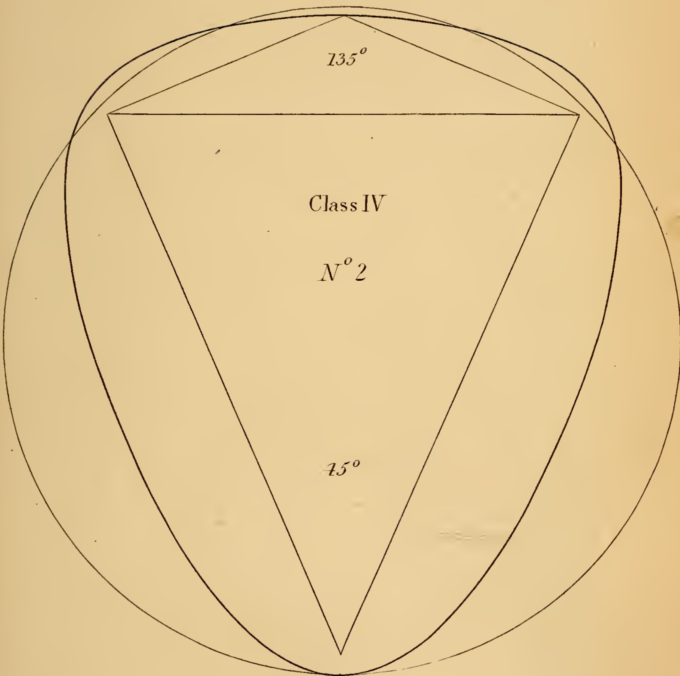

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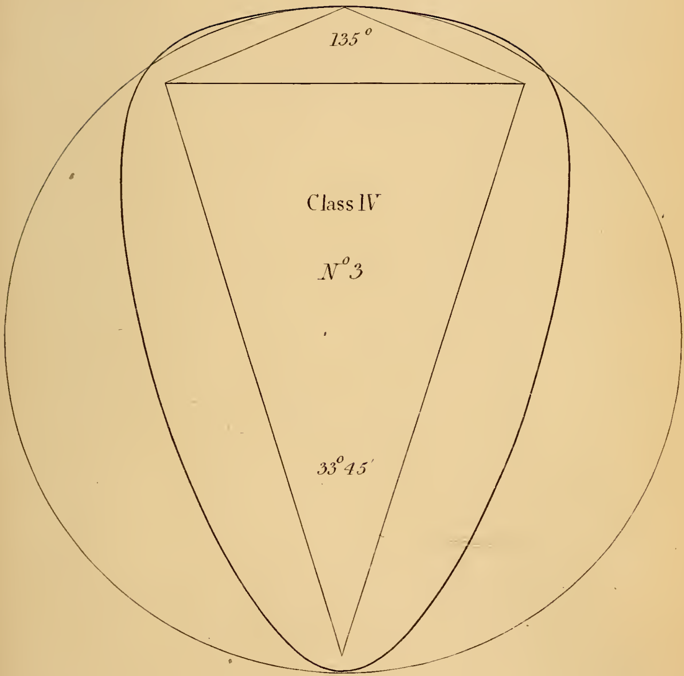

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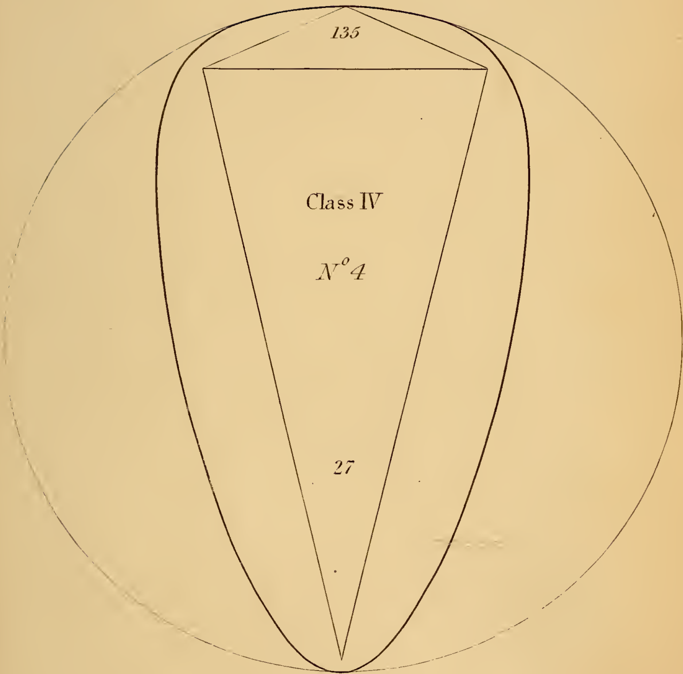

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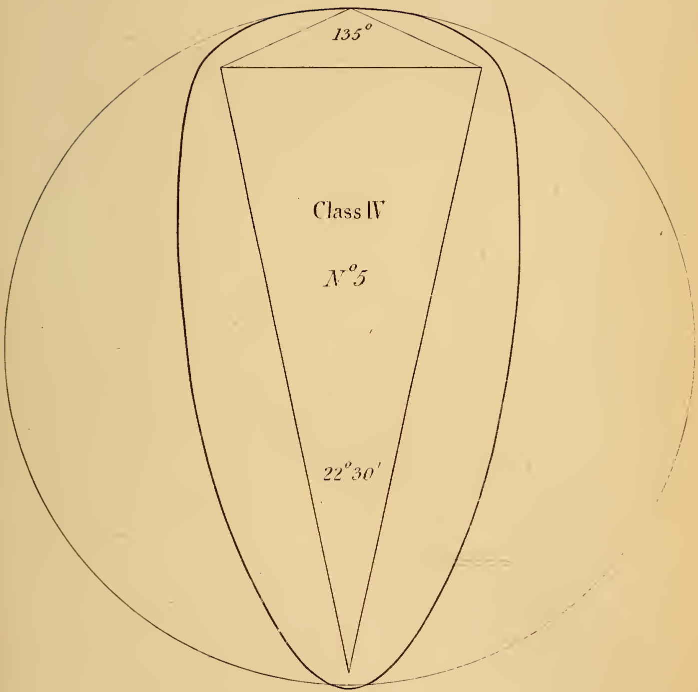

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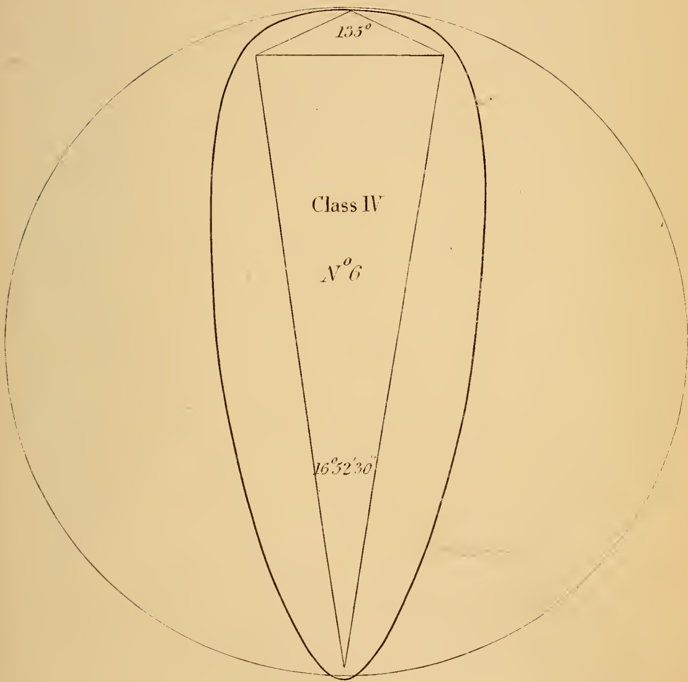

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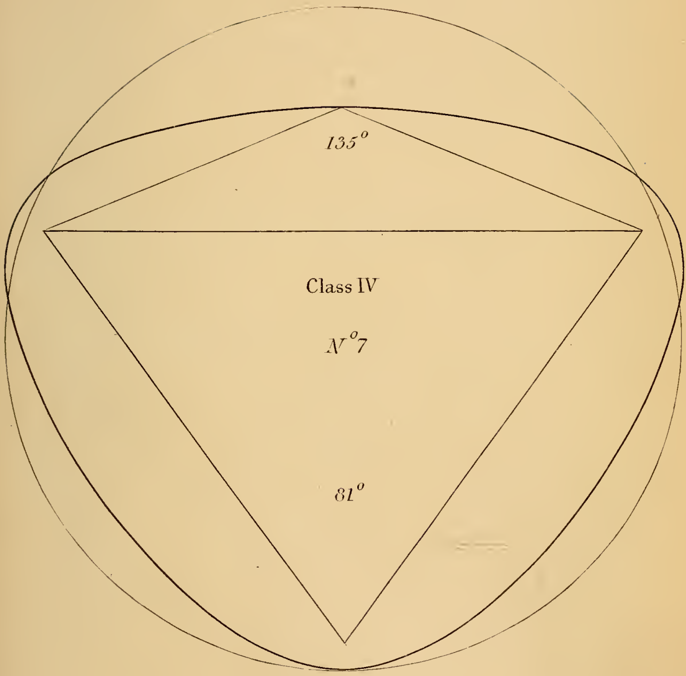

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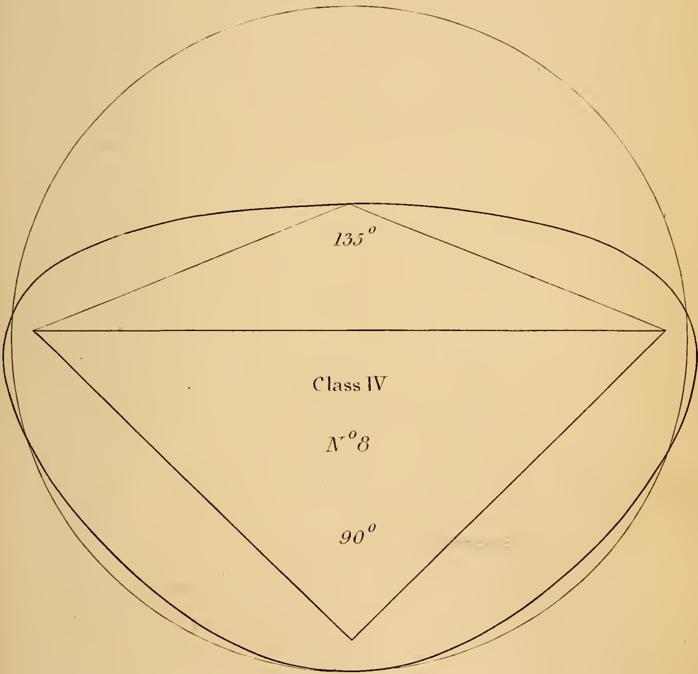

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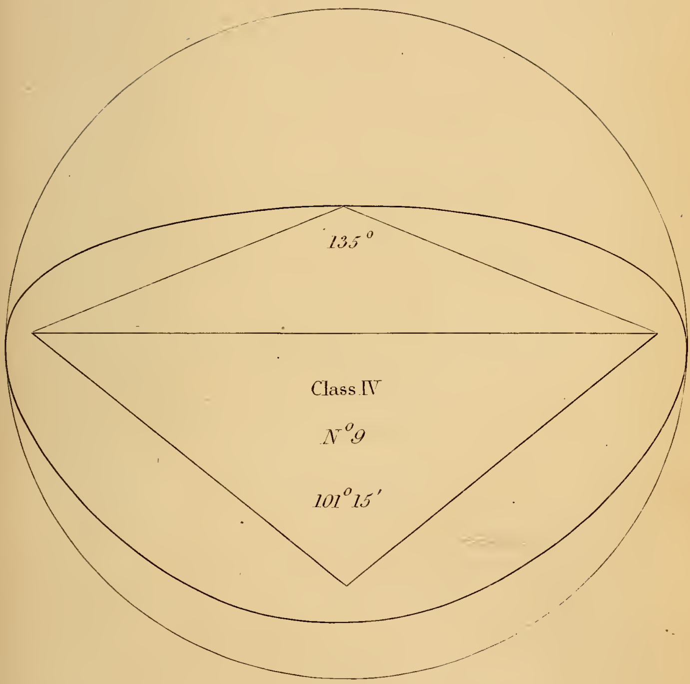

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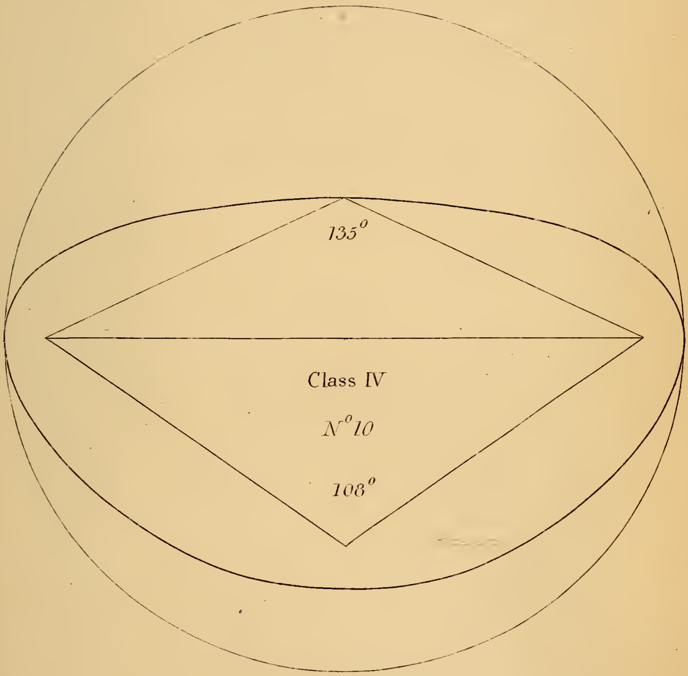

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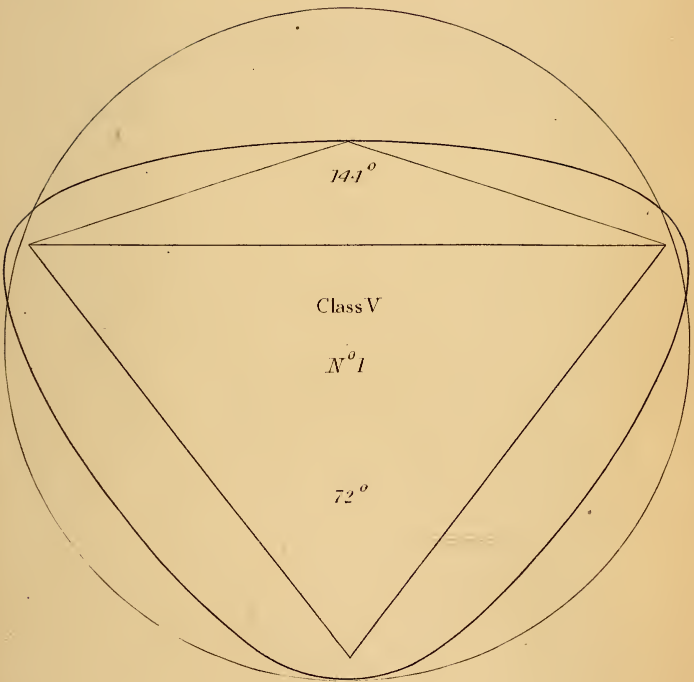

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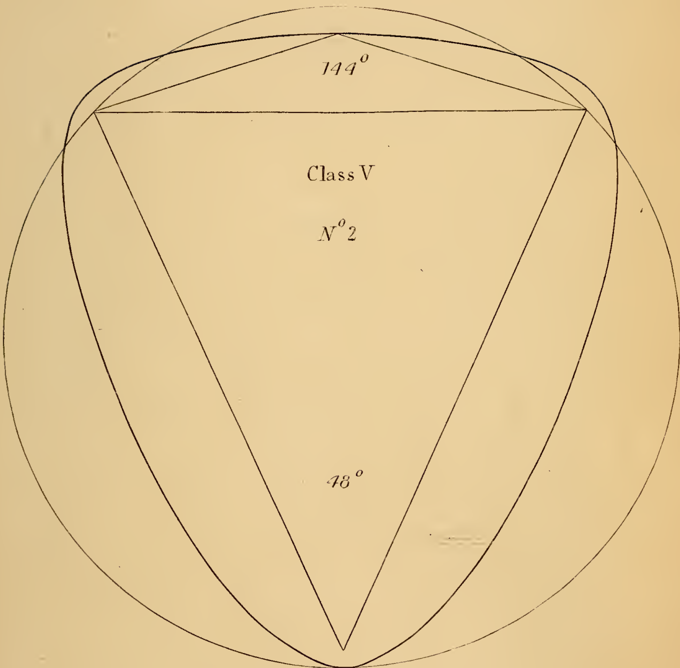

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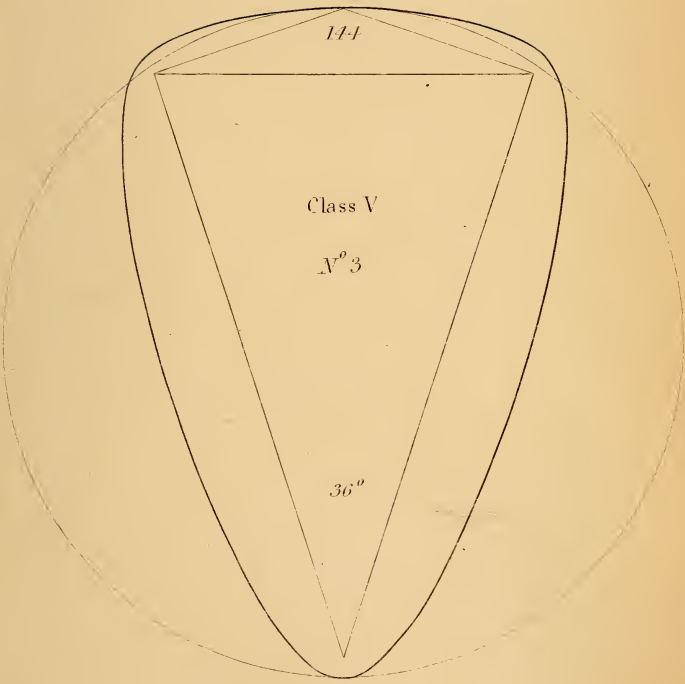

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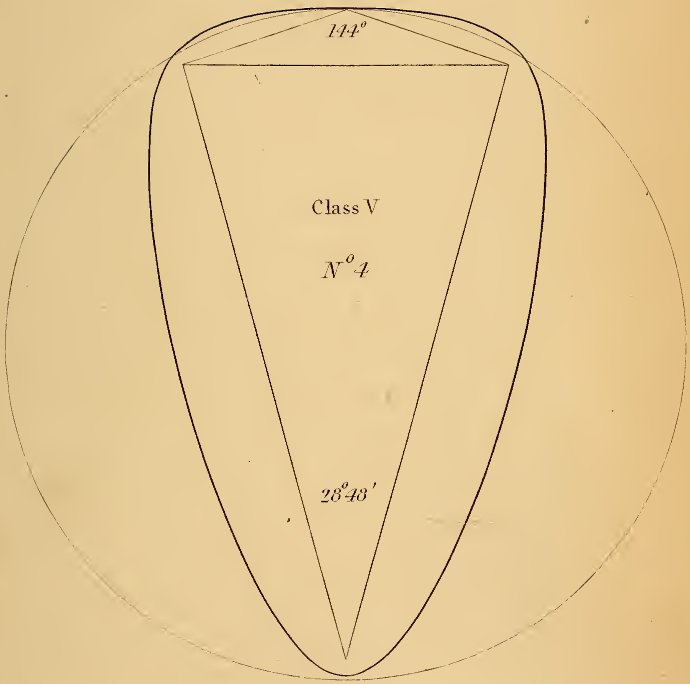

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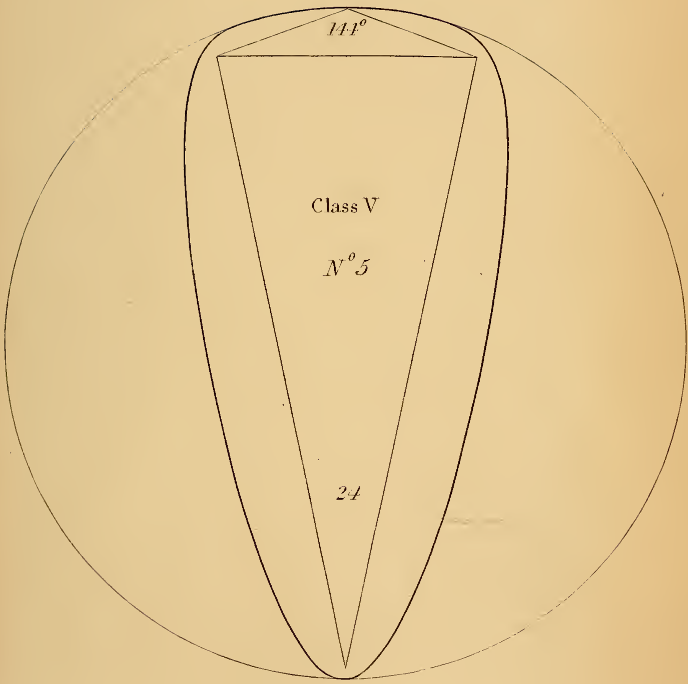

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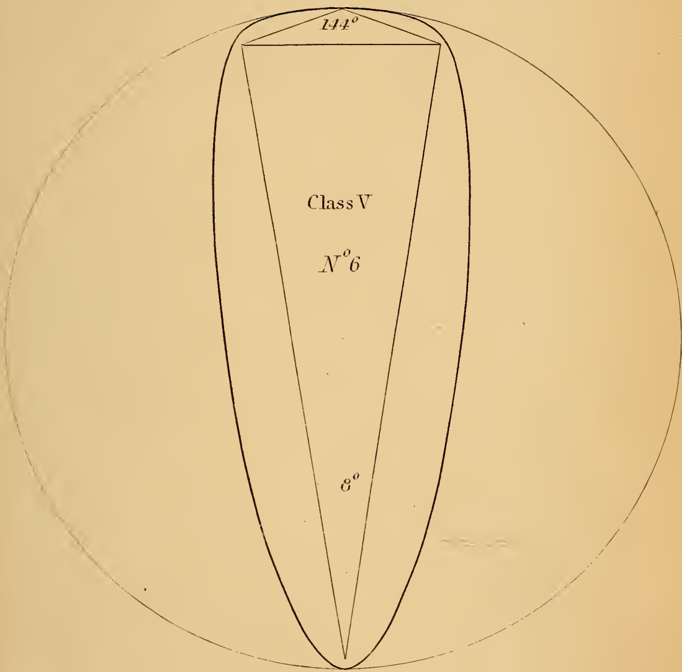

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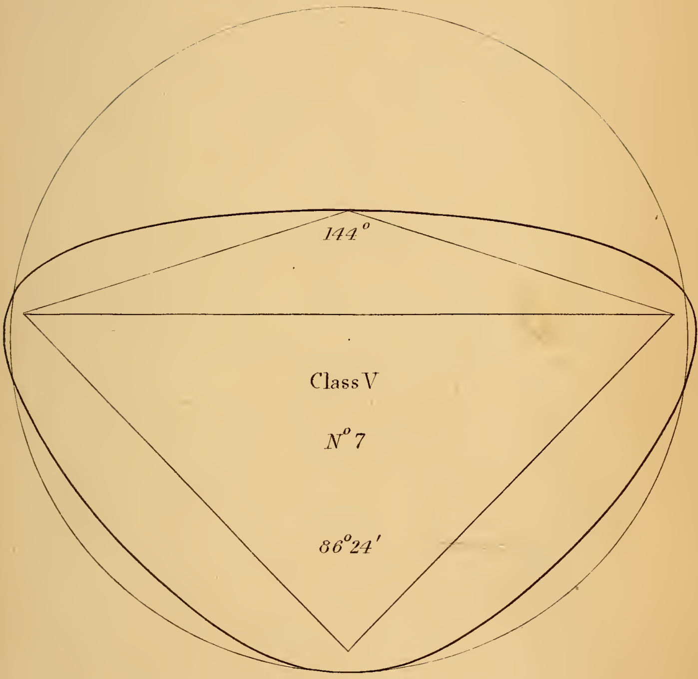

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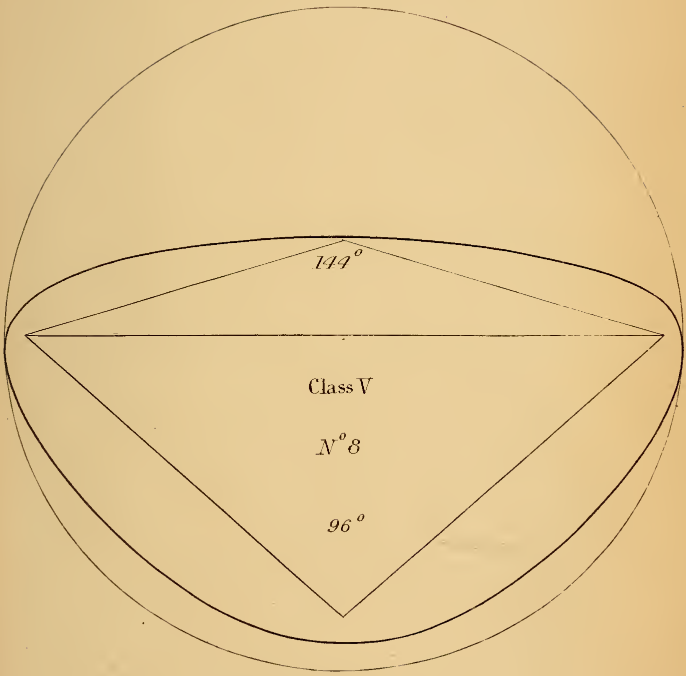

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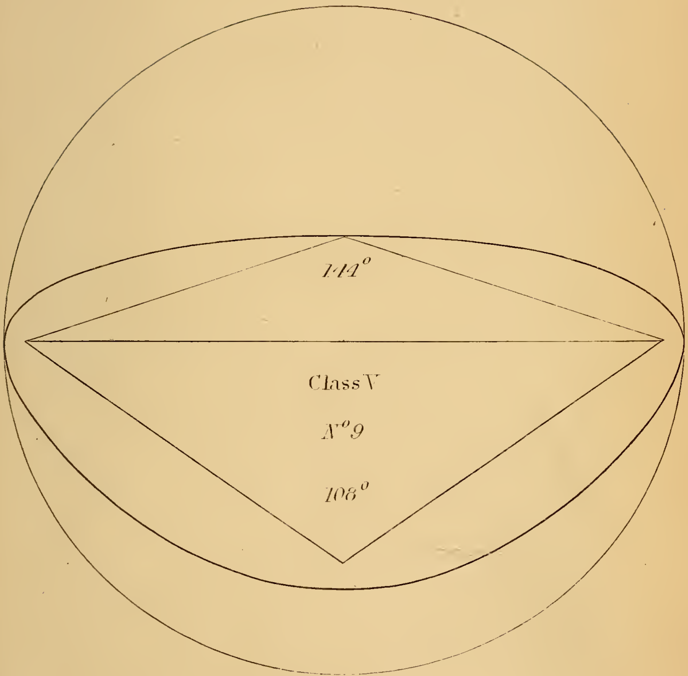

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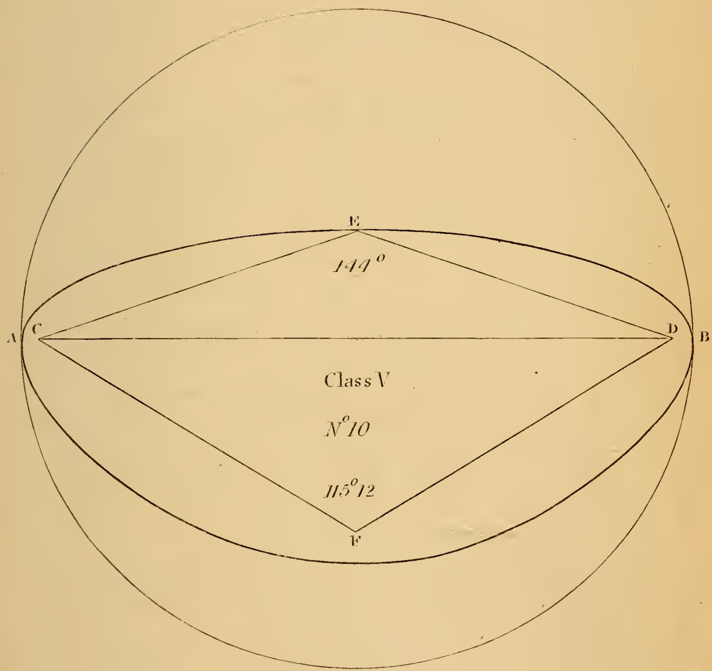


PART IV.

The composite ellipse appears to be fitted for almost universal application in the arts of ornamental design. It seems to supply a method of producing correctly every curve in the outline of the best examples of Grecian architecture from the entases of the columns to the smallest moulding, as also the outlines of the most beautiful of the Grecian and Etruscan vases. It is therefore calculated to afford the means of producing beauty in the curvilinear portions of such of the ornamental works of the architect, the sculptor, and the painter, as depend for their excellence upon the laws of symmetry, and may thus supersede the necessity of copying what remains to us of the ornamental works of antiquity, which evidently have been themselves produced by the application of some fixed principles. But what is of much more importance in this country, where the general diffusion of an appreciation and love of the beautiful is so much required, the use of these figures will improve the most humble works of the potter, without rendering them more expensive than at present, and thus enable the mechanic and cottager to have household utensils as truly beautiful in form as the finest services of plate which the art of the silversmith is capable of producing. These humble utensils, if produced by an adherence to the fixed laws of symmetry, will require none of that efflorescence which, in the works of the silversmith and in those of the higher class of the potter's wares, is as often employed to embellish deformity as to enhance the effect of a good design. Hitherto the works of the silversmith have surpassed those of the potter in beauty of form, because artists of talent have sometimes been employed to give the designs, and consequently the principles of symmetrical beauty have been imparted to them in the ratio of the genius of the artist. But the employment of a high class of genius in such cases is necessarily rare, and can never operate so generally in the improvement of the ornamental productions of a country as the application of first principles, which can be understood and acted upon by men of ordinary capacity.

In regard to the curvilinear decorations of architecture, I believe it is allowed by those best acquainted with the subject, that the most beautiful and most difficult to describe, are those of the ogee, and the ovolo. These curves form the outline of all the mouldings in the best examples of the works of the ancient Grecians; but so far as I am aware, there is not to be found in any of the numerous class of works that treat of the ornamental parts of architecture, a rule for drawing them correctly. Hence we find the perfection with which these mouldings are produced, in the decoration of modern buildings, bearing the same small proportion to the imperfection that true genius bears to mediocre talent amongst those who draw the pattern for the guidance of the workman. If, therefore, a method of applying the principles of symmetrical beauty to the formation of these mouldings were adopted, we might have beauty of the highest order in all such decorations, and at the same time much greater variety than is afforded by the examples handed down to us in the works of the ancients. I shall now give a few examples of a mode of applying these principles in the formation of such mouldings as decorate the best architectural works of the ancient Grecians, beginning with the ogee.

Draw upon thin pasteboard by the process already described, a composite ellipse of a kind and size suitable to the intended work. We shall suppose the kind to be No. 9, Class IV, Plate LXIX, the size any given size, and the intended work a moulding, to be cut by a stone mason for the entablature of a building. Let this composite ellipse be carefully cut out of the pasteboard, preserving upon it the isosceles triangle H F G, Plate LXXXI, formed by the lines which unite its foci. Bisect one of the longest sides of this triangle, by a line C D, at right angles to it. Take a sheet of zinc, such as is generally employed for masons' patterns, and draw upon it the line A B; place the cut out figure E E E upon the sheet of zinc, so that the line C D will meet A B at D, while the line G H will intersect at right angles the line A B, and trace its circumference upon the zinc with a suitable instrument. Invert the cut out figure, so that the line G H may again be at right angles with A B, and that its edge may be tangential to the line already traced, while the lines C D and D C are in the same straight line, and again trace its circumference upon the zinc. Draw the lines I G and G K parallel to A B, and between the points at which they intersect the respective circumferences of the two composite ellipses, will be found the curve of the ogee moulding, as defined by the strong line.

Plates LXXXII and LXXXIII exhibit other two examples of sections of this peculiar moulding, the first described by means of the composite ellipse No. 8, Class V, and the second by means of No. 10, Class V, both being produced by the process just detailed.

Plates LXXXIV and LXXXV exhibit examples of a mode of describing another kind of ogee curve of a more simple nature than the first. This second species when forming the outline of an architectural moulding, is called a cyma recta, being concave in the upper part and convex below; while the moulding produced by the method already explained from being convex above and concave below, is called a cyma reversa.

The cyma recta is described as follows:—Upon any line A B, Plate LXXXIV, place, with its narrowest end upwards, a cut out composite ellipse, so that its diameter K D will be parallel to A B, and that a line E F bisecting at right angles the side of the triangle, C H G, will touch the line A B at the point of intersection with its circumference. Trace round the cut out figure, reverse it, and again trace round it, and between the points K K the curve of the cyma recta will be found. The composite ellipse here used is No. 1, Class II, Plate XLV, and that for the ogee given in Plate LXXXV, No. 3, Class I, Plate XLI.

It will be observed that the cyma recta is composed of arcs of the lower portion of the composite ellipse, and that the cyma reversa was composed of arcs of the upper portion. Also that the composite ellipse on the left of A B is placed with its narrowest end upwards in the one, and downwards in the other, so that the term cyma reversa is quite appropriate, independently of the different positions of the convex and concave portions of this peculiar moulding.

The ovolo moulding seems to derive its name from the composite ellipse or egg form. Plate LXXXVI is an example of a mode of forming a section of one of those mouldings, such as are used in the capitals of antæ in Grecian architecture. It is described as follows:—Draw the line A B, and at right angles with it the lines C D and E F. Place a cut out composite ellipse, say No. 1, Class III, upon A B, so that its longest diameter will coincide with that line, and its narrowest end touch the line E F at B, trace its circumference and then remove it. Draw the line G H at an angle of 45° to E F. Place the cut out composite ellipse No. 3, same class, upon G H, so that its longest diameter will coincide with that line, and with its narrowest end at the point where G H intersects C D, trace its outline and remove it. Draw I K at an angle of 75° to E F, and place upon it, with the narrowest end upwards, the cut out composite ellipse No. 3, so that its longest diameter will coincide with that line, and its circumference touch the line A B where intersected by E F, trace the requisite portion of its outline, and remove it. Draw the line L M parallel to I K, and place upon it the cut out composite ellipse No. 3, with its narrowest end downwards, so that its circumference will touch that of the figure last traced, and again trace its outline. Draw the line N O at an angle of 75° to E F, and the strong lines P P P P are the curves of the moulding.

But perhaps the most direct application of the composite ellipse in the decoration of Grecian architecture is to the formation of the *echinus* or egg and tongue moulding. An example of this is given in Plate LXXXVII, and its curves are described as follows:—Draw the line A B, and at right angles to it the line C D, place a cut out figure of the composite ellipse, No. 1, Class I, (or any other composite ellipse,) upon the line A B, so that its centre will coincide with that line, and its narrowest end be upon the line C D, trace round it and remove it. In the same manner place a cut out figure of the composite ellipse, No. 3, same class, keeping it a little above the line C D, trace its outline and remove it. Parallel to C D draw the line E F, through the upper foci, *a a*, of the composite ellipse No. 1. Draw the lines G H and I K parallel to A B, place the cut out figure No. 3, with its narrowest end upwards, so that C D will bisect its longest diameter at right angles and its circumference be tangential to G H where that line intersects C D, trace round the figure, and repeat the same process at I K, draw the lines L M and O P parallel to A B, and you have a correct outline of one species of the egg and tongue moulding.

It will be observed, that between the composite ellipses Nos. 1 and 3, another figure of the same kind is introduced. I found that No. 2 (ratio 1 to 3) did not answer this purpose, but having in the progress of the work constructed a series of intermediate composite ellipses, having the ratios to the right angle of 4 to 9, 2 to 5, 3 to 8, 3 to 10, 4 to 15, and 2 to 9, I found that the figure whose angles were in the ratio of 3 to 8, harmonized better than that whose angles were in that of 1 to 3, and adopted the former for this intermediate line. The introduction, however, of those intermediate figures would have increased the plates to an inconvenient number, and I have therefore left them out. But this will occasion little inconvenience to the artist or artizan, as the ratios given above will enable him to construct them for himself when required.

The vases of ancient Greece are universally allowed to be beautiful in their general form, and to owe much of this beauty to the nature of the curves which form their outline. These curves are evidently identical with those which I have endeavoured to connect with the architectural decorations of the same period, and the same principles of symmetrical proportion appear to have been applied in their combination. So likewise do those well known and beautiful objects of ancient art, the vases of Etruria, exhibit in their outline a similar species of curve, evidently applied according to some certain and well established rules of symmetrical proportion or harmony. But the use of these peculiar curves, and the principle which regulated their application in the arts of ornamental design, have been lost to succeeding ages. Hence all our attempts to arrive at a like degree of perfection have consisted in copying such specimens of the application of these principles as have been handed down to us from that remote period. Being thus confined within the narrow bounds of precedent, the operations of the ornamental designer have not advanced beyond mere mimicry; for whenever these acknowledged precedents have been departed from, and originality attempted, a lower degree of beauty has been the result. But if we can arrive at a knowledge of the nature of the curve, and of the principles of harmony by which the ancients must have applied it, a new field will be opened to us, and we may then be original with safety; and when genius is thus allowed to follow its native impulse with truth for its guide, we may soon equal or even excel the ancients in the art of ornamental design.

I shall now proceed to show how very easily the finest forms may be produced in the works of the ornamental sculptor, the silversmith, and the potter, by means of symmetrical combinations of the composite ellipse.

Upon any line, A B, Plate LXXXVIII, place a cut out figure of No. 3, Class III, so that its greatest diameter will coincide with that line Trace its circumference, and remove it. Draw the lines C D and D E, forming, with A B, angles of 67° 30′. Place upon C D the same cut out figure, so that its greatest diameter will be upon that line, with its narrowest point towards D, and its circumference tangential to the circumference already traced, and again trace and remove it. Repeat the same on the line D E. Draw the lines F G and G H, forming with A B angles of 72°. Place a cut out figure of the composite ellipse, No. 6, same class, upon F G, with its broadest end towards G, and its circumference tangential to the circumference of the composite ellipse, No. 3, first traced. Repeat the same upon G H. Draw the lines I K and L M, and between their intersection with the curved lines, the outline of the body of a vase will be found, as shown by the strong line.

Plate LXXXIX is the outline of a vase with handles, composed entirely of the composite ellipse, No. 5, Class III.

Plate XC is another outline of a vase with handles, composed entirely of No. 6, Class IV.

Plate XCI is an outline of a jug composed of arcs of the composite ellipse, No. 6, Class IV.

Plate XCII is a second outline of a jug composed of arcs of the composite ellipses, Nos. 3 and 6, Class II.

Plate XCIII is a third outline of a jug composed of arcs of the composite ellipses, Nos. 2 and 6, Class IV.

Plate XCIV is a fourth outline of a jug composed of arcs of the composite ellipses, Nos. 1, 3, and 6, Class III.

Plate XCV is the outline of a decanter or ornamental jar, composed of arcs of the composite ellipse, No. 4, Class I.

Plate XCVI is another outline of a decanter or ornamental jar, composed of arcs of the composite ellipse, No. 5, Class IV.

Plate XCVII is the outline of a two-handled pitcher, composed of arcs of the composite ellipse, No. 3, Class V.

Plate XCVIII is another outline of a pitcher or vase, composed of arcs of the composite ellipses, Nos. 1 and 6, Class V.

Plate XCIX is an outline of a tureen composed of arcs of the composite ellipses, Nos. 1, 3, and 6, Class V.

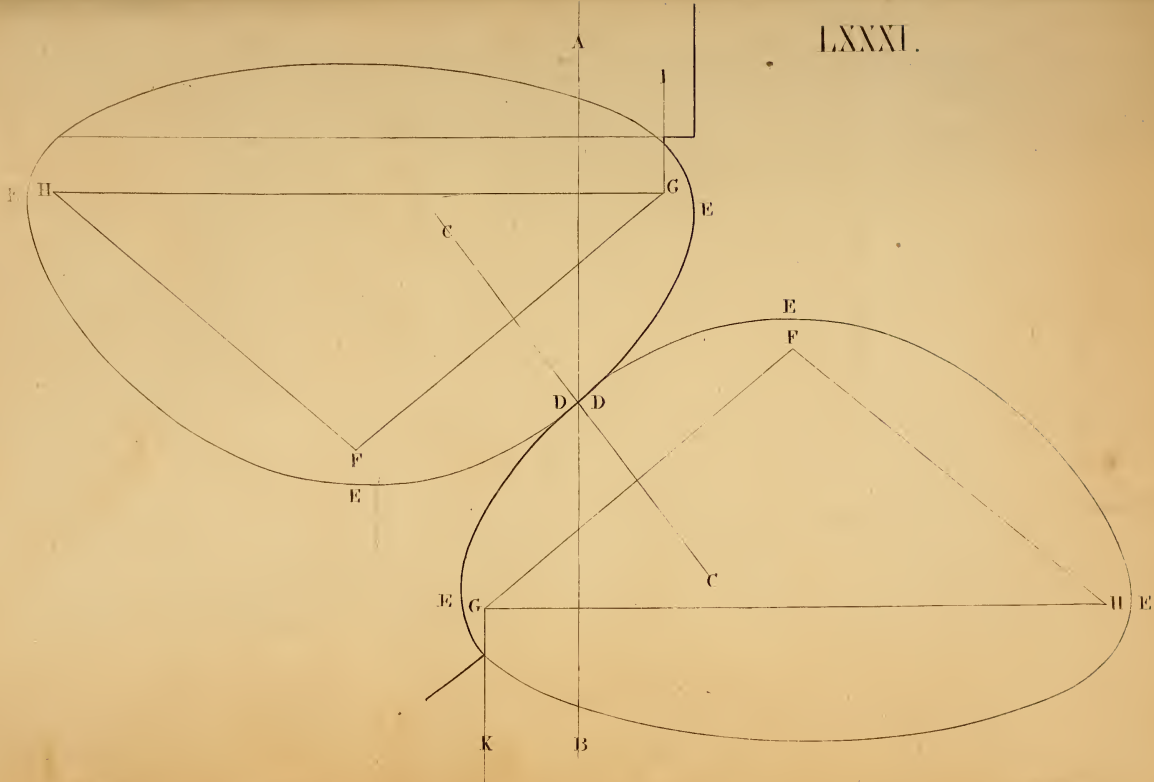
Plate C is an outline of a teapot, composed of arcs of the composite ellipses, Nos. 1 and 2, Class II.

Believing that the description given of the first example of the outline of a vase, Plate LXXXVIII will enable the reader to understand all the others, I have only given the number and class of the composite ellipses of which they are composed.

These examples are given, simply to show how the composite ellipse may be employed in various departments of ornamental art, and are therefore confined to a mere outline of some of the innumerable forms that arise from the various modes of combining that species of figure. I do not give them as designs, but merely to exemplify how the artist, the amateur, or the artizan, may use them in producing an endless variety of similar figures. If these principles of harmony be really, as I have endeavoured to prove, a natural and an inherent quality in geometry, they must be as safe a guide to the student in either architecture or ornamental design, where form is treated in the abstract, as the visible works of nature are to the student in imitative art.

That I have fully developed these principles, either in the foregoing pages, or in any of my former works, I do not pretend, but trust it will be found, that a first step towards their elucidation has been made.

LXXXI.



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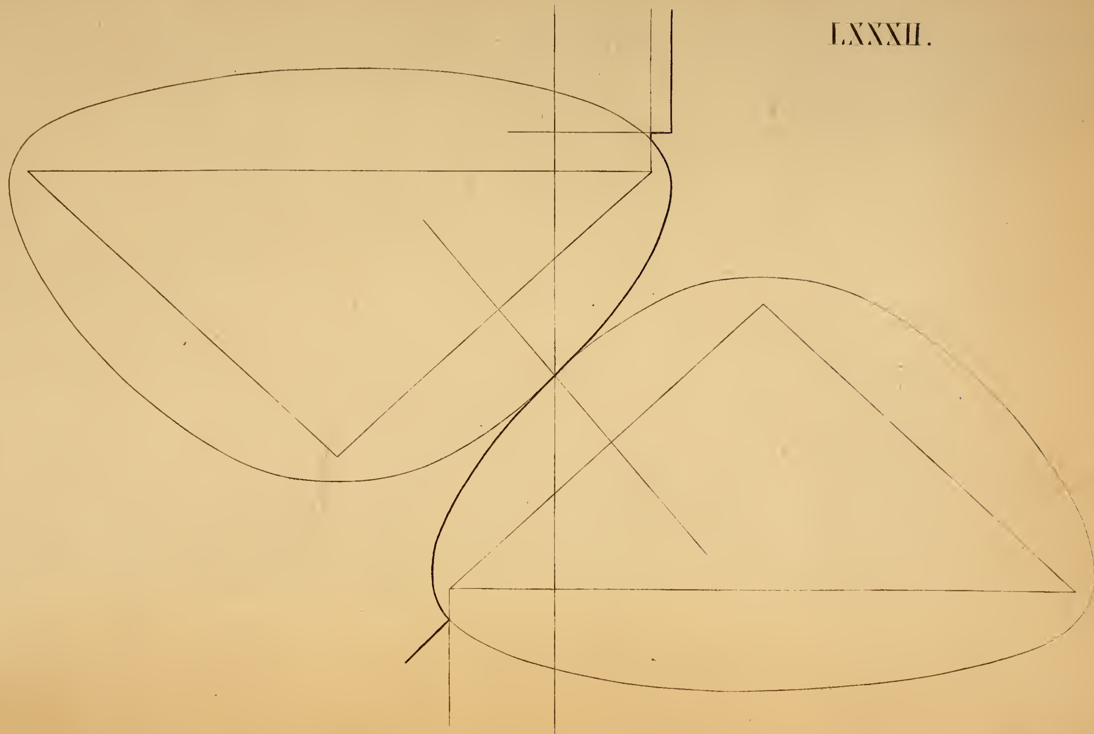

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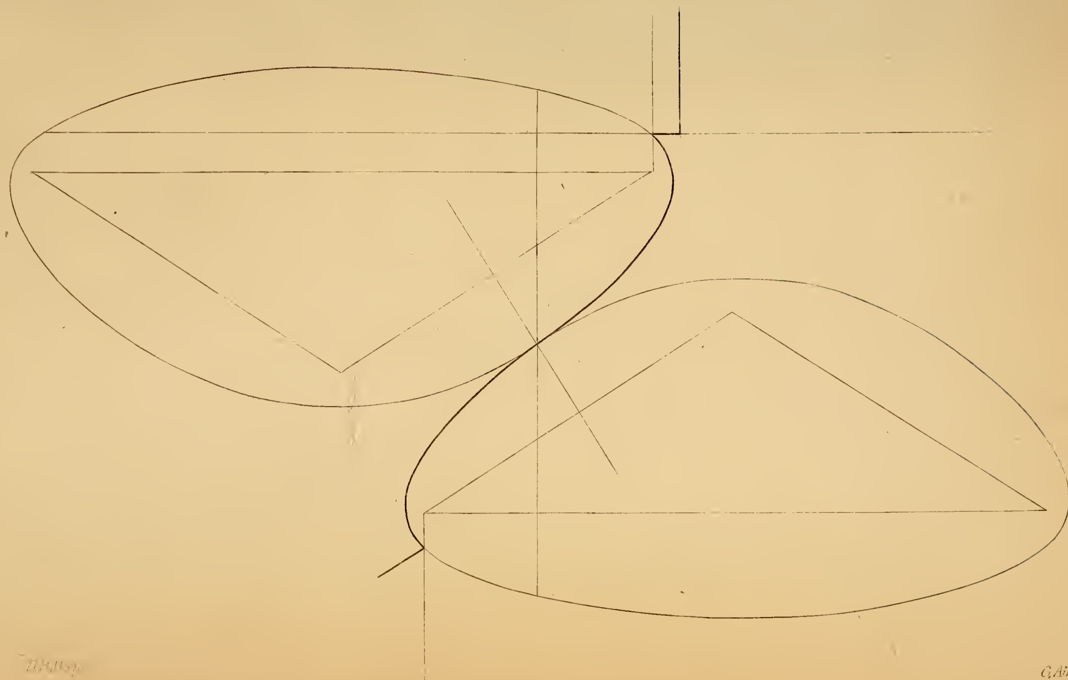
LXXXIII.



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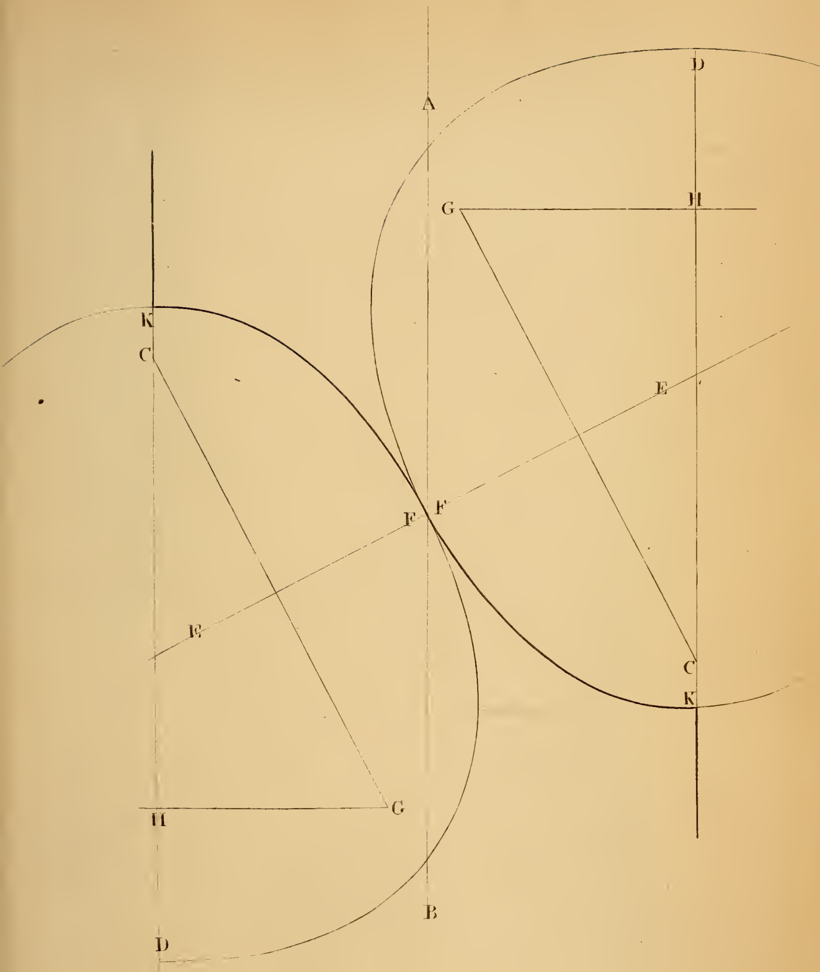

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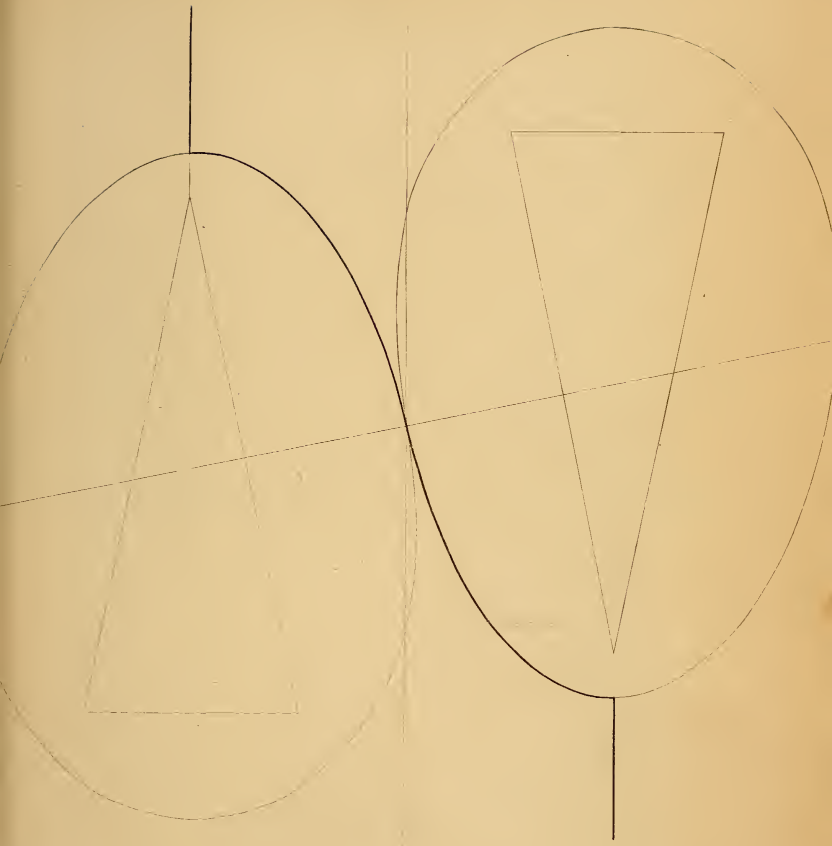
LXXXV.



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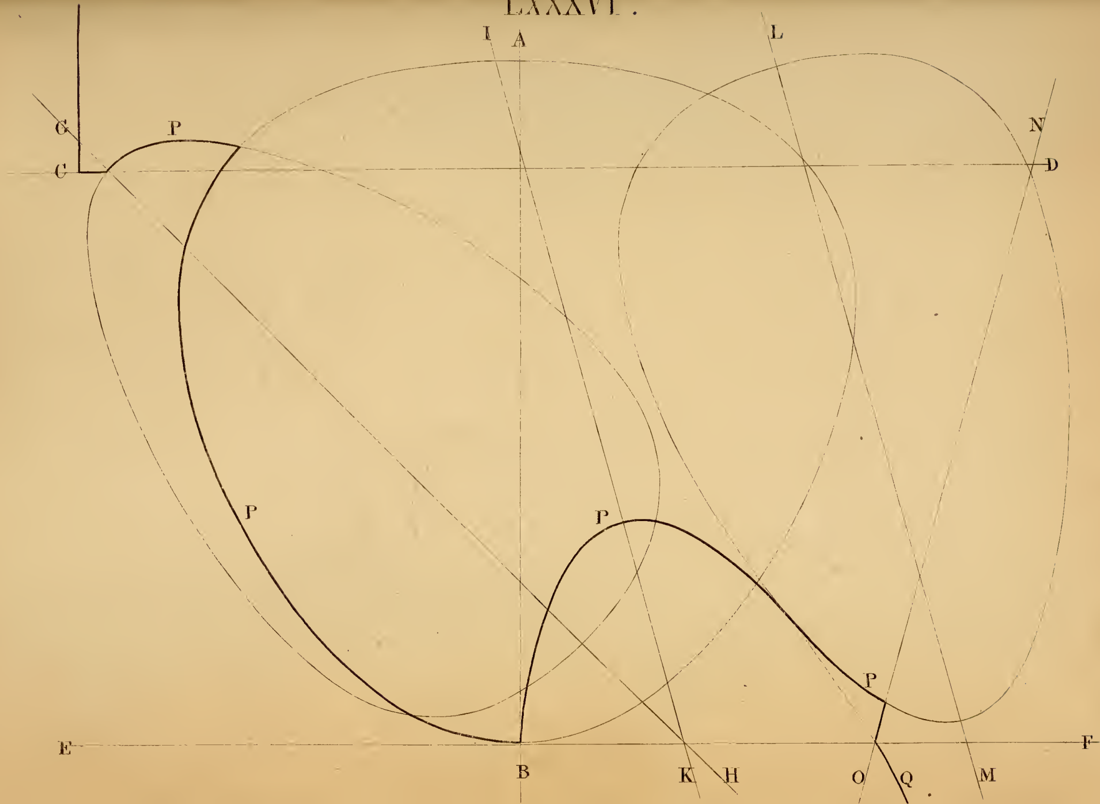
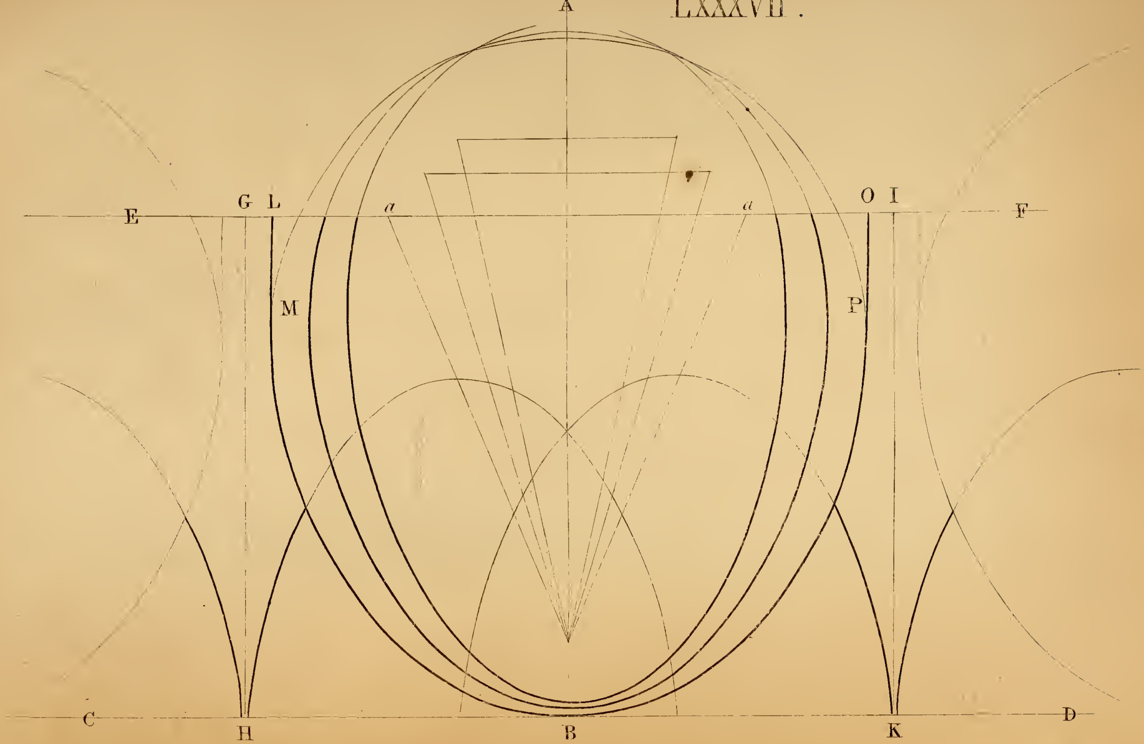
LXXXVI.



LXXXVII.



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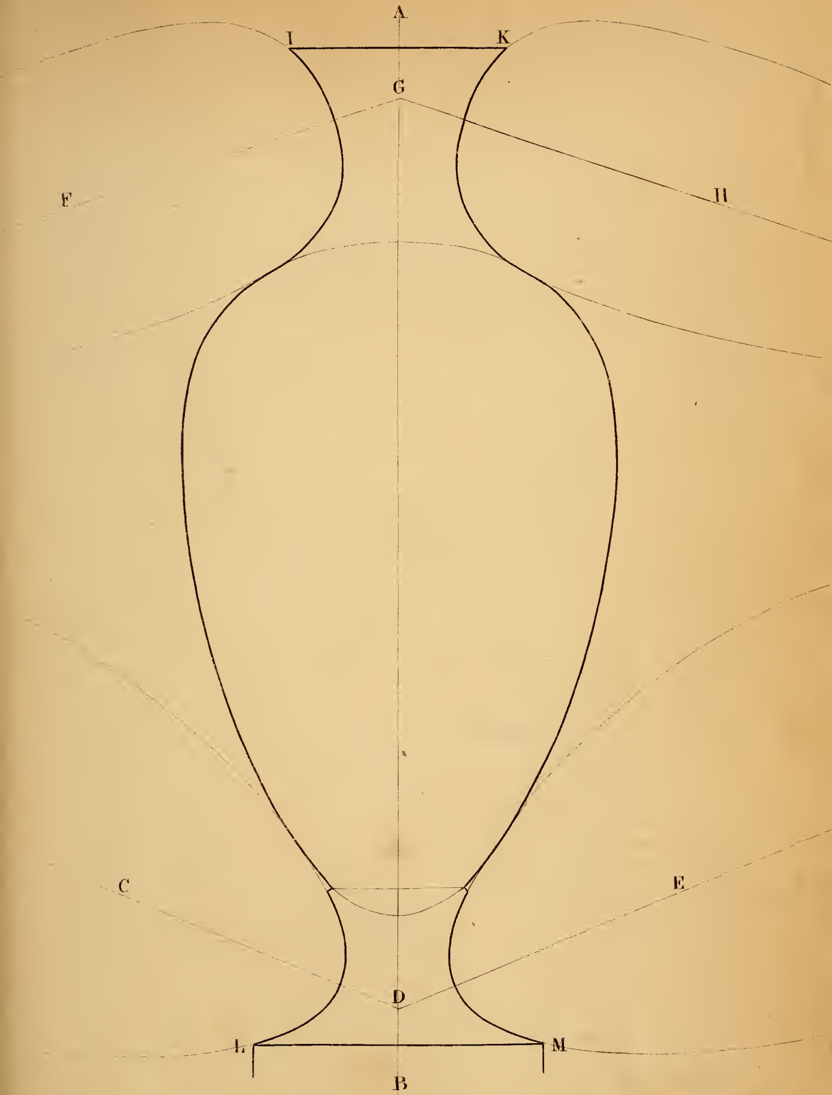
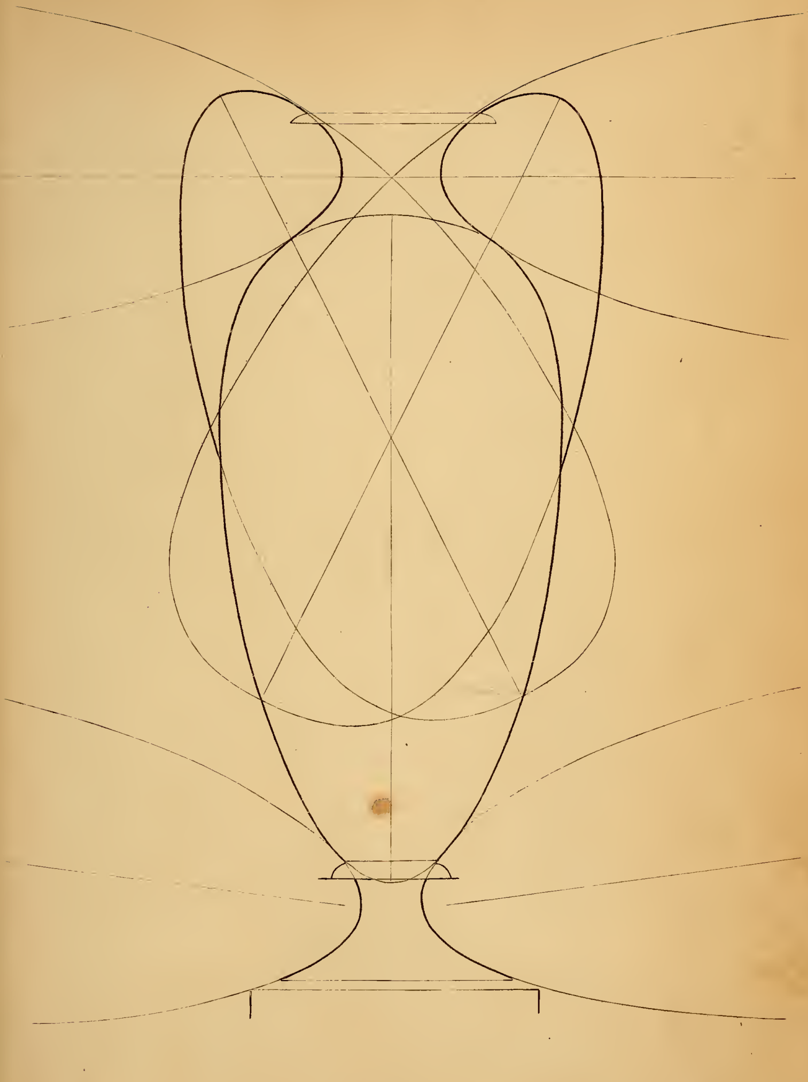
LXXXVIII.



G. Aikman

LXXXIX.



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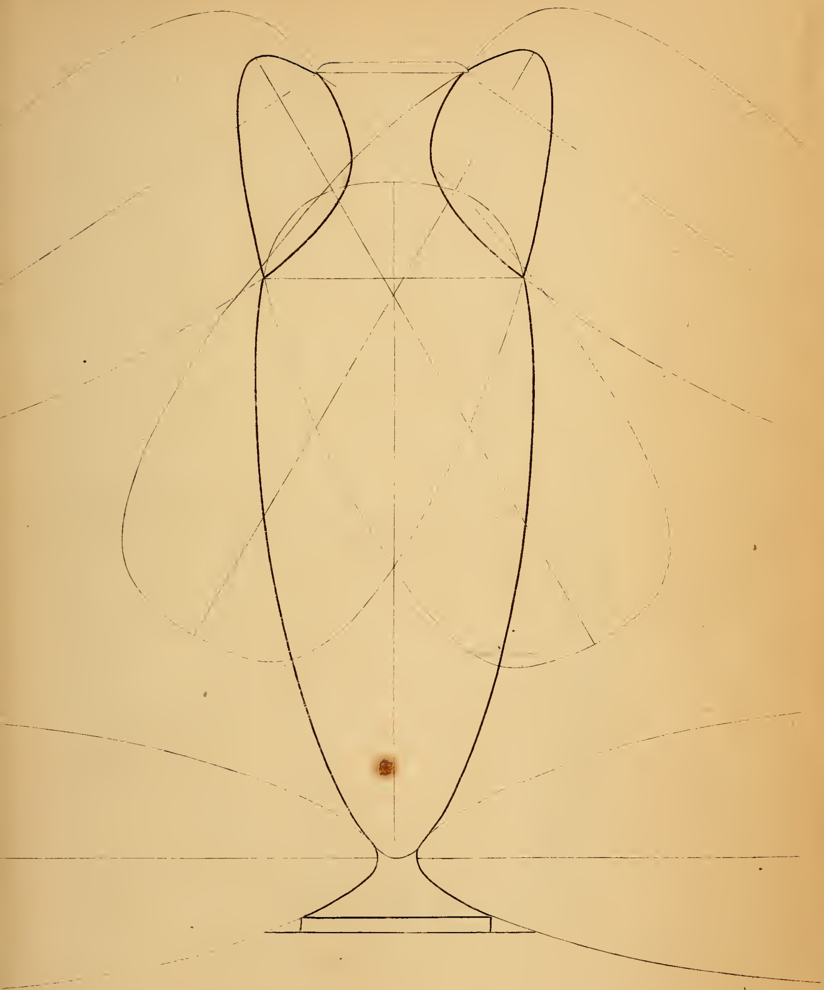
XC.



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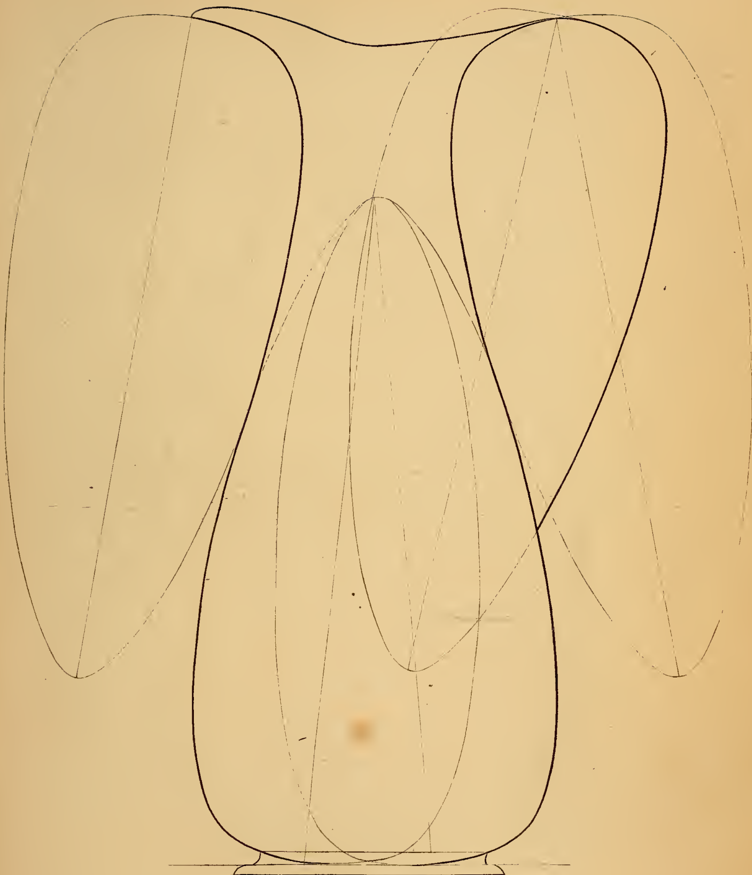

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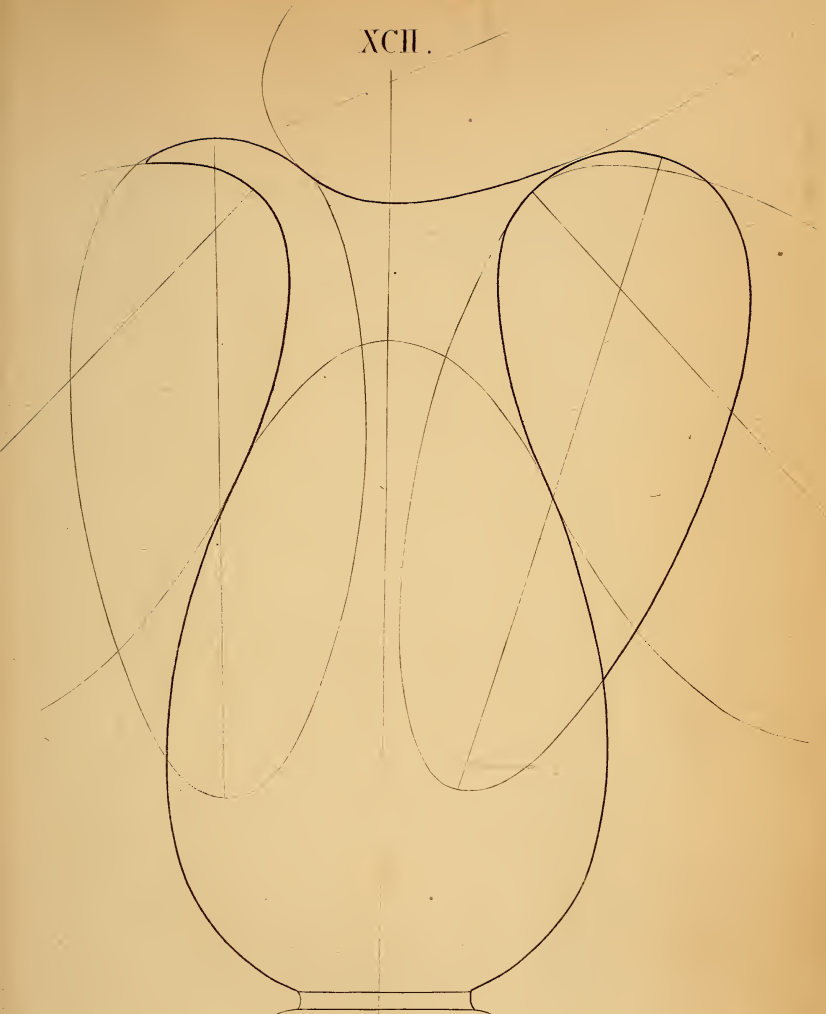
XCII.



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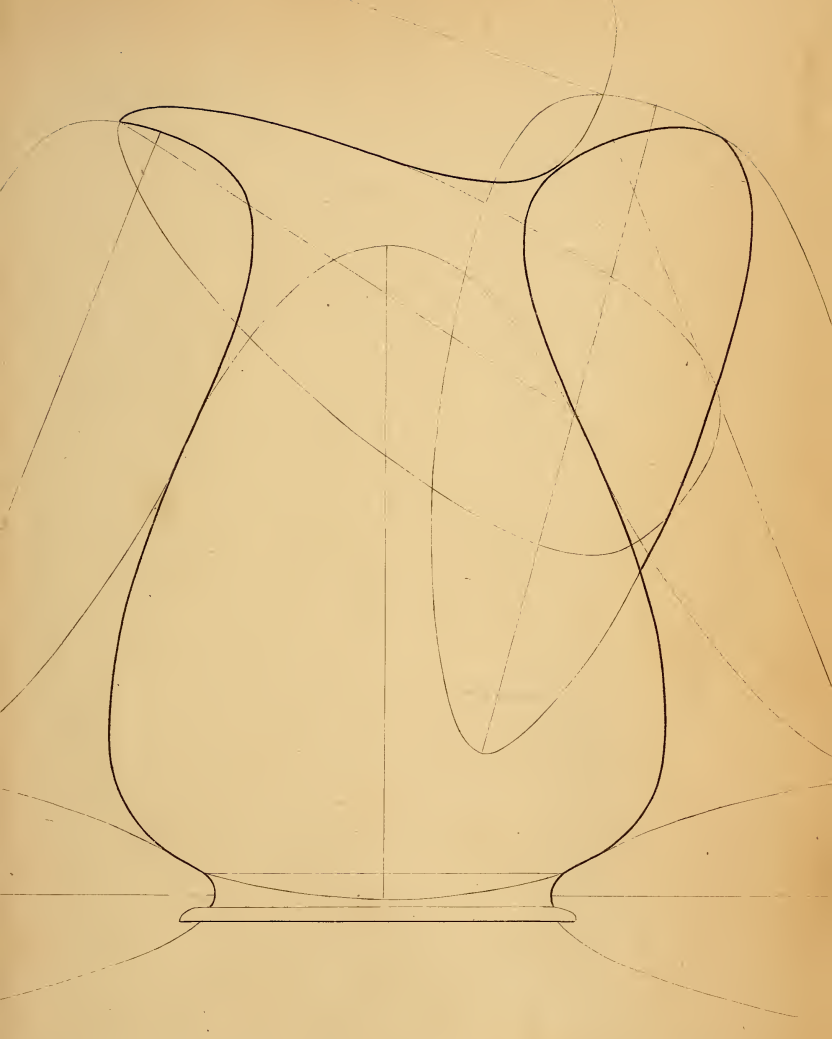

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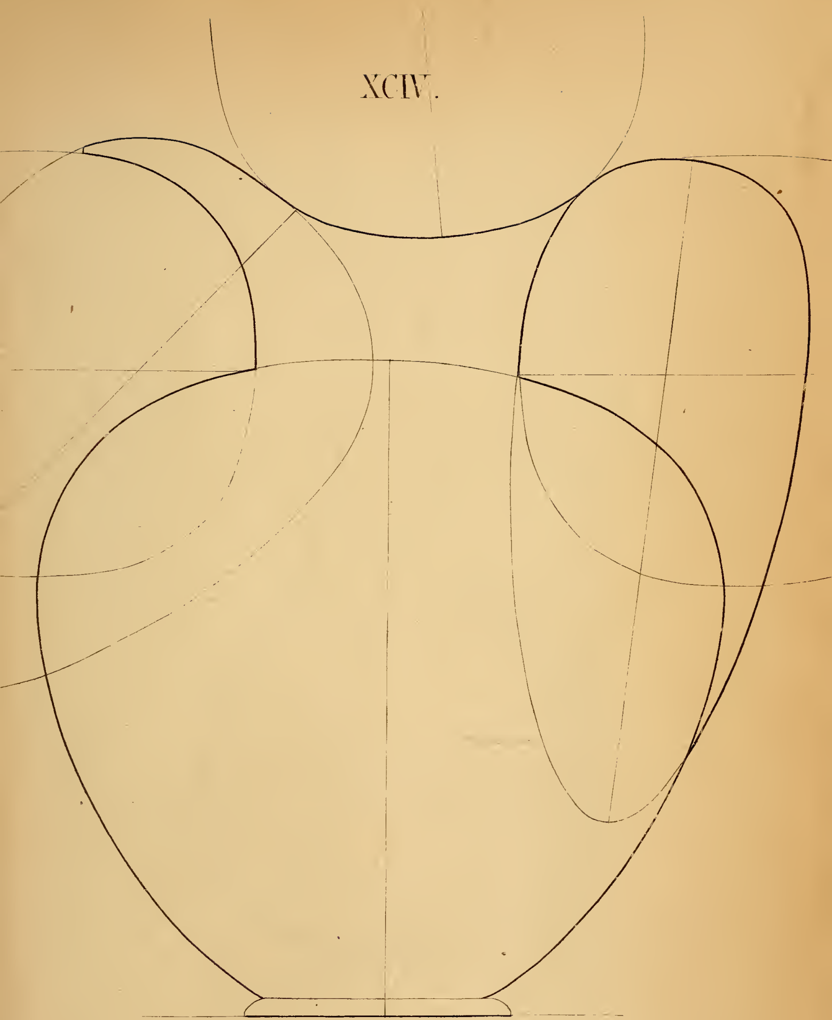
XCIV.



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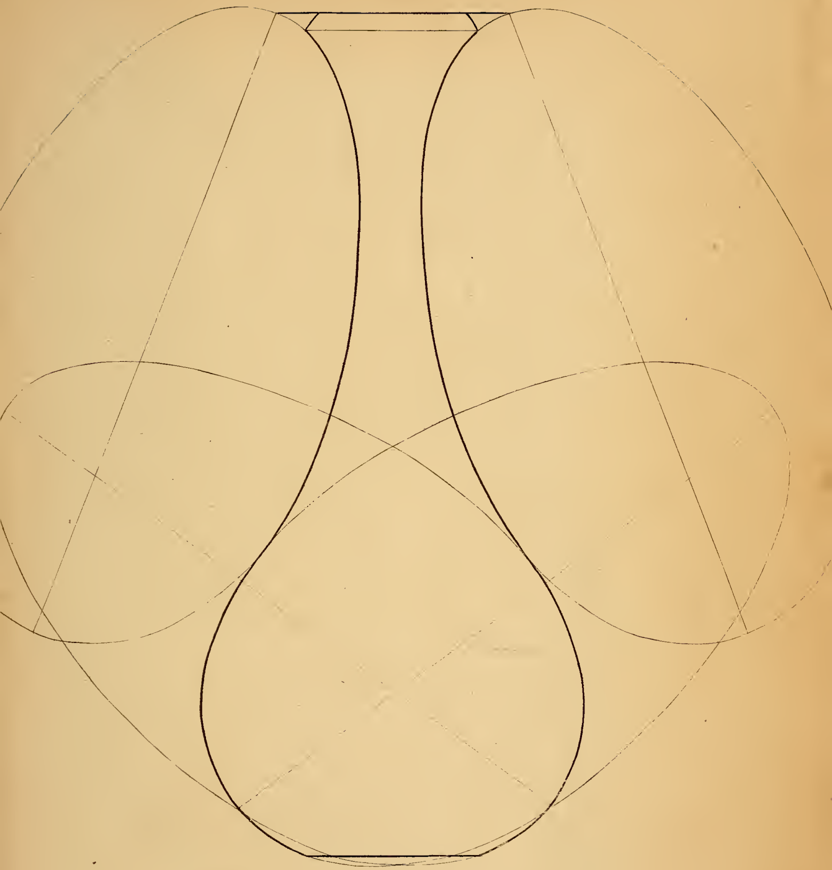
XCV.



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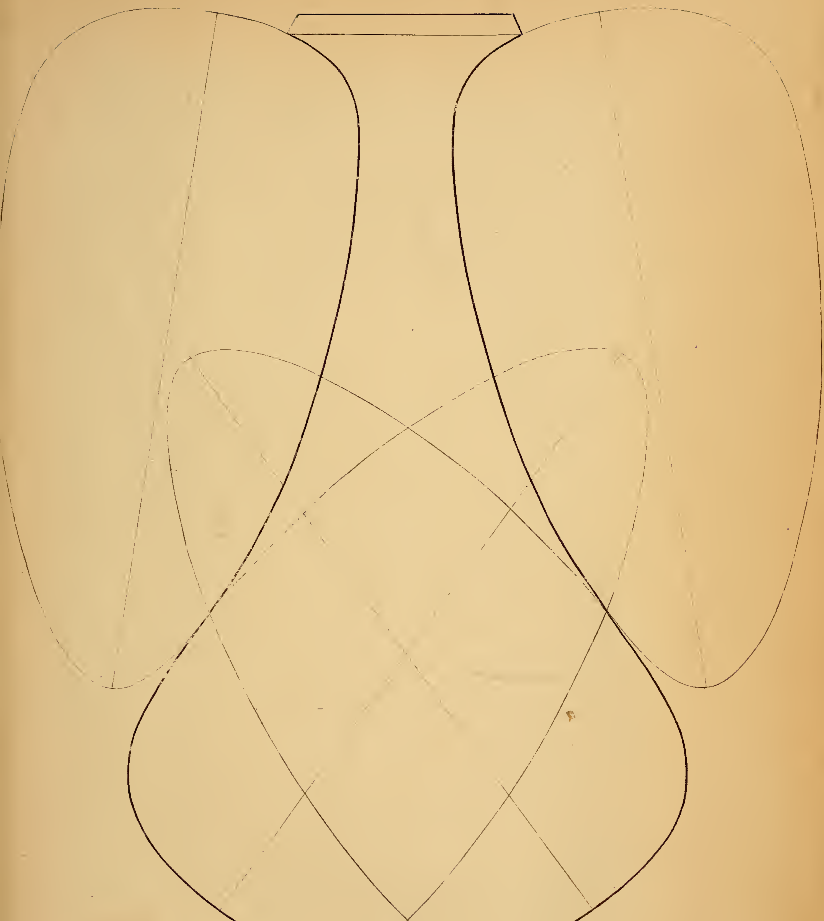
XCVI.



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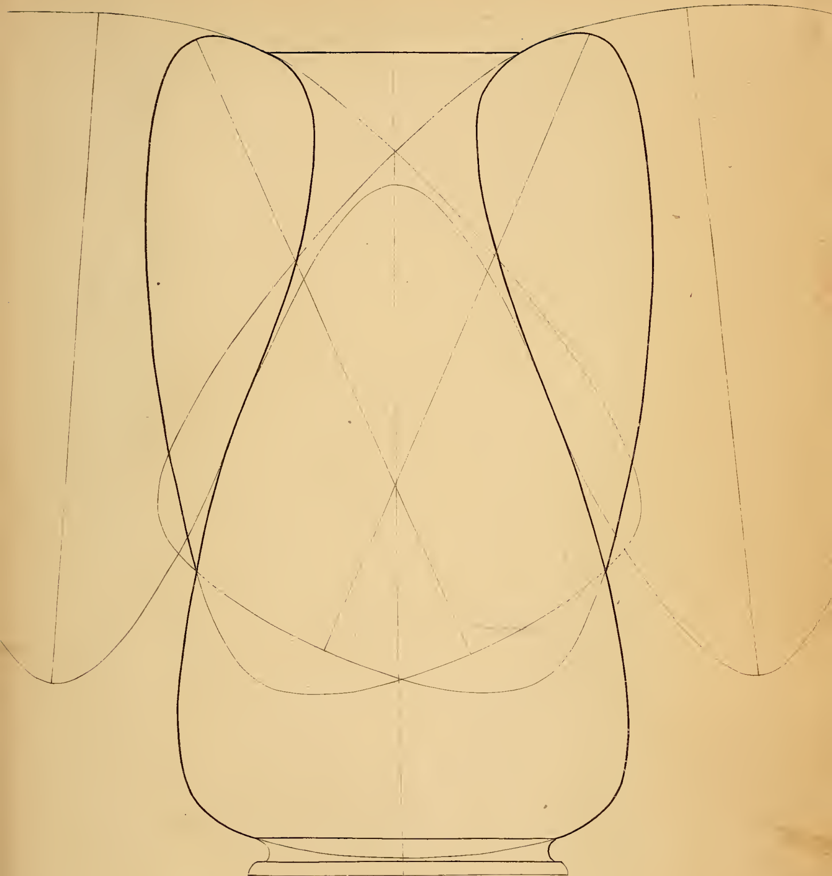
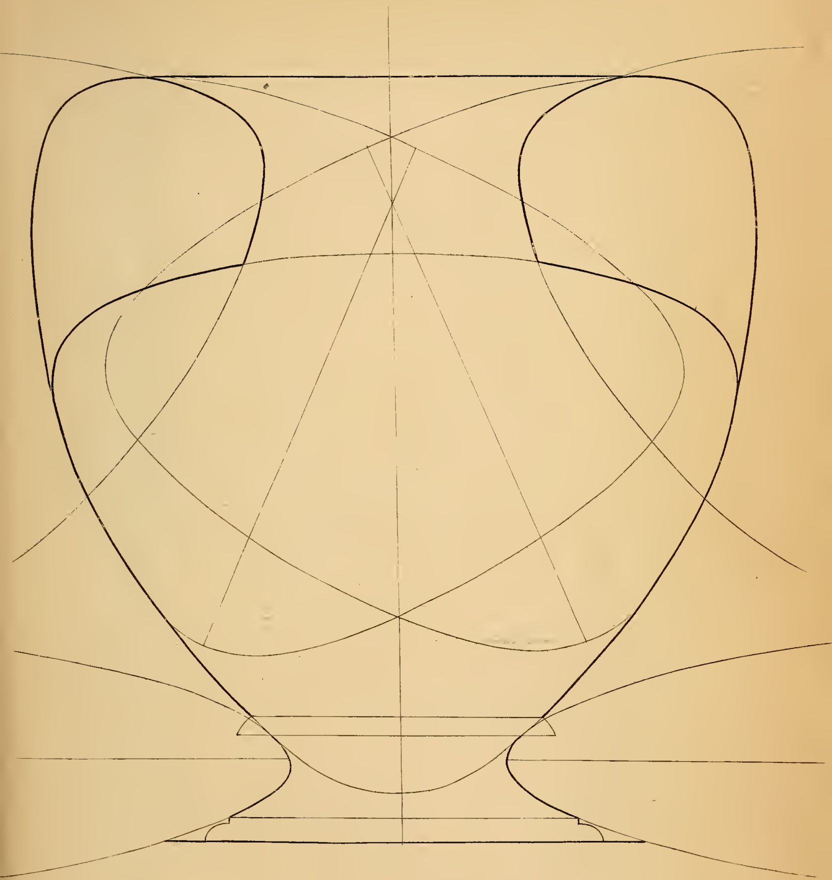
XCVII.



D. R. Hay

C. Aikman

XCVIII.



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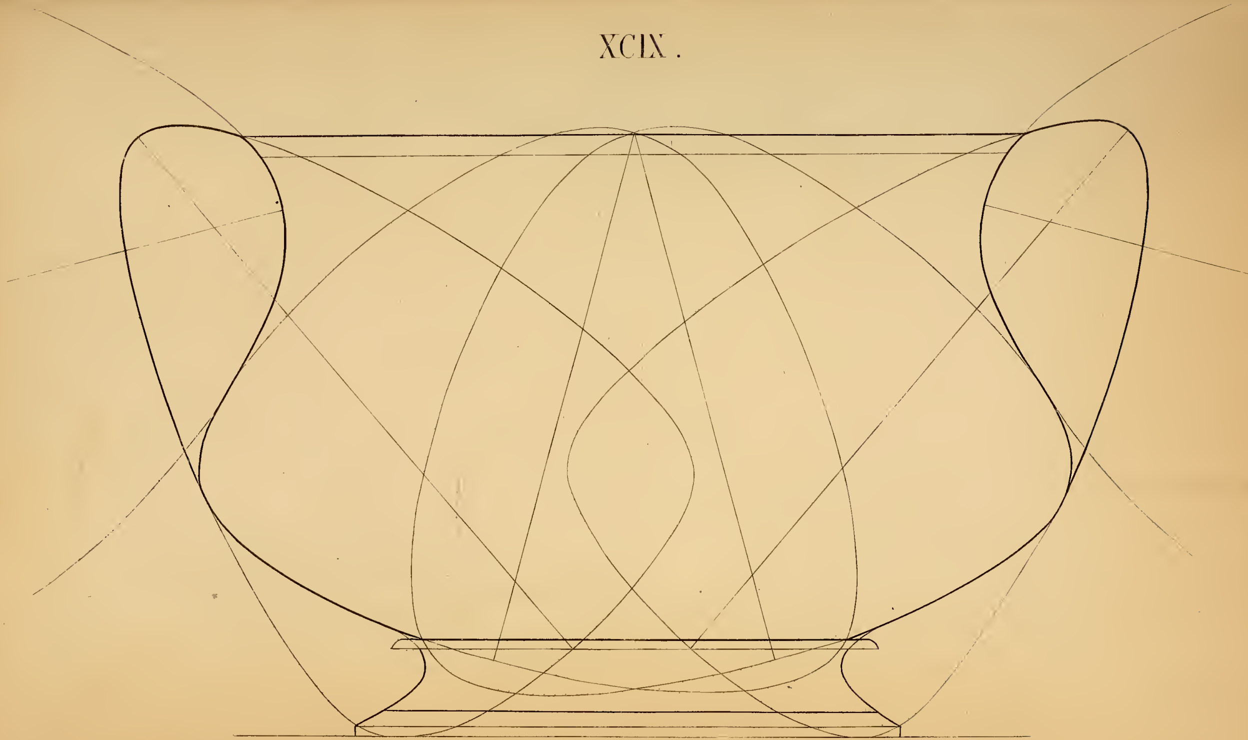
XCIX.



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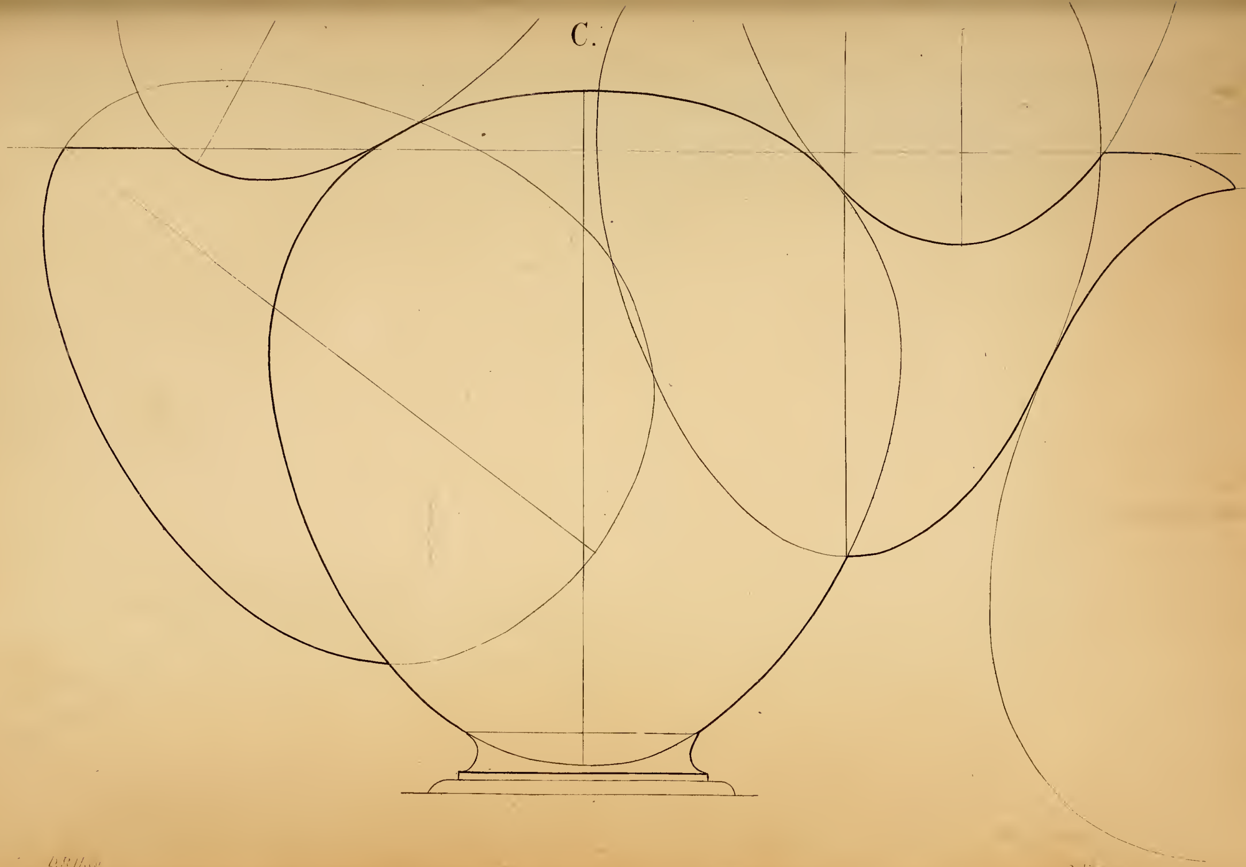
C.



NOTES AND APPENDIX.

NOTES.

NOTE A.

Almost all lecturers and writers upon architecture, refer to the work of Vitruvius; but the important observations with which it commences are seldom noticed, and still more rarely is any attempt made to explain and elucidate the principles of proportion to which they refer. Indeed Vitruvius himself seems to have had but a vague traditionary knowledge of them. He commences as follows:—

"The several parts which constitute a temple ought to be subject to the laws of symmetry; the principles of which should be familiar to all who profess the science of architecture. Symmetry results from proportion, which, in the Greek language is termed analogy. Proportion is the commensuration of the various constituent parts with the whole, in the existence of which symmetry is found to consist. For no building can possess the attributes of composition in which symmetry and proportion are disregarded; nor unless there exists that perfect conformation of parts which may be observed in a well-formed human being."

He then goes into various details of the relative proportions of the human body, and adds,—

"Since, therefore, the human frame appears to have been formed with such propriety that the several members are commensurate with the whole, the artists of antiquity* must be allowed to have followed the dictates of a judgment the most rational, when, transferring to the works of art, principles derived from

* Vitruvius wrote in the age of Augustus, and the artists he refers to were those of Greece, at the period of her highest refinement.

nature, every part was so regulated as to bear a just proportion to the whole. Now, although these principles were universally acted upon, yet they were more particularly attended to in the construction of temples and sacred edifices—the beauties or defects of which were destined to remain as a perpetual testimony of their skill or of their inability."

Now, what were those *laws of symmetry* to which Vitruvius refers—those *principles* that ought to be familiar to all who profess the science of architecture? He says, "they were, by the artists of antiquity, deduced from the proportions of the human body." But how were these principles deduced from the human body, and how applied to architecture? Vitruvius seems himself somewhat at a loss upon the subject, for, after endeavouring to show, that "the standards, according to which all admeasurements are wont to be made, are likewise deduced from the members of the body," he concludes, "If it be true, therefore, that the decennary notation was suggested by the members of man, and that the laws of proportion arose from the relative measures existing between certain parts of each member and the whole body, it will follow that those are entitled to our commendation, who, in building temples to their deities, proportioned the edifices, so that the several parts of them might be commensurate with the whole."

All the remainder of this ancient Treatise consists of architectural details, but there is no elucidation of a general principle of proportion, either as deduced from the human body, or founded upon mathematical laws, and consequently no mode for the application of such a principle in practice.

In a former work,* I have observed that this supposition of Vitruvius is very natural, inasmuch as the human figure is the most truly beautiful work of creation, and the Grecian temple the most scientific specimen of art. But a little investigation will show us that this quality was more likely to have been imparted to the works of the ancient Grecians, through the knowledge of a universal mathematical principle of harmony inherent in the

* An Essay on Ornamental Design. London: D. Bogue. Edinburgh: J. Menzies.

to 6. Upon the conjugate diameter, the eye, the width of the nose and the mouth are as 1 to 5. Every separate portion of the human figure is full of this species of harmony. The eye itself in its division into the parts by which its extraordinary functions are performed, displays it in an eminent degree.

The ratios in the human body are in the order of their simplicity, as follow :— Ratios.		These ratios in the pulsations of the atmosphere produced by similar divisions of the monochord, are called,
1 to 2	....	An octave.
1 ... 3	....	A twelfth.
1 ... 4	....	... fifteenth or second octave.
2 ... 3	....	... fifth.
1 ... 5	....	... seventeenth.
1 ... 6	....	... nineteenth.
3 ... 4	....	... fourth.
1 ... 7	....	
3 ... 5	....	... sixth.
1 ... 8	....	... twenty-second or third octave.
4 ... 5	....	... third.
5 ... 6	....	... third minor.
1 ... 12	....	... twenty-sixth.
6 ... 7	....	
8 ... 9	....	... major tone or second.
8 ... 15	....	... seventh.

Let us now see how this geometric principle of beauty operates in what may be justly considered the finest specimen of symmetrical beauty in the world, namely, the portico of the Parthenon.

The proportions of this portico have for many ages excited the admiration of mankind, and are still referred to as the most perfect example of this kind of beauty known in architecture. It is therefore a subject of some interest to inquire into the nature of those proportions, and especially to ascertain how far they are governed by the same principle of ratio just exemplified in the human figure. The two subjects are quite dissimilar in their general contour, there being no conceivable likeness between a Grecian portico and a human figure. But the beauty of their proportions is traceable to a similar principle differently applied. In the human figure it has been shown that the proportions consist in the division of an imaginary or mathematical straight line passing through the centre of the leading bones in the skeleton; and in the portico the operation of the same principle of harmonic ratio will be found in the inclination of the imaginary line called the diagonal, in each of those rectangles which, combined together, form what may be fairly termed its skeleton. But it is not in the various lengths of these diagonal lines that we are to look for the developement of the harmonic ratios, but to the angle they form with the base of each rectangle, and the following is the result.

The entire portico, from the extreme of the base of the outer columns to the upper point or apex of the pediment, is inscribed in a rectangle, the diagonal of which is 30°, bearing to the angle of 45°, the ratio of 2 to 3, and to the angle of 90°, that of 1 to 3.

The angle of the pediment itself is 15°, bearing to the diagonal of the inscribing rectangle the ratio of 1 to 2; to the angle of 45°, that of 1 to 3; and to the angle of 90°, that of 1 to 6.

The diagonal of the rectangle under the pediment inscribing the columns with their architrave and frieze is 22° 30′, bearing to the diagonal of the inscribing rectangle, the ratio of 3 to 4; to the angle 45° that of 1 to 2; and to the angle 90°, that of 1 to 4.

The diagonal of the rectangle inscribing the columns is 18°, bearing to the diagonal of the inscribing rectangle, the ratio of 3 to 5; to the angle of 45°, that of 2 to 5; and to the angle of 90°, that of 1 to 5.

The diagonal of the rectangle inscribing the architrave and frieze is 5° 37′ 30″, bearing to the diagonal of the inscribing rectangle, the ratio of 3 to 16; to the angle of 45°, that of 1 to 8; and to the angle of 90°, that of 1 to 16.

The rectangles of the six centre columns, which I have taken at their mean diameter, have each a diagonal of 80°, bearing to the angle of 90°, the ratio of 8 to 9; and the five intercolumniations between these have each a diagonal of 75°, bearing to those of the columns, the ratio of 15 to 16, and to the angle of 90°, that of 5 to 6.

The rectangles of the two outer columns and their intercolumniations have diagonals of 78° 45′, being to the right angle in the ratio of 7 to 8.

The two outer columns, with their intercolumniations, are necessarily out of harmony with those that lie between them, for the ratio of 78° 45′ is to 80° as 23 to 24, and that of 78° 45′ to 75° as 63 to 62, ratios too far removed from the primary elements to have a harmonious relation. But the circumstance of these outer columns being of greater diameter than the other six, is well known to have arisen from a knowledge of the fact, that any upright object placed between us and the sky will appear more slender than when placed against a background in shade. As these outer columns of the portico were so situated, while the other six could only be viewed against the inner portion of the building, it became requisite to increase their diameter, the discord being neutralized by this optical illusion.

From the same cause that a solid body appears more slender than it really is when viewed against the light, an open space seen between two solid bodies appears wider; and this assists in harmonizing the two outer intercolumniations.

The harmonic ratios of the Parthenon are in the order of their simplicity as follow:—	Names of those ratios when applied to the vibrations produced by the division of the monochord.
Ratios of	
1 to 2	An octave.
1 ... 3	A twelfth.
1 ... 4	... fifteenth or second octave.
2 ... 3	... fifth.
1 ... 5	... seventeenth.
1 ... 6	... nineteenth.
2 ... 5	... tenth.

The harmonic ratios of the Parthenon are in the order of their simplicity as follow :—	Names of those ratios when applied to the vibrations produced by the division of the monochord.
Ratios of	
3 ... 4	A fourth.
3 ... 5	... sixth.
1 ... 8	... twenty-second or third octave.
5 ... 6	... minor third.
1 ... 16	... twenty-ninth or fourth octave.
8 ... 9	... major second or tone.
3 ... 16	An eighteenth.
15 ... 16	A semitone or minor second.

The ratios in the portico of the Parthenon are agreeable to the dimensions of the elevation in Stewart's Athens, as given in Plate VI. of that work. But as the angle of the pediment in Plates VI., VII., and XV., all differ, I adopted that of the latter, as being the most likely to be correct, because the pediment is there given by itself.

NOTE B.

Dr. Smith in his Harmonics, section 4, article 10, gives the following reasons for confining the harmonic ratios to the operation of those three numbers :—

"If it be asked why no more primes than 1, 2, 3, 5, are admitted into musical ratios ; one reason is, that consonances whose vibrations are in ratios, whose terms involve 7, 11, 13, &c., *ceteris paribus* would be less simple and harmonious than those whose ratios involve the lesser primes only.

"Another reason is this—as perfect fifths, and other intervals resulting from the number 3, make the schism of a coma with the perfect thirds and other intervals resulting from the number 5, so such intervals as result from 7, 11, 13, &c., would make other schisms with both those kinds of intervals."

NOTE C.

To express the relation which subsists between the diagonals AB, AD, AF, &c., in this series of figures, put AC $= a$ when we obtain—

$AB^2 = AC^2 + CB^2 = 2\,a^2$ therefore $AB = a\sqrt{2}$

$AD^2 = AE^2 + ED^2 = AB^2 + ED^2 = 3\,a^2$ ... $AD = a\sqrt{3}$

$AF^2 = AG^2 + GF^2 = AD^2 + GF^2 = 4\,a^2$ ... $AF = a\sqrt{4}$

&c.

It thus appears that if n such triangles be constructed, the hypotenuse of the nth triangle will be $a\sqrt{n+1}$, and that the hypotenuse will be commensurable with the original base AC or a, whenever $n + 1$ is a square number; as, for example, in the 3d, 8th, 15th, 24th triangles.

Putting $a = 1$ we have—

$$AB = \sqrt{2},\ AD = \sqrt{3},\ AF = \sqrt{4} \text{ or } 2,\ AH = \sqrt{5},$$
$$AK = \sqrt{6},\ AM = \sqrt{7}.$$

The angles BAC, DAE, &c. are easily obtained from the values of the diagonals which have now been formed.

For we have BAC $= 45°$.

Tan. DAE $= \frac{1}{\sqrt{2}}$ from which DAE $= 35°\ 15'\ 54''$.

Tan. FAG $= \frac{1}{\sqrt{3}}$ ... FAG $= 30°$ (exactly.)

Tan. HAI $= \frac{1}{\sqrt{4}}$ ... HAI $= 26°\ 33'\ 54''$.

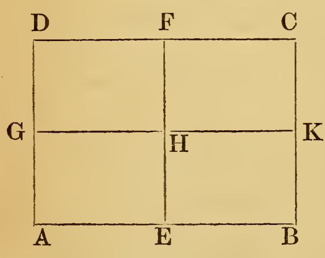
Tan. KAL $= \frac{1}{\sqrt{5}}$ ... KAL $= 24°\ 5'\ 41''$.

Tan. MAN $= \frac{1}{\sqrt{6}}$ ... MAN $= 22°\ 12'\ 28''$.

NOTE D.

"The remarkable property of the divisibility of the rectangles into an infinite number of figures similar to the whole figure is thus shewn.

"The $(n-1)$th rectangle has its sides respectively equal to 1 and $\sqrt{n}$. If the latter side be divided into n equal parts and lines be drawn parallel to the sides of the figure, the rectangle will be divided into n rectangles whose sides are 1 and $\frac{\sqrt{n}}{n} = \frac{1}{\sqrt{n}}$. The ratio of the sides of the whole figure is $\frac{1}{\sqrt{n}}$, and that of the sides of the new rectangles, is $\frac{\frac{1}{\sqrt{n}}}{1} = \frac{1}{\sqrt{n}}$; it is evident therefore that each of these figures is similar to the whole. Now the rectangles thus obtained being similar to the whole, also admit of being divided into n figures similar to themselves; and each of these again into n figures, and the sub-division can never terminate. To illustrate this take ABCD, a parallelogram similar to what two triangles DAE would produce by their union.



If then $AD = 1$, $AB = \sqrt{2}$. Bisect AB in E, and draw EF parallel to AD or BC, then AE and EB are each $\frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$.

Thus $AB : BC :: \sqrt{2} : 1$, and $AD : AE :: 1 : \frac{1}{\sqrt{2}}$; but $1 : \frac{1}{\sqrt{2}} :: \sqrt{2} : 1 \therefore AB : BC ::$

AD : AE, and the figure DE is similar to DB; and, in like manner, BF is similar to DB. Again, since DE is similar to DB, if AD be bisected in G, and GK be drawn parallel to DC, it may be shewn that DH and GE are similar to DE or to DB; and these again being divisible in like manner, it is obvious that the subdivision may be continued *ad infinitum*."

The above, as well as Note C, was supplied by a friend professionally connected with the science of Mathematics.

NOTE E.

Having found that the analogy between the harmony of sound and the harmony of form and colour, which I have pointed out in some of my former works, has been made a stumbling-block to many, inasmuch as they have believed it requisite to the understanding of the harmony of form, that they must first become musicians, therefore, to prevent the like mistake being committed in regard to the present Treatise, I have carefully avoided all reference to this analogy in the text. I have, however, referred to it in these notes, and may here state, that the ratios evolved from the scale of figures and relative scale of degrees of the circle, are precisely those which form the diatonic scale of the musician, and are generally named as follows:—

Ratios.	Names.	Musical Notes.
1 to 2 ...	An eighth or octave,	as from C to c.
2 ... 3 ...	A perfect fifth,	... C ... G.
3 ... 4 ...	... perfect fourth,	... C ... F.
3 ... 5 ...	... major sixth,	... C ... A.
4 .. 5 ...	... major third,	... C ... E.
5 ... 6 ...	... minor third,	... E ... G.
5 ... 8 ...	... minor sixth,	... E ... c.
8 ... 9 ...	... major tone or second,	... C ... D.
9 ... 10 ...	... minor tone or second,	... D ... E.
8 ... 15 ...	... major seventh,	... C ... B.
15 ... 16 ...	... minor seventh or hemitone,	... B ... c.

On referring to Plate V., it will be seen in how small a degree the tempered scalene triangles differ from those which arise naturally from the process of making the hypotenuse of the first the base of the second, the hypotenuse of the second the base of the third, &c., as explained in the text. That there should be a difference is quite in accordance with the musical scale of harmony, which, as presented by nature, requires modification and temperament to adapt it to an extended application of the principles which it evolves. This is clearly pointed out by Dr. Smith in his "Harmonics," Section V. Propositions II. and III.; as also in the article upon "Sound" in the Encyclopedia Metropolitana. P. 798.

NOTE F.

My attention was first directed to the oval or egg form, by the following observations of Mr. J. D. Harding, in his excellent work, "The Principles and Practice of Art." In Chapter IV., p. 43, he says:—

"In the circle we see and feel without preparatory education, and without difficulty, its sameness, and therefore, its want of beauty; but in the egg it requires reflection to see or to feel any great amount of that variety which it possesses; and as, also, by our natural powers, we see, feel, and understand, that all circles must be alike in their properties, so it is only by the acquired powers of a well-practised eye, and feelings rendered sensitive through experience, and a well-informed judgment, that we can perceive by how much one egg differs from another, or which, among many, is the *most* beautiful—is the nearest to perfect beauty in its outline, in consequence of that infinite variety which is the essential constituent of perfect beauty of form."

Mr. Harding then proceeds to show, that the curve of the egg form prevails in the general outline of the human figure, and constitutes the beauty of the human countenance in its general contour as well as in its individual parts. This he exemplifies in Plate II.

by three beautifully drawn female heads, and by the features in detail, contrasting the effects of a circular, an elliptic, and an ovoid arrangement, and adds,—"As we have seen what constitutes the beauty of the egg, so we may now see what constitutes the beauty of the eye; and as in the former, the most beautiful of a number is that which has the greatest variety, so it is with the latter, and with every curve of the human form. It remains now only to give the additional illustration by the drawing of an eye, where every line is the segment of a circle, devoid of the variety which has been proved to be essential to beauty, and inseparable from it; and though nature presents approximations to such forms, the most superficial observer must at once see and feel the striking difference between such forms, and forms of the *greatest* beauty." He next points out the prevalence of this species of curve in the forms of flowers, and in the best specimens of Grecian art, which he amply exemplifies in Plates III. and IV. of his unequalled work, and adds the following observations, the excellence of which will plead my apology for adding them to the extracts already given:—"Decision on beautiful forms, of whatever kind, does not, and ought not, to depend on vague capricious taste, and uncultivated feelings; for unless they be controlled by sound judgment, formed on the observation of the truth of nature, we are not in a condition clearly to distinguish the beautiful, and consequently can have no power either to judge of, or to depict it. Unless from such education no two persons could have the same opinion of the beautiful, and should even the opinion of one of them happen to be right, he would be unable to give a sufficient reason for it; but if the elements of beauty, founded in truth, be understood, beauty may then be demonstrated and distinguished from every shade of deformity which is so often mistaken for it, or set up by fashion in its stead."

Deeply impressed with the truth of Mr. Harding's observations regarding the prevalence of the ovoid curve in all truly beautiful combinations in organic forms, and knowing that no process had as yet been invented for tracing it accurately, I set about the invention of a machine for the purpose. The method described in the text suggested itself in the course of my attempts, but was thrown aside when the machine was completed, for my sole object at the time was, to produce correctly the egg form, or oval, and the machine did produce it in all its varieties. On entering on the present work, however, I found the method just alluded to better adapted to its simple object, and have therefore adopted it in preference to any other process.

For a description of the machine to which I have alluded, see the Edinburgh New Philosophical Journal, No. 81; or the Transactions of the Royal Scottish Society of Arts, for 1846, before a meeting of which Society it was exhibited.

I deem it an act of justice here to state, that since the first sheets of this Treatise were printed off, I have ascertained as a fact, that the author of the valuable papers from which I have made the extracts given in the Introduction, and in which the theory, resolving the beauty of form into the three elements—unity, symmetry, and continuity, is Mr. J. Scott Russell, Secretary to the Society of Arts, London.

APPENDIX.

By the process detailed in Part I. and demonstrated in Notes C and D, it was shown that a series of rectangles was produced, having the peculiar quality of being divisible and subdivisible into 2, 3, 5, &c. other rectangles of the same proportions as themselves. Now, although these rectangles may therefore be individually considered more beautiful than any other figures of the same kind, whose constituent parts do not possess this species of harmony, yet in order to form them into a series, the constituent parts of which would be perfectly harmonious amongst themselves, a slight modification in the proportions of some of them becomes requisite. This species of modification when applied to the musical scale of harmony, is called temperament, and consists in the adjustment of the imperfect concords in keyed instruments, so as to transfer to them part of the beauty of the perfect concords. It has also been shown that the first six rectangles arising from this process, when so modified, relate to each other in the numbers of degrees, which express the inclination of their diagonal lines, in the same ratios of numerical quantity, as the sounds which constitute an octave in music do in the numbers of their vibrations. But the series of angles which arise from the combination of the square with the circle, and of the oblong with the ellipse, as described in Part II., points out a more extended kind of harmony passing from one octave to another like the harmonics which arise from the vibrations of the monochord. It having occurred to me after Part II. was printed, that the results of both processes must, like these harmonics, coincide at certain points, and having found my angles by the use of the protractor only, I could not depend implicitly upon their accuracy, I therefore sent this part to the same learned friend who supplied Notes C and D to Part I., in order that he might also subject the second or curvilinear process to the test of mathematical calculation, and he has favoured me with the following tabular view of the comparative angles obtained by both processes, by which it will be seen that they coincide perfectly at certain definite points. Calling an angle of 45°, term 1, as in the first process, the first coincidence takes place at term 3, a multiple of 1 by 3 ; the second at term 9, a multiple of 3 by 3 ; the third at term 27, a multiple of 9 by 3 ; the fourth at term 81, a multiple of 27 by 3; the fifth at term 243, a multiple of 81 by 3; and the sixth at term 729, a multiple of 243 by 3, and so they would continue to coincide at every term which was a multiple by 3 of the number of the term at which the last coincidence took place.

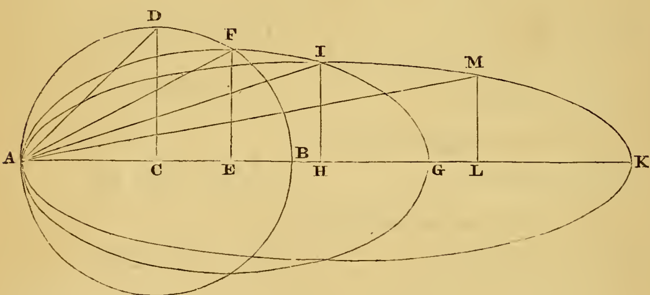
Tan.	First or Rectilinear Process.	Tan.	Second or Curvilinear Process.
		$\sqrt{3}$	60° 0′ 0″ (exact.)
1	45° 0′ 0″ (exact.)	1	45° 0′ 0″ (exact.)
$\frac{1}{\sqrt{2}}$	35° 15′ 53″		
$\frac{1}{\sqrt{3}}$	30° 0′ 0″ (exact.)	$\frac{1}{\sqrt{3}}$	30° 0′ 0″ (exact.)
$\frac{1}{\sqrt{4}} = \frac{1}{2}$	26° 33′ 54″		
$\frac{1}{\sqrt{5}}$	24° 5′ 41″		
$\frac{1}{\sqrt{6}}$	22° 12′ 28″		
$\frac{1}{\sqrt{7}}$	20° 42′ 15″		
$\frac{1}{\sqrt{8}}$	19° 28′ 15″		
$\frac{1}{\sqrt{9}} = \frac{1}{3}$	18° 26′ 6″	$\frac{1}{3}$	18° 26′ 6″
$\frac{1}{\sqrt{10}}$	17° 32′ 54″		
$\frac{1}{\sqrt{11}}$	16° 46′ 45″		
$\frac{1}{\sqrt{12}}$	16° 6′ 8″		
$\frac{1}{\sqrt{13}}$	15° 30′ 5″		

Tan.	First or Rectilinear Process.	Tan.	Second or Curvilinear Process.
$\frac{1}{\sqrt{14}}$	14° 57′ 48″		
$\frac{1}{\sqrt{15}}$	14° 28′ 38″		
$\frac{1}{\sqrt{16}} = \frac{1}{4}$	14° 2′ 10″		
	&c. 17 angles intervene, including the 7 preceding.		
$\frac{1}{\sqrt{27}}$	10° 53′ 40″	$\frac{1}{3\sqrt{3}}$	10° 53′ 40″
	53 angles intervene.		
$\frac{1}{\sqrt{81}}$	6° 20′ 24″	$\frac{1}{9}$	6° 20′ 24″
	161 angles intervene.		
$\frac{1}{\sqrt{243}}$	3° 40′ 12″	$\frac{1}{9\sqrt{3}}$	3° 40′ 12″
	485 angles intervene.		
$\frac{1}{\sqrt{729}}$	2° 7′ 16″	$\frac{1}{27}$	2° 7′ 16″

The column marked "tan." is the value of the trigonometrical tangent from which the angle given in the succeeding column has been found.

In the above table, blanks are left where in the one system there are no angles corresponding to those of the other. It will be seen that the first system is more comprehensive than the second, including every angle of the second system, with a number of intermediate angles besides. The two systems, supposing them both to commence together at 45°, will give the same angles at their 3d, 9th, 27th, 81st, 243d, 729th, &c. terms.

The successive formation of angles by the curvilinear process, of which the values have been given in the above table, has already been fully described in the text. But the exhibition of the first members of the series in a single diagram, may render the relation which they hold to each other more easily deducible; and for this purpose also we shall suppose the curvilinear process to commence at 45°, the first angle which is common to both the rectilinear and curvilinear systems. Let A D B in the following diagram be a circle, of which A B is the diameter, and C the



centre. Draw C D at right angles to A B, bisect C B in E, make E G = A E, draw the perpendicular E F, and describe the ellipse A F G, having A G for its major axis, and passing through F. Next bisect E G in H, make H K = A H, draw the perpendicular H I, and describe the ellipse A I K. By bisecting H K in L, drawing the perpendicular L M, and producing L K until it is equal to A L, three points will in like manner be obtained, through which the next ellipse should pass, and the process may be continued indefinitely. It will easily be seen by comparing the figure now described with the curvilinear process given in the text, that the series of ellipses now obtained is the same with that of the curvilinear process; and that if A D, A F, A I, A M, be joined, the angles D A C, F A E, I A H, M A L, are the first four angles of the same process, supposing it to commence with an angle of 45°. We shall now deduce the law of relation between these angles.

1. Since A C = C D, we have tan. D A C = 1.

2. Referring the circle to A as its origin, making A B the axis of x, and the radius A C = 1; we have by the equation to the circle $y^2 = 2x - x^2$. But at the point E, $x = \frac{3}{2}$, since A C = 1. Therefore E F^2 or $y^2 = \frac{3}{4}$, and E F $= \frac{\sqrt{3}}{2}$.

Hence tan. F A E $= \dfrac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \dfrac{1}{\sqrt{3}}$.

3. Referring the ellipse A F G to the same co-ordinates, we have for its semi-major and semi-transverse axes the lines E A, E F, or $\frac{3}{2}$ and $\frac{\sqrt{3}}{2}$: and the equation to the curve $y^2 = \frac{b^2}{a^2} \times (2ax - x^2)$ where a, b, are the semi-major and semi-transverse axes, will, for this particular ellipse, become $y^2 = x - \frac{x^2}{3}$. Now for the point H, $x = \frac{3}{2}a = \frac{9}{4}$, and therefore I H^2 or $y^2 = \frac{9}{16}$, and I H $= \frac{3}{4}$.

Hence tan. I A H $= \dfrac{\frac{3}{4}}{\frac{9}{4}} = \dfrac{1}{3}.$

4. In like manner, since the semi-major and semi-transverse axes of the ellipse A I K are, A H $= \frac{9}{4}$ and I K $= \frac{3}{4}$, its equation is $y^2 = \frac{x}{2} - \frac{x^2}{9}$; and at the point L, where $x = \frac{3}{2}\ a$ $= \frac{27}{8}$, M L or $y = \frac{3\sqrt{3}}{8}$.

Hence tan. M A L $= \dfrac{\frac{3\sqrt{3}}{8}}{\frac{27}{8}} = \dfrac{1}{3\sqrt{3}}.$

The tangents of the angles D A C, F A E, I A H, &c. have thus been found to be

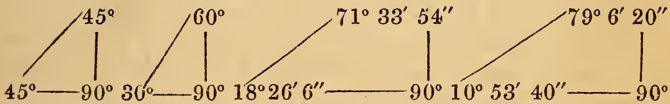
$$1 \qquad \frac{1}{\sqrt{3}} \qquad \frac{1}{3} \qquad \frac{1}{3\sqrt{3}}$$

as given in the table. The law of this series is obvious, its general term being $\frac{1}{\sqrt{3^n}}$; and the repetition of the same construction will always give rise to angles whose tangents belong to the same series. For it will be observed that the tangent of any angle, as F A E expresses the ratio of the semi-transverse to the semi-major axis of one of the ellipses. If then we arrive at an angle whose tangent is $\frac{1}{\sqrt{3^n}}$ we have in the corresponding ellipse, which suppose to be A I K, A H $= \sqrt{3^n}$ and H I $= 1$, and its equation will be $y^2 = \frac{1}{3^n}\left(2\sqrt{3^n}\,x - x^2.\right)$ Then at the point L, $x = \frac{3}{2}\,a = \frac{3\sqrt{3^n}}{2}$, and $y = \frac{\sqrt{3}}{2}$.

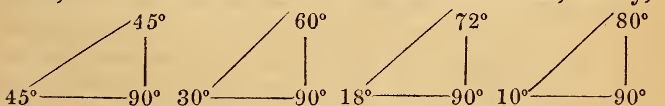
$$\text{Hence tan. M A L} = \frac{\frac{\sqrt{3}}{2}}{\frac{3\sqrt{3^n}}{2}} = \frac{1}{\sqrt{3^{n+1}}},$$

which shews that if the law which has been found for the first four angles includes any number of angles, it extends also to the next greater number, and will therefore be true, however far the series is extended.

The scalene triangles arising from the second or curvilinear process, are consequently not as I have given them in the text, but are as follow :—



or nearly so, the two latter being incommensurable. But as unity of design consists in harmony of parts, and as the proportion which constitutes harmony consists in the commensurability of the various constituent parts with the whole, these latter triangles require the following modification to render them harmonious, and the series will then be as stated in the text, namely,



The acute angles having the relative commensurability to the right angles of 1 to 2, 1 to 3, 1 to 5, 1 to 9, 2 to 3, 4 to 5, and 8 to 9.

These ratios are, in the science of music, the fundamental elements of all harmony. The first is that of an octave to the tonic or key-note; the second that of a twelfth or compound fifth; the third that of a seventeenth or compound third; the fourth that of a twenty-third or compound second; the fifth that of the dominant or perfect fifth; the sixth that of the mediant or major third; and the seventh that of the supertonic or major second. The elements of musical harmony would, however, be complete without the fourth and seventh ratios, but are here given as they occur in the third ellipse. These ratios might be otherwise modified or tempered; for instance, if instead of diminishing 10° 53′ 40″ to 10°, it were augmented to 11-15 by the addition of 21′ 20″, it would give the ratio of 1 to 8, a twenty-second or third octave to the tonic, and 7 to 8 a minor tone. But these are matters for the investigation of the student in analogical philosophy, to whom I would recommend Field's excellent work on that subject.*

The want of some geometric rule of beauty has been felt in the highest quarters; in proof of which I may state the following facts:—When I published in 1842, "The Natural Principles and Analogy of the Harmony of Form," I requested a particular friend in London, a member of the Royal Academy of Arts, to present, in my name, copies of that work to such Institutions in London as he thought might take an interest in the subject of which it treated. Amongst others he presented one to the Royal Institute of British Architects, through the medium of one of its most eminent members and office-bearers, who acknowledged it in the following terms:—" *24th December* 1842.—I have to acknowledge receipt of your letter and book by Mr. D. R. Hay, which I shall have very much pleasure in presenting

* Outlines of Analogical Philosophy by George Field. London: Tilt, 1839.