MATHEMATICS

FOR

TECHNICAL SCHOOLS

WARREN AND RUTHERFORD

TORONTO
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PREFACE

In this book an attempt has been made to present the subject of Elementary Mathematics in a way suitable to industrial students in our technical schools. While it would be manifestly impossible to deal with the mathematics of all the industries in a book of this nature, yet we hope that the fundamentals as herein presented will form a basis for a wide range of industries.

No doubt experts in the various departments will have suggestions to make as to how the book might be improved. We will be very glad to hear from them in this connection.

With respect to the general plan of the work, we are indebted to Dr. F. W. Merchant, Director of Technical Education for Ontario, and Dr. A. C. McKay, Principal of the Toronto Technical Schools. Thanks are due in a special sense to Volney A. Ray, M.A., of the Department of Shopwork in the Central Technical School in connection with the chapter on "Mathematics of the Machine Shop," and to A. J. Stringer, M.S.A., of the Department of Architecture and Design in connection with the chapter on "Application of the Measures to the Trades." The cuts of Quick Change Gears are by courtesy of the R. K. Le Blond Machine Tool Company, Cincinnati, and those of the Planimeters by courtesy of the Hughes Owens Company, Montreal. The drawings were made by James Hanes a former student of our school.

JUNE, 1921.

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CHAPTER I.

THE FUNDAMENTAL OPERATIONS OF ARITHMETIC.

1. **The Symbols of Arithmetic** are 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. These symbols are called numbers, digits or figures. Their values depend on how they are written with respect to each other. When used separately or with commas between them as above they denote one, two, three, four, five, six, seven, eight, nine, zero. When written one after the other with no marks between, their values are determined by their positions. The established method of numeration, the Decimal System (from the Latin word decem, ten) is based on the number ten. For example 534 is read five hundred and thirty four. The figure 4 being in the first place counting from the right indicates 4 units, the figure 3 being in the second place from the right indicates ten times three units or thirty, the figure 5 being in the third place from the right indicates one hundred times five units or five hundred. The following table indicates the values of the figures owing to their positions:

Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Units	Tenths	Hundredths	Thousandths	Ten Thousandths
			7	2	5	6 ·	4	3	8	

in which a point called the decimal point is used to separate the units figure from one having one tenth the value. Thus 7256·438 is read seven thousand, two hundred and fifty six and four hundred and thirty eight thousandths. The figures following the decimal point are read as thousandths because the last figure is in the thousandths place. Thus ·13 would be read thirteen hundredths because the last figure is in the hundredths place. A whole number may be written with a decimal point to the right of the units place.

Exercises I.

Write in words the following numbers:

1.	36	5.	93·4	9.	·34
2.	·734	6.	732·45	10.	·435
3.	43689	7.	43·124	11.	·03
4.	718965	8.	7986·1583	12.	·075

When the meanings of the figures in their relation to the decimal point have been fixed, the figures to the right of the decimal point are not read as above.

For example 134·56 is read one hundred and thirty four decimal five six or more generally one hundred and thirty four point five six, that is the figures to the right of the decimal point are merely named in their order going from left to right.

Exercises II.

1. Read the numbers in Exercises I making use of this notation.

   Express the following numbers in figures:
2. Four hundred and thirty four.
3. Seven hundred and forty eight and twenty six hundredths.
4. Six thousand, four hundred and eighty two and seven tenths.
5. Five million, three hundred and nine thousand five hundred and six and one hundred and twenty five thousandths.
6. Five one-thousandths.
7. Sixty five ten-thousandths.
8. Three hundred and twenty five one-thousandths.
9. Four hundred and seventy eight point three four.
10. Five thousand, three hundred and fifty point seven eight six.

2. The Four Fundamental Operations. All computations in Arithmetic are made by means of the four operations:

Addition or finding the sum.

Subtraction or finding the difference.

Multiplication or finding the product.

Division or finding the quotient.

3. Addition. The sign for addition is + (plus). Thus 6 + 4 means that 6 and 4 are to be added. The result is called the sum. If we wish to add 6 ft. 4 in. and 3 ft. 2 in. we must add in. to in. and ft. to ft. In a similar way when adding numbers it is necessary to place tens under tens, units under units, tenths under tenths and so on. In the case of numbers having no decimal part this may be done by keeping the margin on the right-hand side in a straight line and, in the case of numbers having decimal parts, by keeping the decimal points in a vertical column.

For example to add 9, 75, 18, 324, 9678, 27436, and also 37·5, 124·69, ·75, ·0023, 346·058, 27, the arrangement is as follows:

9	37·5
75	124·69
18	·75
324	·0023
9678	346·058
27436	27·
37540	536·0003

Each column is added beginning at the right. The sum of the figures in the units column of the first case is 40, the 0 is placed in the units column, and the 4 is carried and added to the figures in the second column since 40 units is equal to 4 tens and 0 units. The sum of the figures in the tens column with the 4 carried over is 24, the 4 is placed in the tens column and the 2 is carried to the hundreds column and so on. In a similar way beginning at the right the sum in the second case is found.

Exercises III.

Copy in your work book and add the following:

1.	2.	3.
743	1975	1374
1589	4386	9281
642	721	4962
7593	15935	758
846	420	63

4.	5.	6.
25·72	35·21	328·42
136·01	136·35	736·84
23·54	23·48	39·43
7·28	78·62	100·26
199·71	91·43	702·85

7.	8.	9.
53·92	118·64	321·25
16·81	406·21	76·84
4·25	325·9	1352·41
·85	76·84	·13
3·24	231·35	470·02

10.	11.	12.
1392·6	21985·	2·635
435·84	436·54	18·923
936·815	3985·216	29·712
72·002	798·005	43·002
732·54	·792	1·986
·006	43·841	868·12
13·021	983·521	125·34
4798·058	7648·005	9875·1346

13.	14.	15.
$ 89·25	$1728·36	$320·51
121·63	256·93	192·81
2·87	24·87	568·53
13·42	34·25	402·96
829·78	176·98	768·34

4. **Subtraction.** The sign for subtraction is − (minus). Thus 6−4 means that 4 is to be subtracted from 6. The result is called the difference. In subtraction the numbers are arranged as in addition, that is units under units, tens under tens, and so on. For example, to subtract 872 from 2625 the arrangement is as follows:

$$\begin{array}{r} 2625 \\ 872 \\ \hline 1753 \end{array}$$

2 is taken from 5 leaving 3. Since 7 cannot be taken from 2, 1 hundred or 10 tens is borrowed from 6 hundreds and 7 tens are then subtracted from 12 tens leaving 5. In the third column there are now only 5 hundreds in the upper line. Since 8 cannot be subtracted from 5, 1 thousand is borrowed from the 2 thousands and 8 hundreds are then subtracted from 15 hundreds leaving 7 hundreds. The operation may be performed by adding to the lower line instead of subtracting from the upper line, thus 2 from 5 leaves 3, 7 from 12 leaves 5, 9 from 16 leaves 7, 1 from 2 leaves 1.

Exercises IV.

Copy the following examples in your work book and subtract:

1.	7963−428.	6.	11·423−8·216.
2.	7·48−5·12.	7.	235·48−132·18.
3.	436·2−71·253.	8.	2·415−·0034.
4.	$168·45−$29·25.	9.	21·053−9·821.
5.	34648−239·21.	10.	638·215−421·006.

5. **Multiplication.** The sign for multiplication is × (multiplied by). Thus 6×4 means that 6 is to be multiplied by 4. The number multiplied is called the multiplicand, the number by which it is multiplied is called the multiplier, the result is called the product. Before the operation of multiplication can be performed it is necessary to commit to memory the multiplication tables following:

Multiplication Tables.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100
11	22	33	44	55	66	77	88	99	110
12	24	36	48	60	72	84	96	108	120

In the table the second column gives the products when 1 is multiplied by 2, 2 by 2, 3 by 2 and so on to 12 by 2; the third column gives the products when 1 is multiplied by 3, 2 by 3 and so on to 12 by 3. Similarly the seventh column gives the products when 1, 2, 3 and so on up to 12 are multiplied by 7.

To multiply 8345 by 7 the arrangement is as follows:

$$\begin{array}{r} 8345 \\ 7 \\ \hline 58415 \end{array}$$

5×7 is 35 that is 3 tens and 5 units. The 5 is placed in the units column and the 3 is carried to the tens column. 4×7 is 28 and when the 3 carried over is added the result is 31 tens the 1 is placed in the tens column and the 3 is carried to the hundreds column. 3×7 is 21 and when the 3 carried over is added the result is 24 hundreds. The 4 is placed in the hundreds column and the 2 is carried to the thousands column. 8×7 is 56 and with the 2 carried over the result is 58 thousands. The 8 is placed in the thousands column and the 5 in the ten thousands column.

6. **Powers of 10.** $10 \times 10 = 100$ and may be written 10^2. $10 \times 10 \times 10 = 1000$ and may be written 10^3. 10^2 may be called the second power of 10, 10^3 the third power and so on, the figure placed to the right and above the ten being called the index or exponent of the power. It may be observed that the number of ciphers is the same as the index of the power.

7. **Multiplication by 10 and its Powers.** $436 \times 100 = 43600$. $725 \cdot 26 \times 10 = 7252 \cdot 6$ since 6 hundredths multiplied by 10 becomes 60 hundredths or 6 tenths, 2 tenths multiplied by 10 becomes 20 tenths or 2 units and so on. Also $43 \cdot 568 \times 100 = 4356.800$. The rule may be stated as follows:—*To multiply by 10 or its powers write the number with decimal point moved as many places to the right as the number of ciphers in the power, that is as many places as the index.* Since $400 = 4 \times 100$ it is evident that the product when multiplying by 400 may be obtained by multiplying by 4 and then moving the decimal point two places to the right.

Exercises V.

Copy in your work book the following examples and find the products:

1. 173×9.	9. $78 \cdot 64 \times 100$.	17. $23 \cdot 01 \times 800$.
2. 187×7.	10. $1 \cdot 475 \times 8$.	18. $\cdot 00078 \times 10^5$.
3. 769×8.	11. 298×6.	19. $\cdot 00846 \times 9000$.
4. $34 \times 7 \times 4$.	12. 298×60.	20. $1 \cdot 475 \times 800$.
5. $769 \times 8 \times 9$.	13. $78 \cdot 64 \times 10^2$.	21. $236 \cdot 896 \times 10^6$.
6. 296×10.	14. $\cdot 0067 \times 7$.	22. $\cdot 6345 \times 4 \times 10^3$.
7. 345×100.	15. $\cdot 0067 \times 700$.	23. $12 \cdot 73 \times 8 \times 10^4$.
8. $76 \cdot 45 \times 10$.	16. $4 \cdot 2905 \times 10^3$.	24. $\cdot 765 \times 10^3 \times 9$.

8. **When the multiplier** contains more than one digit the arrangement is as follows: 364×28 =

$$\begin{array}{r} 364 \\ 28 \\ \hline 2912 \\ 728 \\ \hline 10192 \end{array}$$

364 is multiplied by 8 as before. When multiplying by 2 proceed as before but since the 2 is 2 tens the first figure 8 of the partial product is placed in the tens column and so on. The partial products are added and the product 10192 obtained. When the numbers have decimal parts as 13·742×4·3 the arrangement is as follows:

$$\begin{array}{r} 13{\cdot}742 \\ 4{\cdot}3 \\ \hline 41226 \\ 54968 \\ \hline 59{\cdot}0906 \end{array}$$

The number of decimal places in the product is equal to the total number of decimal places in the two numbers multiplied.

Exercises VI.

Copy the following examples in your work book and find the products:

1. 364×9.
2. 793×8.
3. 436×11.
4. 731×43.
5. 936×72.
6. 119×27.
7. 4392×435.
8. 3854×729.
9. 9386×538.
10. 1234×567.
11. 23·54×21.
12. 734·183×36.
13. 98·43×13·2.
14. 93·02×·75.
15. ·754×·028.
16. ·054×·721.
17. 82·9×4·31×·08.
18. ·7854×·09×11·2.
19. 3·009×721·3×23·08.
20.. 5·43×·034×7·18.
21. ·035×·728×436.
22. 43·9×16·8×·002.
23. 143·5×7·25×·075.
24. 12·961×32·4×5·03.
25. 84·21×15·8×·072.
26. 46×·08×·921.
27. 736×·98×4·12.
28. 158×·75×·1625.
29. ·0625×·04×·025.
30. ·1416×3·1416×3·5.

9. **Division.** The sign for division is ÷ (divided by). Thus 35÷7 means that 35 is to be divided by 7, or that it is required to find how many times 7 is contained in 35. The number to be divided is called the dividend, the number by which it is divided the divisor, the result of the division the quotient. When a number is not contained an exact number of times the part left over is called the remainder. Division may also be indicated thus $\frac{35}{7}$.

10. **Short Division.** In general when the divisor is not too large the method of short division is used.

Thus, $5852 \div 7 = 7\underline{/5852}$
$\qquad\qquad\qquad\quad 836.$

7 is contained in 58, 8 times and 2 to carry, 7 is contained in 25, 3 times and 4 to carry, 7 is contained in 42, 6 times. When there is a remainder it is written over the divisor or reduced to decimal form:

$$8\underline{/35826} \qquad \text{or} \qquad 8\underline{/35826\cdot00}$$
$$4478\tfrac{2}{8} \qquad\qquad\qquad 4478\cdot25.$$

11. **Division by 10 and its Powers.** To divide by 10^2 the dividend may first be divided by 10 and the resulting quotient then divided by 10. Since dividing by 10 makes each figure equal to one-tenth its original value owing to position, it is evident that the result may be expressed thus:—to divide by 10 or its powers move the decimal point as many places to the left as the number of ciphers in the power of 10, that is as the index of the power. Since $600 = 6 \times 100$ if 600 is the divisor it is only necessary to divide by 6 and then move the decimal point two places to the left. Hence the rule:—*To divide by a number ending with one or more ciphers move the decimal point in the dividend as many places to the left as the number of ciphers in the divisor and then divide by the part of the divisor preceding the ciphers.*

Exercises VII.

Copy in your work book the following examples and perform the operations indicated:

1. $131948 \div 4$.
2. $2170944 \div 12$.
3. $12 \cdot 348 \div 7$.
4. $\frac{156}{14}$.
5. $\frac{\cdot 00632}{4}$.
6. $176 \div 10^2$.
7. $13 \cdot 25 \div 10^3$.
8. $\frac{732}{600}$.
9. $12 \cdot 5 \div 500$.
10. $7659 \div 90$.
11. $15318 \div 900$.
12. $9\overline{)280765}$.
13. $7\overline{)324 \cdot 94}$.

12. Long Division. The method of long division is indicated by the following example:

```
13/6942/534
   65
   --
    44
    39
    --
     52
     52
     --
```

13 is contained in 69, 5 times. 13×5 is 65 which subtracted from 69 leaves 4. Bring down 4 the next figure of the dividend. 13 is contained in 44, 3 times. 13×3 is 39 which subtracted from 44 leaves 5. Bring down 2 the next figure of the dividend. 13 is contained in 52, 4 times. 13×4 is 52 which subtracted from 52 leaves no remainder. When there is a remainder it may be written over the divisor or changed to a decimal as in short division $62563 \div 39 = 62563 \cdot 00 \div 39$

```
39/62563·/1604 7/39
   39
   ---
   235
   234
   ---
     163
     156
     ---
       7
```

or

```
39/62563·00/1604·17+
   39
   --
   235
   234
   ---
     163
     156
     ---
       70
       39
       --
       310
       273
       ---
        37
```

When the divisor or the dividend or both have decimal figures the position of the decimal point in the quotient may be obtained by paying attention to the following rules:

1. *When the number of decimal places in the dividend exceeds the number in the divisor, divide as if the divisor contained no decimals and point off a number of decimal places in the quotient equal to the number in the dividend minus the number in the divisor.*

2. *When the number of decimal places in the dividend is less than the number in the divisor, annex zeros to the right of the dividend until a sufficient number of decimals has been obtained and proceed as before.*

```
6·79/57·20575/8425
     54 32
     -----
      2 885
      2 716
      -----
        1697
        1358
        ----
         3395
         3395
         ----
```

Since there are 5 places in the dividend and 2 in the divisor, the number in the quotient is 5–2 or 3 and the quotient is therefore 8·425.

```
3430/16·807/4
     13 720
     ------
      3 087
```

and the quotient is ·004 since there are three decimal places in the dividend and none in the divisor. If it is required to carry the division to another decimal place add 0 to the right of the decimal and then divide into 30870 thus:

```
3430/16·8070/49
     13 720
     ------
      3 0870
```

and the quotient is ·0049.

The following examples show a method often used in determining the position of the decimal point.

Example:—Divide 433·652 by 163.

```
       2·660
163/433·652
    326
    ---
    1076
     978
    ----
      985
      978
      ---
       72
```

Explanation:—When the divisor is an integer the point in the quotient should be placed directly above the point in the dividend and the division performed as in whole numbers.

Example:—Divide 27·4289 by 1·24.

```
            22·12
12 4·/27 42·89/
      24 8
      ----
       262
       248
       ---
        148
        124
        ---
         249
         248
         ---
           1
```

Explanation:—When the divisor contains decimal figures move the point in both divisor and dividend as many places to the right as there are decimal places in the divisor. This is equivalent to multiplying both divisor and dividend by the same number, 100 in the above, and does not change the quotient. Then place the decimal in the quotient above the position of the decimal point in the dividend and divide as in whole numbers.

Exercises VIII.

Find results to two decimal places:

1. 5462 ÷ 84.
2. 1024 ÷ 16.
3. 31264 ÷ 46.
4. 746215 ÷ 352.
5. 834 ÷ 6·21.
6. 7342 ÷ 26·4.
7. 1235 ÷ 406.
8. 738·1 ÷ 92·6.
9. 1934·43 ÷ 136·3.
10. 138·42 ÷ ·034.
11. 128·942 ÷ ·4327.
12. $\frac{43\cdot2198}{41\cdot8}$.

13. **Relative Importance of Signs of Operation.** If only + and − signs occur they may be operated in any order. Thus $12+3-2+9-6=16$ in whatever order the signs are used.

If only $\times$ and $\div$ signs occur they must be operated in the order given $12 \div 3 \times 5 \div 2$ means that 12 is divided by 3, the quotient multiplied by 5 and the resulting product divided by 2.

If $+$ and $-$ signs occur together with $\times$ and $\div$ signs the $\times$ and $\div$ signs must be used first and then the $+$ and $-$ signs may be used in any order. Thus $12 \div 3 + 8 \times 2 - 6 \div 2 + 7 = 4 + 16 - 3 + 7 = 24$.

If brackets are used as in $36 \div (4+8)$ the part within the bracket is to be regarded as one quantity and the operation would be $36 \div 12 = 3$.

Exercises IX.

Find the values of:

1. $16 \div 8 + 4 \times 2 \times 3 - 16 \times 2 \div 4$.
2. $60 - 25 \div 5 + 15 - 100 \div 4 \times 2$.
3. $17 \times 3 + 27 \div 3 - 40 \times 2 \div 5$.
4. $864 \div 12 - 124 \div 31 + 54 \div 27$.
5. $13 \times 9 \times 62 + 44 \div 4 - 17 \times 22$.
6. $4963 \div 7 + 144 \div 72 - 14 \times 9$.
7. $1728 \div (36 - 2 \times 12) + (13 \times 12) \div (8 \div 2)$.

14. **Factors—Cancellation.** The factors of the number are the integers (meaning whole numbers) which multiplied together give the number. Thus 3 and 5 are the factors of 15 since $3 \times 5 = 15$.

A number that has no factors but itself and unity (or 1) is called a prime number. If a prime number is used as a factor it is called a prime factor. Thus 2 and 5 are prime factors of 20. When the same number is a factor of two or more numbers it is said to be a common factor of those numbers. Thus 3 is a common factor of 27 and 36. By means of factors it is often possible to shorten the work in division. In $183 \div 15$ since 3 is a factor common to 183 and 15 we can divide by it and then $183 \div 15 = 61 \div 5 = 12\frac{1}{5}$.

This method, cancellation, may be used in finding the value of such an expression as :

$$\frac{4\times3\times14\times32}{3\times2\times3\times21} = \frac{\overset{2}{\cancel{4}}\times\overset{1}{\cancel{3}}\times\overset{2}{\cancel{14}}\times32}{\underset{1}{\cancel{3}}\times\underset{1}{\cancel{2}}\times3\times\underset{3}{\cancel{21}}} = \frac{128}{9} = 14\tfrac{2}{9}.$$

First the 3 below the line is divided into the 3 above the line and since $3\div3=1$ the 3's are cancelled by each other and 1's are placed in their stead. Similarly 2 below the line cancels 2 in the 4 above the line; next since 7 is a common factor of 14 and 21 it is divided into 14 giving 2 and into 21 giving 3. When all common factors are cancelled the remaining numbers are multiplied together giving $\frac{128}{9}=14\frac{2}{9}$.

Exercises X.

Find the values of:

1. $\dfrac{57\times119\times16}{17\times12\times19}$.
2. $\dfrac{20\times56\times12}{21\times10\times18}$.
3. $\dfrac{77\times100\times18\times14}{25\times11\times49\times16}$.
4. $\dfrac{1200\times515\times70\times100}{5\times35\times103}$.
5. $\dfrac{114\times1728\times999}{96\times270\times33}$.
6. $\dfrac{99-25+14\times7}{50\div2\times18}$.
7. $\dfrac{2560\div4+125\times4-14\times76}{17\times27+32\times40-1618}$.
8. $\dfrac{76\cdot5\times9\cdot2\times11}{36\cdot8\times9\times10}$.
9. $\dfrac{32\cdot18\times\cdot006\times3\cdot4}{1\cdot7\times16\cdot09\times\cdot003}$.
10. $\dfrac{42\times\cdot36\times4\cdot8}{1\cdot2\times\cdot7\times1\cdot8}$.
11. $\dfrac{192\times16\cdot8\times4\cdot4}{4\times2\cdot1\times22}$.
12. $\dfrac{10\cdot24\times7\cdot29\times36}{1\cdot44\times9\times1\cdot8}$.
13. $\dfrac{10^2\times8\cdot6\times\cdot0625}{2\cdot5\times4\cdot3\times2}$.
14. $\dfrac{7\cdot2\times12\cdot5\times39}{1\cdot3\times1\cdot2\times10^2}$.

Exercises XI.

Applied Problems.

1. In an electrical shop there were three motors, one weighed 278 lb., another 380 lb., and the third 475 lb. What was the total weight?

2. Three coal sheds contained respectively 6382 lb., 14728 lb., 24725 lb. How many tons in all three?

3. Electric light wire was run around the four sides of two rooms. If the first room was 18 ft. long and 12 ft. wide; the second 20 ft. long and 13 ft. wide, what was the total length of wire required? (Electric lights require two wires).

4. A reel of wire contained 6425 ft. If 3226 ft. were used on a certain job, how many ft. remained on the reel?

5. A reel of wire contained 7280 ft. If 2348 ft. were used on one house and 1425 ft. on another, how many ft. were used on both? How many ft. were left on the reel?

6. In the coal-bin at the school there were 48,720 lb. of coal at the beginning of the week. On Monday 11600 lb. were used; Tuesday 12350 lb.; Wednesday 10718 lb. On Thursday 24600 lb. were received and 11880 lb. used. How much coal was used during these days? How much coal was there in the bin on Friday morning?

7. A machinist sent in the following order for bolts: 15 bolts, 3 lb. each; 21 bolts, 2 lb. each; 14 bolts, 4 lb. each; 9 bolts, 3 lb. each; 11 bolts, 6 lb. each. What was the total weight of the order?

8. A wiring job required the following labour: 3 men for 4 hours each; 6 men for 5 hours each; 8 men for 9 hours each; 2 men for 15 hours each. Find the total number of hours on the job?

9. A rod is 72 in. in length. How many pieces 5 in. in length can be cut from it? Would there be a remainder?

10. An engine requires 90 lb. of coal per mile. How far could it run on 8 tons?

11. If 4 dozen screws weigh one pound, how many cases containing 24 screws could be filled from 30 lb. of screws?

12. A train runs from Toronto to Penetang, a distance of 101 miles, in 4 hours. What is the average rate per hour?

13. The cost of construction of a railway from Toronto to Montreal, a distance of 333 miles, was $3,425,625. What was the average cost per mile?

14. How many gallons of water would be discharged in an hour by two pipes, if one discharged 18 gallons per minute and the other 4 gallons more per minute?

15. If 18 men working 8 hours a day, can do a piece of work in 12 days, how many days will it take 24 men working 9 hours a day?

16. If a horse-shoe weighs 8 oz., how many horse-shoes will 36 lb. of steel produce. (1 lb. =16 oz.).

CHAPTER II.

FRACTIONS—PERCENTAGE.

15. **Definition.** A yard measure is divided, or marked off, into three equal parts called feet so that:

1 foot = one-third ($\frac{1}{3}$) of a yard.
2 feet = two-thirds ($\frac{2}{3}$) of a yard.

A foot rule is divided into twelve equal parts called inches so that:

1 inch = $\frac{1}{12}$ of a foot.
5 inches = $\frac{5}{12}$ of a foot.
9 inches = $\frac{9}{12}$ of a foot.

The symbols $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{12}$, $\frac{5}{12}$, $\frac{9}{12}$ are called fractions because they denote a part or fraction of something which has been divided.

A fraction may then be defined as a number which denotes one or more of the equal parts into which some thing or unit has been divided. In the fraction $\frac{5}{7}$ the number below the line is called the denominator since it denotes or names the parts into which the unit has been divided, the number above the line is called the numerator, since it denotes the number of parts taken. The numerator and the denominator are called the terms of the fraction. A fraction expressed in this notation is called a **vulgar** fraction.

Since $\frac{5}{7} = 5 \div 7$ a fraction may also be regarded as a case of indicated division.

16. **Kinds of Fractions.** When the numerator is less than the denominator the fraction is said to be a proper fraction, Ex. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$. When the numerator is greater than the denominator the fraction is said to be an improper fraction, Ex. $\frac{3}{2}$, $\frac{6}{5}$, $\frac{13}{12}$. A combination of an integer (whole number) and a fraction is called a mixed number, Ex. $3\frac{1}{2}$, $5\frac{7}{16}$, $4\frac{1}{8}$.

17. To change a Fraction to another Equal in Value but with Different Denominator.

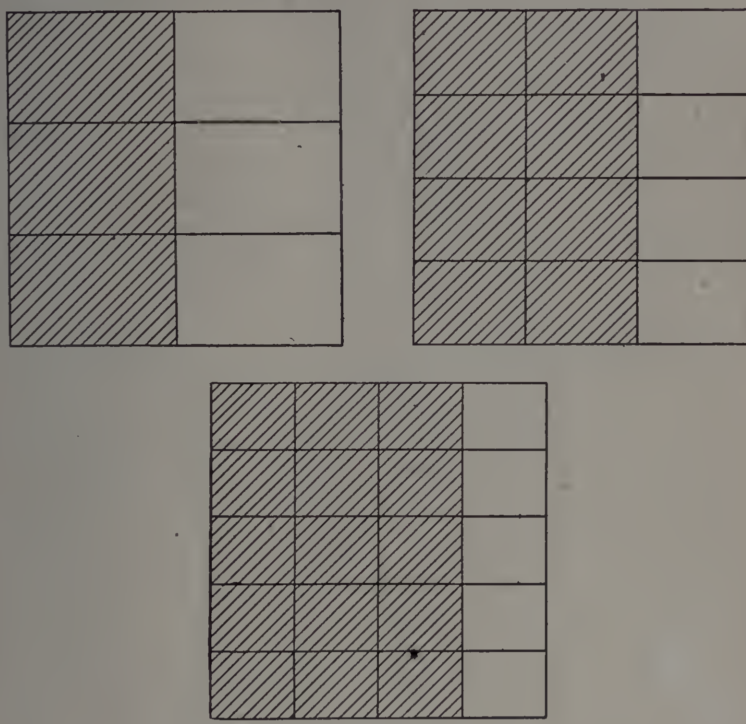


Fig. 1

In the above figure, in each case, we have a square $1\frac{3}{4}$ inches to the side.

In the first case the shaded portion contains three of the six equal parts and is one-half the whole figure so that $\frac{1}{2}=\frac{3}{6}$.

In the second case the shaded portion contains eight of the twelve equal parts and is two-thirds of the whole figure so that $\frac{2}{3}=\frac{8}{12}$.

In the third case the shaded portion contains fifteen of the twenty equal parts and is three-fourths of the whole figure so that $\frac{3}{4}=\frac{15}{20}$. Further, $\frac{3}{6}$ may be obtained from $\frac{1}{2}$ by multiplying numerator and denominator by the same number 3, also $\frac{8}{12}$ may be obtained from $\frac{2}{3}$ by multiplying numerator and denominator by the same number 4, and $\frac{15}{20}$ may be obtained from $\frac{3}{4}$ by multiplying numerator and denominator by the same number 5.

From these illustrations it may be inferred *that a fraction is not changed in value when the numerator and the denominator are multiplied by the same number.*

If the above results are written $\frac{3}{6}=\frac{1}{2}$, $\frac{8}{12}=\frac{2}{3}$, $\frac{15}{20}=\frac{3}{4}$ it may be inferred *that a fraction is not changed in value when the numerator and the denominator are divided by the same number.*

Exercises XII.

Change the following:

1. $\frac{3}{7}$ to an equivalent fraction having 14 as denominator.
2. $\frac{5}{6}$ to an equivalent fraction having 24 as denominator.
3. $\frac{9}{10}$ to an equivalent fraction having 50 as denominator.
4. $\frac{7}{25}$ to 75ths.
5. $\frac{3}{8}$ to 40ths.
6. $\frac{7}{8}$ to 56ths.
7. $\frac{17}{18}$ to 108ths.
8. $\frac{17}{36}$ to 144ths.
9. $\frac{8}{63}$ to 252nds.
10. $\frac{8}{21}$ to 189ths.

18. **Reduction to Lowest Terms.** A fraction is said to be in its lowest terms when no number other than 1 will exactly divide the numerator and denominator or, in other words, when the numerator and denominator have no common factor.

The fraction $\frac{7}{9}$ is in its lowest terms because 7 and 9 have no common factor.

The fraction $\frac{12}{15}$ is not in its lowest terms because 3 is a common factor of 12 and 15 and dividing numerator and denominator by the common factor, $\frac{12}{15}$ becomes $\frac{4}{5}$.

To reduce a fraction to its lowest terms divide both parts by any common factor and continue the process until no further division is possible. Ex. $\frac{120}{144}=\frac{20}{24}=\frac{5}{6}$.

Exercises XIII.

Reduce to lowest terms:

1. $\frac{6}{8}$.
2. $\frac{8}{16}$.
3. $\frac{21}{84}$.
4. $\frac{240}{360}$.
5. $\frac{48}{72}$.
6. $\frac{185}{725}$.
7. $\frac{968}{1244}$.
8. $\frac{428}{1476}$.
9. $\frac{63}{700}$.
10. $\frac{135}{400}$.

19. **To Reduce an Improper Fraction to a Mixed Number.** Example:—Reduce $\frac{126}{5}$ to a mixed number. $126 \div 5$ gives 25 for quotient and 1 for remainder and, as in division, may be written $25\frac{1}{5}$. Therefore $\frac{126}{5}=25\frac{1}{5}$. Hence the rule:—*Divide the numerator by the denominator and express as in division.* Note—any integer may be written in the form of a fraction thus $25=\frac{25}{1}$.

To Reduce a Mixed Number to an Improper Fraction. Example:—Reduce $16\frac{3}{7}$ to an improper fraction.

Since in 1 there are 7 sevenths, in 16 there are 16×7 or 112 sevenths, and $\frac{3}{7}$ is 3 sevenths, then $16\frac{3}{7}$ is $112+3$ or 115 sevenths, therefore $16\frac{3}{7}=\frac{115}{7}$. Hence the rule:—*Multiply the whole number by the denominator and add the numerator to the product. Take this result for the numerator and the original denominator for the denominator.*

Exercises XIV.

Express the following improper fractions as whole or mixed numbers:

1. $\frac{7}{2}$.	4. $\frac{36}{6}$.	7. $\frac{807}{27}$.	10. $\frac{435}{16}$.	13. $\frac{1325}{25}$.
2. $\frac{16}{4}$.	5. $\frac{70}{8}$.	8. $\frac{72}{22}$.	11. $\frac{25000}{10}$.	14. $\frac{729}{81}$.
3. $\frac{64}{9}$.	6. $\frac{125}{7}$.	9. $\frac{369}{33}$.	12. $\frac{360}{39}$.	

Reduce to improper fractions:

15. $3\frac{1}{3}$.	17. $7\frac{3}{11}$.	19. $121\frac{3}{4}$.	21. $431\frac{3}{5}$.	23. $722\frac{8}{9}$.
16. $6\frac{7}{15}$.	18. $115\frac{7}{32}$.	20. $91\frac{2}{3}$.	22. $4000\frac{21}{40}$.	24. $392\frac{7}{9}$.

20. **Addition and Subtraction of Fractions.** When fractions have the same denominator they can be added by adding the numerators, and subtracted by subtracting the numerators.

Exs. $\frac{1}{3}+\frac{2}{3}=\frac{3}{3}$. $\quad \frac{3}{4}-\frac{1}{4}=\frac{2}{4}$.

When the denominators are not alike as $\frac{1}{2}$ and $\frac{1}{3}$, they cannot be added without first changing to equivalent fractions having the same denominator.

$$\frac{1}{2}=\frac{1\times 3}{2\times 3}=\frac{3}{6} \qquad \frac{1}{3}=\frac{1\times 2}{3\times 2}=\frac{2}{6}$$

then, $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$,
also, $\frac{2}{3}-\frac{3}{5}=\frac{10}{15}-\frac{9}{15}=\frac{1}{15}$.

21. Least Common Multiple—Least Common Denominator of Fractions. The fractions $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, can be added only when the denominators are alike. This may be any number of which the different denominators are factors, but in practice it is customary to take the smallest number containing the different denominators. This number is then called the *Least Common Multiple* (*L.C.M.*) of the denominators because it is the least number into which the numbers will divide without remainder. It is also called the *Least Common Denominator* (*L.C.D.*) of the fractions.

In the given case 12 is the L.C.M. of 2, 3, 4, then since 2 is contained in 12, 6 times, the numerator and denominator are multiplied by 6, so that $\frac{1}{2}=\frac{6}{12}$, also $\frac{2}{3}=\frac{8}{12}$, and $\frac{3}{4}=\frac{9}{12}$, then $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}=\frac{6}{12}+\frac{8}{12}+\frac{9}{12}=\frac{6+8+9}{12}=\frac{23}{12}$.

When the L.C.M. cannot be easily determined by inspection the following method may be used:

Find the least common multiple of 12, 14, 15, 16, 18, 20.

2/	12	14	15	16	18	20
2/	6	7	15	8	9	10
3/	~~3~~	7	15	4	9	~~5~~
		7	5	4	3	

L.C.M. $-2\times2\times3\times7\times5\times4\times3=5040$.

Explanation:—Divide through by the least number which is a divisor of two or more of the given numbers. Continue this process until there is no number common to any two as a factor. In the third line 3 and 5 are struck out because 15 is also in that line, and any number which is a multiple of 15 is also a multiple of 3 and 5. The L.C.M. is obtained as indicated.

Exercises XV.

Find the values of the following:

1. $\frac{1}{3}+\frac{2}{5}$.
2. $\frac{1}{7}+\frac{3}{5}$.
3. $\frac{5}{16}-\frac{1}{4}$.
4. $\frac{7}{8}+\frac{2}{3}-\frac{1}{16}$.
5. $\frac{3}{4}+\frac{5}{22}-\frac{3}{16}$.
6. $\frac{5}{13}+\frac{5}{16}-\frac{2}{39}$.
7. $\frac{7}{8}+\frac{3}{24}+\frac{4}{36}-\frac{1}{18}$.
8. $\frac{3}{64}+\frac{5}{8}+\frac{7}{8}+\frac{9}{32}$.
9. $\frac{3}{8}+\frac{5}{64}+\frac{11}{32}-\frac{1}{2}$.
10. $2\frac{1}{2}+4\frac{1}{4}+5\frac{1}{8}$.
11. $3\frac{1}{2}-1\frac{2}{3}+2\frac{1}{8}$.
12. $\frac{15}{8}-\frac{2}{3}+4\frac{1}{9}-2\frac{1}{7}$.
13. $5\frac{3}{5}+7\frac{7}{10}+6\frac{7}{15}$.
14. $\frac{15}{8}-\frac{3}{16}+7\frac{1}{9}$.

Exercises XVI.

1. Four castings weigh respectively $8\frac{7}{8}$ lb., $5\frac{1}{2}$ lb., $11\frac{3}{4}$ lb., and $7\frac{5}{8}$ lb. What is their total weight?

2. A piece of steel on a lathe is 1 in. in diameter. In the first cut $\frac{3}{32}$ in. is taken off, in the second cut $\frac{2}{64}$ in., in the third cut $\frac{1}{16}$ in. Find the diameter of the finished piece.

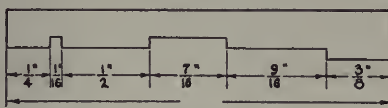


Fig. 2

3. Find the overall length for the template in Figure 2.

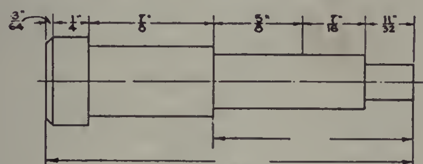


Fig. 3

4. Find the missing dimensions in Figure 3.

5. A drawing calls for the following divisions:
$3\frac{3}{16}$ in., $7\frac{1}{2}$ in., $4\frac{3}{4}$in., $8\frac{7}{8}$ in. Find the overall dimensions.

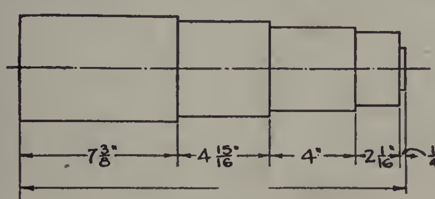


Fig. 4

6. A crank-pin has the dimensions given in Figure 4. If $\frac{1}{4}$ in. is allowed at each end for finishing what must be the length of the rough forging?

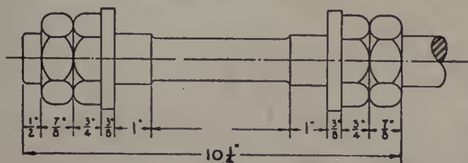


Fig. 5

7. A drawing for a part of the end of a valve rod is given in Figure 5. Find the missing dimension.

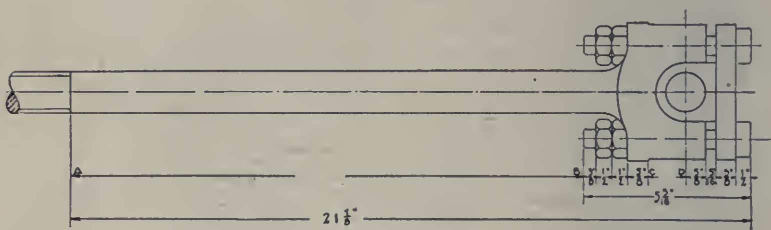


FIG. 6

8. Find the missing dimensions, **AB, CD,** in Figure 6.

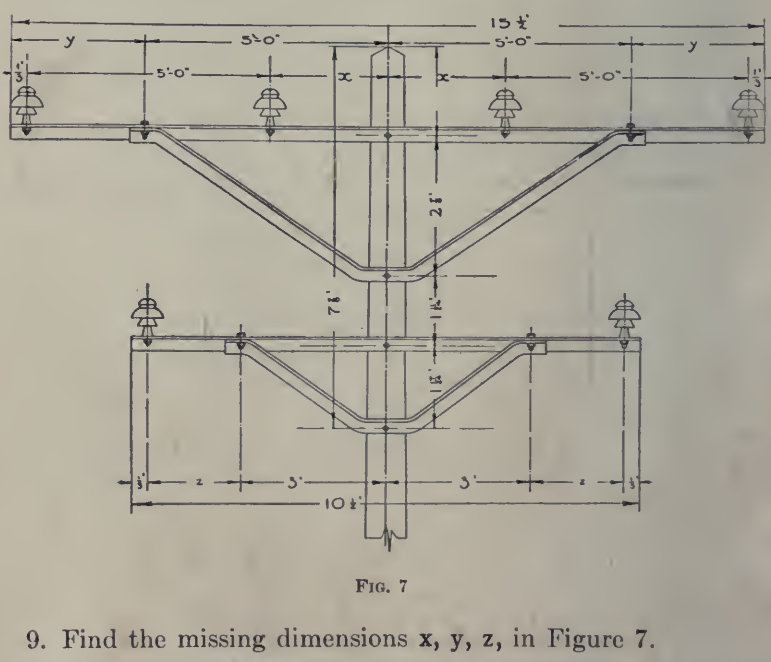


FIG. 7

9. Find the missing dimensions **x, y, z,** in Figure 7.

10. Find the missing dimension in the upper part of the height in Figure 7.

22. **Multiplication of Fractions**—Consider the following example:—A man left $\frac{3}{4}$ of his estate to his children, $\frac{1}{2}$ of this being left to his eldest son. What fraction of the estate did the eldest son receive ?

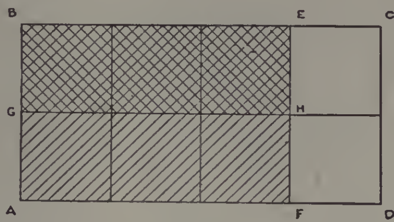


FIG. 8

We might represent this example by the above diagram. **ABCD** represents the whole estate. The shaded part **ABEF,** $\frac{3}{4}$ of the whole, represents the part left to the children. One-half of this is taken, **BEHG,** to represent the eldest son's share, i.e., $\frac{1}{2}$ of $\frac{3}{4}$ or $\frac{1}{2}\times\frac{3}{4}$. We further observe that, of the eight squares in the figure, the eldest son has three or $\frac{3}{8}$ of the whole, $\therefore\ \frac{1}{2}\times\frac{3}{4}=\frac{3}{8}$.

From this illustration we may infer that in order to multiply $\frac{1}{2}$ by $\frac{3}{4}$ we multiply the numerators for a new numerator and the denominators for a new denominator. Hence the rule:—*To multiply two or more fractions together, multiply the numerators for a new numerator and the denominators for a new denominator.*

Thus, $\frac{3}{4}\times\frac{5}{7}=\frac{3\times5}{4\times7}=\frac{15}{28}$.

Frequently cancellation shortens the process.

Thus, $\frac{5}{8}\times\frac{1}{4}\times\frac{4}{9}=\frac{5}{72}$.

Make a drawing to illustrate that $\frac{1}{3}\times\frac{5}{6}=\frac{5}{18}$.

To multiply a mixed number by an integer one of two methods may be used.

Thus to multiply $25\frac{1}{3}$ by 7.

$$25\times7=175$$
$$\frac{1}{3}\times7=\frac{7}{3}=2\frac{1}{3}$$
$$175+2\frac{1}{3}=177\frac{1}{3}$$

or, $25\frac{1}{3}\times7=\frac{76}{3}\times7=\frac{532}{3}=177\frac{1}{3}$.

To multiply two mixed numbers together change each to an improper fraction and then multiply.

Thus to multiply $2\frac{1}{3}$ by $4\frac{2}{5}$. $2\frac{1}{3}\times 4\frac{2}{5}=\frac{7}{3}\times\frac{22}{5}=\frac{154}{15}=10\frac{4}{15}$.

Exercises XVII.

Perform the operations indicated:

1. $\frac{5}{9}\times 4$.
2. $5\times\frac{4}{9}$.
3. $3\times\frac{16}{81}$.
4. $\frac{1}{2}\times\frac{3}{8}$.
5. $\frac{1}{4}\times\frac{5}{16}$.
6. $\frac{1}{2}$ of $\frac{3}{8}$.
7. $\frac{1}{4}$ of $\frac{5}{16}$.
8. $\frac{3}{16}\times\frac{3}{8}\times\frac{2}{5}$.
9. $\frac{4}{13}\times\frac{26}{3}\times\frac{9}{16}$.
10. $\frac{13}{18}\times\frac{7}{26}\times\frac{54}{14}\times\frac{9}{10}$.
11. $\frac{7}{16}\times\frac{3}{64}\times\frac{5}{14}\times\frac{9}{16}$.
12. $\frac{4}{13}\times\frac{51}{6}\times\frac{4}{10}$.
13. $\frac{25}{3}\times\frac{75}{8}\times\frac{64}{10}$.
14. $\frac{15}{16}\times\frac{7}{24}\times\frac{18}{21}$.
15. $15\frac{1}{2}\times 4$.
16. $6\frac{1}{4}\times\frac{4}{1}$.
17. $9\frac{1}{2}\times\frac{3}{19}\times 2$.
18. $3\frac{1}{7}\times\frac{1}{11}\times\frac{7}{2}$.
19. $\frac{4}{3}\times\frac{22}{7}\times 1\frac{1}{4}$.
20. $3\frac{1}{2}\times 2\frac{3}{4}\times 4\frac{1}{8}$.
21. $13\frac{1}{2}\times 1\frac{1}{27}\times 3\frac{1}{2}$.
22. $1\frac{2}{3}\times 2\frac{3}{5}\times 1\frac{1}{13}$.
23. $9\frac{1}{2}\times 3\frac{1}{19}\times\frac{2}{58}$.
24. $6\frac{1}{2}\times 4\frac{3}{13}\times 1\frac{1}{57}$.

23. **Division of Fractions.** Consider Figure 8, page 25, regarding multiplication of fractions.

ABCD represents the whole estate. The shaded part, $\frac{3}{4}$ of the whole, represents the part left to the children. This is divided into two parts to represent the eldest son's share i.e., $\frac{3}{4}\div 2$ or $\frac{3}{4}\div\frac{2}{1}$. We observe that, of the eight squares in the figure, the eldest son has three or $\frac{3}{8}$ of the whole.

$\therefore\ \frac{3}{4}\div\frac{2}{1}=\frac{3}{8}$.

But in the previous illustration $\frac{3}{8}=\frac{3}{4}\times\frac{1}{2}$.

$\therefore\ \frac{3}{4}\div\frac{2}{1}=\frac{3}{4}\times\frac{1}{2}$.

That is we may infer that to divide $\frac{3}{4}$ by $\frac{2}{1}$ we invert $\frac{2}{1}$ obtaining $\frac{1}{2}$ and then multiply $\frac{3}{4}$ by $\frac{1}{2}$.

Hence the rule:—*To divide one fraction by another invert the divisor and proceed as in multiplication.*

Thus to divide $\frac{3}{4}$ by $\frac{5}{6}$. Invert the divisor $\frac{5}{6}$ (i.e., write it $\frac{6}{5}$) and multiply $\frac{3}{4}$ by $\frac{6}{5}$, $\therefore\ \frac{3}{4}\div\frac{5}{6}=\frac{3}{4}\times\frac{6}{5}=\frac{18}{20}=\frac{9}{10}$.

To divide a mixed number by a fraction change the mixed number to an improper fraction and proceed as above.

Thus to divide $16\frac{1}{3}$ by $\frac{1}{5}$. $16\frac{1}{3}\div\frac{1}{5}=\frac{49}{3}\times\frac{5}{1}=\frac{245}{3}=81\frac{2}{3}$.

To reduce a complex fraction say $\frac{5\frac{1}{6}}{7\frac{2}{3}}$ to a simple fraction proceed as follows:

$$\frac{5\frac{1}{6}}{7\frac{2}{3}}=\frac{\frac{31}{6}}{\frac{23}{3}}=\frac{31}{6}\div\frac{23}{3}=\frac{31}{\not{6}_2}\times\frac{\not{3}}{23}=\frac{31}{46}.$$

Exercises XVIII.

Find the results of the following:

1. $\frac{3}{4}\div 5.$
2. $\frac{7}{8}\div 2.$
3. $\frac{3}{64}\div 4.$
4. $\frac{7}{16}\div 3.$
5. $\frac{7}{16}\div\frac{3}{4}.$
6. $\frac{18}{23}\div\frac{6}{5}.$
7. $\frac{7}{8}\div\frac{1}{4}.$
8. $125\frac{1}{2}\div\frac{1}{3}.$
9. $\frac{\frac{3}{4}\times\frac{2}{3}}{\frac{1}{2}\times\frac{1}{6}}.$
10. $\frac{7}{8}\times\frac{2}{3}\div\frac{5}{6}.$
11. $4\frac{3}{4}\times 1\frac{3}{5}-2\frac{1}{2}\times\frac{1}{15}.$
12. $3\frac{3}{7}\div 5\frac{5}{7}\times\frac{3}{40}+\frac{7}{40}.$
13. $12-1\frac{1}{8}\div 3\frac{3}{5}+4\frac{5}{9}\div 4\frac{1}{10}.$
14. $5\frac{5}{7}\times\frac{10}{40}-3\frac{3}{8}\div 2\frac{4}{7}.$
15. $\frac{1\frac{1}{8}\times 21\frac{1}{2}-9\frac{5}{16}}{8\frac{2}{3}+5\frac{3}{16}}.$
16. $3\frac{1}{3}\times 5\frac{1}{5}-\frac{3}{4}\div\frac{7}{8}+1\frac{1}{3}\div 3\frac{1}{9}.$

24. **Decimal Fractions.** The values of the figures in any number depend upon their position with reference to the decimal point.

Thus, $\cdot 2=2$ tenths $=\frac{2}{10}$

$\cdot 25=2$ tenths $+5$ hundredths $=\frac{2}{10}+\frac{5}{100}=\frac{25}{100}$

$\cdot 342=3$ tenths $+4$ hundredths $+2$ thousandths

$=\frac{3}{10}+\frac{4}{100}+\frac{2}{1000}=\frac{342}{1000}.$

In all such cases the decimal parts may be written as fractions with some power of 10 as denominator, and are therefore called decimal fractions.

25. **To change a Decimal Fraction to an equivalent Vulgar Fraction.** It is evident that it is only necessary to write the decimal, after removing the point, as numerator and 1 followed by as many 0's as there are figures in the decimal as *denominator*.

Exercises XIX.

Change to equivalent fractions in their lowest terms:

1. ·43.
2. ·04.
3. ·752.
4. ·7134.
5. ·502.
6. ·004.
7. ·705.
8. ·1234.
9. ·016.
10. ·000155.

26. To change a Vulgar Fraction to its equivalent Decimal Fraction. Example:—Change $\frac{1}{8}$ to its equivalent decimal.

$$\frac{1}{8}=1\div 8=\frac{8/1\cdot 000}{\cdot 125}=\cdot 125.$$

Example:—Change $\frac{11}{16}$ to its equivalent decimal. $\frac{11}{16}=11\div 16=$

```
16/11·0000/·6875
    9 6
    ---
     140
     128
     ---
      120
      112
      ---
        80
        80
        --
```

It is evident that, to change a fraction to its equivalent decimal fraction, it is only necessary to perform the division indicated after 0's have been placed to the right of the decimal point.

Exercises XX.

Change the following fractions to their equivalent decimals:

1. $\frac{1}{4}$.	4. $\frac{11}{16}$.	7. $\frac{31}{32}$.	10. $\frac{127}{128}$.
2. $\frac{1}{2}$.	5. $\frac{3}{5}$.	8. $\frac{15}{16}$.	11. $\frac{37}{50}$.
3. $\frac{3}{8}$.	6. $\frac{124}{125}$.	9. $\frac{24}{25}$.	12. $\frac{127}{250}$.

27. Repeating Decimals. Example:—Change $\frac{1}{3}$ to its equivalent decimal.

$$\frac{1}{3}=1\div 3=3/\underline{1\cdot 000}\quad \cdot 333+$$

The division in this case would never end. $\frac{1}{3}$ therefore produces what is known as a repeating decimal. This is expressed by placing a period above the figure 3 $\therefore \frac{1}{3}=\cdot\dot{3}$.

Example:—Change $\frac{5}{6}$ to its equivalent decimal.

$$\frac{5}{6}=5\div 6=6/\underline{5\cdot 0000}\quad \cdot 8333+$$

In this case the decimal does not begin to repeat until the second figure and is therefore called a mixed repeating decimal.

$$\therefore \frac{5}{6}=\cdot 8\dot{3}.$$

The denominators in Exercises XX contain only 2's or 5's or 2's and 5's as their factors. The fractions can be changed into fractions having some power of 10 as denominators and therefore give terminating decimals. All fractions such as $\frac{1}{3}$, $\frac{5}{6}$, etc., having some factor other than 2 or 5 in the denominator, when expressed in their lowest terms, cannot be changed into fractions having some power of 10 as denominator and therefore give repeating or mixed repeating decimals.

Exercises XXI.

Change the following to their equivalent decimals:

1. $\frac{5}{9}$. 2. $\frac{1}{12}$. 3. $\frac{1}{7}$. 4. $\frac{2}{15}$. 5. $\frac{1}{18}$. 6. $\frac{2}{11}$. 7. $\frac{4}{13}$.

28. To change Repeating and Mixed Repeating Decimals to their equivalent Fractions.

Example:—Change $\cdot\dot{2}\dot{4}$ to its equivalent fraction

$$\cdot\dot{2}\dot{4} = \cdot 242424 \ldots\ldots$$

$$100 \text{ times } \cdot\dot{2}\dot{4} = 24\cdot 242424 \ldots\ldots$$

$$1 \text{ times } \cdot\dot{2}\dot{4} = \cdot 242424 \ldots\ldots$$

$$\text{Subtracting, } 99 \text{ times } \cdot\dot{2}\dot{4} = 24$$

$$\therefore \quad \cdot\dot{2}\dot{4} = \tfrac{24}{99}.$$

That is to change a repeating decimal to its equivalent fraction write the decimal, after removing the point, as numerator and as denominator as many 9's as there are figures in the repeating part.

Example:—Change $\cdot 3\dot{4}$ to its equivalent fraction

$$\cdot 3\dot{4} = \cdot 34444 \ldots\ldots$$

$$100 \text{ times } \cdot 3\dot{4} = 34\cdot 444 \ldots\ldots$$

$$10 \text{ times } \cdot 3\dot{4} = 3\cdot 444 \ldots\ldots$$

$$\text{Subtracting, } 90 \text{ times } \cdot 3\dot{4} = 31$$

$$\therefore \quad \cdot 3\dot{4} = \tfrac{31}{90}.$$

That is to change a mixed repeating decimal to its equivalent fraction subtract the part which does not repeat from the whole giving the numerator, and for denominator take as many 9's as there are figures in the repeating part followed by as many 0's as there are figures which do not repeat.

Exercises XXII.

Express as fractions in their lowest terms:

1. $\cdot\dot{5}$	6. $\cdot\dot{3}6\dot{9}$	11. $2\cdot5\dot{3}0\dot{6}$
2. $\cdot\dot{3}\dot{6}$	7. $3\cdot25\dot{3}$	12. $\cdot04\dot{7}2\dot{6}$
3. $\cdot3\dot{6}$	8. $\cdot251\dot{6}$	13. $\cdot003\dot{6}$
4. $\cdot\dot{1}5\dot{3}$	9. $\cdot\dot{1}4285\dot{7}$	14. $\cdot04\dot{2}\dot{6}$
5. $\cdot3\dot{6}\dot{9}$	10. $2\cdot7\dot{6}$	

29. **Percentage.** The term "percent." usually written %, is an abbreviation of the Latin "per centum" which means by the hundred. Five percent. (5%) would be $\frac{5}{100}$ of the quantity named. Percent. may be changed to a decimal fraction.

$$\text{Thus, } 62\% = \tfrac{62}{100} = \cdot62.$$

$$37\cdot5\% = \tfrac{37\cdot5}{100} = \cdot375.$$

A decimal fraction of a quantity may be expressed as percent.

$$\text{Thus, } \cdot7 = \tfrac{70}{100} = 70\%$$

$$\cdot89 = \tfrac{89}{100} = 89\%$$

$$\cdot375 = \tfrac{37\cdot5}{100} = 37\cdot5\%$$

That is the decimal fraction may be changed to percent. by moving the decimal point two places to the right. Also any fraction may be changed to percent. by changing it to its equivalent decimal fraction, and then moving the decimal point two places to the right.

Exercises XXIII.

1. In the following table supply the missing quantities:

%	Decimal Fraction	Vulgar Fraction	%	Decimal Fraction	Vulgar Fraction	%	Decimal Fraction	Vulgar Fraction
1			$16\frac{2}{3}$			100		
	·02			·25		200		
$2\frac{1}{2}$					$\frac{1}{3}$		1·75	
		$\frac{1}{20}$	$37\frac{1}{2}$					$2\frac{1}{5}$
$6\frac{1}{4}$				·5			3·86	
	·10				$\frac{3}{4}$			$1\frac{7}{12}$
		$\frac{1}{8}$		·9		350		

2. Find 25% of 16, of 8, of 90, of 240.
3. 5 is what % of 10? of 20? of 40?
4. 8 is what % of 16? of 40? of 24?
5. What % of $\frac{3}{5}$ is $2\frac{1}{2}$? $27\frac{2}{5}$ of 600?
6. 20% of what number is 3? 7? 14? 17?
7. 68 is 15% less than what number?
8. 98 is 40% more than what number?

9. A gas bill was 25% higher last month than this. If it is \$6.46 this month how much was it last month?

10. How much water must be added to a 5% solution of a certain liquid to make a 2% solution? (original solution 20 gallons).

30. **Short Methods.** In practical work a large number of decimal places is not needed. In all measurements the accuracy depends upon the instruments, the methods used, and the thing measured. It is only necessary that the error is small compared with the quantity measured; a fraction of an inch in a dimension of several feet would probably not make much difference.

In measuring to ·001 inches it is not necessary to carry the work to say ·00001 inches. In any case of multiplication or division it is only necessary to carry the result to one decimal place more than the measurement. Thus if a measurement of 7·265 inches is multiplied by 3·1416 it is only necessary to carry the work to four places of decimals, care being taken to allow for numbers carried over from the fifth place.

Other short methods of multiplication and division may be used.

To multiply by 5, 50, 500, etc., add 0, 00, 000, etc., to the right of the number and divide by 2. Why?

To multiply by 25, 250, etc., add 00, 000 to the right of the number and divide by 4. Why?

To multiply by 125, add 000 to the right of the number and divide by 8. Why?

To multiply by $33\frac{1}{3}$, $16\frac{2}{3}$, $12\frac{1}{2}$, $8\frac{1}{3}$, $6\frac{1}{4}$. Add 00 to the right of the number and divide by 3, 6, 8, 12, 16. Why?

By using the reverse process division by $33\frac{1}{3}$, $16\frac{2}{3}$, $12\frac{1}{2}$, 125, etc., may be performed. Thus to divide by $33\frac{1}{3}$ multiply by 3 and divide by 100 or mark off two decimal places. Why?

To multiply a number ending in $\frac{1}{2}$ such as $13\frac{1}{2}$ by itself. Multiply the number plus 1 by itself and add $\frac{1}{4}$ to the product.

Thus $13\frac{1}{2}\times 13\frac{1}{2} = 14\times 13+\frac{1}{4}$.

To multiply a number ending in 5 by itself, multiply the number to the left of 5 by a number one greater than itself and place 25 to the right of the number. Thus, 75×75, $7\times8=56$, and the result is 5625.

Exercises XXIV.

Applied Problems.

1. From 2000 lb. of iron bars each weighing 80 lb. $\frac{2}{5}$ is cut up for bolts, $\frac{1}{5}$ for shafts and the remainder for studs. How many bars are used for the different articles?

2. At $2\frac{1}{2}$c. a pound, what will be the cost of 108 castings each weighing 29 lb.?

3. An automobile runs at the average rate of $10\frac{1}{2}$ miles an hour. How long will it take to go from Toronto to London, a distance of 116 miles?

4. A $\frac{3}{4}$ in. steel bar weighs 1·914 lb. per foot. What will be the cost of 5000 ft. of $\frac{3}{4}$ in. steel bars if it cost $1.75 per 100 lb.?

5. Which is cheaper, and by how much, to have a $36\frac{1}{2}$c. an hour man take $12\frac{1}{2}$ hr. on a job or to have a $48\frac{1}{2}$c. an hour man who can do the job in $9\frac{1}{2}$ hr.?

6. The weight of a foot of $\frac{9}{16}$ in. steel bar is 1·06 lb. Find the weight of a 20 ft. bar.

7. At $42\frac{1}{2}$c. an hr. what will be the pay for $21\frac{1}{4}$ days of 8 hours each?

8. If $2\frac{1}{2}$ bundles of shingles are used on $82\frac{1}{2}$ sq. ft. of roof, how many bundles will be used on 325 sq. ft. of roof?

9. How many pieces $5\frac{1}{2}$ in. long can be cut from a rod 27 ft. long?

10. A person spending $\frac{1}{3}$, $\frac{2}{5}$ and $\frac{1}{8}$ of his money has $119 left; how much had he at first?

11. If $\frac{4}{11}$ of a house be worth $1969.92, what is the value of $\frac{5}{16}$ of the house?

12. Three men own a house worth $6250; one owns $\frac{3}{10}$ of it; the second $\frac{1}{5}$ of it; what is the value of the third's share?

13. A man having $271\frac{1}{2}$ acres of land, sold $\frac{1}{3}$ to one man and $\frac{3}{8}$ to another; what was the value of the remainder at $323·68 an acre?

14. I want to mix up a pound of solder to consist of 4 parts zinc, 2 parts tin and 1 part lead; what fraction of a pound of each metal must I have?

15. An apprentice who is drilling and tapping a cylinder for $\frac{7}{8}$ in. studs, tries a $\frac{3}{4}$ in. drill, but the tap binds, so he decides to use a drill $\frac{1}{64}$ in. larger; what size drill will he use?

16. An 8 ft. bar of steel is cut up into 16 in. lengths; what fraction of the whole bar is one of the pieces?

17. The time cards for a certain piece of work show 2 hours and 15 minutes lathe work, 4 hours and 10 minutes milling, 2 hours and 20 minutes bench work; what is the total number of hours charged to the job?

18. A gallon is about $\frac{4}{25}$ of a cubic ft. If a cubic foot of water weighs $62\frac{1}{2}$ lb., how much does a gallon of water weigh?

19. What is the cost of a casting weighing $432\frac{1}{2}$ lb. at $6\frac{1}{4}$c. a pound?

20. How many steel pins to finish $1\frac{1}{8}$ in. long can be cut from an 8 ft. rod if we allow $\frac{3}{16}$ in. to each pin for cutting off and finishing?

21. A machinist whose rate is 67·5 cents per hour puts in a full day of 8 hours and also 3 hours overtime. If he is paid "time and a half" for overtime, how much should he be paid altogether?

22. If an alloy is ·67 copper and ·33 zinc, how many pounds of each metal would there be in a casting weighing 82 lb.?

23. A can do a piece of work in 25 days; B can do it in 30 days; C can do it in 35 days. In what time will they do it, all working together?

24. A man earns \$280 in $2\frac{1}{3}$ months. If he spends in 4 months what he earns in 3 months, how much will he save in a year?

25. From a farm of $125\frac{3}{10}$ acres there were sold at one time 27·63 acres and at another $34\frac{3}{8}$ acres. How many acres remained?

26. From an oil tank containing 375·087 gallons there leaked out each day $2\frac{5}{8}$ gallons. How many gallons remained in the tank at the end of 25 days?

27. If the weight of a brass casting is approximately fifteen and a half times that of its white pine pattern, what will be the weight of a casting if the pattern weighs 15 oz.?

28. Since the shrinkage of brass castings is about $\frac{1}{8}$ in. in 10 in., what length would you make the pattern for a brass collar which is required to be 6 in. long?

29. How long will it take a drill making 134 revolutions per minute (R.P.M.), at the rate of ·012 in. per revolution, to drill a hole $1\frac{1}{2}$ in. deep?

30. A piece of wrought-iron 2·69 in. thick is to have two $\frac{13}{16}$ in. holes drilled through it. If the drill makes 112 R.P.M., what must be the feed to drill each hole in two minutes? (The feed of a drill is the number of revolutions necessary to cause the drill to descend 1 in.).

31. In drilling a bed plate a drill makes 67 R.P.M., and is being fed to the work at the rate of ·015 in. per revolution, how deep will the hole be at the end of $4\frac{1}{2}$ minutes?

32. What will be the R.P.M. of a drill used for drilling a lathe spindle 30·24 in. long, the feed being ·015 in. per revolution, and the time given to the job being 21 minutes?

33. What must be the R.P.M. of a drill, feeding at the rate of ·015 in. per revolution, to drill a hole $2\frac{1}{2}$ in. deep in a casting in 2 minutes?

34. A casting is to have a number of holes drilled in it $2\frac{1}{2}$ in. deep with a high-speed drill making 260 R.P.M. What must be the feed to drill each hole in $\frac{3}{4}$ of a minute?

35. A man who owns $\frac{3}{4}$ of a claim sold $\cdot\dot{6}$ of his share for \$2000. What decimal part of the claim does he still own and what is the claim worth?

36. An engine rated at 1250 horse-power, is found to be 85% efficient. How many horse-power are available for driving the machinery? How many are lost?

37. A board was cut into two pieces, one $8\frac{3}{8}$ in. and the other $5\frac{9}{16}$ in. long. If $\frac{1}{16}$ in. be allowed for waste in cutting, what was the length of the board?

38. A locomotive has a piston displacement of 12656 cu. in. If the clearance space is 6·5% of the piston displacement, what is the clearance space?

39. A merchant bought 15 carloads of apples of 212 barrels each, 3 bushels in each barrel at 90c a bushel. He paid for them in cloth at 25c. a yard. How many rolls of 477 yd. each did he give?

40. A carload of pig-iron weighs 90,000 lb. If $11\frac{1}{9}$% of this is used at once in the foundry, how much is left?

41. The diameter of two holes is $3\frac{7}{8}$ in. and the distance between the sides of the holes is $3\frac{3}{4}$ in. What is the distance from the outside of one hole to the outside of the other?

42. From a steel bar $27\frac{5}{8}$ in. long were cut the following pieces:—one $7\frac{1}{4}$ in., one $6\frac{7}{8}$ in., one $3\frac{3}{4}$ in. long. If the length of the bar was then $8\frac{3}{8}$ in., what was the amount of waste in cutting?

43. A man, buying a house and lot, paid $2200 for the lot and $62\frac{1}{2}\%$ more than that for the house. What did both cost him?

44. A man invested $16,400 as follows:—25% in an automobile, $37\frac{1}{2}\%$ in bank stock, and the remainder in an addition to his house. How much did he invest in each?

45. An electrician has a reel of 300 ft. of copper wire. He used at various times $50\frac{1}{2}$ ft., $32\frac{1}{4}$ ft., $109\frac{2}{3}$ ft. How much wire was left? What percent. was left?

46. If $\frac{2}{3}$ of the shell of a stationary boiler is considered as the heating surface, how many square feet of heating surface are there in a boiler containing $98\frac{7}{16}$ sq. ft.?

47. A pump pumps $3 \cdot 38$ gallons to each stroke and the pump makes $51 \cdot 2$ strokes per minute. How many gallons of water will it pump per hour?

CHAPTER III.

WEIGHTS AND MEASURES—SPECIFIC GRAVITY.

31. **Linear Measure.** Linear Measure is used in measuring lines and distance.

The fundamental unit of English Linear Measure is the yard. It is the distance between two marks on a bronze bar in the Royal Exchange, London, England.

Table.

12 inches (in.)	= 1 foot (ft.).
3 ft.	= 1 yard (yd.).
$5\frac{1}{2}$ yd.	= 1 rod.
320 rods	= 1 mile.

Inches are commonly denoted by two strokes above the figure. Feet are denoted by one stroke. Thus 6 in. is written 6″ and 6 ft. is written 6′.

32. **Surveyor's Measure.** Surveyor's Measure is used in measuring land.

Table.

7·92 in.	= 1 link (li.).
100 li.	= 1 chain (ch.).
80 ch.	= 1 mile.
1 ch.	= 22 yd. = 66 ft.

The chain in this table is known as Gunter's chain. It is the one in general use for country surveys.

Engineers frequently use a chain, or steel tape, 100 ft. long. The feet are usually divided into tenths instead of into inches.

33. **Nautical Measure.**

Table.

6 ft.	= 1 fathom.
120 fathoms	= 1 cable.
6080 ft.	= 1 nautical mile = 1·151 statute miles.
1 knot	= a sailing rate of one nautical mile per hour.

Exercises XXV.

1. How many yards in a mile?
2. How many feet in a mile?
3. One inch is what decimal of a yard?
4. One rod is what decimal of a mile?
5. Reduce 18 yd., 2 ft., 9 in. to inches.
6. Reduce 3 mi., 30 rods, $1\frac{1}{2}$ yd. to feet.
7. Express 1 link as a decimal of a mile.
8. Express 1 in. as the decimal of a chain.
9. Change 4 chains, 15 links to links.
10. Change 26 yd., 1 ft., 2 in. to chains.
11. Change 4356 li. to feet.
12. Change 25 rods, 3 yd., 2 ft. to chains.
13. The world's record (Dec. 1919) for a destroyer was 45·5 knots. What is this in statute miles?

34. **Metric Linear Measure.** Metric is the adjective form of the word metre which is a French word meaning "measure." The earth's quadrant (one fourth of the circumference) was measured by French engineers in 1799. One ten-millionth of this length was taken as the length of the metre.

Table.

10 millimetres (mm.)	= 1 centimetre	(cm.)
10 cm.	= 1 decimetre	(dm.)
10 dm.	= 1 metre	(m.)
10 m.	= 1 decametre	(Dm.)
10 Dm.	= 1 hectometre	(Hm.)
10 Hm.	= 1 kilometre	(Km.)

It may be seen that the prefixes have definite meanings: milli = $\frac{1}{1000}$, centi = $\frac{1}{100}$, deci = $\frac{1}{10}$, deca = 10, hecto = 100, kilo = 1000.

35. **Comparison of English and Metric Linear Measurements.**

1 in.	= 2·5399 cm. (2·54 cm. approx.).
1 cm.	= ·3937 in.
1 mile	= 1·60935 Km. (1·61 Km. approx.).
1 Km.	= ·621 miles.
1 m.	= 39·3707 in. (39·37 in. approx.).

Make calculations to test the accuracy of the above table.

Exercises XXVI.

1. Measure the perimeter of the room with both metre stick and yard stick. Make drawings to scale in your laboratory book. Change the result in the English system to the Metric system and compare.

2. Do the same as in 1 for the door, table, etc.

3. Write all the measurements in the Metric system in terms of the metre.

4. Fill in the omitted entries in the following:

Unit	Equivalent	
	Inches	Feet
1 cm.		
1 dm.		
1 m.		
1 Dm.		
1 Hm.		
1 Km.		

5. A piece of steel bar is laid off to a length of 438 cm. Find this length in feet and inches.

6. The thickness of a steel plate is $\frac{3}{8}''$. Find the thickness in cm. and dm.

7. A speed of 200 ft. per second is how many Km. per second?

8. When a body falls freely from rest it increases in speed each second 32·2 ft. per second. Express this in cm. per second each second.

9. An express train is travelling at the rate of 50 miles per hr. Express this in Km. per minute.

10. Find the difference in cm. between the lengths of two steel rods, one of which is 4·8′ long and the other 4·8″ long.

36. **Square Measure.** In measuring areas or surfaces, the inch, foot, yard, etc., can no longer be used. It is necessary to use the square inch, the square foot, the square yard, etc.

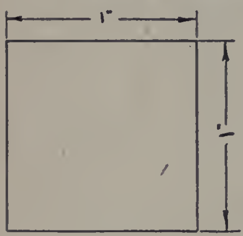


FIG. 9

By a square inch is meant a surface one inch long and one inch wide.

Thus in measuring surfaces two dimensions, **length** and **breadth,** are used.

Table.

144 square inches (sq. in.)	= 1 square foot (sq. ft.).
9 sq. ft.	= 1 square yard (sq. yd.).
$30\frac{1}{4}$ sq. yd.	= 1 square rod (sq. rod).
160 sq. rods	= 1 acre.
10 sq. chains	= 1 acre.
640 acres	= 1 square mile (sq. mi.).

Make drawings to scale in your laboratory book and illustrate the truth of the first three lines in the above table.

37. **Metric Square Measure.**

Table.

100 square mm. (sq. mm.)	= 1 square cm. (sq. cm.)
100 sq. cm.	= 1 square dm. (sq. dm.)
100 sq. dm.	= 1 square m. (sq. m.)
100 sq. m.	= 1 square Dm. (sq. Dm.)
100 sq. Dm.	= 1 square Hm. (sq. Hm.)
100 sq. Hm.	= 1 square Km. (sq. Km.)

Make drawings to scale in your laboratory book and illustrate the truth of each line in the above table.

38. Comparison of English and Metric Square Measure.

Table.

1 sq. in.	= 6·4516	sq. cm.
1 sq. cm.	= ·155	sq. in.
1 sq. ft.	= ·0929	sq. m.
1 sq. m.	= 10·764	sq. ft.
1 sq. yd.	= ·8361	sq. m.
1 sq. m.	= 1·196	sq. yd.

Make calculations to test the accuracy of the above table.

Exercises XXVII.

1. Find the area of the floor of your classroom in square metres and also in square feet. Make drawings to scale in your laboratory book. Change the area in square metres to square yards and compare.

2. Find the area of a page of your laboratory book in sq. in. and also in sq. cm. Test as in preceding question.

3. Perform similar experiments by measuring the school-yard, the door, table, the teacher's desk, etc.

4. Change one acre to sq. yd.

5. Express 4 sq. rods, 25 sq. yd., 7 sq. ft., in sq. ft.

6. Express 5 sq. rods, 8 sq. yd., 5 sq. ft., as the decimal of an acre.

7. Express 5 sq. yd., 3 sq. ft., 18 sq. in., as sq. in.

8. Express 4 sq. ft., 85 sq. in., as the decimal of a sq. yd.

9. A square field measures 20 rods to a side. Find its area in acres.

10. A steel plate in the form of a rectangle is $18\frac{1}{2}''$ long by $6\frac{1}{4}''$ wide. Find the area in sq. ft.

11. A number-plate on an automobile is $21''$ long by $5\frac{1}{4}''$ wide. Find area in sq. ft.

12. A rectangular garden $2\frac{1}{2}$ chains wide contains $\frac{3}{4}$ of an acre. How many feet long is it?

13. How many sq. ft. of glass are there in a box containing 72 panes each $12''$ by $16''$?

14. How many sq. yd. are there in the walls of a room $15'$ $6''$ long, $12'$ wide, and $9'$ $4''$ high?

15. A rectangular piece of land measures 1200 links by 180 links. What is its area in acres?

16. How many bricks 8 in. long and 4 in. wide will pave a yard 116′ long and 46′ wide?

17. Find the cost of laying a concrete walk 400 yd. long and 4 ft. 8 in. wide at 60c. a sq. yd.

18. Find the cost of painting both sides of a tight board fence 80′ long, 5′ 3″ wide at 7c. a sq. yd.

19. How many boards each 12′ long and 10″ wide will be required to build a fence 60 yd. long and 4 ft. high?

20. How many sq. ft. of tin will be necessary to line the inside of an open box whose external measurements are 4′ long, 3′ 8″ wide and 2′ 10″ deep, if the material in the box is 2″ thick and 10% is allowed for cutting and joining the tin?

39. **Cubic Measure.** In the measurement of surfaces in the preceding sections two measurements, length and breadth, were used. The areas resulting were expressed in square inches, square feet, etc.

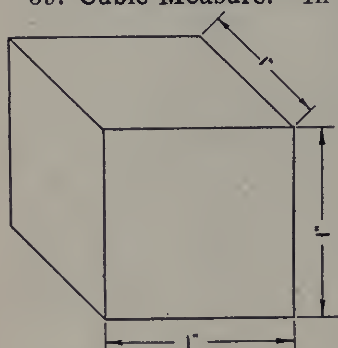


Fig. 10

If it is required to measure the volume of solids, the dimensions, length and breadth must be taken into account and in addition another dimension—**thickness.**

By a cubic inch is meant the volume of a cube, 1 inch on each edge, Figure 10.

Volumes of solids are measured in cubic inches, cubic feet, cubic yards, etc.

Table.

1728 cu. in. = 1 cu. ft.
27 cu. ft. = 1 cu. yd.
128 cu. ft. = 1 cord (8′×4′×4′).

Make drawings to scale in your laboratory book and illustrate the truth of each line in the above table.

40. **Metric. Cubic Measure.**

Table.

1000 cubic millimetres (c.mm.)	= 1 cubic centimetre	(c.c.)
1000 c.c.	= 1 cubic decimetre	(c.dm.)
1000 c.dm.	= 1 cubic metre	(c.m.)
1000 c.m.	= 1 cubic decametre	(c.Dm.)
1000 c.Dm.	= 1 cubic hectometre	(c.Hm.)

41. Comparison of English and Metric Cubic Measure.

Table.

1 cu. in.	= 16·387064 c.c.	= 16·387	c.c.	(approx.)
1. c.c.	= ·06102 cu. in	= ·061	cu. in.	(approx.)
1 cu. ft.	= ·02831 c.m.	= ·028	c.m.	(approx.)
1 c.m.	= 35·3163 cu. ft.	= 35·316	cu. ft.	(approx.)

Make calculations to test the accuracy of the above relations.

Exercises XXVIII.

1. Find the volume of the top of the laboratory table in cu. ft. and in c.m. Make drawings in your laboratory book. Change from one system to the other and compare.

2. Find the volumes of the various rectangular models in the laboratory, in cu. in. and also in c.c. Make drawings in your laboratory book. Change from one system to the other and compare.

3. Change 8 cu. yd., 9 cu. ft., to cubic feet.

4. Change 3 cu. yd., 2 cu. ft., 8 cu. in., to cubic inches.

5. A rectangular vessel is 15″ long, $6\frac{1}{2}$″ wide and $4\frac{1}{4}$″ deep, inside measurements. Find its volume in cubic centimetres.

6. A gravel bed whose surface has an area of 2 acres, contains gravel to a depth of 10″. How many miles of road 12′ wide can be covered with the gravel if it be spread to a uniform depth of 7″?

7. The outside measurements of a cubical box, with a lid, are 3′ 4″ long, 2′ 8″ wide and 1′ 10″ deep. If the box is made of 1″ material, how many cu. ft. of material are there in the box? How many cubic metres will it hold?

8. A cubical cistern, without a lid, 4′ 4″ long, 4′ 4″ wide and 6′ 8″ deep, outside measurements, is made of plank 2″ thick. How many cu. ft. of material are there in the box? How many cu. ft. of water will it hold?

9. A pile of wood 10′ long, 4′ wide and 6′ high was sold for $20.00. What was the price per cord?

10. A wood-yard 20′ long and 18′ wide is filled with cord wood to a height of 6′. What is the wood worth at $8.50 a cord?

11. If 1 cu. yd. of earth make a load, how many loads will be removed in excavating for a foundation 4′ deep, 36′ 3″ long, and 24′ wide?

12. The end of a rectangular bar of iron is a square $\frac{3}{4}''$ to the side. How many c.c. are there in 4′ of the bar?

13. In excavating a tunnel 374,166 cu. ft. of earth were removed. If the length of the tunnel was 492 ft. and the width 39 ft., what was the depth?

14. In making a tender for some excavating a contractor notes that the excavation is in the shape of a rectangle 11′ wide, 86′ long at the top and has a depth of 8′. What will it cost him to excavate it at 40c. a cu. yd.? What must he bid to make a profit of 15%?

15. Rain falling uniformly for 5 hours on a roof, whose dimensions are 30′ by 15′, fills a tank 6′ 3″ by 3′ by 2′ 6″. Find the depth of the rainfall per hour.

16. The ice on a pond whose area is $\frac{1}{3}$ of an acre is 10″ thick. How many cu. ft. of ice may be removed?

42. **Measures of Weight.** The fundamental unit of English weight is the pound. There are the pound Avoirdupois and the pound Troy.

The pound, Avoirdupois, is equal to the weight of 7000 grains (plump grains of wheat) and is used for all ordinary purposes of weighing. The pound, Troy, is equal to 5760 grains and is used in weighing gold, silver and precious stones.

Table—Avoirdupois Weight.

16 drams	= 1 ounce (oz.).
16 oz.	= 1 pound (lb.) = 7000 grains.
100 lb.	= 1 hundredweight (cwt.).
20 cwt.	= 1 ton.
2240 lb.	= 1 long ton.

Table—Troy Weight.

24 grains	= 1 penny weight (dwt.)
20 dwt.	= 1 oz.
12 oz.	= 1 lb. = 5760 grains.

43. **Metric System of Weights.** The fundamental unit of metric weight is the kilogram which is the weight of 1 litre, equal in volume to 1 cubic decimetre, of distilled water under fixed conditions of temperature and pressure.

Table.

10 milligrams	= 1 centigram	(cg.)
10 cg.	= 1 decigram	(dg.)
10 dg.	= 1 gram	(g.)
10 g	= 1 decagram	(Dg.)
10 Dg.	= 1 hectogram	(Hg.)
10 Hg.	= 1 kilogram	(Kg.)

44. **Comparison of English and Metric Systems of Weights.**

Table.

1 gram = 15·432 grains.
1 ounce = 28·35 grams.
1 pound (avoirdupois) = 453·6 grams.
= ·4536 kilograms.
1 kilogram = 2·2046 pounds.
1 metric ton = 1000 kilograms.
= 2204·6 pounds.

Knowing any one of the above relations test the accuracy of the others.

45. **Measures of Capacity.** The fundamental unit of capacity in the English system is the gallon, which contains 10 pounds of distilled water under fixed conditions of temperature and pressure.

46. **Liquid Measure**—used in measuring liquids.

Table.

4 gills	= 1 pint	(pt.)
2 pt.	= 1 quart	(qt.)
4 qt.	= 1 gallon	(gal.)

47. **Dry Measure**—used in measuring grains, vegetables, etc.

Table.

2 pints	= 1 quart	(qt.)
4 qt.	= 1 gallon	(gal.)
2 gal.	= 1 peck	(pk.)
4 pk.	= 1 bushel	(bu.)

48. **Metric System.** The fundamental unit of measurement is the litre and is equal in volume to one cubic decimetre.

Table.

10 millilitres	= 1 centilitre	(cl.)
10 cl.	= 1 decilitre	(dl.)
10 dl.	= 1 litre	(l.)
10 l.	= 1 decalitre	(Dl.)
10 Dl.	= 1 hectolitre	(Hl.)
10 Hl.	= 1 kilolitre	(Kl.)
	= 1 cu. metre	

49. **Comparison of Capacity Tables with Cubic Measure.**

1 litre = 61·024 cu. in. (approx.)
= ·22 gal.
1 gal. = 4·54 l.
1 cu. ft. = 28·38 litres.
= 6·2321 gal.

277·274 cu. in. = 1 gal. 231 cu. in. = 1 gal. (American).

50. **Specific Gravity.** The specific gravity (sp. gr.) of a substance is its weight as compared with the weight of an equal volume of pure water.

Since the weight of a fixed volume of water is known we can find the weight of an equal volume of any substance if we know the specific gravity.

Example:—Find the weight of 8 cu. ft. of steel if its sp. gr. is 7.8.

Solution:—1 cu. ft. water weighs 62·321 lb.
1 cu. ft. steel weighs $62 \cdot 321 \times 7 \cdot 8$ lb.
8 cu. ft. steel weighs $62 \cdot 321 \times 7 \cdot 8 \times 8$ lb. = 3888·83 lb.

Exercises XXIX.

1. How much space will be filled by 14 tons of wrought-iron (sp. gr. 7·7)?

2. Find the average sp. gr. of a piece of brick construction weighing 114 lb. per cu. ft.

3. If 13 litres of milk weigh 13·39 kilograms, what is the sp. gr. of milk?

4. A tunnel 625 yd. long having a cross-section of 64 sq. yd. is excavated through rock of sp. gr. 2·7. Find the weight of rock removed.

5. If 3 litres of alcohol weigh 2·37 kilograms, what is the sp. gr. of alcohol?

51. Measure of Time:

Table.

60 seconds (″)	= 1 minute (1′).
60 minutes	= 1 hour.
24 hours	= 1 day.
7 days	= 1 week.

365 days, 5 hours, 48 minutes, 48 seconds = 1 year.

As the calendar year of 365 days is nearly 6 hours less than the above, correction is made as follows:—Every year whose number is divisible by 4 is a leap year and contains 366 days, the other years containing 365 days, except that the century years are leap years only when the number of the year is divisible by 400.

The year is divided into 12 months:—January (Jan.), February (Feb.), March, April, May, June, July, August (Aug.), September (Sept.), October (Oct.), November (Nov.), December (Dec.).

" Thirty days hath September, April, June and November." The other months, except February, have 31 days each. February has 29 days in leap years and 28 days in all other years.

Exercises XXX.

1. Compute the actual number of days from Sept. 23, 1919, to April 6, 1920.

2. A note bearing interest from March 8, 1899, was paid on July 5, 1900. Compute the interest period.

3. Reduce to the lowest denomination named:—4 weeks, 3 days, 15 hr. 23 min.

4. How many hours between 10 A.M. Jan. 1, 1920, and 6 P.M. March 3, 1920.

52. Miscellaneous Measures:

Counting Tables.

12 things = 1 dozen (doz.).
12 doz. = 1 gross.
12 gross = 1 great gross.
20 units = 1 score.

Stationers' Tables.

24 sheets = 1 quire.
20 quires = 1 ream.
3 reams = 1 bundle.
5 bundles = 1 bale.

Exercises XXXI.

1 Calculate the volumes of a number of the rectangular solid models in the laboratory and estimate their weights in both systems. Change from one system to the other and check.

2. Fill in the omitted entries in the following:

QUANTITY	VOLUME		WEIGHT	
	cu. in.	c.c.	wt. in lb.	wt. in Kg.
2 pints water				
3 qt. water				
1 cu. ft. water				
1 gal. water	277·274		10	
10 c.c. water		10		
1. Kl. water				

3. A rectangular tank is 2·5 m. long, 1·4 m. wide, and ·98 dm. deep. Find its capacity in litres. Find the weight of water it will hold in grams.

4. The thickness of a steel plate is $\frac{5}{8}''$. If the plate has an area of 400 sq. dm., find its volume in cu. in. and its weight in lb. if 1 cu. in. of steel weighs ·283 lb.

5. A block of granite weighs $2\frac{1}{2}$ tons. Find its weight in kilograms.

6. Find the weight in grams of the air in a room $16' \times 10'$ and 9′ high, if the air is ·00128 times as heavy as water.

7. Find the number of litres in a rectangular tank $8' \times 6'\ 6'' \times 4'\ 3''$.

8. How many gallons of water are contained in a tank 6 metres long, 3·4 metres wide, and 2·7 metres deep?

9. A concrete watering trough is $3\frac{1}{2}'$ wide, 8′ long and 2′ deep outside while inside the basin is 2′ 10″ wide, 7′ 4″ long and 1′ 6″ deep. What is its weight if a cu. ft. of concrete weighs 145 lb.? If the concrete was mixed in the proportion of 1 cement, 2 sand, 3 stone, and $1\frac{1}{2}$ cu. yd. dry material makes 1 cu. yd. concrete, how many bags of cement were used. (1 bag = 1 cu. ft.)?

CHAPTER IV.

SQUARE ROOT.

53. **The Square of a Number** is the product obtained by multiplying the number by itself. Thus the square of $5 = 5 \times 5 = 25$.

The square root of a given number is that number whose square is the given number. Thus the square root of 25 is 5 because $5 \times 5 = 25$.

Square root is indicated by prefixing the symbol $\surd$ to the given number. Thus $\sqrt{64}$ denotes the square root of 64.

When a number is small the square root may be found by inspection or by means of the factors of the number. Thus $1225 = 5 \times 5 \times 7 \times 7 = 5^2 \times 7^2$ so that $\sqrt{1225} = \sqrt{(5^2 \times 7^2)} = 5 \times 7 = 35$.

The following general method may be used for finding the square root. To find the square root of 1326·4164.

	1326·4164/36·42
	9
66	426
	396
724	3041
	2896
7282	145 64
	145 64

Explanation:—Beginning at the decimal point, separate the number into groups of two figures each, counting both to the right and the left. Find the greatest square in the left-hand group and write its square root as the first figure of the root.

In the example, 9 is the greatest square in 13, and 3 is the first figure in the root.

Subtract the square from the left-hand group and to the remainder bring down the next period to the right, thus forming a new dividend.

In the example, 9 is subtracted from 13 and along with the remainder 4 the next group 26 is brought down, giving 426 as the new dividend. Divide the new dividend, with its right-hand figure omitted, by twice the part of the root already obtained and annex the result to both the root and the divisor.

Multiply the complete divisor by the last figure of the root obtained, subtract, and bring down the next group to form a new dividend as before.

In the example the 3 in the root is doubled giving 6, 6 is now divided into 42 giving 6, and this figure is placed to the right of the 6 already in the divisor and also as the second figure of the root. Although 6 divided into 42 gives 7, if this result is taken the result 67×7 gives a quantity too great to subtract from 426, so that 6 must be taken instead. Proceed in this manner until all the groups are used.

For every group to the right of the decimal point there must be a decimal figure in the root.

When the number is not an exact square the root may be obtained to any number of decimal places.

Exercises XXXII.

Find the square root (correct to four decimal places) of:

1. 2025. 2. 39601. 3. 15129. 4. 106929. 5. 1369.
6. 3. 7. 12·186. 8. 143·2041. 9. ·5432. 10. ·06285.

11. Find the length in yards of the side of a square 10 acre field.

12. A square pipe has an area of 136·0752 sq. in. What is the length of its side?

13. An outlet on a heating system is 4′ 4″ wide and 18″ high. A pipe leading from it must have the same area and must be square. Find the size of the square pipe.

14. Would it be cheaper to build the square pipe or one of the same dimensions as the outlet? Why?

15. A steel plate is rectangular in shape, $18'' \times 14''$. Find the side of a square plate of the same area.

54. One of the most Valuable Practical Uses of Square Root is in finding the third side of a right-angled triangle, when two of its sides are given.

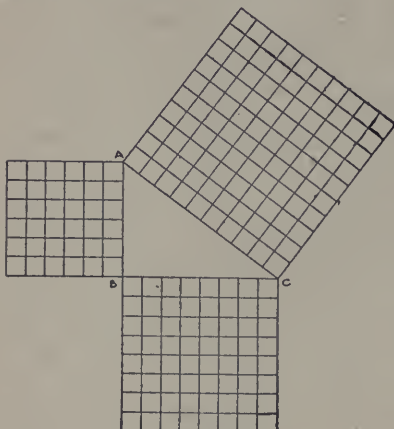


Fig. 11

In the adjoining figure **ABC** is a right-angled triangle with the sides **AB** and **BC** 3 in. and 4 in. respectively (to scale).

Squares are described on the three sides and divided into smaller squares as indicated. If we make tests with dividers we will find that the small squares are equal throughout the figure. We will also notice that the number of small squares in the square on **AC** is equal to the total of the number of small squares in the squares on **AB** and **BC**.

From this experiment we derive:—*In a right-angled triangle the square on the side opposite the right angle* (*hypotenuse*) *is equal to the sum of the squares on the other two sides.*

Exercises XXXIII.

1. Find the distance from corner to corner of a square piece of tin which contains 100 sq. in.

2. A room is $40' \times 28'$. Find the length of a diagonal.

3. If the above room is 16′ high, find the distance from any corner to the diagonal corner of the ceiling.

4. A baseball diamond is in the form of a square 90′ to the side. Find the distance from "first" to "third."

5. A boy was flying a kite with a string 650′ long. If the distance, from where the boy was standing, to a point directly under the kite was 450 ft. how high was the kite?

6. A tree broke in such a way that the top struck the ground 30′ from the base of the tree. What was the height of the tree, the broken part being 60 ft. long?

7. A ladder, 42′ long, placed with its foot 24′ from a wall, reached within 2′ of the top. How near the wall must the foot of the ladder be brought in order that it may reach the top?

CHAPTER V.

APPLICATION OF MEASURES TO THE TRADES.

55. Stone Work. In stone work it is difficult to get any fixed method of estimating the cost. One job will have a set of conditions which do not exist in another, hence the contractor will make an allowance in one that he would not regard as necessary in another. There are two kinds of stone work, **rubble** or **rough** and **ashlar** or **squared.** In rubble work the **toise** is the common unit of measurement. This is used both in estimating the amount of stone required and in the cost of the work.

Table.

10 tons rubble = 1 toise (approx.)
1 toise—in wall = 162 cu. ft. (approx.)
1 toise—measured loose = 216 cu. ft. (approx.)

In ashlar work the unit for estimating either the amount of stone necessary, or the cost of laying, is the **cubic foot.** The labour for dressing the stone is figured by the **square foot.** The minimum thickness of stone work for facing is 4 in. increasing in thickness as requirements demand.

Exercises XXXIV.

1. How many toise of rubble will be required for the foundation of a house 40′ 0″ × 32′ 0″, the stone work being 5′ 0″ high and 18″ thick?

2. A cellar is 23′ 6″ wide by 35′ 8″ long and 6′ 6″ high. If the wall is 16″ thick and has two openings each 3′ 3″ × 2′ 3″, find the number of toise of stone required.

3. The basement walls for a house 26′ 0″ wide and 38′ 0″ long are to have 6 windows each 3′ 0″ × 2′ 0″. The walls are to be 7′ 0″ high and 18″ thick. (*a*) Find the cost at $20.00 a toise if the actual volume be estimated and 5% be allowed for extra work on openings. (*b*) Find the cost at $18.00 a toise if corners be doubled and only 50% of the openings be deducted.

4. A foundation wall for a building 28′ 0″ × 40′ 0″ is to be 7′ 0″ high and 1′ 6″ thick. There are to be 4 openings, two 3′ 0″ × 2′ 6″ and two 3′ 0″ × 5′ 0″. Concrete is to be used in the construction and is to be mixed in the following proportions:—1 cu. ft. (1 bag) of cement, $2\frac{1}{2}$ cu. ft. sand and 5 cu. ft. broken stone. If $1\frac{1}{2}$ cu. yd. of dry material will make 1 cu. yd. of concrete, find the number of cu. ft. of cement, of sand, and of broken stone.

5. A building 24′ 6″ wide, 36′ 0″ long and 20′ 0″ high, above the foundation, is to be of stone with walls 16″ thick. The foundation, 6′ 0″ high, 16″ thick, is to be concrete and to have 6 windows 1′ 10″ × 3′ 4″. If a cu. yd. of concrete requires 25 cu. ft. of stone, 12 cu. ft. of sand, and 4 cu. ft. of cement, find the number of cu. ft. of each in the foundation. In the walls of the house there are to be 8 windows 2′ 0″ × 5′ 0″, 3 windows 3′ 6″ × 5′ 0″ and 3 doors 3′ 6″ × 7′ 0″. How many cu. ft. of stone will be necessary?

56. **Brick Work.** There is the same lack of uniformity in methods of estimating cost in brick work as in stone work. In measuring up the cost of the work some contractors make no deduction for openings less than 2 ft. square. Usually, however, the exact volume of the brick work is estimated and, in fixing the cost, allowance is made for extra labour and material for arches, cuttings, etc.

Since bricks are of varying size no fixed rule for the volume of laid brick can be given. If we consider an ordinary stock brick as $8\frac{5}{8}'' \times 2\frac{1}{2}'' \times 4''$ and add a $\frac{3}{8}''$ joint to thickness, length and width we get $9'' \times 2\frac{7}{8}'' \times 4\frac{3}{8}''$ or approximately $9'' \times 3'' \times 4\frac{1}{2}''$. The number of bricks for 1 cu. ft. of masonry would then be $\frac{1728}{9 \times 3 \times 4\frac{1}{2}} = 14\frac{2}{9}$.

Table—(Based on above calculation).

Per cubic foot,	15 bricks.
Superficial foot of 9″ wall,	11 bricks.
Superficial foot of 13″ wall,	$16\frac{1}{2}$ bricks.
Superficial foot of 18″ wall,	22 bricks.

The labour and material for brick work are usually estimated by the 1000 brick, if in a straight wall.

Exercises XXXV.

1. Make drawings to scale, in your laboratory book, of bricks of different sizes. Allowing a $\frac{3}{8}''$ joint calculate the number of bricks that will be required for a wall 20′ 0″ long, 8′ 0″ high, and 18″ thick.

2. A house is to have 27′ 0″ frontage, 30′ 0″ in depth, and 20′ 0″ in height above foundation. It is to have 8 windows 4′ 6″ × 5′ 6″ and 4 doors 4′ 3″ × 7′ 0″. The wall is to be 9″ thick; allowing 15 bricks to the cu. ft., how many bricks will be required?

3. If the wall in the preceding question is 13″ thick, find the number of bricks.

4. What will it cost to lay the brick in each of the two preceding questions if a bricklayer lays an average of 700 a day and received 90c. an hour for an eight hour day?

5. The walls of a building 40′ 0″ wide and 100′ 0″ long are to be 18′ 0″ high. There are 4 doors 8′ 0″ × 8′ 0″, 4 doors 3′ 3″ × 7′ 0″, 30 windows 4′ 0″ × 5′ 0″. Making use of the table for superficial area find the number of bricks required, if the wall is 13″ thick?

6. Reckoning 15 bricks per cu. ft., find the cost at $30 a thousand for the walls of a building 30′ 0″ wide, 50′ 0″ long and 24′ 0″ high with the following specifications:—the lower 14′ 0″ is to have a wall 18″ thick and is to have 4 doors 2′ 10″ × 6′ 10″ and 5 windows 3′ 0″ × 7′ 0″; the upper 10′ 0″ is to have a wall 13″ thick, and is to have 6 windows 3′ 0″ × 5′ 0″.

57. **Lumber.** The common unit of measurement in lumber is the **board** foot.

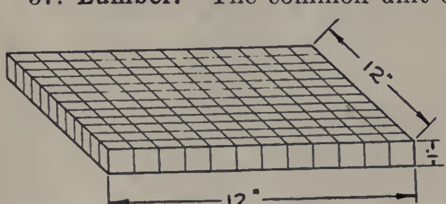


Fig. 12

It is a piece of lumber 1 ft. long, 1 ft. wide, and 1 in. thick.

If we take a board 12 ft. long, 12 in. wide, and 1 in. thick, we readily see that it will contain 12 board feet.

This might have been obtained as follows:—length (in feet) $\times$ width (in feet) $\times$ thickness (in inches), thus $12 \times 1 \times 1 = 12$.

This rule is applicable for finding the board feet of all kinds of lumber. Example:—Find the number of board feet of lumber in a floor joist 2″ $\times$ 10″, 18′ 0″ long.

Solution:—Number of board feet = length (in ft.) $\times$ width (in ft.) $\times$ thickness (in in.) $= 18 \times \frac{10}{12} \times 2 = 30$.

Lumber is billed in different ways, (1) per thousand (M) board feet, (2) per thousand (M) sq. ft., (3) per foot run.

Speaking generally we may say that, in dealing with material 1″ thick and up, the board foot is the unit, although special sizes up to 2″ $\times$ 3″ are frequently charged as per foot run. Below 1″ in thickness material is reckoned in sq. ft., except "trim" which is sold as per foot run.

FIG. 13

The following data for estimating the amount of allowance for dressing and working the tongue in flooring is furnished by one of the large lumber companies of Toronto:

$1\frac{1}{2}$″ wide, $\frac{7}{8}$″ thick, add 50%

$1\frac{1}{2}$″ wide, $\frac{3}{8}$″ thick, add 33%

2″ wide, $\frac{7}{8}$″ thick, add $37\frac{1}{2}$%

2″ wide, $\frac{3}{8}$″ thick, add 25%

$2\frac{1}{4}$″ wide, $\frac{7}{8}$″ thick, add $33\frac{1}{3}$%

Example:—Find the cost of flooring a room 20′ $\times$ 10′ with No. 1 red oak flooring, $1\frac{1}{2}$″ $\times$ $\frac{7}{8}$″, at \$160 per M sq. ft.

Solution:—Area of floor $= 20 \times 10$ sq. ft.

Lumber required $= \frac{150}{100} \times 20 \times 10$ sq. ft.

Cost $= \frac{150}{100} \times 20 \times 10 \times \frac{160}{1000} = \$48.00$.

The following is a sample bill of lumber:

	Feet	Price	Amount	
125 ft. lineal, $1\frac{3}{8}'' \times 6''$, Pine D4S..	125	\$7.25	\$ 9.06	
400 ft. lineal, $1'' \times 1''$, Pine Rgh..	400	1.00	4.00	
130 ft. lineal, $\frac{7}{8}'' \times 10''$, Pine D4S..	130	8.00	10.40	
130 ft. lineal, $\frac{7}{8}'' \times 2\frac{1}{2}''$, Bed Mldg.	130	3.00	3.90	
310 ft. lineal, $\frac{7}{8}'' \times 1\frac{3}{4}''$, Fir Picture Mldg....................	310	3.00	9.30	
500 ft. B.M., 1″ No. 1 H. D1S....	500	62.00	31.00	
2000 ft. Strip 6″ H. Decking.......	2000	66.00	132.00	
2000 ft. Strip $\frac{7}{8}''$ Spruce Flg.........	2000	68.00	136.00	
14 pieces, $2'' \times 4'' \times 12'$, H. Szd...	112	63.00	7.06	
70 pieces, $2'' \times 10'' \times 10'$, No. 1 H. Szd.....................	1167	65.00	75.86	
100 ft. lineal, $2'' \times 2''$, H. Rgh.....	100	2.00	2.00	
100 ft. lineal, $3'' \times 2''$, H. Rgh......	50	65.00	3.25	
5 pieces, $2\frac{3}{4}'' \times 5\frac{3}{4}'' \times 10'$, Oak Sill.	50	.75	37.50	
			Total.	461.33

D4S—dressed on four sides.
Rgh.—rough.
Mldg.—moulding.
Flg.—flooring.
Szd.—sized.
No. 1—No. 1 (best quality).
H.—hemlock.
Lin.—per foot run.

Exercises XXXVI.

1. Take measurements of a number of pieces of lumber obtained from the woodworking shop. Make drawings in your laboratory book and estimate the board feet in each.

2. Measure the top of a laboratory table, the top of the teacher's desk, etc. Make drawings in your laboratory book and estimate the board feet in each.

3. Take measurements of the floor of your classroom and make a drawing in your laboratory book. Find the cost of flooring with birch $2\frac{1}{2}''$ wide and $\frac{7}{8}''$ thick at \$140 per thousand square feet.

4. By means of a drawing in your laboratory book, show the number of board feet in a cubic foot.

5. Find the number of cu. ft. in a stick of timber 6″ × 8″ × 18′ 0″. Change to board feet and check by rule.

6. An oak stick is 8″ × 8″ × 30′ 0″. Find its volume by cubic measure. Change to board feet and check by rule.

7. A lot 60′ 0″ frontage and 120′ 0″ in depth is to be enclosed on two sides and an end by a tight board fence 6′ 0″ high. The posts are to be placed 6′ 0″ apart and to cost 40c. each; there are to be two string pieces 2″ × 4″ from post to post on which to nail the boards; the boards are to be 1″ thick. If lumber is worth $56 per M, find the total cost of same.

8. What will it cost at $52 per M to cover the floor of a barn 32′ 0″ × 42′ 0″ with 2″ square plank?

9. A room is 12′ 0″ wide and 16′ 0″ long. Find the cost, at $175 per thousand square feet, of laying a No. 1 red oak floor, the material being $1\frac{1}{2}$″ wide and $\frac{3}{8}$″ thick.

10. Complete the following bill of lumber:

	Feet	Price	Amount
500 ft. lineal, $\frac{7}{8}$″×$3\frac{3}{4}$″, Pulley stile......		$4.00	
500 ft. lineal, $\frac{7}{8}$″×$3\frac{3}{4}$″, Lining..........		2.50	
500 ft. lineal, 7″×$3\frac{1}{4}$″, Lining..........		2.50	
500 ft. lineal, Parting stop.............		1.00	
150 ft. lineal, 2″×6″, Sash sill.........		9.50	
125 ft. lineal, $1\frac{3}{8}$″×6″, Pine D4S........		7.25	
400 ft. lineal, $\frac{1}{2}$″ ×6″, Backing.........		2.00	
200 ft. lineal, 2″ ×6″, Door jamb......		9.50	
		Total.	

Note—Prices are for 100 ft. lineal.

11. Complete the following bill of lumber:

	Feet	Price	Amount
44 pieces, 2″ × 12″–20′ 0″ long, Red Pine.		$84.00	
10 pieces, 8″ × 14″–16′ 0″ long, Red Pine.		86.00	
105 pieces, 2″ × 4″–10′ 0″ long, Hem. Szd		63.00	
15 pieces, 2″ × 4″–12′ 0″ long, Hem. Szd		63.00	
32 pieces, 2″ × 4″– 8′ 0″ long, Hem. Szd		62.00	
17 pieces, 2″ × 6″–16′ 0″ long, Com. Pine		86.00	
23 pieces, 2″ × 6″–12′ 0″ long, Hem. Szd.		65.00	
9 pieces, 2″ × 8″–14′ 0″ long, Hem. Szd.		66.00	
		Total.	

12. Complete the following bill of lumber:

	Feet	Price	Amount
12 pieces, $\frac{7}{8}$″ × 7″–16′ 0″ long, Pine D4S .		$ 6.00	
2 pieces, 6″ × 6″–16′ 0″ long, Pine D4S..		85.00	
310 ft. lineal, 8″, Fir base.		8.75	
680 ft. lineal, $\frac{7}{8}$″ × $2\frac{3}{4}$″, Fir base D4S.		5.50	
46 pieces, $\frac{7}{8}$″ × $5\frac{1}{4}$″–14′ 0″ long, Door jamb sanded.		8.00	
x100 ft. lineal, $1\frac{3}{4}$″ × $3\frac{3}{4}$″, Pine D4S.		123.00	
x9 pieces, $1\frac{3}{4}$″ × $5\frac{3}{4}$″–10′ 0″ long, Pine D4S.		163.00	
5 pieces, $2\frac{3}{4}$″ × $5\frac{3}{4}$″–12′ 0″ long, Oak sill. .		.75	
5 pieces, $2\frac{3}{4}$″ × $5\frac{3}{4}$″–10′ 0″ long, Oak sill. .		.75	
2 pieces, $2\frac{3}{4}$″ × $5\frac{3}{4}$″–14′ 0″ long, Oak sill. .		.75	
65 ft. lineal, 3″, Crown moulding.		3.75	
x120 ft. lineal, $\frac{7}{8}$″ × $9\frac{3}{4}$″, Clear Pine D4S. . .		190.00	
x125 ft. lineal, $\frac{7}{8}$″ × $5\frac{3}{4}$″, Clear Pine D4S. . .		160.00	
x125 ft. lineal, $\frac{7}{8}$″ × $3\frac{3}{4}$″, Clear Pine D4S. . .		150.00	
		Total.	

x Dressed out of material even inch above.

58. Roofs, Rafters, Pitch.

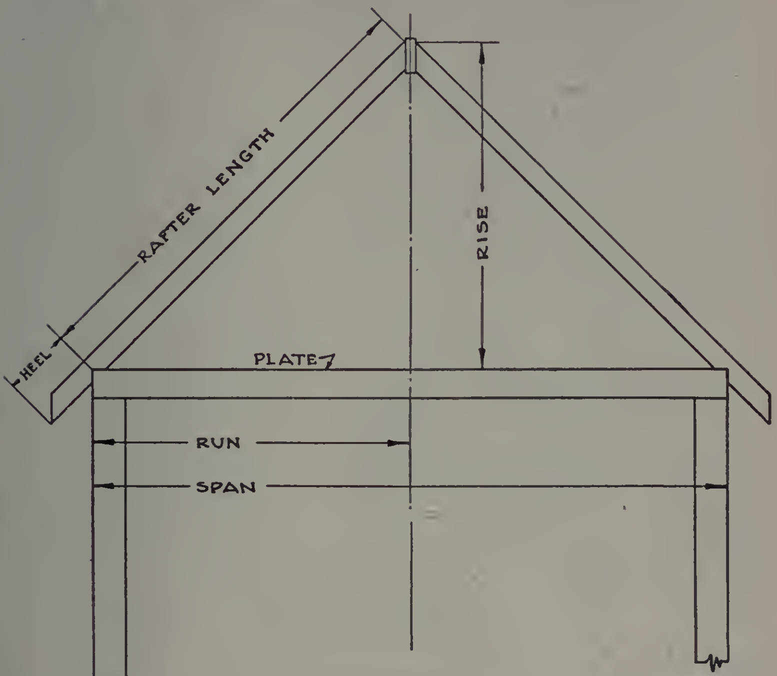


FIG. 14

In the above section of an ordinary gable roof, some of the terms used in connection with roofs are indicated. The span of a roof is the same as the width of the building. The run is one-half the span, and the rise is the vertical distance from the top of the plate to the top of the ridge. The pitch of a rafter is given by dividing the number of feet in the rise by the number of feet in the span. Thus if the rise is 6 ft. and the span 12 ft. the roof would have a one-half pitch. The rafter length is the distance from the outside corner of the plate to the centre of the ridge. The heel is the distance from the outside corner of the plate to the end of the rafter. The length of the heel would have to be added to the rafter length if the above method were used for the construction of the eaves.

The accompanying figures illustrate the method of finding the lengths of the different rafters in a Hip or Cottage roof. Figure

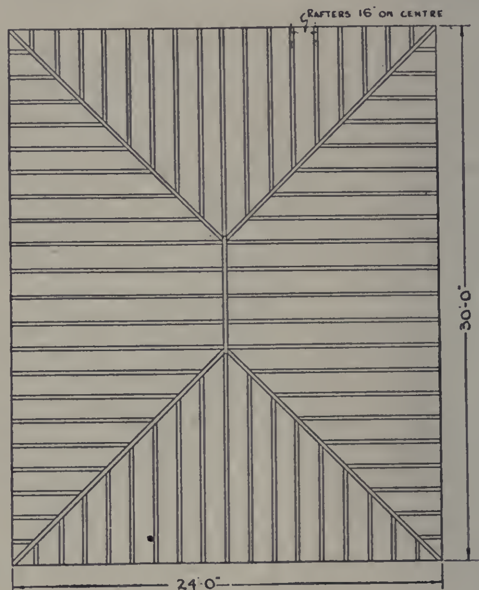


FIG. 15

15 shows a plan of the roof, Figure 16 a right side elevation, Figure 17 a plane at plate level.

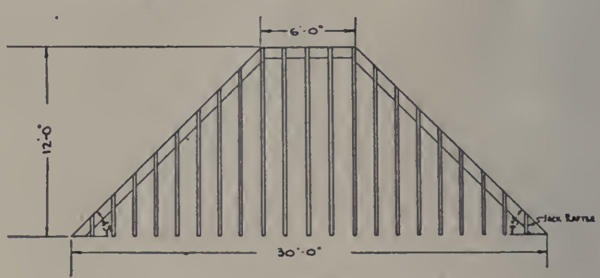


FIG. 16

In order to find the length of a hip rafter it would first be necessary to find the length of HK in Figure 17. Using this length and the perpendicular distance from **H** to the ridge the length of the hip rafter may be found as in Figure 18.

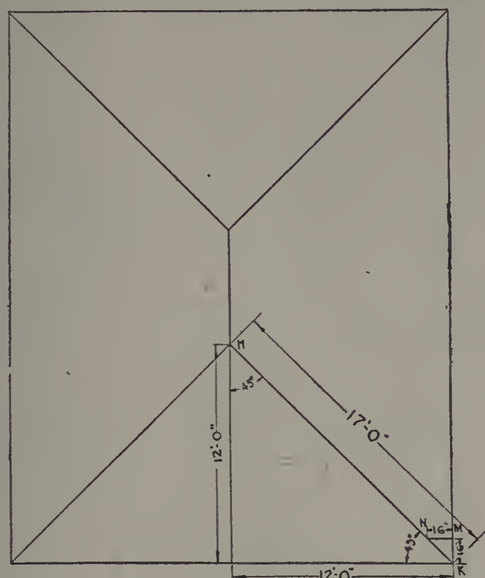


Fig. 17

To find the length of the jack rafter we observe in Figure 17 that, if the rafters be 16″ on centre, **MN** would also be 16″.

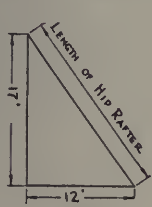


Fig. 18

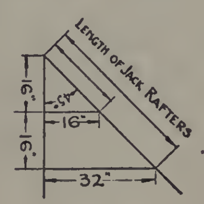


Fig. 19

Also since the roof has a ½ pitch, the perpendicular distance from the hip to **N** would also be 16″, hence Figure 19. If the roof has other than a ½ pitch, similar triangles would give the lengths of the jack rafters.

59. **Roofing—Shingles.** Shingles for roofing are estimated as being 16″ long and averaging 4″ wide. They are put up in bundles of 250 each, four bundles making a square of shingles.

The unit in measuring for roofing is the square. A square contains 100 sq. ft. If shingles are laid 4″ to the weather, each shingle would on an average cover an area of 16 sq. in. This would give for 100 sq. ft. $\frac{14400}{16}$ or 900 shingles. In this result, however, no allowance has been made for waste in cutting or for defective shingles.

The following table has been found useful in practice (Kidder's Pocket Book):

Inches to the Weather	Area Covered by 1000 Shingles	Number to Cover a Square
4	100 sq. ft.	1000
$4\frac{1}{4}$	110 sq. ft.	910
$4\frac{1}{2}$	120 sq. ft.	833
5	133 sq. ft.	752
$5\frac{1}{2}$	145 sq. ft.	690
6	156 sq. ft.	637

60. **Roofing—Slate.** Slate for roofing is also measured by the square (100 sq. ft.). In estimating either the amount required or the cost of laying, eaves, hips, valleys, etc., are measured extra—1 ft. wide by the whole length. The sizes of slates range from 9″ × 7″ to 24″ × 14″. "Each slate should lap the slate in the second row below, 3 inches", Kidder.

The gauge of a slate is the portion exposed to the weather, which should be one-half of the remainder obtained by subtracting 3 in. from the length of the slate.

The following table is taken from Kidder's Pocket Book:

Size of Slates in Inches	Inches Exposed to Weather	Number to a Square
14×24	$10\frac{1}{2}$	98
12×24	$10\frac{1}{2}$	115
12×22	$9\frac{1}{2}$	126
11×22	$9\frac{1}{2}$	138
12×20	$8\frac{1}{2}$	142
10×20	$8\frac{1}{2}$	170
12×18	$7\frac{1}{2}$	160
10×18	$7\frac{1}{2}$	192
9×18	$7\frac{1}{2}$	214
12×16	$6\frac{1}{2}$	185
10×16	$6\frac{1}{2}$	222
9×16	$6\frac{1}{2}$	247
8×16	$6\frac{1}{2}$	277
10×14	$5\frac{1}{2}$	262
8×14	$5\frac{1}{2}$	328

Exercises XXXVII.

Note.—In working the following problems take the actual quantity of lumber used, not allowing for waste due to having to buy stock lengths of material. In case of fractional inches take the inch above in each separate piece.

Lumber is cut in lengths of 10′ 0″, 12′ 0″, 14′ 0″, 16′ 0″, 18′ 0″, and will be charged on that basis.

1. Find the number of shingles for a square of roof for each line in the table if no allowance be made for waste.

2. A shed 9′ 0″ wide and 18′ 0″ long is to have a "lean to" roof, $\frac{1}{3}$ pitch. If the rafters are 2″ × 4″ at 16″ centre and have a 12″ heel, find their cost at $52 per M. If the roof extends 12″ on each end, find the cost of covering with 1″ square sheeting at $56 per M. Find the cost of shingling the above with shingles laid $4\frac{1}{2}$″ to the weather, if material and labour cost $14 a square of shingles.

3. A garage 10′ 0″ wide and 16′ 0″ long is to have a gable roof, $\frac{1}{2}$ pitch. The rafters are 2″ × 4″ at 2′ 0″ centre and have a 15″ heel. The rafter ties are 2″ × 4″ × 10′ 0″. Find the cost at $50 per M. If the roof extends 10″ on the ends and

6″ more on each end be allowed for waste, find the cost of covering with 1″ square sheeting, at $48 per M.

Find the cost of shingling the above with shingles laid $4\frac{1}{2}$″ to the weather if material and labour cost $13.50 per square of shingles.

4. A stable 15′ 0″ wide and 20′ 0″ long is to have a gable roof, $\frac{1}{2}$ pitch. The rafters are 2″ × 4″ at 20″ centre and have an 18″ heel, the ridge board being 1″ × 6″. The roof is supported at every second rafter by a brace 2″ × 4″ (see Figure 20) and collar ties 2″ × 6″ × 15′ 0″. Find the cost of lumber at $52 per M. If the extension on the ends is 12″, find the cost of sheeting with 6″ tongued and grooved lumber at $55 per M sq. ft., allowing 10% for the tongue and groove and 6″ on each end for waste.

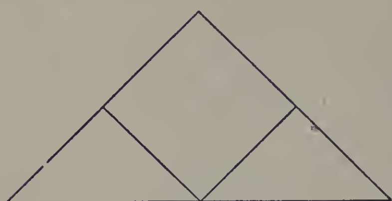


FIG. 20

Find the cost of shingling the above roof with shingles laid 5″ to the weather if material and labour cost $13 per square of shingles.

5. A house 25′ 0″ wide and 32′ 0″ long is to have a gable roof, $\frac{2}{3}$ pitch. The rafters are 2″ × 6″, 16″ on centre, with an 18″ heel. The roof is supported by braces 2″ × 4″, 4′ 0″ on centre, 6′ 0″ long, and tied with ceiling rafters 2″ × 6″ × 25′ 0″. Find the cost of the above lumber at $55 per M.

If the extension on the ends be 12″, find the cost of covering with $\frac{7}{8}$″ × 6″ tongued and grooved sheeting at $66 per M sq. ft., allowing 10% for the tongue and groove and 8″ on each end for waste.

Find the cost of roofing the above with slate, the gauge being $8\frac{1}{2}$″, if material and labour cost $30 a square.

6. A building 20′ 0″ wide and 28′ 0″ long is to have a hip roof, $\frac{1}{2}$ pitch, the ridge being 1″ × 8″ × 8′ 0″ long. The hip rafters are 2″ × 6″, the jack rafters 2″ × 6″, at 16″ centre. Ceiling joist 5′ 0″ from plate level, 2″ × 6″; act as ties. Find the cost of the above lumber at $56 per M.

Note.—In estimating the amount of material in rafters, find the length of a common rafter and multiply by the number of rafters on both sides.

7. A building 22′ 0″ wide and 32′ 0″ long is to have a hip roof, $\frac{1}{2}$ pitch, the ridge being 1″ × 8″ × 10′ 0″. The hip rafters are 2″ × 6″, the jack rafters 2″ × 6″, at 16″ centre; ceiling joist 2″ × 6″ × 22′ 0″ act as ties. There are also 14 braces 2″ × 4″, at 16″ centre, from centre of ceiling joist to centre of common rafters. Find the cost of lumber at $55 per M.

61. **Lathing and Plastering.** In lathing and plastering the square yard is the unit of measurement.

Standard laths are 4 ft. long, $1\frac{1}{4}$ in. wide, and are laid $\frac{1}{4}$ in. apart. They are usually put up in bundles of 50. It requires approximately 18 laths to cover a square yard. In estimating both the amount and the cost of lathing and plastering, the percentage of the openings deducted from the total area will depend upon the job. In small openings no deduction will be made; in medium openings about 40% or 50%; in very large openings from 75% to 90%.

Exercises XXXVIII.

1. What will it cost to lath and plaster the walls and ceiling of the following rooms at 70c. per square yard? (*a*) 17′ 0″ × 13′ 0″ × 9′ 0″ high, with a door 3′ 0″ × 7′ 0″, and 3 windows each 3′ 3″ × 5′ 0″. Deduct 40% of the openings. (*b*) 20′ 0″ × 18′ 0″ × 11′ 0″ high, with 2 doors 3′ 0″ × 9′ 0″, and 4 windows 3′ 0″ × 5′ 0″. Deduct 50% of the openings. (*c*) 40′ 0″ × 22′ 0″ × 15′ 0″ high, with 2 doors 5′ 6″ × 8′ 0″, and window space 15′ 0″ × 9′ 0″. Deduct 90% of the openings.

2. A room is 16′ 0″ wide, 18′ 0″ long, and 10′ 0″ high. There are 2 doors 4′ 6″ × 7′ 0″, 3 windows 3′ 8″ × 4′ 9″, the sills being 2′ 3″ from the floor. Find (*a*) the cost of lathing and plastering at 90c. a sq. yd., deducting 50% of the openings; (*b*) the cost of laying and finishing a $\frac{3}{8}$″ × 2″ No. 1 red oak floor at 32c. per sq. ft.; (*c*) the cost of paneling the walls to a height of 4′ 0″ at 80c. a sq. ft. (Note—plastering is carried to floor behind paneling.)

3. Find the cost of lathing and plastering a room 27′ 0″ wide, 30′ 0″ long, and 12′ 0″ high at 80c. a sq. yd., if there are 2 doors 3′ 6″ wide and 7′ 0″ high, and 6 windows 3′ 4″ wide and 6′ 0″ high. Deduct 50% of the openings and also deduct 12c. a sq. yd. for the area paneled (see No. 4), on account of the finishing coat being unnecessary.

4. Find the cost of paneling the walls in the preceding question to a height of 4′ 6″ if the sills of the windows be 2′ 6″ from the floor, at 85c. a sq. ft.

5. A hall is 50′ 0″ wide, 90′ 0″ long, and 20′ 0″ high. There are 4 doors 5′ 6″ × 10′ 0″, 2 windows 5′ 0″ × 11′ 0″, 7 windows 5′ 0″ × 8′ 3″. Find the cost of lathing and plastering at 75c. a sq. yd., deducting 80% of the openings.

62. **Decorating and Painting.** Wall paper is put up in rolls, the number of yards in the roll and the width of the paper varying. The kinds chiefly in use are:

(1) Paper 18″ wide and in single rolls 8 yd. in length, or double rolls 16 yd. in length.

(2) Paper 21″ wide and in rolls 12 yd. in length.

(3) Paper 30″ wide and in rolls 5 yd. in length; frequently put up in 15 yd. rolls.

To estimate for the walls.

(1) Find the perimeter of the room, less the width of doors and windows.

(2) Find the number of strips required by dividing the result in (1) by the width of the paper.

(3) Find the number of strips that can be cut from a roll by dividing the length of the roll by the height to be papered.

(4) Find the number of rolls by dividing the number of strips required for the room by the number of strips in a roll.

To estimate for the ceiling.

If the strips are to run lengthwise, find the number of strips by dividing the width of the room by the width of the paper, then proceed as in case of walls.

To estimate for the border.

Find the total perimeter of the room. Estimate cost per running yard.

When double rolls are available they would be used, if more economical in cutting.

Example:—A room 16′ 0″ wide, 20′ 0″ long and 9′ 0″ high from base-board, has two doors each 4′ 0″ wide and three windows each 3′ 6″ wide. Find the cost of paper for the walls and ceiling, the wall paper being 18″ wide and costing $2 a double roll, the ceiling paper being 18″ wide and costing 80c. a double roll.

Perimeter of room $=(16' + 20')\ 2=72'$.

Perimeter—Width of doors and windows $=72'-18\frac{1}{2}'=53\frac{1}{2}'$.

Number of strips required $=\dfrac{53\frac{1}{2}\times 12}{18}=35\frac{2}{3}\ \therefore 36$.

Number of strips in a double roll $=\frac{48}{9}=5\frac{1}{3}\ \therefore 5$.

Number of rolls $\frac{36}{5}=7\frac{1}{5}\ \therefore 8$. Cost $=\$16$.

Number of strips required for the ceiling if running lengthwise $=\dfrac{16\times 12}{18}=10\frac{2}{3}\ \therefore 11$.

Number of strips in a double roll $=\frac{48}{20}=2\frac{2}{5}\ \therefore 2$.

Number of rolls $=\frac{11}{2}=5\frac{1}{2}\ \therefore 6$. Cost $=\$4.80$.

Total Cost $20.80.

Painting. The area is usually estimated in sq. yd.

The following is a common method of reckoning the area of doors, windows, etc.:

Doors are taken to average 3′ 0″ × 7′ 0″, windows 3′ 0″ × 6′ 0″. If the window be divided into 12 lights the area is doubled, if divided into 6 lights one-half the area is added, and so on. The base-board is taken as 1′ 0″ by total perimeter, picture moulding 3″ by total perimeter, and dado rail 6″ by total perimeter.

Exercises XXXIX.

1. A room is 13′ 0″ wide, 15′ 0″ long and 8′ 6″ high. There are two doors each 2′ 8″ wide, and three windows each 3′ 0″ wide. Two of the windows have 6 lights and the other 2 lights. A picture moulding 2″ wide and a base-board 8″ wide run around the room. Find (1) the cost of tinting the ceiling and 1′ down to picture moulding at 25c. a sq., yd., (2) the cost of painting the interior woodwork at 50c. a sq. yd.,

(3) the cost of papering the walls with paper 21″ wide at $1.25 a roll, the decorator charging 40c. a roll for the work.

2. A room 10′ 8″ wide, 11′ 4″ long and 8′ 6″ high, has two doors each 2′ 10″ wide, one window 4′ wide, 2 lights, two windows each 3′ wide, 12 lights, base-board 10″ wide running around the room. Find (1) the cost of painting the woodwork at 25c. a sq. yd., (2) the cost of papering the ceiling with paper 18″ wide at 25c. a single roll, (3) the cost of papering the walls with paper 30″ wide at 90c. a roll, using a border 4″ wide at 20c. a yard. The decorator charges 30c. a roll for the work in both walls and ceiling.

3. A room 12′ 0″ wide, 18′ 6″ long and 10′ 0″ high, has two doors each 3′ 10″ wide, two windows each 2′ wide, 4 lights in each, one window 4′ 6″ wide, 12 lights, a fire-place 5′ 6″ wide, a picture moulding 3″ wide and a base-board 1′ 0″ wide running around the room. Find (1) the cost of tinting the ceiling and 16″ on wall to picture moulding at 30c. a sq. yd., (2) the cost of painting the woodwork at 40c. a sq. yd., (3) the cost of papering the walls with paper 18″ wide at $1.20 a double roll, the decorator charging 50c. a roll for the work.

CHAPTER VI.

ALGEBRAIC NOTATION.

63. **In Arithmetic** we denote quantities by numbers, each number having a fixed value. By 5 in. we mean that the line, or pencil, or bolt, is 5 in. in length. For this purpose we have the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. These symbols, in whatever way they are combined, have definite fixed values. **In Algebra** there is no limit to the number of symbols employed, the letters from our own alphabet being the ones chiefly used.

64. **Algebra—generalized Arithmetic.** These symbols:—a, b, c, d, etc., in contrast to the symbols of Arithmetic, have not fixed values but may be given any values required by the conditions under discussion. Thus in Arithmetic 2×4 is always 8, whereas $2\times a$, or more briefly $2a$, will have different values according to the numerical values assigned to the symbol a. When $a=4$, $2a=2\times 4=8$, when $a=8$, $2a=2\times 8=16$, and so on. Here the $2a$ is called an **Algebraic Expression,** the 2 being called the **Numerical Coefficient,** denoting the number of times a is taken in the sum. When no numerical coefficient is placed in front of the symbol, 1 is understood, thus a means $1a$.

65. **Arithmetic Laws Applicable.** Since the symbols a, b, $c \ldots . x$, y, z stand for numerical quantities we may apply the ordinary Arithmetic laws in using them. In Arithmetic $2\times 6+3\times 6=5\times 6=30$. So in Algebra $2a+3a=5a$, $6a-2a=4a$. In Arithmetic $2\times 6=6\times 2$, so in Algebra $a\times b=b\times a$. Also $2\times 4\times 6=4\times 6\times 2=6\times 2\times 4$, so in Algebra $a\times b\times c=a\times c\times b=b\times a\times c=abc=acb=bca$. If we then wish to add $3abc+2acb+7cab$ we should rearrange the terms thus, $3abc+2abc+7abc=12abc$. An important difference between

the notation of Arithmetic and that of Algebra should be noted. *In Arithmetic 34 means thirty-four or* $3\times10+4$; *in Algebra ab means* $a\times b$.

Exercises XL.

Find the values of the following:

1. $3x+9x$.	8. $2ab+3ba$.	15. $3ab+2ab+4ab$.
2. $a+a$.	9. $ab-ba$.	16. $5xy+6xy+3xy$.
3. $2a-a$.	10. $11xy-7xy$.	17. $3abc+2bca+10cab$.
4. $7x-3x$.	11. $9xy-3yx$.	18. $x+x+x+x$.
5. $11x-4x$.	12. $6ab-ba$.	19. $3x+4x+x+6x$.
6. $x-x$.	13. $8abc-3cab$.	20. $9b+3b+5b+6b$.
7. $3ab+5ab$.	14. $3x+4x+5x$.	

What is the value of $8x$ when:

21. $x=2$.	23. $x=\frac{1}{2}$.	25. $x=\frac{3}{4}$.
22. $x=4$.	24. $x=\cdot4$.	26. $x=2\frac{1}{2}$.

What is the value of $\frac{x}{2}$ when:

27. $x=4$.	29. $x=5$.	31. $x=\cdot5$.
28. $x=16$.	30. $x=\frac{1}{2}$.	32. $x=2\cdot5$.

What is the value of $\frac{x}{3}$ when:

33. $x=6$.	35. $x=7\cdot5$.	37. $x=\cdot6$.	39. $x=\cdot036$.
34. $x=18$.	36. $x=2\cdot7$.	38. $x=\cdot9$.	40. $x=\cdot0024$.

Exercises XLI.

1. What is the number which is 2 greater than x?

2. What is the number which is 3 less than x?

3. If an article costs x cents what is the cost of three articles? of seven articles? of eleven articles?

4. Express x sq. ft. in sq. in.

5. Express x sq. in. in sq. ft.

6. Express x metres in (1) decimetres, (2) in centimetres, (3) in millimetres, (4) in kilometres.

7. Express x millimetres (1) in centimetres, (2) in decimetres, (3) in metres, (4) in kilometres.

8. If there is an average of x trains leaving Toronto every day and an average of y cars per train, how many cars leave Toronto per day?

9. In a rectangle $ABCD$ if AB is c ft. in length and BC, b ft. in length, find (1) the perimeter of the rectangle, (2) the area of the rectangle.

10. If the side of a square is b feet, find its perimeter.

11. A can do a piece of work in m days and B in n days; write down (1) the amount of work each can do in 1 day, (2) the amount of work both can do in 1 day.

12. The sides of a triangle measure x, y, z ft. Write down an expression for (1) the perimeter, (2) the semi-perimeter.

13. A merchant mixes x lb. tea worth z c. a lb. with n lb. worth y c. a lb. Find the value of one lb. of the mixture.

14. If a man works x hr. per day and handles y castings per hour, how many castings does he handle each day?

15. If there are x cars in a railroad yard, how many trains will there be if there is to be an average of b cars per train?

16. What is the length of the casting in the accompanying figure?

FIG. 21

17. What is the length of the casting in the accompanying figure?

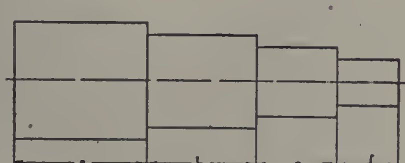


FIG. 22

18. Find the length of the crank-pin in the accompanying figure.

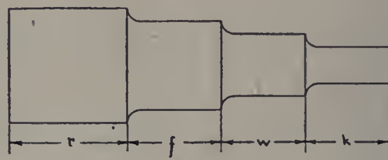


FIG. 23

19. If l is the length of the crank-pin in the accompanying figure what is the length of the last step?

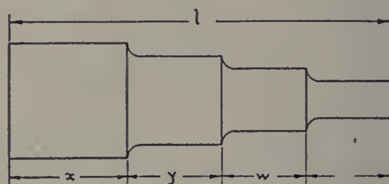


FIG. 24

20. If l is the length of the cylinder and saddle shoulder bolt in the accompanying figure what is the length of the shoulder?

FIG. 25

66. **Index or Exponent, Power.** In Arithmetic 3×3 may be written 3^2 or 9.

In Algebra if we multiply a by a we cannot write the product as a single symbol, since we do not know the value of a; but we may express it as a^2. In a similar way $a\times a\times a=a^3$, $a\times a\times a\times a=a^4$. The small figure placed to the right and above the symbol is called the **Index or Exponent, and the product** a^4 **is called the fourth power of** a **or more commonly** a **to the fourth.**

67. Index Laws. $x^4 \times x^5 = x \times x \times x \times x \times x \times x \times x \times x \times x = x^9$.

From this example we have the law:—*The index of the product of two powers of the same symbol is equal to the sum of the indices of the factors.*

Examples:—$a^6 \times a^5 = a^{6+5} = a^{11}$. $x^3 \times x^7 = x^{10}$.

$$a^2b \times b^2a = a^{2+1} \times b^{1+2} = a^3b^3.$$

$$a^5 \div a^3 = \frac{a^5}{a^3} = \frac{a \times a \times a \times a \times a}{a \times a \times a} = a^2.$$

If we cancel 3 of the factors in the numerator by the 3 factors of the denominator, the above expression becomes $a \times a = a^2$. From this example we infer the law:—*The index of the quotient of one power of a symbol divided by another power of the symbol is obtained by subtracting the index of the divisor from the index of the dividend.* Examples:

$$a^7 \div a^4 = a^{7-4} = a^3. \quad b^{25} \div b^{14} = b^{25-14} = b^{11}.$$

68. Some expressions in detail. a^3 means that a is taken three times as a product.

ab^2 means that a is taken once, b is taken twice as a product, and the two results are multiplied together.

$6a^2b^3$ means that a is taken twice as a product, b is taken three times as a product, the two results are multiplied together, and the resultant product is taken six times.

Exercises XLII.

What is the:

1. second power of a?
2. third power of 4?
3. fifth power of 5?
4. sixth power of b?
5. product of x and x^2?
6. product of a^2 and a^3?
7. product of $4a$ and $3b$?
8. product of $4a^2$ and $5a^3$?
9. product of $12abc$ and $3abc$?
10. product of $3a^2b$ and ab^2?
11. Express the product abx^2 in different forms.
12. Do the same with $3x^2y^3$, $6a^2b^3c^4$, $12ab^3x$.
13. Write in detail what is meant by the following:

$$4a^3b,\ 5a^2b^2,\ 6abc,\ 7a^3bc.$$

Find the results of the following expressions in the most simplified form:

14. $a+3a+6a$.
15. $7a-2a+4a$.
16. $3abc+6abc$.
17. $7ab+10ba$.
18. $13abc+6bca-2cab$.
19. $4xyz+6yxz+2zxy-4zyx$.

What is the result of:

20. $a^6 \div a^2$.
21. $x^{16} \div x^3$.
22. $x^3y^3 \div xy$.
23. $x^2 \div x^6$.
24. $xy \div x^4y^4$.
25. $3a^2b^2 \div ab$.
26. $15x^3y^3 \div x^2y^2$.
27. $x^2y^2 \times x^3y^3 \div xy$.
28. $a^2b^2 \div ab \times a^3b^3$.
29. $3x^3y^3 \div xy \times 2x^2y^2$.
30. $4m^3n^3 \times mn \div m^2n^2$.

31. The side of a square is b in. What is its area?

32. The edge of a cube is b in. What is the area of a face? What is the area of all the faces? What is the volume of the cube?

33. The volume of a cube is $8x^3$. What is the area of a face? What is the area of all the faces?

34. If a train travels l hr. at k miles per hr. and c hr. at d miles per hr., find the total distance travelled.

35. Represent three consecutive numbers, (1) if x is the first one, (2) if x is the middle one, (3) if x is the last one.

36. If the length of a stick is b ft. find its length in in., in yd., in rods.

37. If a rod is x yd. b ft. and c in., how many inches in length is it?

38. If x is the price per quart for beans, what is the price per gallon? What is the price per bushel?

39. A man earned \$$x$ per day and his son \$$y$. How many dollars did they both earn in a month if the man worked 25 days and the son 20 days?

69. **Roots. As in Arithmetic** the square root of x, or the expression whose second power is x, is indicated by $\sqrt{x}$. Similarly the cube, fourth, fifth, etc., roots of x, or the expressions whose third, fourth, fifth, etc., power is x, are indicated by $\sqrt[3]{x}$, $\sqrt[4]{x}$, $\sqrt[5]{x}$, etc.

Thus, $\sqrt[3]{a^6} = a^2$ Since $a^2 \times a^2 \times a^2 = a^6$.
$\sqrt[4]{a^{12}} = a^3$ Since $a^3 \times a^3 \times a^3 \times a^3 = a^{12}$.
$\sqrt[5]{32} = 2$ Since $2 \times 2 \times 2 \times 2 \times 2 = 32$.

The symbol $\sqrt{}$ is called the radical sign.

Exercises XLIII.

Find the square root of:

1. x^2.
2. x^6.
3. $16x^2$.
4. x^{12}.
5. $64x^4$.
6. $16a^6$.
7. $49x^2y^2$.
8. $81a^4b^4$.
9. $144a^6b^6$.
10. $169a^6y^4$.
11. $\frac{a^2}{4}$.
12. $\frac{a^6}{9}$.
13. $\frac{x^8}{16}$.
14. $\frac{a^2b^2}{9}$.
15. $\frac{x^4y^4z^4}{16}$.

Find the value of:

16. $\sqrt{a^4b^4}$.
17. $\sqrt{a^6}$.
18. $\sqrt{49} - \sqrt{36}$.
19. $\sqrt{49} + \sqrt{4}$.
20. $\sqrt{x^6} - x^3$.
21. $\sqrt{x^8} - x^4$.
22. $\sqrt[3]{x^8}$.
23. $\sqrt[4]{x^{16}}$.
24. Square of a^4b.
25. Square of $4xy$.
26. Cube of x^2.
27. Fourth power of y^2.
28. Cube of $2\,a^2y^4$.
29. Cube root of x^6.
30. Cube root of $8a^3$.
31. Cube root of $27a^6$.
32. $\sqrt{25-16}$.
33. $\sqrt{49-33}$.
34. $\sqrt{25a^4-16a^4}$.
35. $\sqrt{\frac{49a^2}{b^2}}$.
36. $\sqrt{\frac{1}{a^2}}$.
37. $\sqrt{\frac{1}{a^6}}$.
38. $\sqrt[4]{\frac{1}{a^{16}}}$.

70. **Like and Unlike Terms.** Two terms which contain the same letters involved in the same way are called like terms. Thus $6a$ and $3a$ are like terms. $3ab$ and $4ab$ are like terms. $7x^2$ and $9x^2$ are like terms.

Since ab and ba both mean $a \times b$, ab and ba are also like terms, also $5ab$ and $7ba$ are like terms.

Like terms may then be defined as terms that differ only in their numerical coefficients.

Unlike terms may be defined as terms that differ in other than their numerical coefficients.

Thus $6a$ and $4b$ are unlike terms. x^2y and xy^2 are unlike terms. $7a^2$ and $9b^2$ are unlike terms.

If we wish to add such terms all we can do is to write them down with a plus sign between them, thus $6a+4b$, x^2y+xy^2, $7a^2+9b^2$.

When we wish to simplify an algebraic expression such as $3a+4b-2a+6b$ we can combine the like terms $3a$ and $-2a$, giving a, and the like terms $4b$ and $6b$, giving $10b$, and write the result $a+10b$.

Examples:

$$10a+6b-3a+4c-2b-c=10a-3a+6b-2b+4c-c$$
$$=7a+4b+3c.$$
$$9xy+4x^2y^2+2xy-3x^2y^2=9xy+2xy+4x^2y^2-3x^2y^2$$
$$=11xy+x^2y^2.$$

Exercises XLIV.

Simplify by combining like terms:

1. $4a+3a+6a-2a$.
2. $3a+2a+6b-4b$.
3. $3ab+4ba+3bc-bc$.
4. $6abc+3a^2b^2-2bca$.
5. $7xy+6x^2y^2-3yx+4y^2x^2+3xy-2x^2y^2$.
6. $3m+2n+2m-m-n+3mn-n+2mn$.
7. $6p+2q+4r-3p+6q-2r+4p-2q$.
8. $3a+2x-4y+7a+8y+5x$.

If $a=8$, $c=0$, $k=9$, $x=4$, $y=1$, find the value of:

9. $\sqrt{2ak^2}$.
10. $\sqrt[3]{3k}$.
11. $\sqrt[3]{cy^5}$.
12. $2x\sqrt{2ay}$.
13. $5y\sqrt{4kx}$.
14. $3c\sqrt{kx}$.
15. $\sqrt{\frac{8x^3}{ak}}$.
16. $\sqrt{\frac{25a}{2k}}$.
17. $\sqrt[3]{\frac{3a}{k^3}}$.
18. $\sqrt{\frac{kax^2}{18y^3}}$.

71. **Brackets. In Arithmetic** when a number of terms is included within a bracket it is understood that these terms are to be regarded as a whole.

Thus, $10+(5+4)$ means that we first add 5 and 4 and then add the result to 10. Also $10-(5+4)$ means that we first add 5 and 4 and then subtract the result from 10. So in **Algebra**, $a+(b+c)$ means that we first add b and c and then add the result to a.

Certain rules are necessary with respect to the signs of the terms within the bracket when the bracket is removed. These rules may be obtained by an analysis of a few type cases.

By $a+(b+c)$ we mean that the quantity $b+c$ is to be added to a. We may first add b and then afterwards add c, giving $a+b+c$. By $a+(b-c)$ we mean that the quantity obtained by subtracting c from b is to be added to a. It is evident that if we add b to a, obtaining $a+b$, our result will be too great by c; we must therefore subtract c from $a+b$, obtaining $a+b-c$ as a result. From these illustrations we infer the rule:—*When a group of terms is contained within a bracket preceded by the sign + the bracket may be removed without changing the signs of the terms within.*

In $a-(b+c)$ we have to subtract the sum of b and c from a. If we subtract b from a, giving $a-b$, it is evident that the result is too great and that it is too great by c; therefore we must subtract c from $a-b$, giving $a-b-c$. In $a-(b-c)$ we have to subtract the result $b-c$ from a. If we subtract b from a, giving $a-b$, it is evident that we have taken away too much, for we were required to take away only $b-c$. The result $a-b$ is therefore too small by c, and we must add c to $a-b$, giving $a-b+c$. From these illustrations we infer the rule:—*When a group of terms is contained within a bracket preceded by the sign − the bracket may be removed by changing the signs of the terms within.*

$3x$ means $x+x+x$, similarly $3(a+b)$ means $(a+b)+(a+b)+(a+b)=3a+3b$. This would lead us to the rule:—*The product of an expression, consisting of two or more terms and a single factor, is the sum of the products of each term of the expression multiplied by the single factor.*

Examples:

1. $3x-(a+b)=3x-a-b.$
2. $7a+(b+c)=7a+b+c.$
3. $9x^2-(x-y)=9x^2-x+y.$
4. $6(a+b+c)=6a+6b+6c.$

It is necessary to note the difference between $3a^2$ and $(3a)^2$. In $3a^2$ we have to multiply a by a and take the result three times. In $(3a)^2$ we have to square the whole quantity $3a$, giving $3a \times 3a$ or $9a^2$.

Examples: 1. $(7ab)^2 = 7ab \times 7ab = 49a^2b^2$,

2. $(2a^3)^4 = 2a^3 \times 2a^3 \times 2a^3 \times 2a^3 = 16a^{12}$.

It is sometimes necessary to enclose with brackets part of an expression already enclosed within brackets. In such cases the pairs of brackets are made of different shapes—( ), { }, [ ]. Thus $a - \{b + (c - d)\}$.

The same rules with respect to the removal of brackets apply, it being usually best to begin with the inside pair and remove one pair at a time. In the example given we would first simplify thus, $a - \{b + c - d\}$. We would then remove the remaining pair and write the expression $a - b - c + d$.

Exercises XLV.

Simplify:

1. $3a + (4a - 2a)$.
2. $15x - (6x + 3x)$.
3. $3b - (2a + 4a)$.
4. $6a - (4a + 2a)$.

Prove the following by removal of brackets:

5. $6 + (x - 2) - (3 + 4x) + (6x + 1) = 3x + 2$.
6. $(3x - 2) - (4x + 5) + (x + 7) = 0$.
7. $(9a - b) + (3a - 2b) - (6a - 5b) = 6a + 2b$.
8. $(x + 6a) - (2x - 3a) - (a - 6x) = 5x + 8a$.
9. $2(x + 1) + 3(1 + x) + 2(2 + 3x) = 9 + 11x$.
10. $3(2 - a) + 6(2a + 7) + (a - 42) = 10a + 6$.
11. $2(a + b) - (2a - b) = 3b$.
12. $3(a + b + c) - (b + a - c) - (2c - 2a - b) = 4a + 3b + 2c$.
13. $2(3x + 12) + 3(x + 4) - (8x - 12) = x + 48$.

Simplify:

14. $3\{x - (2x - 6x)\}$.
15. $\{3a + (6a - 2a) + 4a\}$.
16. $x + \{2x + 3(x + 2x)\}$.
17. $3x^2 + x(x + 3) + x^2$.
18. $a - \{b + (c - d)\}$.
19. $a - [b - \{a - (b - a) + b\} - a]$.

20. Enclose $a-b+c-d-e+f$ in alphabetical order in brackets, two letters in each; three letters in each.

72. Negative Quantities. We have in Arithmetic found the value of an expression such as $6-5$. In every case however the number to be subtracted was less than the number from which it was subtracted.

A difficulty is presented if we are asked to find the value of $5-6$. This is arithmetically impossible. We cannot take \$6 from \$5. We may, however, by making use of brackets, write $5-6$ thus, $5-(5+1)=5-5-1$. Here $5-5$ is 0, and the value appears as 1 to be subtracted with nothing from which to subtract it. We shall say that the result is the negative number 1 or minus 1, and denote it by -1. The idea of negative numbers may be made clearer by means of a graphical representation.

−	−	−	−	−	−	|	+	+	+	+	+	+
6	5	4	3	2	1	0	1	2	3	4	5	6

In the above diagram we have represented numbers to the right of the vertical line as positive, and numbers to the left of the vertical line as negative. The two series of numbers may be considered as forming but a single series consisting of a positive branch, a negative branch, and zero.

If then we wish to subtract 4 from 2 we begin at 2 in the positive series, count 4 units in the negative direction (to the left) and arrive at -2 in the negative series, that is, $2-4=-2$.

A few examples may be added to show the practical value of negative quantities.

Example 1:—If the temperature is 30° below zero it may be recorded $-30°$. If it rises 5° it is then 25° below zero or $-25°$. If it increases 10° more it is 15° below zero or $-15°$.

Example 2:—If a merchant during a day's transactions gains \$80 on one class of goods and loses \$100 on another class we can represent the result of the day's business as $\$80-\$100=-\$20$.

Example 3:—If a man rowed 50 yards up stream and then drifted down 60 yards, his position relative to the starting point would be 50 yards $-$ 60 yards $= -10$ yards.

Exercises XLVI.

1. A man has $500 and owes $500. How much is he worth?

2. A man has $500 and owes $700. How much is he worth?

3. A man goes 5 miles north of Barrie, then 9 miles south. How many miles north of Barrie is he? How many miles has he travelled? Make a diagram showing his route and his last position.

4. The temperature at 6.00 A.M. is $+14°$ and during the morning it grows colder at the rate of 4° an hour. Find the temperature at 9.00 A.M., at 10.00 A.M., and at noon.

5. A freight engine is switching in front of a station. If it runs 400 ft. to the right of the station (+400 ft.) and then backs 525 ft. ($-$ 525 ft.), how many feet is it from the station?

6. In drilling a well the drill is raised 8 ft. (+8 ft.) above the surface. It is then dropped 15 ft. ($-$15 ft.). Where is it then with respect to the surface?

7. A boy is fishing in deep water with a line 20 ft. long. If the tip of the pole is +6 ft. above the water, how far is the sinker from the surface of the water, if it is 3 ft. from the hook?

8. A man who was $350 in debt contracted another debt of $200. He then earned $1000. How much was he then worth?

9. A boat, that runs 16 miles an hour in still water, is going against a stream flowing 4 miles an hour. What is the rate at which the boat travels?

10. If a mine is opened 200 ft. above the base of a mountain and a shaft is sunk 700 ft., how much is the base of the shaft above or below the base of the mountain?

11. A man starts from a point 0 on a road running north and south, and walks c miles north and then b miles in the opposite direction. How far is he now from the starting point? How far has he travelled?

Illustrate from the following cases:—(1) $c=8$, $b=6$. (2) $c=5$, $b=9$. (3) $c=8$, $b=8$.

12. The thermometer stands at $x°$; in the course of an hour there is a fall of $y°$ and in the course of the next hour a rise of $z°$. Find the reading at the end of this time.

Illustrate for the following cases:—(1) $x=6$, $y=4$, $z=5$; (2) $x=-8$, $y=4$, $z=9$; (3) $x=-2$, $y=3$, $z=6$.

Simplify:

13. $3a+2b-4a+6b-8a-9b$.
14. $2s-3s+s-s-5s+5s$.
15. $4x^3-5x^3+3x^3-8x^3+7x^3$.
16. $-5b+\frac{1}{2}b-\frac{3}{2}b+2b-\frac{1}{2}b+\frac{7}{2}b$.
17. $\frac{2}{3}x-\frac{1}{2}y+\frac{3}{4}x+\frac{2}{3}y-\frac{1}{6}x+\frac{1}{3}y-y$.
18. $6a^2-3a^2+2b^2+3a^2-2b^2$.
19. $x^2+xy+y^2-3x^2-2xy+4y^2$.
20. $3p+2q-2p+4q-6p$.

CHAPTER VII.

SIMPLE EQUATIONS.

73. We might say that the greater part of a student's work in Arithmetic has been concerned with equations. The statement that 3 added to 4 is 7 might be expressed in the form $3+4=7$. This is **an equation or a statement of equality between two expressions,** $3+4$ being one and 7 the other.

All such equations involving only simple Arithmetical operations may be called **Arithmetical** equations, to distinguish them from equations of the form $3x=9$ which we will call **Algebraic equations.**

The x in this equation is called the **unknown** and the process of finding its value is called **solving** the equation.

An equation, in which the unknown quantity is involved to the first power only, is called a **simple equation.**

In the given case if $3x=9$, then $x=3$; the value 3 is said to **satisfy** the equation.

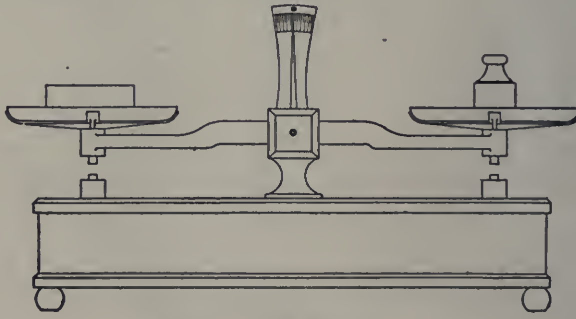


FIG. 26

74. **Operations on the equation.** The two sides of an equation must always balance, just as the weights in the two pans of the scales above must be equal if the scales are to

balance. In the equation $3x=9$ if we add 2 to the left-hand side we must of necessity add 2 to the right-hand side. The equation then becomes $3x+2=9+2$ or $3x+2=11$.

In the same way, if we subtract 2 from the left-hand side we must subtract the same quantity from the right-hand side. The equation then becomes $3x-2=9-2$ or $3x-2=7$.

Further, if we multiply the left-hand side by 2 we must multiply the right-hand side by 2, giving $2\times 3x=2\times 9$ or $6x=18$.

We might also divide the left-hand side by 2 giving $\frac{3}{2}x$, but we would also have to divide the right-hand side by 2, giving $\frac{9}{2}$ or the equation $\frac{3}{2}x=\frac{9}{2}$.

75. **Transpositions.** Let us consider the equation $3x+4=16$. By the previous paragraph we could subtract 4 from both sides of the equation, giving $3x+4-4=16-4$ or $3x=16-4$. We then observe that the equation $3x+4=16$ is equivalent to $3x=16-4$, or that the 4 has been moved from the left-hand side to the right-hand and that its sign has been changed.

Next let us consider the equation $3x-2=10$. We could add 2 to both sides giving $3x-2+2=10+2$ or $3x=10+2$. We then observe that the equation $3x-2=10$ is equivalent to $3x=10+2$, or that the 2 has been moved from the left-hand side to the right and that its sign has been changed. These two examples would lead us to make the statement:—*A quantity may be transferred from one side of an equation to the other without altering the balance, provided we change the sign of the quantity transferred.*

Examples:

1. Solve the equation $3x-2+5x-4=3x-10-7x+16$. Transposing so that we have all the terms containing x on the left and the other terms on the right we get

$$3x+5x-3x+7x=-10+16+2+4 \text{ giving } 15x-3x=22-10.$$

$$\text{or,}\quad 12x=12.$$

$$x=1.$$

2. Solve the equation $3(3x+1)-(x-1)=6(x+10)$.

Multiplying out $\quad 9x+3-x+1=6x+60.$

Transposing $\quad 9x-x-6x=60-3-1.$

$$2x=56.$$

$$x=28.$$

Verification:—Substitute the value 28 for x in the equation and we get $3(84+1)-(28-1)=6(28+10)$

$$\text{or,}\quad 3\times 85-27=6\times 38$$

$$255-27=6\times 38$$

$$228=228.$$

76. **Need of the equation.** Let us work the following problem, first without employing Algebraic symbols and then by making use of the Algebraic symbols, and compare the methods.

Example:

A shopper bought three articles, the second costing three times as much as the first and the third \$3 more than the second; find the cost of each if the total cost was \$10.

First solution:—Suppose that the third article had cost as much as the second, then the total cost would have been \$10 − \$3 or \$7. Then for every share allotted to the first article we must allot three to the second and three to the third. This makes seven shares into which we must divide \$7, giving \$1 for one share.

∴ the first article cost \$1,
the second article cost \$3,
the third article cost \$3 + \$3 = \$6.

Second solution:—Let $x=$ No. of dollars in cost of first,
then $3x=$ No. of dollars in cost of second,
and $3x+3=$ No. of dollars in cost of third,

$$\text{then } x+3x+3x+3=10$$

$$7x+3=10$$

$$7x=10-3$$

$$7x=7$$

$$x=1$$

$$3x=3$$

$$3x+3=3+3=6.$$

or the first article cost \$1, the second \$3, the third \$6. If we compare the two solutions it is evident that the latter method has the advantage in both directness and clearness.

Additional examples:

1. A man works a full day of 8 hours, and in addition works 3 hours overtime, for which he receives time and a half. If he is paid \$8.75 for the entire time, what is his regular rate per hour?

Let x c. = regular rate per hour,

then $\frac{3}{2}x$ c. = rate per hour for overtime,

then $8x$ c. = pay for 8 hours' work,

and $3\times\frac{3}{2}x$ or $\frac{9}{2}x$ c. = pay for overtime,

$$\therefore 8x+\tfrac{9}{2}x=875.$$

Multiplying both sides of the equation by 2 we get

$$16x+9x=1750$$
$$25x=1750$$
$$x=70,$$

or the regular rate is 70c. per hour and the overtime rate is $\frac{3}{2}\times70$ or \$1.05 per hour.

2. How much water must be added to a quart of alcohol, which already contains 5% of water, so that the mixture may contain 50% of alcohol? (No allowance being made for contraction).

Let x = the number of quarts of water to be added,

then $1+x$ = total number of quarts of mixture,

and $\frac{1}{2}(1+x)$ = number of quarts of alcohol in the mixture.

Since no alcohol has been added, this must equal the number of quarts of alcohol in the mixture at the beginning.

$$\therefore \tfrac{1}{2}(1+x)=\tfrac{95}{100}.$$

Multiplying both sides through by 100 we get:

$$50(1+x)=95$$
$$50+50x=95$$
$$50x=45$$
$$x=\tfrac{45}{50}=\tfrac{9}{10}$$

$\therefore \frac{9}{10}$ quarts of water must be added.

3. The sum of \$1100 is invested, part at 5% and part at 6% per annum. If the total income is \$59, how much was invested at each rate?

Let \$$x$ = amount invested at 5%

then income = $\frac{5}{100}$ \$$x$

and \$$(1100-x)$ = amount invested at 6%

then income = $\frac{6}{100}$ \$$(1100-x)$

or $\frac{5}{100}x+\frac{6}{100}(1100-x)=59.$

Multiplying through by 100 we get:

$$5x+6600-6x=5900$$

$$6600-5900=6x-5x$$

$$700=x$$

∴ \$700 invested at 5% and \$400 at 6%.

Exercises XLVII.

Solve the following and verify:

1. $3x+x=64.$
2. $5x+4x=81.$
3. $8x-3x=50.$
4. $13x-3x=100.$
5. $4x+7=3x+10.$
6. $9x-6=7x-4.$
7. $7x+3=3x+67.$
8. $2x-7=11-4x.$
9. $27-3x=68-4x.$
10. $42-3x=48-9x.$
11. $6x-18=4x-8-3x+5.$
12. $10x-10-6x-27=3.$
13. $24x+10-20x+100=5x+96.$
14. $5x=\cdot05.$
15. $8x=\cdot24.$
16. $\cdot7x=2\cdot1.$
17. $\cdot2x=4.$
18. $\frac{x}{3}=\frac{1}{2}.$
19. $\frac{7x}{9}=21.$
20. $\frac{3}{4}=\frac{x}{12}.$
21. $\frac{11x}{13}-\frac{19x}{31}=0.$
22. $\frac{x-3}{5}=0.$
23. $\frac{2x-1}{3}=3.$
24. $\frac{3x+5}{7}=2.$
25. $\frac{2}{3}(x-10)=0.$
26. $\frac{4}{9}(6x-15)=0.$

27. $3(3x+1)-(x-1)=6(x+10)$.

28. $3(2x+5)-(4x-12)=5(3x+1)-4$.

29. $(11x-22)-(8-6x)-(4-8x)=17x+7$.

30. $x(x+4)=x^2+36$.

31. $x^2+2x=x^2+4$.

32. $3x^2-5-(3x^2-x)=0$.

33. $\frac{x}{4}+\frac{3}{5}=\frac{1}{4}-\frac{x}{5}+\frac{7}{2}$.

34. $\frac{x+3}{4}+\frac{x+5}{2}=\frac{x+9}{8}+\frac{x+4}{3}+\frac{13}{12}$.

35. $\frac{x+3}{4}-6+\frac{x+1}{5}=\frac{x+5}{3}-1$.

36. $\frac{x}{3}+\frac{x+2}{5}=\frac{3+x}{4}$.

37. $\frac{3x}{4}+3x=\frac{7x}{8}+2x+9$.

38. $\cdot09x-\cdot01x=\cdot14-\cdot06x$.

39. $\cdot03x+\cdot02=\cdot17$.

40. $\cdot007x-\cdot008=\cdot004x+\cdot412$.

41. $\frac{x}{\cdot5}-\frac{x}{\cdot75}=\cdot46$.

42. $\frac{\cdot25x+\cdot025}{\cdot125}=\frac{2x+\cdot45}{1\cdot25}+\cdot6$.

Exercises XLVIII.

1. A tree 84 ft. high is broken so that the length of the part broken off is five times the length of the part standing. What is the length of each part?

2. After selling $\frac{1}{2}$ of his farm and then $\frac{1}{8}$ of what was left a man still has 140 acres. How many acres had he at first?

3. The length of a rectangular building is b ft., the width is 70 ft., and its area is 43,400 sq. ft. Find the value of b.

4. A rectangular building is 84 ft. long and x ft. wide. Find x if the area is 13,440 sq. ft.

5. A rectangular shop is x ft. long and y ft. wide. If $x=120$ and $y=48$ what is the area?

6. The desired area of a new rectangular boiler shop is x sq. ft. Owing to the space available the width is limited to b ft. What must be the length c, if $b = 64$ and $x = 9648$?

7. The length of a rectangular machine shop is x ft., the width 50 ft., and the floor space must be capable of accommodating 20 machines, each occupying an average of 300 sq. ft. Find the value of x.

8. The front section of an engine frame is required to have 60 sq. in. area, the width is 5 in., and the depth is x in. Find x.

9. A man saves \$100 more than $\frac{1}{6}$ of his salary, spends 4 times as much for living expenses as he saves, and pays the remainder which is \$500 for rent. What is his salary?

10. If air is a mixture of 4 parts of nitrogen to 1 part of oxygen, how many cubic feet of each are there in a room 20 ft. by 30 ft. by 10 ft.?

11. The length of a room is to its width as 4 is to 3 and its perimeter is 70 ft. Find the width of the room.

12. The number-plate on an automobile has a perimeter of 48 in., and its length is to its width as 3 is to 1. Find its length and width.

13. Sirloin steak costs $1\frac{1}{2}$ times as much as round steak. Find the cost per lb. of each if 3 lb. sirloin and 5 lb. round steak cost \$3.04.

14. If 2 lb. butter cost as much as 5 lb. lard, and $4\frac{1}{2}$ lb. lard and 6 lb. butter cost \$5.07, find the cost of each per pound.

15. The interest on \$138 for a certain time at 6% per annum is \$16.56. Find the time.

16. A can do a piece of work in 6 days, B can do the same work in 8 days, and C in 24 days. In how many days can they do the work if they all work together?

17. A tank is emptied by two pipes; one can empty the tank in 30 min., the other in 25 min. If the tank is $\frac{2}{3}$ full and both pipes are opened, in what time will it be emptied?

18. Four pipes discharge into a cistern; one fills it in one day, the second in two days, the third in three days, the fourth in four days. If all run together how soon will they fill the cistern?

19. A train runs 100 miles in the same time as a second train runs 120 miles. If the rate of the first train is 5 miles an hour less than that of the second train, find the rate of each.

20. How many quarts of water must be mixed with 250 quarts of alcohol 80% pure to make a mixture 75% pure? (No allowance for contraction).

21. A man sells $\frac{1}{2}$ his interest in a factory and later sells $\frac{1}{4}$ of what he has left. His interest is then worth $75,000.00. How much was his original interest worth?

22. A lot of brass scrap weighing 500 lb. contains 25% zinc. How many pounds of zinc must be added in melting to increase the percentage of zinc to 34%?

23. From a tank one-half full of crude oil, 500 gallons are drawn and 25 gallons are lost by evaporation and leakage. If the tank is then one-quarter full, how much does it hold when full?

24. The sales of a firm increased 10% the second year over the first, and the third year they were 20% more than they were the second year. If the sales total $235,125.00 the third year, how much were they for the first year?

25. What is the value of the property of a person whose income is $645.00, when he has two-thirds of it invested at 4%, one-fourth at 3%, and the remainder at 2%?

26. If a boy weighing 75 lb. sits 6 ft. from the fulcrum, where should a boy weighing 100 lb. sit to balance the beam?

27. A weight of 200 grams is placed 25 centimetres from the fulcrum. How far from the fulcrum must a weight of one-half a kilogram be placed to balance the beam?

CHAPTER VIII.

FUNDAMENTAL OPERATIONS.

77. Addition. In a previous section we dealt with the addition of simple expressions such as $6a$ and $3a$, $4x^2$ and $3x^2$, etc. We now wish to deal with compound expressions such as $2a+5b$, $6a-4x+3b^2$, etc. If we wish to add a number of these compound expressions we must recall what was stated with respect to like and unlike terms. It was there pointed out that like terms may be added, as for example, $6a$ and $2a$, giving $8a$. It was also stated that unlike terms could not be added in the above way but merely written with a plus sign between them. Thus $6a^2$ plus $7b^2$ would be written $6a^2+7b^2$. If then we wish to add $3a-5b+c$, $2a+4b-c$ and $6b+7a-2c$ greater accuracy may be secured by arranging so that the like terms would be in the same vertical columns.

Thus, Example 1:

$$\begin{array}{r} 3a-5b+c \\ 2a+4b-c \\ 7a+6b+2c \\ \hline \text{Sum}=12a+5b+2c \end{array}$$

Example 2: Add $5ax-7by+cz$, $ax+2by-cz$, $-3ax+2by+3cz$.

Arrange as above giving:

$$\begin{array}{r} 5ax-7by+cz \\ ax+2by-cz \\ -3ax+2by+3cz \\ \hline 3ax-3by+3cz \end{array}$$

Exercises XLIX.

Add:

1. $x^3-3x^2,\ 3x^2-4x,\ 4x+1.$
2. $3(x-1),\ 4\ (x-1).$
3. $x-2y+3z,\ 2x+y-3z,\ x-2y+z.$
4. $a+b,\ a-b.$
5. $\frac{a}{2}+\frac{b}{2},\ \frac{a}{2}+\frac{b}{2}.$
6. $a-c,\ b-c.$
7. $x^2-2xy+y^2,\ x^2+2xy+y^2.$
8. $x+y-z,\ 3x-2y+4z.$
9. $x-(y+z),\ y-(x-z).$
10. $4(x-y),\ 5(x-y),\ 6(x-y).$

Find the values of the following sums when $x=\frac{1}{2}$, $y=\frac{1}{4}$, $z=\frac{1}{3}$, $a=3$, $b=2$, $c=\frac{1}{5}$.

11. $\frac{1}{2}a+\frac{1}{2}b-c,\ a-\frac{1}{4}b-\frac{2}{3}c,\ 5a-\frac{2}{5}b+2c.$
12. $5xy-5x^2y-5xy,\ \frac{1}{2}xy+\frac{8}{3}x^2y.$
13. $\frac{2}{3}a-\frac{3}{4}b+\frac{5}{2}c,\ \frac{3}{2}a-\frac{1}{2}b+\frac{2}{5}c.$
14. $12yz-8xy+\frac{1}{4}a+\frac{5}{2}bc.$

78. Subtraction. In its most elementary form subtraction has already been dealt with in connection with like terms.

Thus, $6a-2a=4a.$

$$7a-9a=-2a.$$

Also the rules for the removal of brackets would deal with an expression such as $6a-(-3a)$. We could write this expression $6a-(0-3a)=6a-0+3a=6a+3a.$

also, $-7x-(-5x)=-7x-(0-5x)=-7x-0+5x$
$=-7x+5x.$

An examination of the operation and the result in the two latter examples brings us to a very important result with respect to subtraction. In the first example we see that the subtracting of $-3a$ from $6a$ is equivalent to adding $+3a$ to $6a$; in the second that the subtracting of $-5x$ from $-7x$ is the same as adding $+5x$ to $-7x$. This gives us the fundamental principle with respect to subtraction:—*To subtract one* *expression from another we change the sign of the quantity to be subtracted and add it to the other expression.*

An examination of the following examples in subtraction placed as in Arithmetic would illustrate this:

$$\begin{array}{ccccc} 6 & 4a & 7x^2 & 3ab & -6x^2 \\ \underline{9} & \underline{2a} & \underline{8x^2} & \underline{-2ab} & \underline{-3x^2} \\ -3 & +2a & -x^2 & +5ab & -3x^2. \end{array}$$

If we wish to subtract one compound expression from another we arrange as in addition. Thus to subtract $3a-2b+c$ from $4b-6a-3c$ we write

$$\begin{array}{r} 4b-6a-3c \\ \underline{-2b+3a+c} \\ 6b-9a-4c. \end{array}$$

Exercises L.

Subtract:

1. $4a-3b+c$ from $2a-3b+c$.
2. $a-3b+5c$ from $3a-6b+2c$.
3. $2x-8y+z$ from $15y-6x+4z$.
4. $-4xy+2yz-10zx$ from $3xy-6yz+7zy$.
5. $4x^2-6x+2$ from $7x^2-3x-4$.
6. From the sum of $3a+2b$ and $7a-3b$ subtract $3a-b$.
7. Subtract $5x^2+3x-1$ from $6x^3$ and add the result to $3x^2+2x+1$.
8. Add the sum of $2y-3y^2$ and $1-4y^3$ to the remainder obtained when $1-4y^2+2y$ is subtracted from $8y^3+3$.

79. **Multiplication.** The method of representing the product of two simple expressions has already been given, thus the product of a and $b=ab$, the product of a, b, and $c=abc$, the product of x, y, z, and $k=xyzk$.

Combining this with our index laws we can find the product of expressions like x^2y^2 and xy giving $x^2y^2\times xy=x^3y^3$.

$$\text{Also, } 3x^2\times 7x^2=3\times 7\times x^2\times x^2=21x^4$$

$$\text{and, } 4x^3\times\frac{2}{x^2}=4\times 2\times x^3\times\frac{1}{x^2}=8x^{3-2}=8x.$$

In the section dealing with brackets it was seen that $3(a+b)=3a+3b$. In this case one of the expressions, 3, is a simple expression while the other $a+b$ is a compound expression.

If now we wish to multiply two compound expressions together, say $x+a$ by $x+b$, we may write it in the form $(x+a)(x+b)$.

The work may be conveniently arranged thus,

$$\begin{array}{l} x+b \\ \underline{x+a} \\ x^2+bx \\ \underline{\qquad +ax+ab} \\ x^2+bx+ax+ab. \end{array}$$

Multiply $x+b$ by x, then multiply $x+b$ by a and add the results.

Example: Multiply $x+2$ by $x+3$

$$\begin{array}{l} x+2 \\ \underline{x+3} \\ x^2+2x \\ \underline{\quad +3x+6} \\ x^2+5x+6. \end{array}$$

80. **Rule of Signs in Multiplication.** In the examples given above all the signs are plus. It is necessary to consider cases where the signs are minus, or some plus and some minus.

We might first recall the meaning of multiplication as understood in Arithmetic. The fundamental unit was $+1$ and all numbers were obtained from this unit.

Thus, $3=1+1+1$.

Also, $3\times4=3+3+3+3$.

From this multiplication might have been defined as follows:—*To multiply one number by a second is to do to the first what was done to unity to obtain the second.*

This law applies with equal force to the multiplication of fractions. Thus to multiply $\frac{5}{6}$ by $\frac{3}{4}$ we do to $\frac{5}{6}$ what was done to unity to get $\frac{3}{4}$: that is, we divide $\frac{5}{6}$ into four equal parts and take three of them. Each part would be $\frac{5}{6\times 4}$, and by taking three of these parts we get $\frac{5}{6\times 4}\times 3=\frac{5}{6}\times\frac{3}{4}$.

We will, therefore, make the above definition the basis of the rule of signs in multiplication.

(1) To multiply $+3$ by $+4$,

$$+3\times +4=+3+3+3+3=+12,$$

or generally $+a\times +b=+ab$.

(2) To multiply -3 by $+4$.

If we do to -3 what was done to unity to obtain 4 we have $-3\times +4=-3-3-3-3=-12$,

or generally $-a\times +b=-ab$.

(3) To multiply $+3$ by -4.

To obtain -4 from the fundamental unit we changed its sign and took it four times. If this be done with $+3$ then

$$+3\times -4=-3-3-3-3=-12,$$

or generally $+a\times -b=-ab$.

(4) To multiply -3 by -4.

Explaining -4 as in (3) and applying definition we have

$$-3\times -4=+3+3+3+3=+12,$$

or generally $-a\times -b=+ab$.

The results of (1), (2), (3), (4) may be stated in words giving the following rule for signs in multiplication:—*The product of two numbers with like signs is positive and with unlike signs is negative.*

Exercises LI.

Multiply:

1. $3a$ by 2.
2. $3x$ by -2.
3. $-2b$ by -4.
4. $-3a^2$ by a^2.
5. $-3ab$ by $2ab$.
6. $3x$ by $4y$.
7. $7x^3$ by $-3x$.
8. a^2b by $-ab$.
9. $4x^2$ by $-2x$.
10. p^3 by $-p^2$.
11. a^3b by $-ab^3$.
12. p^{11} by $-p^3$.
13. $x^2y^2z^2$ by $-xyz$.
14. $\frac{1}{2}x$ by $-\frac{1}{3}y$.
15. $\frac{3}{4}a^2$ by $-\frac{4}{3}b^3$.
16. $\frac{5}{8}x^3$ by $-\frac{8}{3}x^2$.
17. $\frac{4}{3}x^2y$ by $-\frac{9}{16}xy^2$.
18. $-\frac{3}{11}ab^2$ by $\frac{22}{9}a^2b$.
19. $\frac{1}{4}x^2y^2$ by $-\frac{4}{x^2y^2}$.
20. $-4x^2y$ by $-5x^3y$.

Write down the continued product of:

21. $-3, -4, 6$.
22. $a, -b, c$.
23. $a^2, -b^2, c^2$.
24. $-b^2, -c^2, a$.
25. $2a, 3b, -a$.
26. $2x, -3x, -4x$.
27. a^2x, x, y.
28. $-2x, -2x, -2x$.
29. $x, -x, x, -x$.
30. $3p^2, 2pq, 4qp$.
31. $2x, -3x^2, -2x^4, -x^5$.
32. $a^2, b^3, 2c$.

Write down the values of:

33. $(-x)^3$.
34. $(-a)^4$.
35. $(-2a)^3$.
36. $(x^2)^3$.
37. $(-a)^6$.
38. $(-x^2)^3$.
39. $(2xy)^3$.
40. $(-1)^2$.
41. $(-1)^3$.
42. $(-1)^4$.
43. $(-1)^5$.
44. $(-x^2)^7$.
45. $(-x^3)^5$.
46. $(-2a^2b)^2$.
47. $(-3x^2y)^3$.
48. $(-3x^2y)^4$.
49. $(-7x^2y^2)^2$.
50. $(-xyz)^3$.

Exercises LII.

Multiply:

1. $a+b-c$ by 4.
2. $2a-3b+c$ by -2.
3. $x+y+z$ by $2x$.
4. $3x^2+y^2$ by $-2x$.
5. $x^2+2xy+y^2$ by x.
6. a^2-ab+b^2 by $-a$.
7. $3x^4-2x^3+6$ by $-5x$.
8. $-3a^2-2ab+b^2$ by $-2b^2$.
9. $1-2x+x^2$ by $-2x$.
10. x^2-y^2 by $-xy$.

Find the continued product of:

11. $a+b, a, b$.
12. $a^2-2ab+b^2, a, b$.
13. x^2-5x+3, x^2, x.
14. $a-b, a, -b$.
15. $x^4-3x^3+2x^2-1, -3x, -2x$.
16. $a^3-a^2b+ab^2-b^3, -a, -b$.

When $a=-2$, $b=-3$, find the value of:

17. a^2-2.	22. b^4-81.	27. a^2+b-b^2.
18. $2a^2-a+2$.	23. b^2-a^2+2a.	28. a^4-b^4.
19. a^2-b^2.	24. a^3+8.	29. a^5-b^5.
20. $a^2-2ab+b^2$.	25. a^3+b^3.	30. a^3-3b.
21. $2a^3+16$.	26. $8a^2-b^3$.	31. a^4-1.

Exercises LIII.

Find the product of:

1. $x+a, x-b$.	11. $x^2-a^2, x+a$.
2. $ay-b, cy-d$.	12. $x+2y, 3x+1$.
3. $5+3x, 7-2x$.	13. $7a-2b, a^2-b^2$.
4. $x-5y, 2x+3y$.	14. $ax^2-bx, ax+b$.
5. $a+3x, a-5x$.	15. $6a-2b, a-b$.
6. $ax+1, bx+1$.	16. $a+b, c-d$.
7. $4a^2-3b, 2a^2-b$.	17. x^2+a, x^3-b.
8. $x^3-1, x+1$.	18. $bx-ay, ax-cy$.
9. a^2+6b, a^2-4b.	19. $xyz-1, xy+2$.
10. $a+3x, a-5x$.	20. x^5-1, x^4+1.

81. **There is a number** of types of products in which the results can be written down by inspection if a few typical examples are examined.

(1) $(x+a)(x-a)=$
$$\begin{array}{l} x+a \\ \underline{x-a} \\ x^2+ax \\ \underline{\quad\;\; -ax-a^2} \\ x^2 \qquad\quad -a^2. \end{array}$$

That is, the product of the sum and difference of two quantities is equal to the difference of their squares.

Thus, $(x+3)(x-3)=x^2-9$.

$(a+b)(a-b)=a^2-b^2$.

$(xy+1)(xy-1)=x^2y^2-1$.

(2) $(a+b)(a+b)$ or $(a+b)^2=$
$$\begin{array}{l} a+b \\ \underline{a+b} \\ a^2+ab \\ \underline{\quad\;\; +ab+b^2} \\ a^2+2ab+b^2. \end{array}$$

That is $(a+b)^2=$ the square of a, plus the square of b, plus twice the product of a and b. Any expression consisting of two terms is called a binomial, so that we may state as a general rule:—*The square of a binomial is equal to the sum of the squares of the terms plus twice their product.*

Thus, $(x+3)^2=x^2+9+6x.$

$$(x-4)^2=x^2+16-8x.$$

$$(xy-1)^2=x^2y^2+1-2xy.$$

Also, $(a+b+c)^2=\{a+(b+c)\}^2=a^2+(b+c)^2+2a(b+c).$

$$=a^2+b^2+c^2+2bc+2ab+2ac.$$

$$=a^2+b^2+c^2+2ab+2ac+2bc.$$

This method may be used for the square of an expression containing any number of terms so that the rule may be given thus:—*The square of an expression consisting of any number of terms is equal to the sum of the squares of each of the terms plus twice the product of each term multiplied by each of the terms that follow it.*

(3) $(x+2)(x+3)=x+2$

$$\begin{array}{l} \quad x+3 \\ \hline x^2+2x \\ \quad\ +3x+6 \\ \hline x^2+5x+6. \end{array}$$

Here we observe that the first term in the product x^2 is obtained by multiplying the first terms in each of the factors, the second term $5x$ is obtained by adding the 3 and the 2 and multiplying by x, the third term is obtained by multiplying the 2 and the 3 together.

Thus, $(x+4)(x+5)=x^2+9x+20.$

$$(x-4)(x+3)=x^2-x-12.$$

$$(x-6)(x-4)=x^2-10x+24.$$

Exercises LIV.

Write down the results of the following:

1. $(c+d)(c-d)$.
2. $(2x+3)(2x-3)$.
3. $(x^2-2a^2)(x^2+2a^2)$.
4. $(x^2+2)(x^2-2)$.
5. $(a+3b)(a-3b)$.
6. $(px+q)(px-q)$.
7. $(x^2-3y^2)(x^2+3y^2)$.
8. $(2x-3y)(2x+3y)$.
9. $(a^2-4b)(a^2+4b)$.
10. $(x+y)(x-y)(x^2+y^2)$.
11. $(c+d)^2$.
12. $(a-2b)^2$.
13. $(2x-y)^2$.
14. $(2x+3y)^2$.
15. $(xy+1)^2$.
16. $(x^2-1)^2$.
17. $(a+b-c)^2$.
18. $(2a-b-c)^2$.
19. $(a+b+c-d)^2$.
20. $(2a-3b+c)^2$.
21. $(x+3)(x+4)$.
22. $(a+5)(a-2)$.
23. $(z+8)(z-5)$.
24. $(p+3q)(p-6q)$.
25. $(ab+4)(ab-5)$.
26. $(xy-b)(xy+c)$.

Use the rule for the square of a binomial to find the value of:

27. 99^2.
28. 102^2.
29. 105^2.
30. 95^2.
31. $(100-6)^2$.
32. $(99\cdot5)^2$.

82. **Division.** The **Rule of Signs in Division** may be readily deduced from the rule in **Multiplication.**

Thus, (1) $+xy=+x\times+y \therefore +xy\div+x=+y$ or $\dfrac{+xy}{+x}=+y$.

(2) $-xy=-x\times+y \therefore -xy\div-x=+y$ or $\dfrac{-xy}{-x}=+y$.

(3) $+xy=-x\times-y \therefore +xy\div-x=-y$ or $\dfrac{+xy}{-x}=-y$.

(4) $-xy=+x\times-y \therefore -xy\div+x=-y$ or $\dfrac{-xy}{+x}=-y$.

From these results we have the following rule of signs in division:—*Terms with like signs when divided give plus* (+). *Terms with unlike signs when divided give minus* (−).

Examples: $\frac{+6}{+2}=+3.$ $\qquad \frac{-6}{-2}=+3.$

$\frac{+6}{-2}=-3.$ $\qquad \frac{-6}{+2}=-3.$

$\frac{+21a^2}{-3a}=-7a.$ $\qquad \frac{-3x^2y^2}{+xy}=-3xy.$

$\frac{-35a^3b^2c}{-7abc}=5a^2b.$ $\qquad \frac{+5x^7}{-5x^2}=-x^5.$

Exercises LV.

Divide:

1. $3x$ by 3.
2. $-3x$ by 3.
3. $-3x$ by -3.
4. $-3x$ by x.
5. $6xy$ by $6x$.
6. a^2 by $-a^2$.
7. $8a^2$ by $-4a$.
8. $-b^4$ by b.
9. $8a^2$ by $-4a^2$.
10. $-54a^2bc$ by $6abc$.
11. $24a^2b^2c^2$ by $-40bc$.
12. $-21x^3y^4$ by $-7x^3y^2$.
13. $-49a^3b^3$ by $7a^2b^2$.
14. $-x^5$ by $+x^2$.

Simplify:

15. $\frac{15x}{5}$.
16. $\frac{-21x^3y^3}{-3xy}$.
17. $\frac{-8xy^2}{-xy}$.
18. $\frac{24y^2z^2}{-4y}$.
19. $\frac{49pq^2r}{-7pqr}$.
20. $\frac{-32l^2m^2n^2}{4lm}$.
21. $\frac{121x^3y^6}{11x^3y^2}$.
22. $\frac{-16a^3b^3}{-8a^2b}$.
23. $\frac{\frac{1}{4}abc}{\frac{1}{8}abc^2}$.

Divide:

24. $3x-6y$ by 3.
25. $3x-9$ by -3.
26. $3x^2-6x$ by $-3x$.
27. $-b^2+ab$ by b.
28. $4a^2b-8ab^2$ by $-2ab$.
29. $-x^3+x^2$ by $-x^2$.
30. $6a-9b+12c$ by -3.
31. x^3+3x^2-3x by x.
32. $15y^4-5y^3x^3-30y^3$ by $5y$.
33. $-5m^3n+20m^2n^3$ by $-5mn$.
34. $a^2bc-ab^2c+abc^2$ by $-abc$.
35. $-a^2b^2c^2+abc^2-cab^2$ by abc.

83. To divide one compound expression by another the work may be arranged by following the method of long division in Arithmetic:

Example. Divide x^2+5x+6 by $x+2$.

$$
\begin{array}{l}
x+2)\,x^2+5x+6\,/\,\underline{x+3}. \\
\quad\quad\ \underline{x^2+2x}\,\ldots\ldots\ldots(1) \\
\quad\quad\quad\quad\ 3x+6\ldots\ldots(2) \\
\quad\quad\quad\quad\ \underline{3x+6}\ldots\ldots(3)
\end{array}
$$

$x^2 \div x = x$ $\therefore$ x is the first term in the quotient, $(x+2)$ multiplied by x gives x^2+2x and we obtain (1). Line (2) is obtained by subtracting x^2+2x from the expression and bringing down $+6$. $3x$ divided by $x=3$, $\therefore$ 3 is the second term of the quotient.

$(x+2)$ multiplied by $3=3x+6$ and we obtain line (3). This when subtracted leaves no remainder and the quotient is $x+3$.

This method may be applied to an expression of any number of terms, if care is taken to arrange the divisor and dividend in descending or ascending powers of some common letter, and to keep the remainder in each case in the same order.

Exercises LVI.

Divide:

1. $x^2+7x+12$ by $x+3$.
2. a^2+3a+2 by $a+2$.
3. a^2-3a+2 by $a-1$.
4. $x^2-5x-14$ by $x+2$.
5. $15x^2-26x+8$ by $5x-2$.
6. $6-13a+6a^2$ by $2-3a$.
7. $4+4x+x^2$ by $2+x$.
8. $x^2+2xy+y^2$ by $x+y$.
9. $25-30a+9a^2$ by $5-3a$.
10. $4x^4-49$ by $2x^2-7$.
11. $x^2+ax+bx+ab$ by $x+a$.
12. $x^4+x^2y^2+y^4$ by x^2-xy+y^2.
13. a^3+b^3 by $a+b$.
14. $x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+y^5$ by $x^2+2xy+y^2$.
15. $a^3+b^3+c^3-3abc$ by $a+b+c$.

CHAPTER IX.

FORMULAS.

84. **One of the most Valuable Uses** for algebraic symbols is to express a scientific law in a short form. When such a law is expressed in algebraic form it is called a **formula.** For example, the area of a rectangle is equal to the length multiplied by the breadth. If we let A represent the area, l the length, and b the breadth, we could briefly represent this relation by the equation $A = lb$.

If $l = 15$ in., $b = 10$ in., then $A = 15 \times 10 = 150$ sq. in.

Again if $A = lb$., then $l = \frac{A}{b}$. This is called solving for l.

If $A = 200$ sq. in., $b = 25$ in., then $l = \frac{200}{25} = 8$ in.

Further if $A = lb$., $b = \frac{A}{l}$. This is called solving for b.

If $A = 400$ sq. in., $l = 40$ in., then $b = \frac{400}{40} = 10$ in.

If a scientific law be stated in detail it is important to be able to express it as a formula.

Example.

To find the number of revolutions of a driven pulley in a given time, multiply the diameter of the driving pulley by its number of revolutions in the given time, and divide by the diameter of the driven pulley.

Using D and d for the diameters, and N and n for the number of revolutions respectively, express the above as a formula.

Exercises LVII.

1. The cutting or surface speed, that is the number of linear feet measured on the surface of the work that passes the edge of a cutting tool in a minute, is found by multiplying the circumference of the work being turned by its R.P.M. Express the rule as a formula.

2. The current flowing along a conductor is given by the formula $I=\frac{E}{R}$, where I is the current in amperes, E the electromotive force in volts, and R the resistance in ohms.

Solve for E and R.

If $E=110$, $R=220$, find I.
If $I=2\frac{1}{2}$, $E=220$, find R.
If $I=\cdot 5$, $R=75$, find E.

3. The resistance of a wire in an electric circuit is given by $R=K\frac{L}{A}$, where R is the resistance, L the length of the wire, A its area in circular mils, K the resistance of 1 mil foot in ohms.

Solve for K, L and A.

If $L=3$ ft., $A=1000$, $K=10\cdot 5$, find R.
If $R=220$, $K=10\cdot 5$, $L=1000$ ft., find A.
If $R=10$, $K=10\cdot 5$, $A=250$, find L.

4. The work done by any force is given by $W=FS$, where W is the work done, F the force in pounds, S the distance in feet through which the force acts.

Solve for F and S.

If $F=525$ lb., $S=51$ ft., find W.
If $W=150$, $F=5$ oz., find S.
If $W=500$, $S=800$ ft., find F.

5. The total resistance of a series circuit is given by $R=R_1+R_2+R_3$, where R is the total resistance and R_1, R_2, R_3 are the resistances of the separate parts of the circuit respectively.

Solve for R_1, R_2 and R_3.

If $R_1=2$, $R_2=4$, $R_3=9$, find R.
If $R=12$, $R_1=2$, $R_2=7$, find R_3.

6. The indicated horse-power of a single acting engine is given by:

$$H.P.=\frac{PLAN}{33000},$$

where P is the mean effective pressure on the piston in pounds per sq. in., L the length of the stroke in feet, A the effective area of piston in sq. in., N the number of strokes per minute.

Solve for P, L, A and N.

If $P = 80$, $L = 2$ ft., $A = 30$ sq. in., $N = 60$, find $H.P.$

If $H.P. = 4$, $L = 1\frac{1}{2}$ ft., $A = 24$ sq. in., $N = 50$, find P.

If $H.P. = 10$, $P = 50$, $A = 30$ sq. in., $N = 100$, find L.

If $H.P. = 20$, $P = 60$, $L = 2$ ft., $N = 100$, find A.

If $H.P. = 16$, $P = 60$, $L = 2$ ft., $A = 40$, find N.

7. The formula $D = QIT$ is used in electrolysis, where D is the weight of the deposit, Q the electro-chemical equivalent, T the time in seconds, I the current.

Solve for Q, I and T.

If $Q = \cdot 001118$, $I = 40$, $T = 600$, find D.

If $D = 2$, $Q = \cdot 000328$, $I = 250$, find T.

If $D = \cdot 5$, $I = 10$, $T = 153$, find Q.

8. The diameter of a rivet is given by $d = 1\cdot 2\sqrt{t}$, where d is the diameter in inches and t the thickness of the plate in inches.

If $t = \cdot 75$, find d.

If $d = \frac{3}{16}$ in., find t.

9. The space through which a body falls from rest is given by $s = \frac{1}{2}gt^2$, where s is the space in ft., g the acceleration due to gravity, t the time in seconds.

Solve for g and t.

If $t = 12$, $g = 32\cdot 2$, find s.

If $s = 3155.6$, $g = 32\cdot 2$, find t.

10. In a machine $E = \frac{W}{PV}$, where E is the efficiency, W the weight, V the velocity ratio, P the horizontal force.

Solve for W, P and V.

If $W = 112$, $P = 20$, $V = 12\cdot 5$, find E.

If $E = \cdot 55$, $P = 25$, $V = 18\cdot 5$, find W.

If $E = \cdot 74$, $W = 350$, $V = 23$, find P.

If $E = \cdot 346$, $W = 799\cdot 26$, $P = 20$, find V.

11. The allowable working pressure in a steam boiler is given by:

$$B = \frac{2T\,sk}{DF},$$

where T is the thickness of the plate in inches, s the tensile strength of plate in pounds per sq. in., k the efficiency of the joint, D the inside diameter of shell in in., F the factor of safety.

Solve for T, s, k, D and F.

If $T=\frac{1}{4}$, $s=35000$, $k=\cdot 45$, $D=30$, $F=4$, find B.
If $B=75$, $s=40000$, $k=\cdot 5$, $D=40$, $F=5$, find T.
If $B=38$, $T=\frac{3}{16}$, $k=\cdot 5$, $D=50$, $F=4$, find s.
If $B=150$, $T=\frac{1}{2}$, $s=60000$, $D=60$, $F=5$, find k.
If $B=200$, $T=\frac{3}{4}$, $s=70000$, $k=\cdot 75$, $F=5$, find D.
If $B=40$, $T=\frac{1}{2}$, $s=65000$, $k=\cdot 8$, $D=120$, find F.

12. The horse-power of an electric current is given by $H.P.=\frac{EI}{746}$, where E is the electromotive force and I the current in amperes.

Solve for E and I.

If $E=110$, $I=30$, find $H.P.$
If $H.P.=6$, $E=200$, find I.
If $H.P.=10$, $I=40$, find E.

13. The heat generated by a current is given by $H=\cdot 24\,EIT$ (Joule's Law) where H is the heat in calories, E the electromotive force, I the current in amperes, T the time in seconds.

Solve for E, I and T.

If $E=110$, $I=2$, $T=30$, find H.
If $H=500$, $E=6$, $I=10$, find T.
If $H=1000$, $I=\cdot 5$, $T=160$, find E.

14. The space traversed by a body starting from rest and moving with a uniform velocity is given by $s=vt$, where s is the space, v the velocity and t the time.

Solve for v and t.

If $v=12$ ft. per sec., $t=25$ sec., find s.
If $s=300$ ft., $t=15$ sec., find v.
If $s=500$ ft., $v=20$ ft. per sec., find t.

15. The width of a single belt to transmit a given horse-power is given by $W=\frac{33000\times H}{P\times S}$, where W is the width of the belt in in., H the horse-power transmitted, P the allowable pull per in. of width of belt, S the speed of the belt in feet per min.

Solve for H, P and S.

If $W=13$, $P=30$, $S=3000$, find W.
If $W=8$, $P=40$, $S=3500$, find H.
If $W=6$, $H=20$, $S=3200$, find P.
If $W=6$, $H=22$, $P=30$, find S.

16. The brake horse-power of an engine is given by $B.H.P.=\frac{2\pi PRN}{33000}$, where $B.H.P.$ is the brake horse-power, P the reading of the scale beam, R the length of the arm in ft., N the revolutions per min.

Solve for P, R and N.

If $P=180$ lb., $R=48$ in., $N=250$, find $B.H.P.$
If $B.H.P.=30$, $R=54$ in., $N=120$, find P.
If $B.H.P.=32$, $P=200$ lb., $N=180$, find R.
If $B.H.P.=36$, $P=220$ lb., $R=5$ ft., find N.

CHAPTER X.

MENSURATION OF AREAS.

85. **Mensuration** is that part of Mathematics which deals with the length of lines, the areas of surfaces, and the volumes of solids.

86. **To Find the Area of a Rectangle or of a Square.**

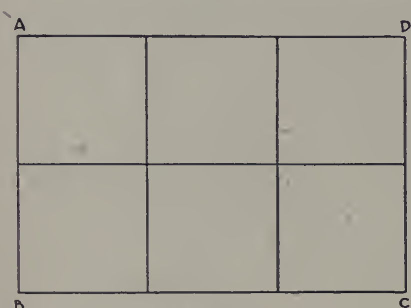


FIG. 27

In Figure 27, $ABCD$ is a rectangle, i.e., a quadrilateral with its opposite sides parallel and its angles right angles. If we divide each side into inches and join as above we see by actually counting the small squares that the area of the rectangle is six square inches (to scale). This result might have been obtained by multiplying the number of inches in the length (3) by the number of inches in the width (2). From this example we infer a formula for the area of a rectangle. If A represents the area, b the length, and h the breadth, then $A = bh$, *or the area of a rectangle* $=$ *length* $\times$ *breadth.*

Note.—A correct statement of the above formula would manifestly be—the *measure* of the area of the rectangle = the *measure* of the length multiplied by the *measure* of the breadth, but for the sake of brevity the word "measure" will be omitted throughout.

Make drawings in your laboratory book to test the accuracy of the above.

87. To Find the Area of a Parallelogram.

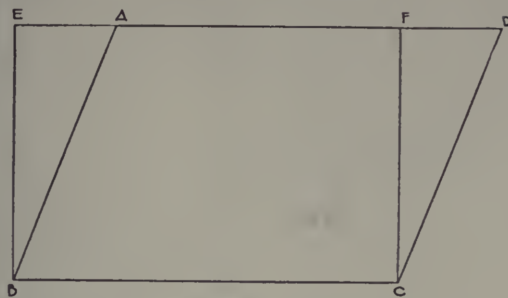


FIG. 28

In Figure 28, *ABCD* is a parallelogram ($\|^{gm}$), i.e., a quadrilateral with its opposite sides parallel.

If the right-angled triangle *DFC* be cut out and placed on *EBA*, it will coincide with *EBA*. The $\|^{gm}$ *ABCD* is therefore equal in area to the rectangle *EBCF*. If the area of the $\|^{gm}$ is A, the base b, and the perpendicular height or altitude h, then $A = bh$, *or the area of a parallelogram = base × perpendicular height.*

Make drawings in your laboratory book to test the accuracy of the above.

88. To Find the Area of a Triangle in Terms of its Base and Altitude.

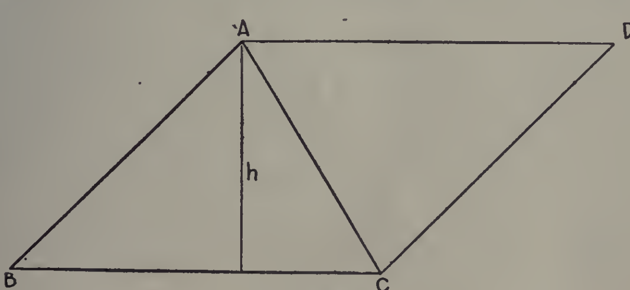


FIG. 29

In Figure 29, *ABC* is a triangle. If we draw *CD* parallel to *AB* and *AD* parallel to *BC*, we have the $\|^{gm}$ *ABCD*. Since the area of the $\|^{gm}$ $ABCD$ is bisected by its diagonal AC, we have the area of the triangle ABC as one-half the area of the $\|^{gm}$ $ABCD$. If A is the area of the triangle, b its base, and h its altitude, then $A = \frac{1}{2}bh$, *or the area of a triangle* $= \frac{1}{2}$ *base* $\times$ *altitude.*

Make drawings in your laboratory book to test the accuracy of the above.

89. To Find the Area of a Triangle in Terms of the Sides.

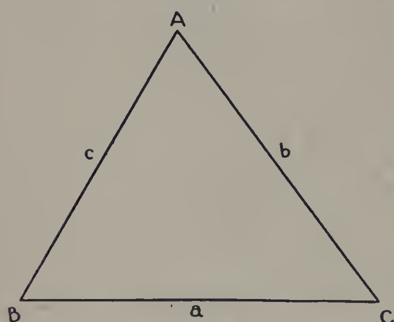


FIG. 30

In Figure 30 we have a triangle ABC and have denoted the sides by a, b, c; a being opposite angle A, b opposite angle B, and c opposite angle C.

The area of the triangle is given by the formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ *where* a, b, c *are the sides, and* s *is one-half their sum.*

Example:—If the sides in Figure 30 are 13 ft., 14 ft., 15 ft. respectively, then $s = 21$, $s-a = 8$, $s-b = 7$, $s-c = 6$.

$$\therefore A = \sqrt{21 \times 8 \times 7 \times 6} = \sqrt{7056} = 84 \text{ sq. ft.}$$

Exercises LVIII.

1. Supply the missing quantities in the following rectangles:

AREA	LENGTH	BREADTH
sq. ft.	4 ft.	3 ft.
444 sq. ft.	37 ft.	ft.
360·5 sq. ft.	ft.	18·9 ft.
sq. yd.	24 ft. 9 in.	15 ft. 6 in.
$\frac{3}{4}$ acre	ft.	$2\frac{1}{2}$ chains

2. Supply the missing quantities in the following parallelograms:

Area	Base	Altitude
sq. ft.	4 ft.	3 ft.
48400 sq. yd.	352 yd.	yd.
sq. ft.	2 ft. 3 in.	8 in.
378 sq. in.	3 ft. 6 in.	in.
sq. yds.	5 yd. 1 ft.	3 yd. 2 ft.

90. To Find the Area of a Trapezium.

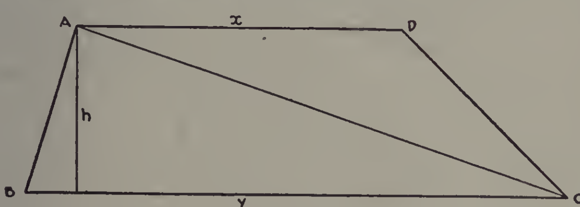


Fig. 31

Figure 31 represents a trapezium, i.e., a quadrilateral with a pair of sides parallel.

The diagonal AC divides the trapezium into the two triangles ABC and ADC.

$$\text{Area of } ABC = \tfrac{1}{2}\, yh.$$

$$\text{Area of } ADC = \tfrac{1}{2}\, xh.$$

$$\therefore\ A = \tfrac{1}{2}\, yh + \tfrac{1}{2}\, xh = \tfrac{1}{2}h(x+y),$$

or the area *of a trapezium = sum of parallel sides* $\times$ $\tfrac{1}{2}$ *the perpendicular distance between them.*

91. A Practical Application of the Triangle and the Trapezium is found in the Measurement of Land.

The following represents an entry in a surveyor's field book and the corresponding plan:

Field Book

	Links	
	To *B* 460	
to *E* 120	340	
	180	90 to *C*
to *D* 80	100	
From	*A*	go North.

Plan

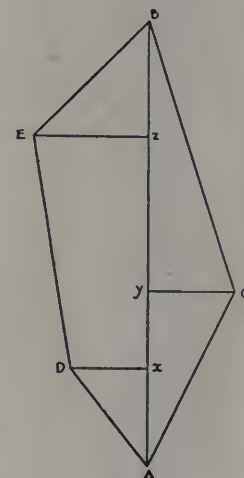


FIG. 32

The field book entry is read upwards, which in the case above indicates that the chain line runs north from *A*. The centre column refers to measurement from *A* along the chain line to points from which the offsets are taken. Offsets to the right are indicated on the right and offsets to the left are indicated on the left.

In the plan the area of $ADX = \frac{1}{2} \times 100 \times 80 = 4000$ sq. li.

the area of $EDXZ = \frac{1}{2} \times 240(120+80) = 24000$ sq. li.

the area of $BEZ = \frac{1}{2} \times 120 \times 120 = 7200$ sq. li.

the area of $AYC = \frac{1}{2} \times 180 \times 90 = 8100$ sq. li.

the area of $BYC = \frac{1}{2} \times 280 \times 90 = 12600$ sq. li.

Total Area $= 55900$ sq. li.

$= \cdot 559$ acres.

Exercises LIX.

1. Find the missing quantities in the following triangles:

Area	Base	Altitude
sq. in.	24 in.	13 in.
sq. ft.	2 ft. 6 in.	3 ft. 4 in.
120 sq. in.	in.	15 in.
22 sq. ft.	5 ft. 6 in.	ft.
$5\frac{1}{2}$ acres	320 rods	yards

2. Find the missing quantities in the dimensions of the following boiler plates in the form of trapeziums:

Area	x	y	h
sq. in	6 ft.	62 in.	102 in.
sq. in.	6 ft. 9 in.	77 in.	83 in.
8505 sq. in.	11 ft. 10 in.	101 in.	in.
sq. in.	8 ft. 5 in.	$79\frac{1}{2}$ in.	93 in.
$9841\frac{3}{4}$ sq. in.	$98\frac{1}{2}$ in.	$90\frac{1}{2}$ in.	
549 sq. ft.	25 ft.		18 ft.

3. Find the areas of the following triangles:

Sides 3 ft., 4 ft., 5 ft.; answer in sq. feet.

Sides 4 yd., 2 ft., 3 yd., 2 ft., 1 yd., 1 ft.; answer in square yards.

Sides 17 in., 18 in., 19 in.; answer in sq. inches.

Exercises LX.

1. Find the areas of the triangular faces of a number of the models in the laboratory, using both methods. Make drawings in your laboratory book.

2. Find the areas of trapeziums available in the laboratory. Make drawings in your laboratory book.

3. A rhombus is a quadrilateral with all its sides equal. Construct a rhombus in your laboratory book having each side 2 in. Employ both experiment and equality of triangles to establish how one diagonal divides the other, and also the magnitude of the angle contained by the diagonals. Write out the details and derive a formula for the area of a rhombus in terms of the diagonals.

4. Take a series of measurements in the school grounds and enter in your laboratory book as suggested. Draw a plan to scale from your measurements and calculate the area.

5. What is the area of the surface of a boiler plate 3′ 8″ by 1′ 6″?

6. How many square pieces of zinc 6″ × 6″ can be cut from a zinc plate 3′ × 6′?

7. What is the value of copper in an open copper tank measuring $4\frac{3}{4}$″ long, $3\frac{1}{2}$″ wide and $2\frac{1}{4}$″ deep; copper weighing 12 lb. per sq. ft. and costing 40c per lb.? (No allowance being made for laps, seams or waste).

8. The diagonals of a sheet of zinc in the form of a rhombus are 24″ and 16″. Find the area of the sheet.

9. If a sheet of copper 5′ × 10′ weighs 500 lb., what is the weight per sq. ft.?

10. How many sq. ft. of sheet copper will be required to make an open rectangular tank 7′ long, 3′ wide, and $1\frac{1}{2}$′ deep, allowing 12% extra for waste?

11. Find the cost of shingling the roof in the diagram on page 62 with shingles laid $4\frac{1}{2}$ in. to the weather if material and labour cost $14 a square of shingles, the eaves projecting 2′ (equivalent to a roof 34′×28′ on plan).

12. Find the cost of putting a slate roof on the building in the diagram on page 62, gauge $8\frac{1}{2}$″, at $30 a square.

13. Find the cost of covering with 1″ square sheeting the gable ends of the building in the diagram on page 61, width 20′, rise 10′, material to cost \$50 per M, allowing 8% for waste.

14. Find the cost of covering with 1″ square sheeting the gable ends of the roof represented in Fig. 33 at \$52 per M, allowing 10% for waste.

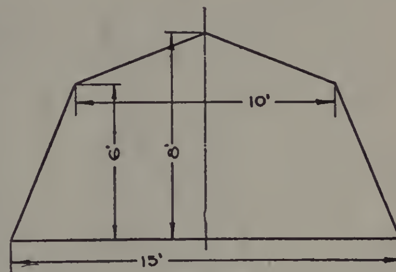


FIG. 33

15. In the map of a district it is found, that two of its boundaries are approximately parallel and equal to 13 miles and 18 miles. If the breadth is 8 miles find the area.

16. In the quadrilateral $ABCD$ the diagonal AC is 62″ long, and the perpendiculars on AC from B and D are 15″ and 12″ respectively, find the area of the quadrilateral in sq. ft.

17. Construct an equilateral triangle 2″ to the side. Find its altitude.

18. Construct an equilateral triangle with the length of the side taken at random. Denote it by x and find the altitude. What is the relation between the altitude and half the base?

19. Find the area of a regular hexagon, if one of the sides be 3″. (Divide into six equilateral triangles).

20. Find the side of an equilateral triangle equal in area to a triangle with sides 13″, 14″ and 15″ respectively.

21. How many sq. ft. are there in the surface of a board 18′ long, 6″ wide at one end and 14″ wide at the other?

22. How many sq. in. are there in a triangular plate, if one of its sides be 15″, and the perpendicular on it from the opposite vertex be 8″?

23. If the sides of a triangular plot of ground be 26′, 28′ and 30′ respectively, find the length of the perpendicular from the opposite vertex on the 30 ft. side.

24. One side of a triangular plate, containing 45 sq. in., is 8″. Find the length of the perpendicular on this side from the opposite vertex.

25. The sides of a right-angled triangle are 5″, 12″ and 13″ respectively. Find the areas of the equilateral triangles described on its sides. Do these areas bear any relation to each other?

26. A column having a cross-shaped section has two opposite arms of the cross $4\frac{1}{2}''$ long, and the other two arms 4″. The arms are $\frac{1}{2}''$ wide. What is the area of the section?

27. A *T*-shaped section has the top flange 8″ long and $\frac{5}{8}''$ wide, the other flange measuring 4′ long by $\frac{3}{4}''$ wide. What is the area of the *T*?

28. The two parallel sides of a trapezium measure 13 chains 60 links, and 6 chains 40 links; the other sides are equal, each being 8 chains 50 links. Find the area.

29. $ABCD$ is a quadrilateral in which the following measurements have been taken: $AB=30''$, $BC=17''$, $CD=25''$, $DA=28''$, the diagonal $BD=26''$. Find the area in sq. ft.

30. $ABCD$ is a quadrilateral in which the angles ABC, CDA are right angles, and $AB=36$ chains, $BC=77$ chains, $CD=68$ chains. Find the area in acres.

31. Find the area of a quadrilateral $ABCD$ in which the diagonal AC measures 30′, and the perpendiculars on it from B and D are $3\frac{1}{2}'$ and 6′ respectively.

32. Draw the plan and calculate the area, in acres, of a plot of ground from the following notes:

	Links	
	To *B*	
	530	
to *E* 75	400	
	240	120 to *C*
to *D* 100	150	
From	*A*	go North.

33. Draw a plan and calculate the area, in acres, from the following notes:

CHAINS

	to *B*	
	24·5	
To *F* 2	15·26	
	10·1	3·16 to *E*
To *D* 2·4	8·6	
	4·3	1·5 to *C*
From	*A*	go North.

34. Draw a plan and calculate the area, in acres, from the following notes:

LINKS

	to *B*	
	1200	
250	100	
	760	324
0	400	
50	360	200
From	*A*	Go N. 30° W.

35. How many 6 in. sq. tiles should be supplied to cover the courtyard shown in Figure 34, an allowance of 5% being added to cover cutting and breakage?

36. Figure 35 shows a gusset-plate for a girder. What is its weight if the plate is of mild steel $\frac{1}{2}''$ thick, weighing 20·4 lb. per sq. ft.?

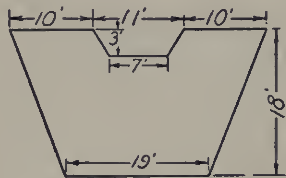


Fig. 34

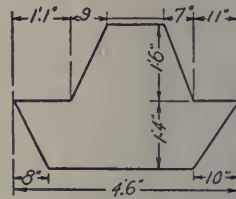


Fig. 35

37. Calculate the length of the rafters on each pitch and the total area of the entire gable end of the building in Figure 36.

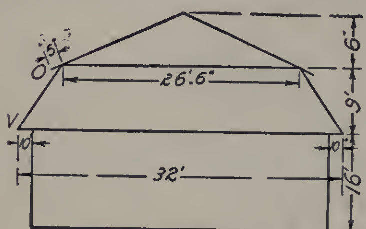


Fig. 36

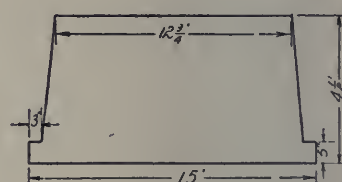


Fig. 37

38. Determine the area of the cross-section in Figure 37.

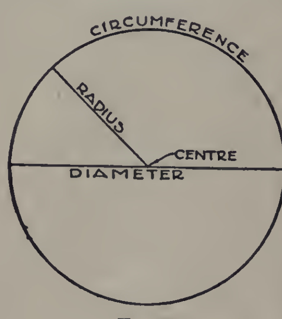


Fig. 38

92. **The Circle.** A circle is a plane figure bounded by a line called the circumference and such that every point on it is equidistant from the centre.

93. To Find the Circumference of a Circle, the following measurements of a series of circular models were made and tabulated as follows:

CIRCUMFERENCE	DIAMETER	$\frac{\text{Circumference}}{\text{Diameter}} = \pi$
11·78 in.	3·75 in.	3·1413
6·54 in.	2·08 in.	3.1442
10·98 in.	3·5 in.	3.1371
6·35 in.	2·02 in.	3·1435
16·85 in.	5·36 in.	3·1436
5·15 in.	1·64 in.	3·1402
30·0 in.	9·54 in.	3·1446
10·71 in.	3·41 in.	3·1407
8·64 in.	2·75 in.	3·1418
	Average.....	3·1418

The value of π has been determined to a great number of decimal places but 3·1416 is a close approximation. Since the fraction $\frac{22}{7} = 3 \cdot 142$ when carried out to the third place, it is commonly used as the value of π.

From the above experiment we infer that *Circumference* = π *Diameter* or $C = \pi D$. If in the formula $C = \pi D$ we substitute for diameter its value in terms of the radius, we obtain $C = \pi\,(2r)$ or $C = 2\pi r$.

94. To Find the Area of a Circle. If we take a circular board and divide it into sections as shown in Figures 39, 40, and place them as in Figure 41, we practically have a rectangle whose length is one-half the circumference and whose width is one-half the diameter of the circle.

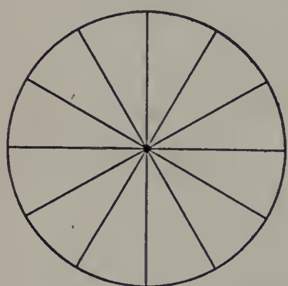


FIG. 39

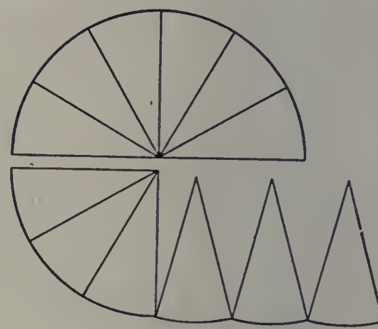


FIG. 40

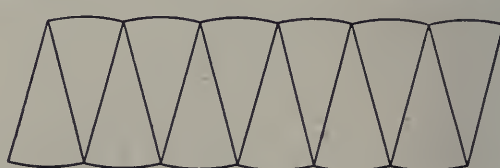


FIG. 41

Hence, to find the area of a circle we multiply one-half the circumference by the radius, i.e., $\pi r \times r = \pi r^2$.

$$\therefore A = \pi r^2.$$

In Figure 41, how would you to some extent overcome the difficulty of the length of the rectangle not being a straight line?

In the formula $A = \pi r^2$ if we write for r its value in terms of

$$D \text{ we get } A = \pi\left(\frac{D}{2}\right)^2 = \frac{\pi D^2}{4} = \frac{3 \cdot 1416 D^2}{4} = \cdot 7854 D^2.$$

This formula for the area of a circle is commonly used by engineers and machinists.

Exercises LXI.

1. Supply the missing quantities in the following circles: ($\pi = 3 \cdot 1416$)

Radius	Diameter	Circumference	Area
5 ft.			
	14 ft.		
		3·1416 ft.	
			150 sq. ft.

95. To Find the Area of a Circular Ring or Annulus.

The area of the outer circle is πR^2 and the area of the inner circle is πr^2. $\therefore$ area of the ring $= \pi R^2 - \pi r^2 = \pi(R^2 - r^2) = \pi(R+r)(R-r)$.

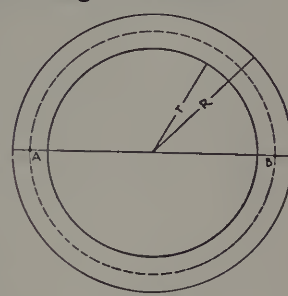


Fig. 42

If we examine this latter formula in relation to the figure we see that $R+r$ is the length of the mean diameter AB, and that $R-r$ is the width of the ring. The formula $\pi(R+r)(R-r)$ may, therefore, be written π(mean diameter) $\times$ (width of ring). $\therefore$ *area of ring = mean circumference $\times$ width of ring.*

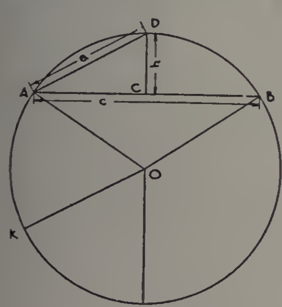


Fig. 43

96. To Find the Length of the Arc of a Circle.

In Figure 43 the chord AB divides the circumference of the circle into two arcs ADB and AHB.

If the angle AOB, that is the angle subtended at the centre of the circle by the arc ADB, is 120° then the length of the arc

ADB is $\frac{120}{360}$ × circumference or in general the *length of the arc* ADB *is* $\frac{n}{360}$ × *circumference, where* n *is the number of degrees in the angle subtended at the centre.*

The length of an arc may be found approximately by the formula:—*Length of arc* $= \frac{8a - c}{3}$ *where* c *is the chord of the arc and* a *is the chord of half the arc.*

97. **To Find the Area of a Sector of a Circle.** A sector of a circle is that part contained by two radii and the arc cut off by them.

In Figure 43, KOH represents a sector of a circle. If the angle KOH be 60° the area of the sector will be $\frac{60}{360}$ of the area of the circle, or in general the *area of the sector is* $\frac{n}{360}$ × *area of the circle, where* n *is the number of degrees in the angle contained by the two radii.*

By a method similar to that used in finding the area of a circle it may be shown that the *area of the sector* $= \frac{1}{2}$ *arc of sector* × *radius of circle.*

98. **To Find the Area of the Segment of a Circle.** A segment of a circle is that part of the circle contained by an arc and its chord.

In Figure 43 the chord AB divides the area of the circle into two segments, the area above AB being called the minor segment, and the area below AB the major segment. It is evident that the area of the segment ADB is equal to the area of the sector AOB minus the area of the triangle AOB, so that if sufficient data is given the area may be found by this method. The area of the minor segment in Figure 43 may be found approximately from the formula.

Area of Segment $= \frac{h^3}{2c} + \frac{2}{3}\,ch$. *Where* c *is the length of the chord* AB *and* h *is the height* CD.

Exercises LXII.

1. Measure a number of circular objects as suggested on page 119. Tabulate in your laboratory book, find the ratio of the circumference to the diameter in each case, and take the average of these results.

2. What length of steel sheet would be needed to roll into a drum 42″ in diameter?

3. What is the circumference in feet of an 18″ emery wheel?

4. The 72 in. drivers on a locomotive make 245 turns per min. How many feet will the locomotive go per min.? How many miles will it travel per hour?

5. Find the diameter of a driving wheel measuring 15′ $8\frac{1}{2}$″ around the outside?

6. If a point on the rim of a fly-wheel should not travel over a mile a minute, what should be the maximum speed, in revolutions per min., of a fly-wheel 7′ in diameter?

7. A pulley 3′ in diameter makes 200 revolutions per min. Through how many feet does a point on its rim travel in 2 minutes?

8. If a belt connects the pulley in the preceding question to a 15″ pulley, through how many feet will a point on the rim of the latter travel in 1 minute?

9. If a speed of a mile a minute is desired, what size emery wheel should be ordered to go on a spindle running 1320 R.P.M.?

10. A degree of latitude in Toronto measures 264613·31 ft. Find in miles the length of the parallel of latitude passing through Toronto.

11. The perimeter of a semi-circle is 72″, find its radius.

12. A pulley 36″ in diameter drives another pulley 14″ in diameter. The belt velocity is 22′ per second. What are the R.P.M. of the pulleys?

13. The fly-wheel of an engine is 12′ 6″ in diameter and revolves at 96 R.P.M. It is belted to a 48″ pulley on the main line shaft. Find the speed of the shaft.

14. A 36″ pulley making 143 R.P.M. is belted to another making 396 R.P.M. Find the diameter of the latter.

15. Describe a circle of given radius on your cross-section paper. Count the squares as accurately as possible. Construct a square on the radius and count the squares. Divide

the former result by the latter. Repeat this experiment changing the radius in each case. Tabulate in your laboratory book and find the average of your results.

16. To establish $A = \cdot 7854 D^2$ experimentally, cut a circular piece of cardboard 1′ in diameter and also a square piece of the same material 1′ to the side. Weigh both and find the value of $\frac{\text{weight of square}}{\text{weight of circle}}$. Repeat this with pieces of board, pieces of zinc, etc., taking care that the materials, in any one case, have the same thickness and density, and that the diameter of the circular part is the same as the side of the square. Tabulate in your laboratory book and find the average of the results.

17. Fill in the omitted entries in the following:

No.	Diameter	Circumference	Area
1	7		
2		44	
3			154
4		176	
5			$201\frac{1}{7}$
6			55·44
7	14·8		
8		264	
9	15·4		

18. The piston of a locomotive is 20″ in diameter. Find its area in sq. in. If the highest pressure carried is 205 lb. per sq. in., what would be the total pressure tending to blow off the cylinder head?

19. A workman finds the circumference of a shaft to be 11″. In order to find the strength of the shaft he must know the area of a cross-section. Find this area.

20. Which has the greater capacity, one 4″ pipe or two 2″ pipes?

21. The area of an 8″ circle is how many times the area of a 4″ circle? The area of a 12″ circle is how many times the area of a 4″ circle?

22. Employ the relation between the sides of a right-angled triangle to find the diameter of a pipe equal in carrying capacity to two pipes 2″ and 3″ in diameter respectively. Illustrate by means of a diagram. Extend this method of illustration to find the diameter of a pipe equal in carrying capacity to three pipes 2″, 3″, 4″ respectively, in diameter.

23. The total pressure in a cylinder is to be 6000 lb. If the pressure per sq. in. is 50 lb., what is the diameter of the piston?

24. A circular duct in a heating system is to supply air for four rectangular outlets 6″ by 8″. What must be the diameter of the duct so that its capacity will be equal to the combined capacity of the four outlets?

25. Establish experimentally the formula—Area of ring = mean circumference × width—by considering the ring as a trapezium.

26. The inner and outer diameters of a ring are 9″ and 10″ respectively, find the area of the ring.

27. A hollow cast-iron column has inside and outside diameters of 12″ and 16″ respectively, find the area of the end of the pipe.

28. What is the area of a circular race track 378 yd. inside diameter and 16′ wide?

29. What is the area of the end of a cast-iron pipe that is 12″ outside diameter and 1″ thick?

30. What is the area of the end of a rod that is $4\frac{1}{2}$″ outside diameter, and has a $1\frac{1}{4}$″ hole running through the centre of it?

31. A circular court 150 yd. in diameter is to have a walk 10′ wide around it on the inside. The remainder is to be sodded. Find the total cost if the walk costs $2.00 a sq. yd. and the sodding 40c a sq. yd.

32. The radius of a circle is 8′. Find the area of a sector of the circle, the angle of which is 36°.

33. Find the radius of a circle such that the area of a sector whose angle is 60° may be 182·5 sq. in.

34. Find the area of the sector of a boiler supported by a gusset-stay, the radius of the boiler being 42″ and the length of the arc 25″.

35. The centres of two circles which intersect are 12′ apart. The radius of the one circle is 9′, and that of the other 8′; find the area of the part which is common to both circles.

36. Find the area of the segment of a circle if the chord be 15″ long and the height of the arc 6″.

37. Construct an arc of a circle by tracing part way around any circular object. Join the ends of the arc to form the segment of a circle. Find the centre of the circle and determine the length of the arc by treating as part of the total circumference. Also find the length of the arc by the formula $\frac{8a - c}{3}$, and hence determine the percentage error in this formula.

38. Find the area of the segment in the above by finding the area of the sector and subtracting the triangle. Also, find area by the formula $\frac{h^3}{2c} + \frac{2}{3}ch$, and hence determine the percentage error in this formula.

99. **The Ellipse.** An ellipse is a plane figure bounded by a curved line, such that the sum of the distances of any point in the bounding line from two fixed points is constant. Each of these fixed points is called a focus (plural foci).

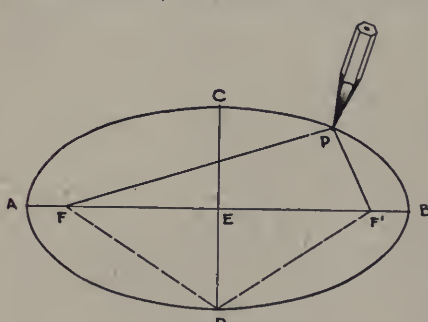


Fig. 44

Figure 44 shows an ellipse for which the sum of the distances of the point P from the foci F and F' is equal to the sum of the distances of any other point in the bounding line from F and F'. AB is called the major axis and CD the minor axis.

To Construct the Ellipse. Place the given diameters AB and CD at right angles to each other at their centres E. From D with radius AE cut the major axis at F and F'. This gives the foci. Place pins at F and F' and also at D. Place a string around the three pins forming a triangle of string FDF'. Take out the pin at D and, substituting a pencil, trace as in Figure 44. If we represent the major axis by $2a$ and the minor axis by $2b$ the circumference of the ellipse is given by the formula:

$$Circumference = \pi\ (a+b)\ \ (approximately).$$

The area of the ellipse is given by the formula $Area = \pi\, ab$.

100. **Regular Polygons.** A regular polygon is a figure having all its sides equal and all its angles equal.

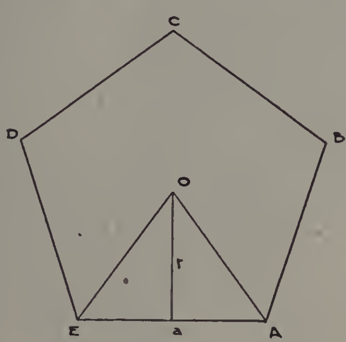


FIG. 45

ABCDE in Figure 45 is a regular pentagon (5 sides).

If we bisect the angles A and E, the point of intersection of the bisectors will give us the centre O of the inscribed circle of the pentagon, and hence a point equidistant from all the sides. This perpendicular from the centre on the side is called the apothem. The area of the triangle $AOE = \frac{1}{2}ar$, $\therefore$ area of pentagon $= 5 \times \frac{1}{2}\ ar$.

Generally, then, the area of the polygon with n sides is $\frac{1}{2}nar$.

Since it is necessary to employ Trigonometry to find the apothem, the following table is given:

Number of Sides.	When Side is a Multiply a^2 by	When Area is a^2 Multiply a by
3	·433013	1·519671
4	1	1·
5	1·720477	·762387
6	2·598076	·620403
7	3·633912	·524581
8	4·828427	·455090
9	6·181823	·402200
10	7·694209	·360511
11	9·365640	·326762
12	11·196150	·298858
15	17·642360	·238079
18	25·520770	·197949
20	31·567876	·177980

Explanation of table. The first column gives the number of sides, the second gives the area when the side is known, the third gives the side when the area is known.

Thus, if the side of a five-sided regular polygon is 6 in. then the area is obtained by multiplying 6^2 by 1·720477; also if the area of a ten-sided regular polygon (decagon) is 256 sq. in. the length of a side is obtained by multiplying $\sqrt{256}$ by ·360511.

Example:—The side of a twelve-sided regular polygon is 7″. Find the area.

From the second column of the table—

$$\text{Area} = 7 \times 7 \times 11 \cdot 196150 = 548 \cdot 61135 \text{ sq. in.}$$

Example:—The area of a nine-sided regular figure is 726 sq. ft. Find the length of a side.

From the third column the length of the side $= \sqrt{726} \times \cdot 402200$
$= 10 \cdot 8353'$.

101. Irregular Figures. Simpson's Rule for Finding Area. The area of an irregular figure may be accurately determined by the use of a planimeter, a description of which is given on page 131. When great accuracy is not required, a sufficiently accurate measurement may be made by the use of Simpson's Rule.

FIG. 46

Figure 46 represents an irregular figure. The base line is divided into eight equal parts. The perpendiculars to this base line, d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8, d_9, are called ordinates, and since there is an *even* number of divisions there will be an *odd* number of ordinates. The rule applies only when there is an odd number of ordinates.

Consider Figure 47 consisting of the first two sections of Figure 46.

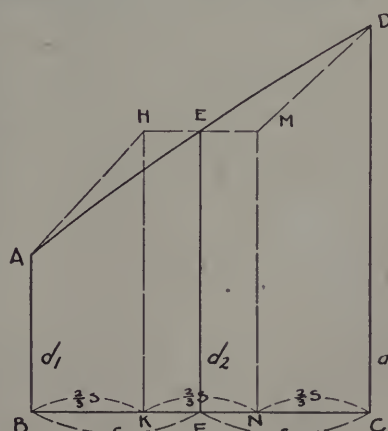


FIG. 47

HM is drawn through E parallel to BC and such that $BK = KN = NC$.

The area of $ABCD$ = area of $ABKH$ + area of $HKNM$ + area of $MNCD$ approximately.

$$\text{Area of } ABKH = \tfrac{1}{3}\, s(d_1+d_2)$$
$$\text{Area of } HKNM = \tfrac{2}{3}\, sd_2$$
$$\text{Area of } MNCD = \tfrac{1}{3}\, s(d_2+d_3)$$
$$\text{Area of } ABCD = \tfrac{1}{3}\, s(d_1+4d_2+d_3)$$

Similarly the area of the third and fourth sections

$$= \tfrac{1}{3}\, s\,(d_3+4d_4+d_5)$$

Similarly the area of the fifth and sixth sections

$$= \tfrac{1}{3}\, s\,(d_5+4d_6+d_7)$$

Similarly the area of the seventh and eighth sections

$$= \tfrac{1}{3}\, s\,(d_7+4d_8+d_9)$$

Adding these we get the total area

$$= \tfrac{1}{3}\, s\left\{(d_1+d_9)+4(d_2+d_4+d_6+d_8)+2\,(d_3+d_5+d_7)\right\}$$
$$= \tfrac{1}{3}\, s(A+4B+2C)$$

Where A = sum of first and last ordinates.

B = sum of the even ordinates.

C = sum of the odd ordinates, omitting the first and last.

s = common interval.

102. **The Planimeter.** The name of the instrument comes from "planus" meaning flat, and "meter" meaning measure. As the principle of recording area is the same in both of the types shown, Figures 48, 49, we will confine our description to the compensating planimeter. Its use consists in tracing the contour of the figure to be measured with the tracer f as shown. When doing so the wheel M is made to revolve, and it is by the extent of these revolutions that the area of the traced figure is ascertained. The various parts of the planimeter are so dimensioned as to bring about one complete revolution of the wheel when an area of 10 sq. in. has been traversed.

Attached to the wheel is a white drum, divided into 100 parts, one of which indicates an area of ·10 sq. in. By means

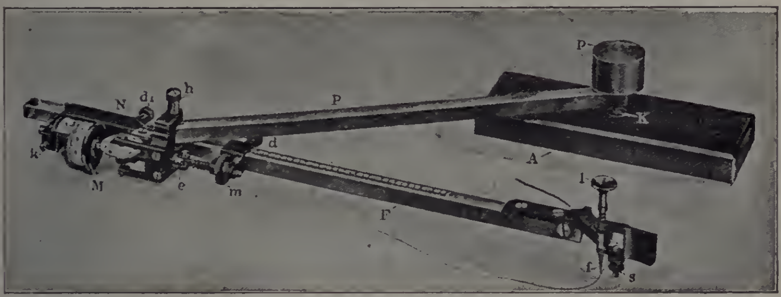


FIG. 48—AMSLER PLANIMETER

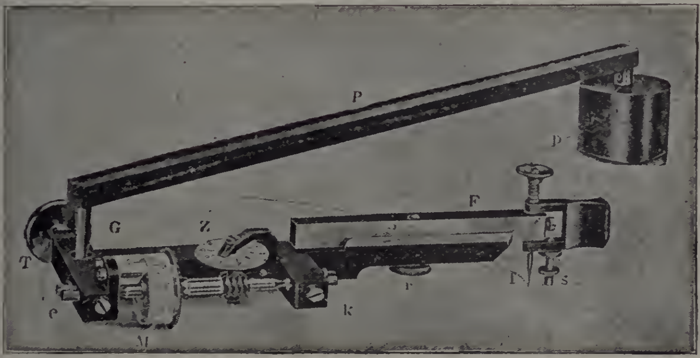


FIG. 49—OTT'S COMPENSATING PLANIMETER

of the vernier (see page 173) the single parts of the drum can be read to tenths, giving an area of ·01 sq. in. To keep a record of the number of revolutions of the wheel a counting disc Z is attached to it by means of a worm-gear. Each mark on the disc corresponds to one revolution of the wheel, therefore 10 revolutions of the wheel corresponds to one revolution of the disc.

Applying this to the reading in Figure 50 we have—the last number on the disc is 3, ∴ 30 sq. in.; the last number on the drum is 5, ∴ 5 sq. in.; the last division between the 5 and 6 is 8, ∴ ·8 sq. in.; the 4th division of the vernier is opposite a division on the drum, ∴ ·04 sq. in. Total reading 35·84 sq. in.

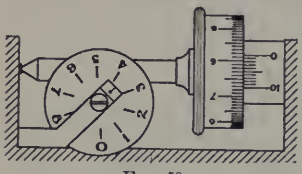


FIG. 50

Exercises LXIII.

1. Construct an ellipse in your laboratory book as suggested. Write a note as to why your construction fulfils the requirements. Find the area by counting the squares and check by formula for area.

2. Construct an ellipse on cardboard having major and minor axes 4″ and 2″ respectively, and also a rectangle having length 4″ and breadth 2″. Cut out both, weigh, and find the value of $\dfrac{\text{wt. of rectangle}}{\text{wt. of ellipse}}$. Do the same with different materials and find the average of your results.

3. A plot of ground in the form of an ellipse has major and minor axes, 200′ and 150′ respectively. Draw to scale in your laboratory book and find the perimeter and area.

4. An elliptic man-hole door has major and minor axes of 3′ and 2′ respectively. It is made of cast iron $\frac{1}{4}$″ thick. Find weight if 1 cu. in. weighs ·26 lb.

5. At what distance from the end of the major axis should the hole for the centre of revolution be drilled in an elliptic gear whose axes are $1\frac{1}{4}$″ and 2″? (Elliptic gears will mesh when revolving about their foci).

6. The area of an elliptic sheet of zinc is 88 sq. in. If its minor axis is 4″, find its major axis.

7. The head of a hexagonal bolt is $\frac{1}{2}$″ to the side; find the area of the head.

8. A square is 4″ to the side. An octagon is formed by cutting off the corners of the square. Find the side of the octagon and hence its area. Find the area by subtracting the areas of the four corners from the square and compare with previous result.

9. Ten hurdles, each 4′ long, are placed to form a regular decagon. Find the area enclosed.

10. A steel plate in the form of a regular pentagon measures $1\frac{3}{4}''$ on each side and is $\frac{1}{4}''$ thick. Find its weight, if a cu. in. of steel weighs ·283 lb.

11. The area of a regular hexagon is 284·112 sq. in. Find a side of the hexagon.

12. Regular polygons of 6 sides are inscribed in and circumscribed about a circle of radius 1′. Find the difference of their areas.

13. Construct a semicircle 4″ in diameter in your laboratory book. Find its area by Simpson's rule. Check by means of formula for the area of a circle and thus calculate the percentage error in Simpson's rule.

14. Construct an ellipse with major and minor axes 4″ and 2″ respectively. Proceed as in the preceding question.

15. Make a drawing of Figure 51 in your laboratory book. Common interval $\frac{1}{4}''$. If the scale be $\frac{1}{32}''$ to the foot, find the area in square ft. by Simpson's rule. Check by planimeter and estimate percentage error. Note—areas of similar figures are to one another as the squares on corresponding sides.

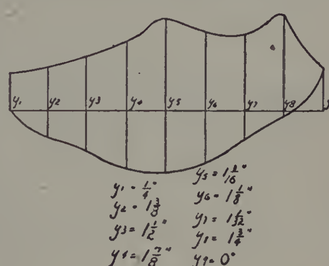


FIG. 51

16. The ordinates of a curved piece of sheet lead in inches are 20, 30, 29·9, 29·5, 28·4, 25·7, 14·2. The common distance between them is 3·65″; find the area.

17. The half-ordinates of a transverse section of a vessel are in feet 12·2, 12·2, 12·1, 11·8, 11·2, 10, 7·3 respectively. The common interval is 18″; find the area.

CHAPTER XI.

RATIO AND PROPORTION.

103. **Ratio.** We are constantly comparing weights, distances, sizes, etc. If one piece of metal weighs 50 lb. and another 10 lb., we say that the first is five times as heavy as the second, or that the second is one-fifth as heavy as the first. If one board is 8 ft. long and another 2 ft. long, we say that the first is four times as long as the second, or that the second is one-fourth the length of the first.

This Relation between Two Quantities of the same Kind is called Ratio.

Note.—In the above definition of ratio it is important to notice "of the same kind." It would clearly be absurd to compare bushels and feet.

A ratio may be written in two different ways. For example, the ratio of the diameters of two wheels which are 10 in. and 16 in. in diameter can be written as a fraction $\frac{10}{16}$. Again, since a fraction indicates division, i.e., $10 \div 16$, the line in the division sign is sometimes left out and the ratio is written 10 : 16. In either case the ratio is read "as ten is to sixteen."

Since a ratio may be expressed as a fraction it may be reduced to lower terms without changing its value. For example, if one casting weigh 600 lb. and another 150 lb., the ratio of the weight of the first to the weight of the second is $\frac{600}{150} = \frac{4}{1}$.

Example 1:

The diameter of the cylinder on an engine is 18″ and the diameter of the piston rod is 3″. What is the ratio of the cylinder diameter to the piston rod diameter?

$$\frac{\text{Diameter of cylinder}}{\text{Diameter of piston rod}} = \frac{18}{3} = \frac{6}{1}.$$

Example 2:

A concrete mixture is made of cement, sand, and gravel in the ratio of $1 : 2\frac{1}{2} : 5$. If 25 bags of cement be used (1 bag = 1 cu. ft.), how many cu. ft. of sand and gravel will be required?

25 (cu. ft. cement) $\times 2\frac{1}{2} = 62\frac{1}{2}$ cu. ft. sand.

25 (cu. ft. cement) $\times 5 = 125$ cu. ft. gravel.

104. Proportion. When two Ratios are Equal, the Four Terms are said to be in Proportion.

The two ratios $3 : 9$ and $12 : 36$ are evidently equal, since we can reduce $12 : 36$ to $3 : 9$.

When written $3 : 9 = 12 : 36$, these numbers form a proportion.

Further we observe in the above proportion that the product of 3 and 36, i.e., the first and last, is equal to the product of 9 and 12, i.e., the second and third.

The first and last are called the **extremes,** and the second and third are called the **means.** We, therefore, have :—*The product of the means is equal to the product of the extremes.*

This relation may be expressed generally.

If a, b, c and d represent the four terms of any proportion then:

$$\overbrace{a : \underbrace{b \quad = \quad c}_{\text{means}} : d}^{\text{extremes}}$$

Then in accordance with the above $ad = bc$.

Application of this principle to some practical problems.

Example 1:

If it requires 60 men to turn out 200 shells in a day, how many men will be required to turn out 360 shells in a day?

If x be the required number of men, then $60 : x = 200 : 360$

$$\text{or } 200x = 60 \times 360$$

$$x = \frac{60 \times 360}{200} = 108.$$

Example 2:

If the diameter of a pulley is 40″, and it makes 120 R.P.M., what is the R.P.M. of a second pulley belted to the first if its diameter is 16″?

Note.—When two pulleys are belted together, the larger of the two is the one that makes the least R.P.M. The proportion formed from their diameters and revolutions is, therefore, called an **inverse** proportion.

Let x = R.P.M. of the second pulley, then $40:16=x:120$

$$\text{or } 16x=40\times 120$$

$$\text{or } x=\frac{40\times 120}{16}=300$$

After some practice in proportion we might write this directly—R.P.M. of 16 in. pulley $=\frac{40}{16}$ of $120=300$.

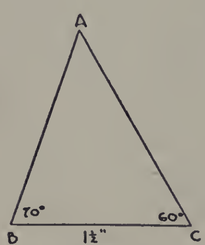


Fig. 52

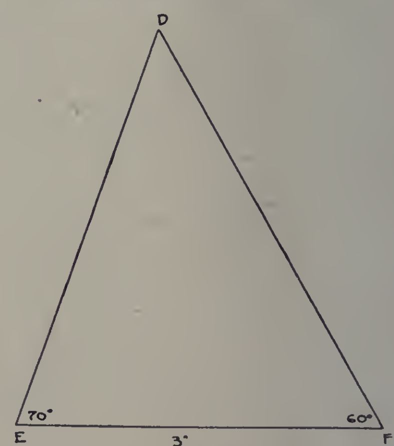


Fig. 53

105. Proportion in Similar Triangles:

In triangle ABC and DEF, BC and EF are $1\frac{1}{2}''$ and $3''$ respectively, also $\angle B=\angle E$ and $\angle C=\angle F$.

The triangles ABC and DEF are, therefore, equiangular and are called similar triangles. If we compare corresponding sides with dividers we observe that $DE=2AB$ and $DF=2AC$.

This experiment would suggest that, when triangles are equiangular, their corresponding sides are proportional.

$$\text{Thus,}\quad \frac{BC}{EF}=\frac{AB}{DE}=\frac{AC}{DF}.$$

Make drawings in your laboratory book to verify the above.

Observe this principle in the following Example :

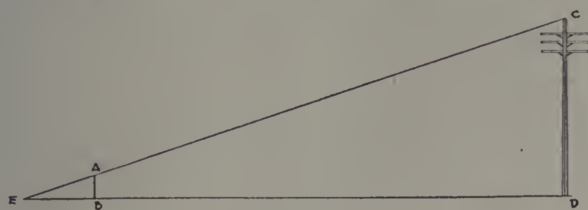


FIG. 54

The top of a telegraph pole, Figure 54, is sighted across a 5′ pole placed 100′ from the foot of the telegraph pole, the observer sighting from the ground at a distance of 15′ from the foot of the 5′ pole. Find the height of the telegraph pole.

The triangles ABE and CDE are equiangular and therefore similar.

$$\therefore\quad \frac{CD}{AB}=\frac{DE}{BE}$$

$$\text{or,}\quad \frac{CD}{5}=\frac{115}{15}$$

$$\text{or,}\quad CD=\frac{5}{15}\times\frac{115}{1}=38\tfrac{1}{3}'.$$

Exercises LXIV.

1. A room is 16′ by 12′. What is the ratio of the length to the breadth?

2. Two gear-wheels have 100 teeth and 40 teeth respectively. What is the ratio of the number of teeth?

3. Detroit has a population of 1,000,000 and Toronto 600,000. What is the ratio of the population of Toronto to that of Detroit?

4. A man rode 280 miles partly by rail and partly by boat. What distance did he travel by each, if the ratio is as 3 to 2?

5. A locomotive has a heating surface of 1340 sq. ft. and a grate area of 24 sq. ft. What is the ratio of the heating surface to the grate area?

6. The steam pressure in a locomotive is 196 lb. and the mean effective pressure in the cylinders is found to be 80 lb. What is the ratio of the mean effective pressure to the boiler pressure?

7. Two pulleys connected together have diameters of 36″ and 22″. If the first makes 120 R.P.M., what is the R.P.M. of the second?

8. The ratio of two gears connected together is as 5 to 3. The first makes 105 R.P.M., how many does the second gear make?

9. If 15 tons of coal cost $120, what will 18 tons cost?

10. If the freight charges on a shipment be $40.50 for 216 miles, what should it be for 300 miles?

11. A pump discharges 20 gal. per min., and fills a tank in 24 hrs. How long would it take to fill the tank with a pump discharging 42 gal. per min.?

12. A machinist gets $7.50 a day and a helper $5.00 a day. How much would the helper receive when the machinist gets $80.00, providing they both work the same number of days?

13. If a yard-stick held upright casts a shadow 3′ 9″ long, how long a shadow would be cast at the same time by a chimney 66′ 8″ high? Check by drawing to scale.

14. What is the height of a signal pole whose shadow is 12′, when a 10′ pole at the same time casts a shadow of 2′ 4″? Check by drawing to scale.

15. The length of shadow of a telegraph pole to the first cross-arm is 24′. A 6′ pole at the same time casts a shadow of 2′ 8″. What is the height of the first cross-arm? Check by drawing to scale.

16. In a mixture of copper, lead and tin there are 4 parts copper, 3 parts lead and 1 part tin. How many lb. of each would there be in 248 lb. of the mixture?

17. A solder is made of 5 parts zinc, 2 parts tin, and 1 part lead. How many parts of each metal in 96 lb. of the mixture?

18. A man 6′ high, standing 8′ from a lamp-post, observes his shadow to be 6′ in length. Find the height of a boy who casts a shadow of 4′ when he stands 9′ from the lamp-post.

19. What will be the diameter of a gear that is to make 60 R.P.M., if it is to mesh with a gear 24″ in diameter, which makes 72 R.P.M.?

20. What is the percent. grade of a road bed that rises 1·2′ in a horizontal distance of 40′?

21. The roof of a house rises $1\frac{1}{2}'$ in a run of 2′. How far will it rise in a run of 20′?

22. A road bed rises $2\frac{1}{2}'$ in 200′, what is the percent. grade? In how many feet will it rise 1′?

23. A cable railway up the side of a mountain has at places on the route a grade of 20%. What rise does this represent for every mile of horizontal distance?

CHAPTER XII.

SIMULTANEOUS EQUATIONS.

FORMULAS—(*continued*).

106. **In the Discussion of the Simple Equation,** we learned that it contained only one unknown quantity and that, therefore, the value of the unknown could be definitely determined.

$$\text{Thus, if } 3x+4=16$$
$$3x=12$$
$$x=4.$$

If, however, a single equation contains two unknown quantities, we cannot find a definite value for either of them, and are restricted to finding the value of one in terms of the other.

Thus, in the equation $x+2y=13$, if we transpose $2y$ we have $x=13-2y$. The value of x in this equation will depend upon the values assigned to y.

$$\text{If, } y=1,\ x=13-2=11.$$
$$y=2,\ x=13-4=9.$$
$$y=5,\ x=13-10=3.$$

Similarly, if we give values to x we may obtain corresponding values for y. It is evident, then, that we cannot find definite values for x and y but have merely a statement of relation between them.

If, however, another relation between x and y be obtained from the same problem, we can, from the two equations, determine the definite values of x and y. If we also find that $3x+y=14$, then transposing and dividing by 3 we have $x=\frac{14-y}{3}$.

If, as above, $y=1,\ \ x=\frac{14-1}{3}=\frac{13}{3}$.

$$y=2,\ \ x=\frac{14-2}{3}=4.$$

$$y=5,\ \ x=\frac{14-5}{3}=3.$$

Comparing the two sets of values for x and y we observe that there is only one pair of values that will satisfy both equations, namely, $x=3$ and $y=5$. These values of x and y for the two equations are called **simultaneous** values and the equations are known as **simultaneous equations.**

107. In the following Problem we will illustrate Two Methods of Solution. One alloy contains 4% copper and another 10% copper. How many pounds of each should be used to make a 100 pound mixture containing 6% copper?

Solution by Substitution:

Let $x=$ No. of lb. from the first alloy.

And $y=$ No. of lb. from the second alloy.

Then $x+y=100$. (1)

also, $\frac{4x}{100}+\frac{10y}{100}=\frac{6}{100}\times 100$, which reduces to $4x+10y=600$

or, $2x+5y=300$. (2)

From (1) $x=100-y$

Substituting in (2)

$$2\ (100-y)+5y=300$$

$$200-2y+5y=300$$

giving $y=33\frac{1}{3}$.

Substituting in (1) $x=66\frac{2}{3}$.

Therefore we must use $66\frac{2}{3}$ lb. from the first alloy and $33\frac{1}{3}$ lb. from the second.

Solution by Elimination:

$$x + y = 100 \quad (1)$$
$$2x + 5y = 300 \quad (2)$$
$$(1) \times 2 = 2x + 2y = 200$$
$$(2) \quad = 2x + 5y = 300$$

Subtracting

$$-3y = -100$$
$$y = 33\tfrac{1}{3}.$$
$$\text{and } x = 100 - y = 66\tfrac{2}{3}.$$

In simultaneous equations it frequently happens that one of the unknowns can be readily expressed in terms of the other, and the solution obtained by means of the simple equation. The problem given above affords an example of this.

It has been observed that, *if the equation contains only one unknown, the value of this unknown may be found from one equation*; *also that, if the equation contains two unknowns, two equations are necessary to find the values of the unknowns.*

This may be extended to three or more, and we say, generally, *that we must have as many distinct equations as there are unknowns to be found.*

Exercises LXV.

Solve the following equations:

1. $3x + 4y = 10.$
   $4x + y = 9.$
2. $4x + 7y = 29.$
   $x + 3y = 11.$
3. $8x - y = 34.$
   $x + 8y = 53.$
4. $2x + 5y = 25.$
   $3x - y = 9.$
5. $3x - 5y = 6.$
   $4x + 3y = 37.$
6. $\frac{2x}{3} + y = 16.$
   $x + \frac{y}{4} = 14.$
7. $\frac{x}{5} + \frac{y}{2} = 5.$
   $x - y = 4.$
8. $\frac{x}{8} + \frac{y}{3} = 15.$
   $\frac{x}{4} - \frac{y}{5} = 4.$
9. $\frac{2}{x} + y = 1.$
   $\frac{1}{x} + 2y = 1\tfrac{1}{4}.$
10. $\frac{x}{5} - \frac{y}{4} = 0.$
    $3x - \tfrac{1}{2}y = 17.$

Exercises LXVI.

1. If scrap-iron contains 4% silicon and we have a pig-iron containing 9% silicon, how many pounds of each must be used to make a ton of mixture containing 6% silicon?

2. The law of a machine is given by $R=aE+b$ and it is found that when $R=100$, $E=25$, and when $R=250$, $E=60$, find a and b.

3. One mixture for casting contains 20% copper and another mixture, for the same purpose, contains 8% copper. How many pounds of each should be taken to make 200 pounds of a mixture containing 12% copper?

4. The distance between the centres of two parallel shafts is 8″. It is required to connect these shafts by a pair of gears so that one shaft will turn twice as fast as the other. Calculate the diameters of the gears.

5. In a pulley-block lifting tackle, a force of 15 lb. will lift a load of 100 lb., and a force of 35 lb. will lift a load of 300 lb. If the force (P lb.) and the load (W lb.) are related by an equation of the form $P=mW+k$ find the values of m and k, and hence the law of the machine.

6. If E represents the fixed expenses of a manufacturing company, V the variable expenses for each machine manufactured, N the number of machines per year, and C the total cost of operating, then $C=E+VN$. In 1918 the company built 600 machines at a total cost of \$18,000, and in 1919, 800 machines at a total cost of \$22,000. Calculate E and V.

7. The law of a machine is given by $E=aR+b$ and it is found that when $R=10$, $E=5 \cdot 46$ and when $R=100$, $E=9 \cdot 6$; find a and b.

8. The total cost C of a ship per hour is given by $C=a+bs^3$ where s is the speed in knots. When s is 10, C is found to be \$26.00 and when $s=15$, C is found to be \$36.50. Find a and b and express the relation between C and s.

9. At an election there were two candidates and 3478 votes were cast. The successful candidate had a majority of 436. How many votes were cast for each?

10. The moment of a force is the tendency of that force to produce rotation of a body, and is measured by the product of the force (in pounds) and the perpendicular distance (in feet) from the axis to the line of the applied force. For

equilibrium in the case of parallel forces, the algebraic sum of the forces is 0 and the algebraic sum of the moments is 0.

A uniform plank 20′ long, weight 90 lb., rests on supports at its ends. A load of 500 lb. rests 8′ from one end. Find the reactions of the supports.

11. A uniform beam 16′ long weighs 300 lb. It is supported at one end and at a point 4′ from the other end. Calculate the reactions of the supports.

12. A uniform beam, 12′ long, is supported at each end and carries a distributed load, including its own weight, of ½ ton per foot run. A concentrated load of 1 ton rests 5′ from one end and another of 3 tons, 4′ from the other end. Calculate the reactions of the supports.

13. If $\frac{E}{240+r}=\cdot 34$ and $\frac{E}{r}=1\cdot 47$ find E and r.

14. If $V_1=V_0(1+Bt)$ and $V_1=12\cdot 4$ when $t=21\cdot 5$ and $V_1=17\cdot 3$ when $t=75\cdot 0$, find V_0 and B.

15. The receipts of a railway company are divided as follows:—40% for cost of operating; 10% for the reserve fund; a 6% dividend on the preferred stock which is ¼ of the capital; and the remainder, $630,000, as dividend on the common stock, being at the rate of 4% per annum. Find the capital and receipts.

Exercises LXVII.

Formulas—(*continued*).

1. The time taken by a pendulum for a complete oscillation is given by $t=2\pi\sqrt{\frac{l}{g}}$, where t is the time in seconds, l the length in ft. and g the acceleration due to gravity in ft. per sec.

Solve for l and g:

If $l=1\frac{1}{2}$, $g=32\cdot 2$ find t.
If $t=2$, $g=32\cdot 2$ find l.
If $t=1\cdot 57$, $l=2$ find g.

2. The resultant of two forces at right angles is given by $R=\sqrt{P^2+Q^2}$, where P and Q are the forces at right angles and R the resultant.

Solve for P and Q:

If $P=8$, $Q=5$, find R.
If $R=17$, $Q=8$, find P.

3. The velocity of a body at the end of a specified time is given by $v=u+at$, where v is the final velocity in ft. per sec., u the initial velocity in ft. per sec., a the acceleration in ft. per sec. per sec., t the time in seconds.

Solve for v, u, a, t:

If $u=12$, $a=15$, $t=6$, find v.
If $v=750$, $a=30$, $t=18$, find u.
If $v=10\cdot9$, $u=45$, $t=16$, find a.
If $v=215$, $u=75$, $a=10$, find t.

4. The space traversed by a body is given by $s=ut+\frac{1}{2}at^2$, where s is the space in ft., u the initial velocity in ft. per sec., t the time in sec., a the acceleration in ft. per sec. per sec.

Solve for u and a:

If $u=20$, $a=32\cdot2$, $t=8$, find s.
If $s=300$, $a=16$, $t=5$, find u.
If $s=750$, $u=25$, $t=10$, find a.

5. The thickness of plate required in a boiler is given by $t=\frac{pd}{2fe}$, where t is the thickness in in., p the pressure in lb. per sq. in., d the diameter of the boiler in in., f the tensile stress in lb. per sq. in., e the efficiency of the joint.

Solve for p, d, f, and e:

If $p=160$, $d=8$ ft., $f=20000$, $e=\cdot7$, find t.
If $t=\cdot5$, $d=90$, $f=16000$, $e=\cdot6$, find p.
If $t=\frac{3}{8}$, $p=150$, $f=18000$, $e=\cdot7$, find d.
If $t=\frac{5}{16}$, $p=140$, $e=\cdot75$, $d=72$, find f.
If $t=\frac{7}{16}$, $p=120$, $d=48$, $f=5$ tons, find e.

6. The Kinetic energy of a falling body in foot-pounds is given by $K=\frac{wv^2}{2g}$, where K is the energy, w the weight of the body in lb., v the velocity in ft. per sec., g the acceleration due to gravity.

Solve for w, v, g:

If $w=1000$, $v=44$, $g=32\cdot2$, find K.
If $K=11\cdot2$, $w=5000$, $g=32\cdot2$, find v.
If $K=12\cdot4$, $v=35$, $g=32\cdot2$, find w.

7. The effort of friction (measured in lb.) in diminishing the load lifted is given by $E = PV - W$, where E is the effort of friction, P the effort in lb., W the weight in lb., and V the velocity ratio $= \frac{\text{motion of effort}}{\text{motion of weight}}$.

Solve for P, V, W:

If $P = 2 \cdot 4$, $V = 16$, $W = 25$, find E.
If $E = 52$, $V = 16$, $P = 4 \cdot 2$, find W.
If $E = 82$, $V = 16$, $W = 30$, find P.
If $E = 104$, $P = 9$, $W = 40$, find V.

8. The magnetic lines of force (Flux) is given by $Q = \frac{\cdot 4\pi NI}{R}$, where Q is the total flux, N the number of turns of wire in the coil, R the reluctance of the magnetic circuit, I the current in amperes.

Solve for N, I, R:

If $N = 200$, $I = 5$, $R = \cdot 0002$, find Q.
If $Q = 30000$, $N = 500$, $I = 15$, find R.
If $Q = 100000$, $N = 50$, $R = \cdot 00005$, find I.

9. For a single riveted lap-joint, the efficiency in tension is given by $K_t = \frac{P - d}{P}$, where K_t is the efficiency in tension, P the pitch in in., d the diameter in in. of the rivet.

Solve for P and d:

If $P = 1\frac{1}{2}$ in., $d = \frac{5}{8}$ in., find K_t.
If $K_t = \cdot 5$, $d = \frac{9}{16}$ in., find P.
If $K_t = \cdot 6$, $P = 1\frac{3}{4}$ in., find d.

10. The relation between a Centigrade and a Fahrenheit scale is given by $C = \frac{5}{9}(F - 32)$, where C represents the Centigrade and F the Fahrenheit reading.

Solve for F:

If $F = 63°$, find C.
If C $= 72°$, find F.
If C $= -4°$, find F.

11. The counter electromotive-force (E.M.F.) of a motor is given by $E = E_c + IR$, where E is the impressed E.M.F., E_c the counter E.M.F., I the current in amperes, R the resistance in the armature.

Solve for E_c, I, R:

If $E_c = 108$, $I = 40$, $R = \cdot 08$, find E.
If $E = 220$, $I = 60$, $R = \cdot 05$, find E_c.
If $E = 110$, $E_c = 108 \cdot 5$, $I = 25$, find R.
If $E = 110$, $E_c = 109$, $R = \cdot 15$, find I.

12. The approximate length of an open belt connecting two wheels is given by $L = 3\frac{1}{4}(R+r) + 2d$, where L is the length, R and r the radii of the large and small wheels respectively, d the distance between the centres.

Solve for R, r, d:

If $R = 18$ in., $r = 10$ in., $d = 40$ in., find L.
If $L = 15$ ft., $R = 16$ in., $r = 12$ in., find d.

13. The approximate length of a crossed belt connecting two wheels is given by $L = 3\frac{3}{8}(R+r) + 2d$, where L, R, r, and d have values as in question 12.

Solve for R, r, d:

If $R = 18$ in., $r = 12$ in., $d = 6$ ft., find L.
If $L = 12$ ft., $R = 16$ in., $r = 8$ in., find d.

14. The horse-power transmitted by belts is given by $H.P. = \frac{(T_1 - T_2)V}{33000}$, where T_1 is the tension on the tight side of the belt in lb., T_2 the tension on the slack side of the belt in lb. V the velocity in ft. per minute of the driver.

Solve for $T_1 - T_2$, V:

If $T_1 = 120$, $T_2 = 50$, $V = 3141 \cdot 6$, find $H.P.$
If $H.P. = 81$, $V = 2500$, find $T_1 - T_2$.

15. The width of a single belt required to transmit a given horse-power at a given speed of the belt is expressed by $W = \frac{H.P. \times 33000}{S \times 60}$, where W is the width in in., $H.P.$ the horse-power, S the speed in ft. per min.

Solve for $H.P.$ and S:

If $H.P. = 100$, $S = 3000$, find W.
If $S = 3200$, $W = 6$ in., find $H.P.$
If $H.P. = 40$, $W = 5$ in., find S.

16. For a double belt the formula in question 15 becomes $W = \frac{H.P. \times 33000}{S \times 100}$.

Solve for $H.P.$ and S:

If $H.P.=100$, $S=3000$, find W.
If $S=3200$, $W=6$ in., find $H.P.$
If $H.P.=50$, $W=5$ in., find S.

17. The length of belting in a closely rolled coil is given by $L=\cdot 1309\,N\,(D+d)$, where L is the length in ft., D the diameter of the roll in in., d the diameter of the eye in in., N the number of turns in the coil.

Solve for N, D, d:

If $N=15$, $D=16\frac{1}{2}$, $d=5$, find L.
If $L=80$, $D=14$, $d=3$, find N.
If $L=200$, $D=44$, $N=30$, find d.

18. For a single riveted lap-joint the efficiency in shear is given by $K_s=\dfrac{aS_s}{PTS_t}$, where K_s is the efficiency in shear, a the cross-section area of the rivet in sq. in., P the pitch of the rivet in in., T the thickness of the plate in in., S_s the strength of rivet steel in shear (lb. per sq. in.), S_t the strength of plate in tension (lb. per sq. in.).

Solve for a, S_s, P, T, S_t:

If $a=\cdot 7854$, $S_s=30000$, $P=1\frac{1}{2}$ in., $T=\frac{3}{8}$ in., $S_t=40000$, find K_s.
If $K_s=1\cdot 5$, $S_s=32000$, $P=1\frac{3}{4}$ in., $T=5$ in., $S_t=35000$, find a.
If $K_s=1\cdot 75$, $a=\cdot 5$, $P=1\cdot 3$ in., $T=\frac{1}{2}$ in., $S_t=38000$, find S_s.

19. The horse-power of a boiler is given by $B.H.P.=\dfrac{W(H-t+32)}{34\cdot 5\times 965\cdot 7}$, where $B.H.P.$ is the boiler horse-power, W the number of pounds of water evaporated per hour, H the total heat of steam above 32° F., t the temperature of the feed water.

Solve for W, H, t:

If $W=20000$, $H=1180$, $t=100°$, find $B.H.P.$
If $B.H.P.=600$, $H=1175$, $t=120°$, find W.
If $B.H.P.=650$, $H=1200$, $W=20000$, find t.

20. The quality of steam (%) as determined by the throttling calorimeter is given by $x=100\left\{\frac{H-h-C_p(T_s-T_e)}{L}\right\}$, where x is the moisture in steam, H the total heat of steam at main pressure, h the total heat of saturated steam at pressure in calorimeter, T_e the temperature of saturated steam at pressure in the calorimeter, T_s the observed temperature in the calorimeter, C_p the specific heat of superheated steam at constant pressure, L the latent heat of steam at main pressure.

Solve for H, h, C_p, T_s, T_e, L:

If $H=1180$, $h=1150$, $T_s=220$, $T_e=215$, $C_p=\cdot 48$, $L=920$, find x.

If $x=\frac{1}{50}$ (2%), $h=1160$, $T_s=225$, $T_c=218$, $C=\cdot 48$, $L=930$, find H.

CHAPTER XIII.

GRAPHS.

108. **If we wish to fix the position of a point P** on the page of this book, one way would be to find its perpendicular distance from the left of the page and also its perpendicular distance from the bottom of the page. If these distances were 3 in. and 4 in. respectively, then the point P would be definitely fixed with respect to the plane of the paper.

Consider a sheet of paper ruled as in Figure 55. If we know that a point P is 6 divisions to the right of OY and 4 divisions above OX, we can, at once, locate the position of the point by counting 6 divisions along OX and then counting 4 divisions vertically to the point P. This method of fixing the point is called **plotting the point.** The lines OX and OY are called **Axes of Reference,** the point of intersection O is called the **Origin,** and the distances 6 and 4, which locate the point, are called **Co-ordinates.** We would now say that the co-ordinates of P are 6 and 4, and would write it P (6, 4), the first number always giving the distance along OX and the second the distance along OY. OX is usually spoken of as the axis of X and OY as the axis of Y. Distances along OX are called **abscissae** and distances along OY are called **ordinates.** We see from the above that any point can be plotted on the squared paper if we know its distances from the axes OY and OX.

109. **Let us use this for a Practical Purpose.** A sewer runs across a rectangular lot and it is necessary to know its exact location in case of trouble later.

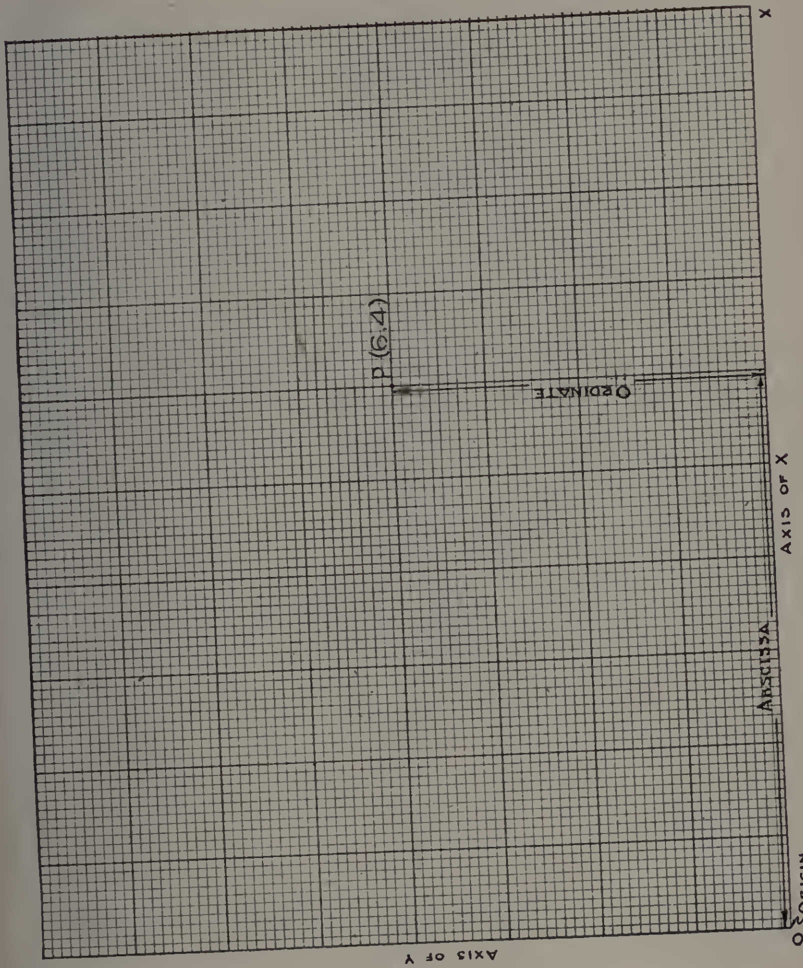


FIG. 55

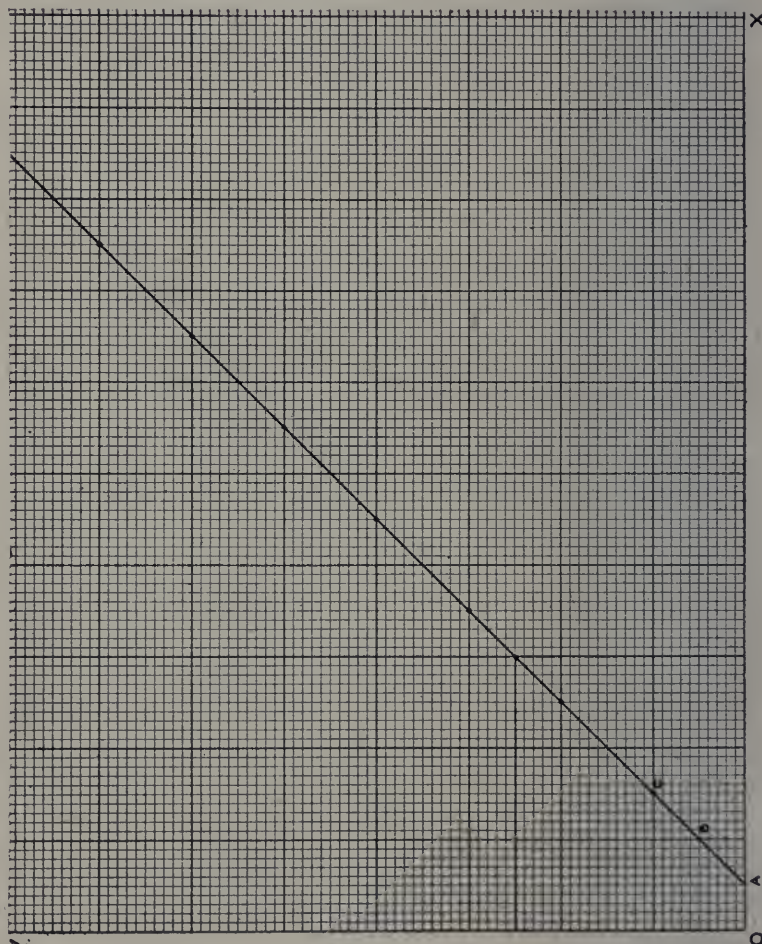


Fig. 56

Measurements are taken according to the following plan:

Distance from OY	5	10	15	25	35	45	55	65	75
Distance from OX	0	5	10	20	30	40	50	60	70

A graphical representation of the position of the sewer is shown in Figure 56. Each small division of the squared paper represents 1 ft. The first point A has for its co-ordinates (5, 0), the second point $B(10, 5)$, the third point $C(15, 10)$ and so on. If we join these points we have a graph of the position of the sewer. At some subsequent date it is necessary to make an excavation for the footings of a building on this lot, and the contractor wishes to know if a particular footing will come too near the sewer. He takes measurements and finds that the distance from the side corresponding to OY is 30 ft., and the distance from the side corresponding to OX is 25 ft. While these distances are not actually recorded in the data previously taken, yet by going out 30 ft. (30 spaces) from OY and up 25 ft. (25 spaces) from OX, he would find that he is directly over the sewer. This illustration brings out one of the most important functions of a graph: **It gives results for data not actually recorded at the outset.**

110. It usually happens in practice that we require to make a record of two corresponding sets of measurements, in which the unit of measurement in one is entirely different from the unit in the other.

The following will illustrate:

The observations below were taken of the loads on a lighting plant from 3 P.M. to 12 P.M. at intervals of one hour.

Time in hours. . . .	3	4	5	6	7	8	9	10	11	12
Load in Kilowatts.	50	60	76	120	140	150	142	100	45	30

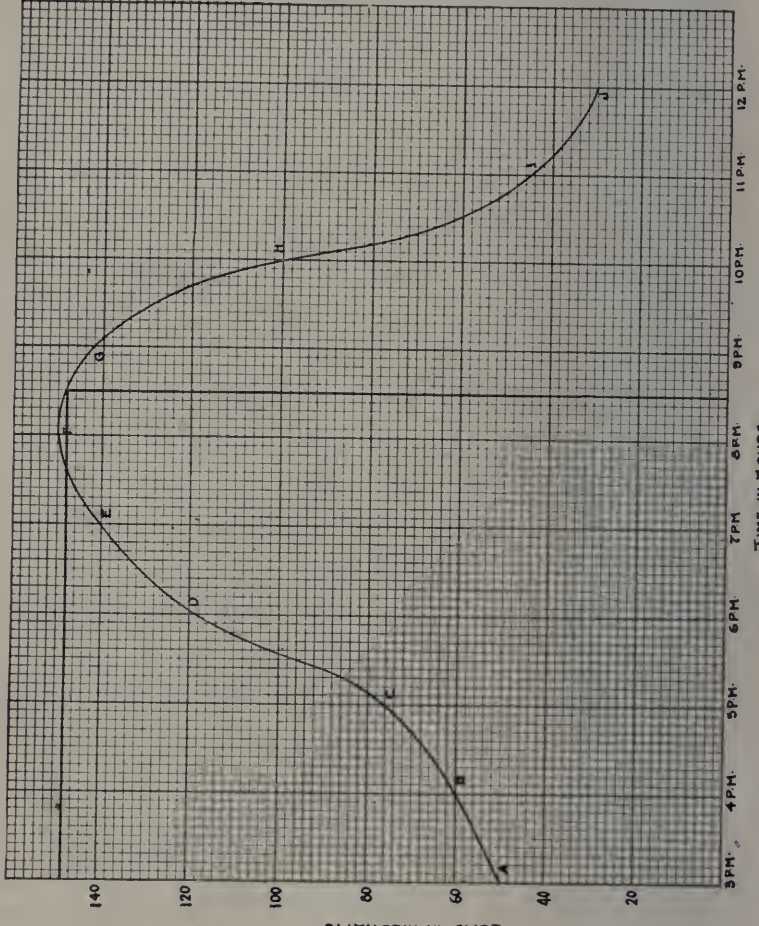

A
B
C
D
E
F
G
H
I
J
3 P.M.
4 P.M.
5 P.M.
6 P.M.
7 P.M.
8 P.M.
9 P.M.
10 P.M.
11 P.M.
12 P.M.
TIME IN HOURS
140
120
100
80
60
40
20
LOAD IN KILOWATTS

FIG. 57

Figure 57 shows how the relation of time and load may be represented graphically. Along the axis of X we let each main division represent one hour, while along the axis of Y we let each main division represent 20 kilowatts. The letters, A, B, C, D, E, F, G, H, I, J, represent the locations of the observations. By drawing a curve through these points, we have a graph which will show at a glance the variations in load. We see, further, that it will give us the probable load for times in between those recorded in the data above.

For example, if we wished to know the load at 8.30 P.M. we would take a point half-way between 8 and 9 on the axis of X and draw a perpendicular to it, represented by the heavy line in the figure.

From the point where this meets the curve, draw a line perpendicular to the axis of Y as represented, and we have 148 kilowatts as the probable load at 8.30 P.M.

111. Frequently a Relation between Measurements is expressed by an Algebraic Equation. Suppose we wished to find the Fahrenheit reading corresponding to a Centigrade reading in degrees. Since 180 degrees Fahrenheit, measuring the range from 32° to 212°, are equal to 100 degrees Centigrade, measuring the range from 0° to 100° we have:

100° Centigrade = 180° Fahrenheit.

$a°$ Centigrade $= \frac{180}{100}\ a°$ Fahrenheit $= \frac{9}{5}\ a°$ Fahrenheit.

If $b°$ represent the Fahrenheit reading corresponding to $a°$ Centigrade, then the relation is given by the equation $b = \frac{9}{5}a + 32$.

By giving different values to a in the equation we can obtain the corresponding values of b. These may be tabulated as follows:

Values of a	0	10	20	30	40	60	80	100
Corresponding values of b	32	50	68	86	104	140	176	212

Figure 58 is a graphical representation of this algebraical relation. Along the axis of Y we have represented the values of a, while along the axis of X we have the corresponding values of b. Corresponding values other than those recorded may be read off from the graph. Thus 77° Fahrenheit equals 25° Centigrade as represented in the drawing.

Another value of graphs is here illustrated, that is, **they act as checks on computations.**

112. It is often important to represent more than one Set of Relations on the same Sheet.

The following are the results obtained with a wheel and axle mounted on ordinary plain bearings. W represents the load lifted in pounds, P the effort applied in pounds, F the friction measured in pounds, E the efficiency percent.

W	P	F	E
0	·8	1·60	0
5	4·30	3·60	58·2
10	7·14	4·28	70·1
15	9·91	4·82	75·7
20	12·81	5·62	78·0
25	15·63	6·26	80·0
30	18·50	7·00	81·2
35	21·50	8·00	81·4
40	24·45	8·90	81·8

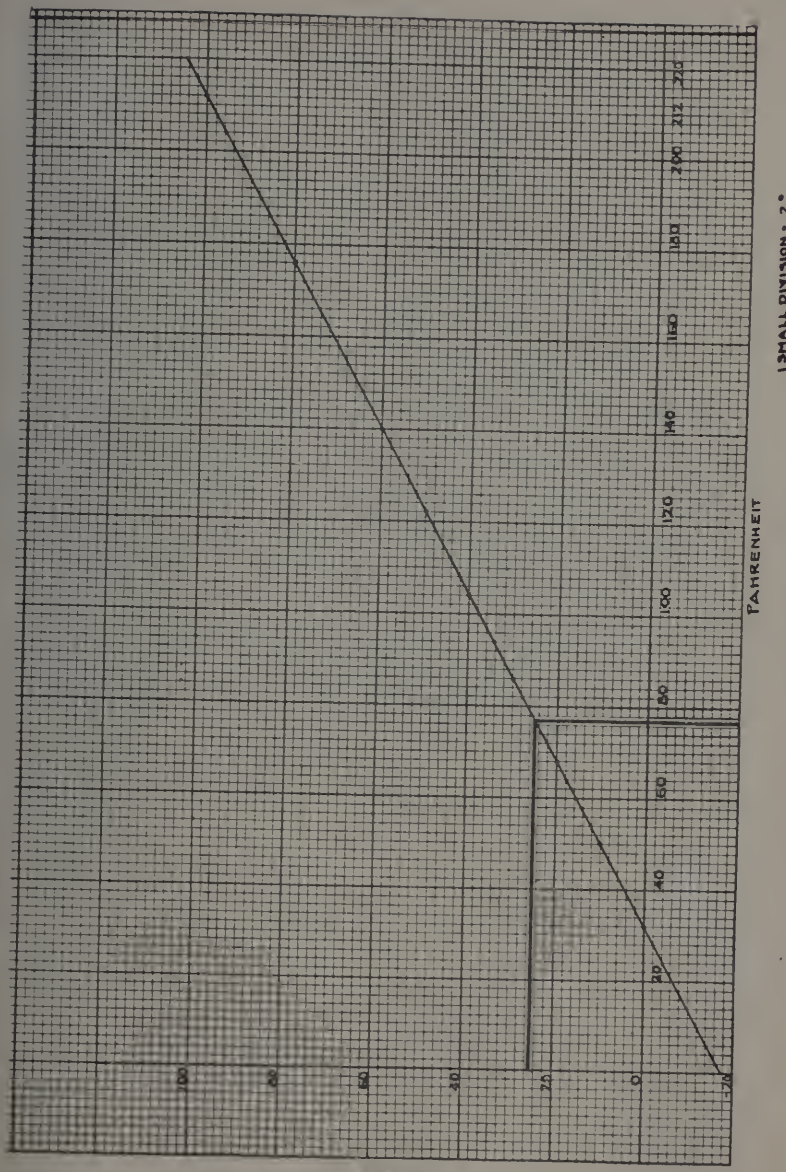

1 SMALL DIVISION = 2°
FAHRENHEIT
220
212
200
180
160
140
120
100
80
60
40
20
CENTIGRADE
100
80
60
40
25
0

Fig. 58

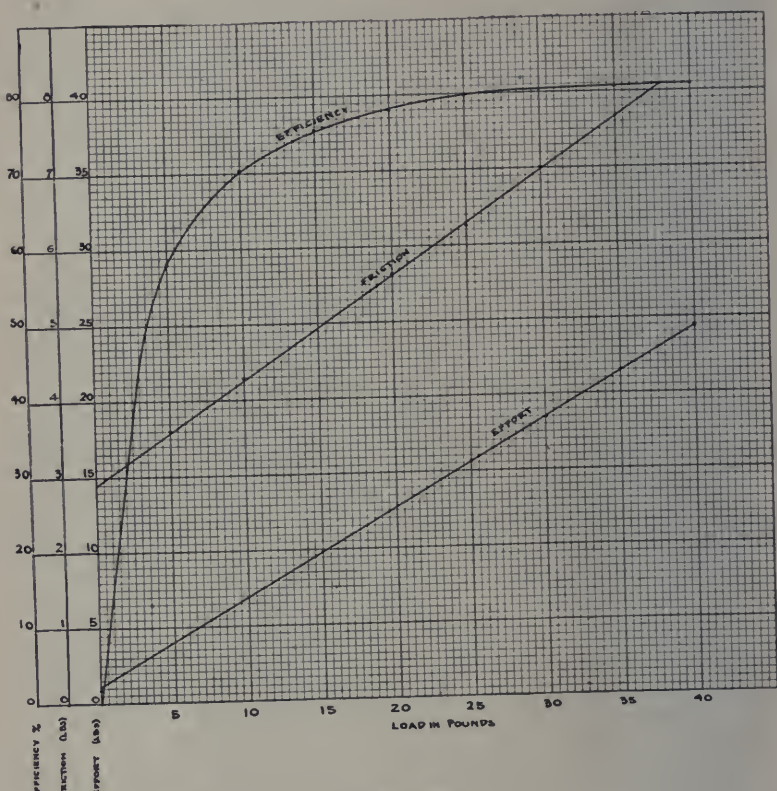


FIG. 59

In Figure 59, the lower line represents the relation between the load and the effort. The middle line represents the relation between the load and the friction. The top line represents the relation between the load and the efficiency percent.

Note.—In the drawing of a graph relating to machines it often happens that the points are not absolutely on a straight line. It is necessary, in such a case, to take the line which lies most nearly along the path of the points.

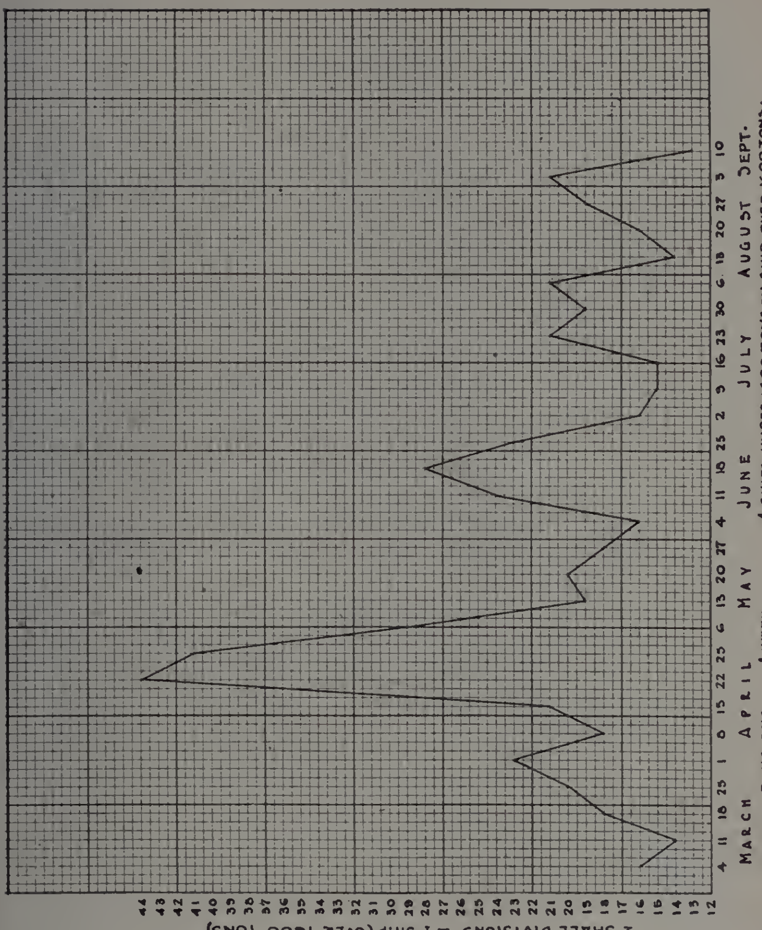


FIG. 60

113. Sometimes the sole Purpose of a Graph is to picture in a concise and striking way the relation between two measurements.

Figure 60 is a graph (from a Toronto daily paper) based on the official weekly figures of losses sustained by the British merchant fleet during the height of the submarine warfare.

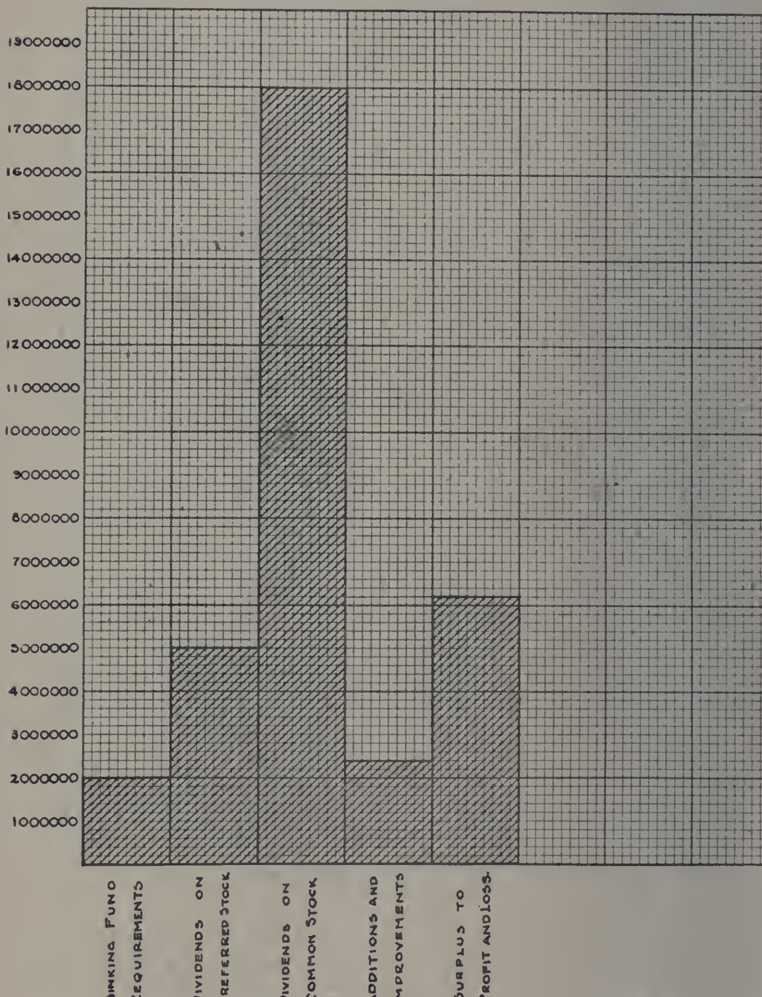

19000000
18000000
17000000
16000000
15000000
14000000
13000000
12000000
11000000
10000000
9000000
8000000
7000000
6000000
5000000
4000000
3000000
2000000
1000000
Sinking Fund Requirements
Dividends on Preferred Stock
Dividends on Common Stock
Additions and Improvements
Surplus to Profit and Loss.

Fig. 61

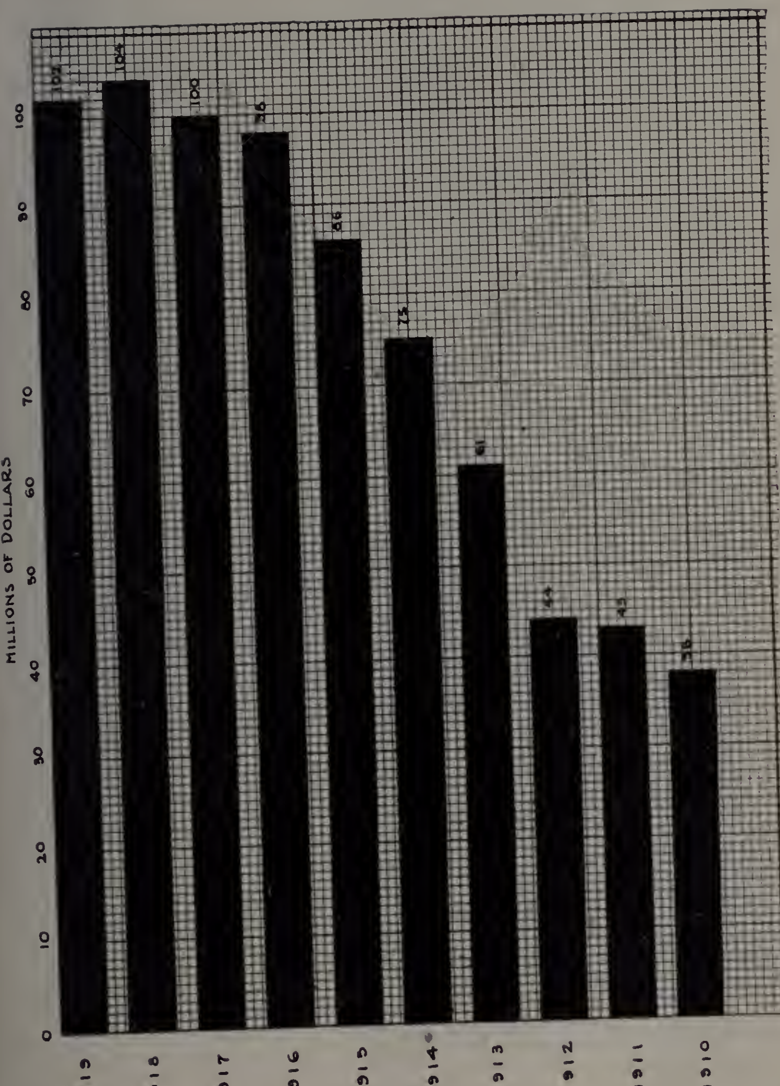

MILLIONS OF DOLLARS
0
10
20
30
40
50
60
70
80
90
100
1919
1918
1917
1916
1915
1914
1913
1912
1911
1910
102
104
100
96
86
75
61
44
43
38

FIG. 62

114. **In Business Transactions frequent use is made of the Pictograph.** This kind of representation takes a variety of forms—varying sized men may represent populations, varying sized bales of cotton may represent the export of cotton, and so on.

Figure 61, called the bar pictograph, is of frequent use.

Example:

The net income of a certain railway company for a recent year was divided as follows:

Sinking Fund Requirements,	$2,000,000.
Dividend on Preferred Stock,	$5,000,000.
Dividend on Common Stock,	$18,000,000.
Additions and Improvements,	$2,400,000.
Surplus to Profit and Loss,	$6,200,000.

In Figure 61, the above amounts are represented by a series of parallel bars, each main division on the vertical line representing $2,000,000. When the division of the income was presented in this form, the directors saw at a glance the relative division of the returns from the road.

This form of pictograph is also extensively used to represent a decline or growth in business.

Example:—The graph on page 161 is from a report *re* Toronto's gross funded debt 1910-1919.

It illustrates clearly the rapidity of the growth in recent years and its arrest in 1919.

When it is necessary to represent a percentage division, the circular pictograph is of common use.

Example.—The following figure is also from a report *re* Toronto's debt for 1919:

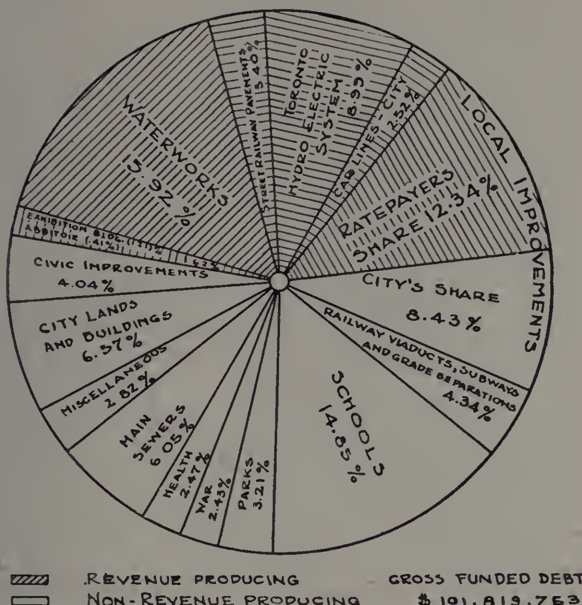


FIG. 63.

The object of this figure is to show the percentage of the debt, which is due to each investment mentioned.

Exercises LXVIII.

Note.—Before attempting to draw a graph of any relation, it is important to make a careful study of the squared paper at our disposal for the work. The larger the graph the greater accuracy and range of readings; we should, therefore, draw our graph to cover, if possible, all the sheet. Further, we should so divide the horizontal and vertical distances that as many as possible of our readings may come exactly on the lines of the paper.

1. The distances along a road from a certain point and the height of the road above sea-level at these distances are shown as follows:

Distance from starting point in miles.	0	1	2	3	4	5	6	7	8
Height above sea-level in feet.....	60	75	90	140	175	230	260	290	330

Represent the above relations by means of a graph. Estimate the probable height above sea-level $7\frac{1}{2}$ miles from the starting point.

2. The following table represents the output of an automobile firm for the past ten years:

Year.......	1910	1911	1912	1913	1914	1915	1916	1917	1918	1919
Number....	700	780	850	900	1000	950	960	1020	1050	1850

Represent the above relations graphically.

3. The following table gives the revolutions per minute of a 60 in. diameter locomotive driver and the corresponding speed of the locomotive in miles per hour:

Revolutions per min..........	0	60	90	100	150	200	250	275	300
Miles per hour..	0	10·5	15·7	17·5	26·3	35	39·3	48·2	52·5

Represent the above graphically and find the revolutions for a speed of 30 miles an hour.

4. The following observations of temperature were recorded on July 25, 1919:

Hour of the day	4 A.M.	6 A.M.	8 A.M.	10 A.M.	12 NOON	2 P.M.	4 P.M.	6 P.M.	8 P.M.	10 P.M.
Temperature in degrees.......	40	45	60	70	75	85	90	80	64	60

Draw a graphical representation of this variation in temperature.

5. If a cu. in. of steel weighs ·28 lb. construct a graph showing relation between volumes and weights.

6. If 1 inch = 2·54 centimetres, construct a graph showing relation between the two systems of measurement.

7. The following is an extract from a table giving breaking strength of steel, in pounds per sq. in., in relation to the percentage of carbon in the steel:

% Carbon	.09	.16	.20	.31	.39	.50	.57	.71	.79
Breaking Strength	53000	64000	65000	77000	90000	97000	110000	124000	127000

Represent the above graphically and estimate the percentage carbon for a breaking strength of 100,000 pounds per sq. in.

8. The prices charged by a manufacturing concern for a certain motor of different horse-powers is given by the following table:

H.P.	1	2	3	4	5	$7\frac{1}{2}$	10	15
Price	100	140	165	180	200	250	275	325

Represent graphically the relation between H.P. and price.

9. The quotations of a certain industrial stock at intervals of a week, were, 48, 49, $52\frac{1}{2}$, 53, $56\frac{1}{2}$, 58, $56\frac{1}{2}$, $55\frac{1}{2}$, 53.

Represent graphically the probable fluctuations in price.

10. The record of a patient's temperature for a certain time at intervals of a half-hour, is 97, 97·5, 98, $98\frac{1}{2}$, $99\frac{1}{2}$, 101, $101\frac{1}{2}$, 102, 101, 100. Represent the fluctuations graphically.

11. A tram-car is found to travel the distance y feet in x seconds, the distance moved in different times being measured and recorded as follows:

Distance in feet	(y) 0	7·5	13	20	27	34	42	49·5	57·5
Time in seconds	(x) 0	1	2	3	4	5	6	7	8

Represent this relation graphically.

12. A company finds that the buying expenses are 15% of its gross income; office expenses 5%; management 10%; other overhead 25%; selling expenses 30%; interest 10%; dividends 3%; incidentals 2%. Use the pictograph to represent the division of the gross income.

13. The value of the exports and imports of the United States for a given period is as follows:

Year.......	1830	1840	1850	1860	1870	1880	1890	1900
Value in Millions..	134	222	318	687	829	1504	1647	2100

Use the pictograph to represent this growth in commerce.

14. The following results were obtained by hanging a series of weights on the free end of a spiral spring and thereby stretching it:

Weight in lb.	0	1	2	3	4	5	6	7	8	9	10
Stretch in in.	0	·2	·4	·6	·8	1·0	1·2	1·4	1·6	1·8	2·0

Represent this relation graphically and indicate the probable stretch for a load of $5\frac{1}{2}$ lb.

15. The following results were obtained as in the preceding except that the stretch in inches is given by differences:

Load in lb.....	0	10	20	30	40	50	60	70	80	90	100	105	110	118	128	135	149	154	
Stretch in in. by differences	0	.01	.022	.035	.05	.062	.075	.10	.20	.40	.75	1.04	1.29	1.78	2.18	2.91	5.4	7.0	broke

Plot the above in two parts—the first for loads up to 60 lb., the second for loads above. Compare the two graphs.

16. A car starting from rest is drawn by a varying force F pounds, which, after t seconds, is as shown in the following table:

t (seconds).....	0	2	5	8	11	13	16	19	20
F (pounds).....	1280	1270	1220	1110	905	800	720	670	660

If the frictional resistance is constant and equal to 500 lb., draw a graph of the above relation and indicate the force after 10 seconds.

17. The elasticity of a wire may be found by twisting. The following readings were taken in experimenting with a steel wire:

Load in lb.......	0	1	2	4	.5	6
Angle of twist in degrees........	0	6	12	24	29	26

Represent graphically and indicate the probable twist for a load of $4\frac{1}{2}$ lb.

18. The law of a machine is given by the relation $P = \cdot 08 W + 1 \cdot 4$. P being the force in pounds required to raise a weight W. The following values of W are given:—21, 36·25, 66·2, 87·5, 103·75, 120, 152·5. Find the corresponding values of P and plot the relation. Find the force necessary to raise a weight of 310 lb. from your graph.

19. In a certain machine, P is the force in pounds required to raise a weight W. The following corresponding values of P and W were obtained experimentally:

P........	2·8	3·7	4·8	5·5	6·5	7·3	8·0	9·5	10·4	11·75
W.......	20·0	25·0	31·7	35·6	45·0	52·4	57·5	65·0	71·0	82·5

Draw the graph connecting P and W, and read the value of P when $W = 70$. Also determine the law of the machine, and from it the weight that could be raised by a force of 45 pounds. $(P = aW + b)$.

20. The length of one degree on a parallel of latitude is given for certain latitudes as follows:

Latitude...	0	10°	20°	30°	40°	50°	60°	70°	80°	90°
Length in Miles....	69·2	68·1	65	60	53·1	44·6	34·7	23·7	12·1	0

Draw a graph of the above and employ it to estimate the length of a degree in latitudes 15°, 45°, 73°.

21. The following are the results obtained with a set of rope pulleys:

Load in lb...........	0	5.5	12.2	17.1	25.0	31.0	37.5	44.8	50.8	62.0	76.0
Effort in lb..........	.94	3	5.5	7.1	10.2	12.2	14.5	17.2	19.7	25.5	30.0
Friction in lb.........	3 76	6.5	9.8	11.3	15.8	17.8	20.5	24.0	28.0	40.0	43.2
Efficiency in %.......	0	45.8	55.8	60.2	61.3	63.5	64.6	65.2	64.5	60.8	63.4

On the same sheet of paper draw graphs of the relation between load and effort, load and friction, load and efficiency. From your graph estimate the effort necessary to lift a load of 40 lb., also the friction and efficiency for this load.

22. From a series of tests on an oil engine the following values of the weight of oil used per hour (W) and the Brake Horse Power ($B.H.P.$) were obtained:

$B.H.P.$.....	1·0	2·1	3·0	4·2	4·70	5·3
W lb......	1·07	2·16	2·85	3·91	4·40	4·90

Represent the above graphically and estimate $B.H.P.$ when $W=4$ lb.

23. Toronto required $30,080,000 during 1920, to meet civic expenses. This was obtained as follows:

General taxes................	$13,074,312
School taxes.................	6,396,788
Water rates..................	2,840,066
Surplus from 1919...........	2,415,345
Hydro........................	606,069
Local improvements.........	1,605,675
Street railway...............	1,098,651
Abattoir	130,000
Rentals......................	186,600
Licenses.....................	113,000
City car lines...............	445,000
C.N.E........................	100,000
Fines........................	150,000
Other revenues..............	917,120

Employ the circular pictograph to represent the above.

24. The following are the results obtained with a screw-jack:

Load in lb.....	0	5	10	15	20	25	30	35	40	45
Effort in lb....	.172	.282	.359	.469	.578	.688	.797	.960	1.000	1.100
Friction in lb..	19.86	27.48			46.77		62.04		75.50	83.13
Efficiency %...	0	15.4			29.9		32.6		34.6	35.1

On the same sheet of paper draw graphs of the relation between load and effort, load and friction, load and efficiency. Estimate the missing quantities from your graph.

25. The results shown in the following table were obtained experimentally from a lifting machine. Plot the two curves connecting P and W and F and W:

Load (W) lb..........	0	5	10	15	20	25	30	35	40
Effort (P) lb..........	.094	.45	.81	1.17	1.53	1.88		2.61	2.97
Friction (F) lb.........	2.34	6.32	10.31	14.29	18.28	22.26			34.21

Estimate the missing quantities from your graph.

26. The tax rate in Toronto in 1919 was $28\frac{1}{2}$ mills, divided as follows:

General City purposes.........10.89 mills
Schools.........................7.90 mills
Public Library..................0.25 mills
Administration of Justice.......2.27 mills
Street Maintenance.............3.93 mills
War Expenditure................3.26 mills

Employ the circular pictograph to represent the above.

27. The increase in wages of the employees of a railway company from 1913 to 1916, based on \$1 a day, is given as follows:

Trackmen from...........\$1.15 to \$1.30
Station Agents from...... 1.75 to 2.25
Office Clerks from........ 2.10 to 2.50
Trainmen from........... 1.80 to 2.80
Machinists from.......... 2.25 to 3.20
Conductors from.......... 3.15 to 4.25
Enginemen from.......... 3.60 to 4.75

Represent these increases graphically.

28. The following table gives the edible portions of various kinds of fish and the price per pound:

Kind	Halibut	Haddock	Whitefish	Bass	Herring	Perch	Pike	Canned Salmon
Edible portion in %........	72	49	56	45	57	37	42	86
Price per lb....	24	18	20	22	16	12	18	32

Express graphically the edible portions of these various kinds of fish that can be bought for \$1.

29. THE INCREASE IN THE LENGTH OF OCEAN LINERS IS GIVEN BELOW:

The lengths given are approximate

Britannia	Hibernia	America	Asia	Arabia	Persia	Gt. Eastern	Washington	Oceanic	Britannic	Arizonia	Umbria	Campania	Deutschland	Baltic	Lusitania	Olympic	Imperator
210′	230′	260′	262′	280′	360′	675′	350′	425′	450′	450′	480′	560′	650′	700′	750′	860′	919′
1840–	......	–1850*	....	1850–	....	..–1860*	1860–70*	1870–	....	–1880*	1880–	90* 1890	..–1900*–	....	..1900–	1910*	1913
—	Wood	en pad	dle	wheel	....	......*	....iron,	1 scr	ew....	......	....*	...steel,	..2 screws	...*	..steel,	4 scr	ews

(World's Work)

Represent by means of the bar pictograph

CHAPTER XIV.

MATHEMATICS OF THE MACHINE SHOP.

115. Machinist's Scale. A machinist's scale is made of steel and is usually either 6″ or 12″ in length. There are markings on the four edges, 8ths, 16ths, 32nds, and 64ths. In measuring machinists prefer to "split" a 32nd, instead of attempting to read to 64ths.

116. Try Square. The try-square is used for testing if surfaces are at right angles to one another. The diagram illustrates its use for testing a piece of work.

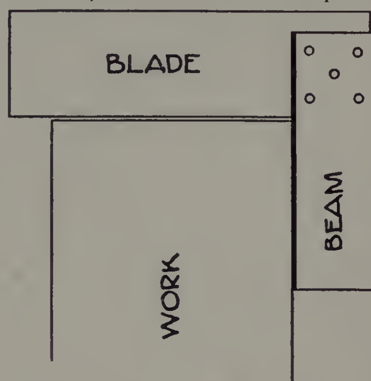


FIG. 64

117. Calipers. Frequently it is not possible to obtain an accurate measurement with the scale, for example the outside diameter of a cylinder, or the inside diameter of a pipe. For such purposes calipers are used. These are of three types—**outside calipers, inside calipers,** and **hermaphrodite calipers.**

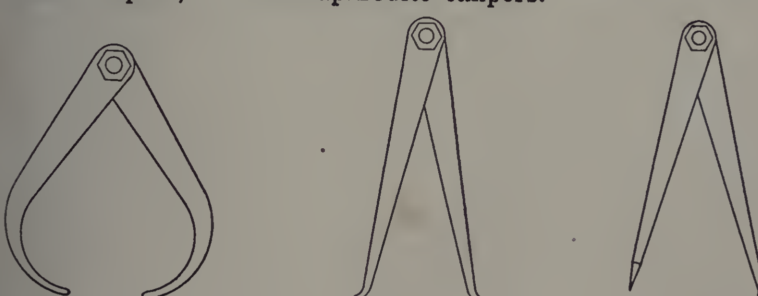


FIG. 65

Outside calipers are used for taking outside dimensions as the diameter of a cylindrical piece of work, inside calipers for measurements such as the bore of a pipe, and hermaphrodite calipers for finding the centre of a piece of work and for scribing.

The calipers must be finely adjusted so that they will just touch the sides of the work as they pass over it. Care must also be taken to keep them at right angles to the work.

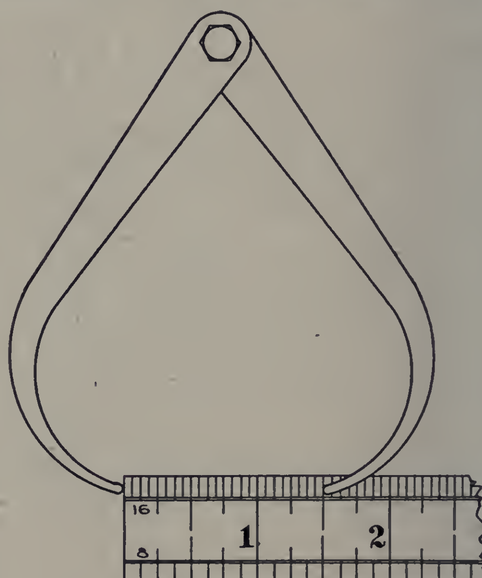
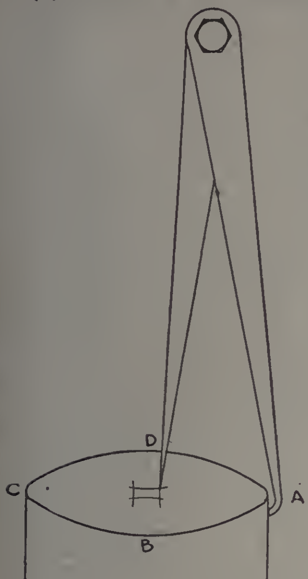


Fig. 66

In laying the calipers on the scale to find the length, place one leg at the end of the scale and read the mark on the scale where the other leg touches (see Figure 66).

118. Centring. Work is frequently held in a lathe between two points called centres. In order to accommodate these, small holes must be drilled in the ends of the work. These holes are countersunk to the same angle as the centres, usually 60°.

(1) **Centring by hermaphrodite calipers.** The calipers are set so that the pointed leg reaches approximately the centre of the work. The calipers are then placed at A, B, C and D and arcs are described as shown. The centre of the work will be the centre of the figure thus obtained.



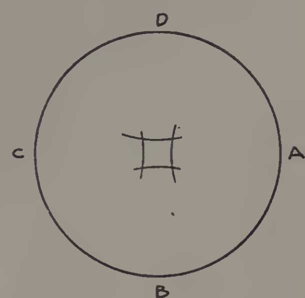


Fig. 67

(2) **Centring by the centre square.** The centre square consists of a head and blade. The head is so adjusted that the edge of the blade comes across the diameter of a piece of round stock placed in the head as shown.

A line is drawn along the blade on the work. The work is then turned to some other position and another diameter is drawn. Where these diameters cross will be the centre.

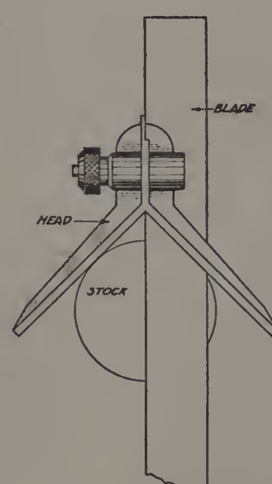


Fig. 68

119. **Vernier.** With the scale, Figure 69, we could measure to a certain degree of accuracy. If the length came between 7 and 8, we could estimate the amount, say

7·6. For obtaining greater accuracy in this part between the 7 and 8 a device known as a **vernier** is used (Pierre Vernier —1631).

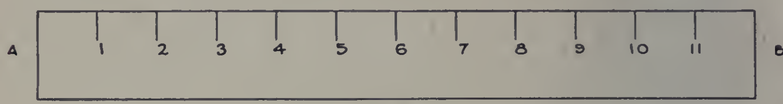


FIG. 69

In Figure 70 a second scale *CD*, called a vernier, is placed alongside of the scale *AB* and we observe that 10 divisions on the vernier is equal to 9 divisions on the scale. Obviously

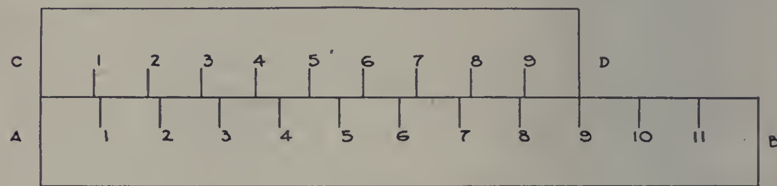


FIG. 70

each division on *CD* is $\frac{1}{10}$ less than a division on *AB*. Therefore the length between the 1 mark on *AB* and the 1 mark on *CD* will be $\frac{1}{10}$ of a division on *AB*. Also the length between the 2 mark on *AB* and the 2 mark on *CD* will be $\frac{2}{10}$ of a division on *AB*, and so on.

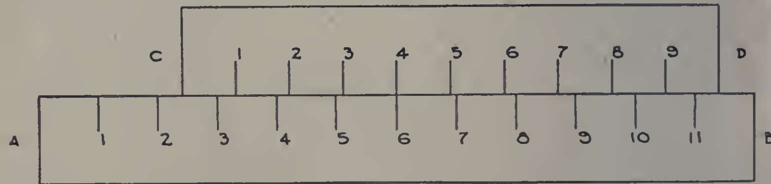


FIG. 71

In Figure 71 the reading on AB is 2 plus a decimal. To get the decimal part we observe that division 4 of the vernier coincides with a division on the scale. Evidently the excess of the reading over 2 is the difference between 4 divisions on AB and 4 vernier divisions, which as above explained is $\frac{4}{10}$ of a division on AB. Therefore the reading is 2·4.

For more accurate readings the vernier sometimes has 25 divisions corresponding to 24 divisions on the scale.

120. **Micrometer.** A micrometer is an instrument for measuring to a greater degree of accuracy than can be measured with a scale.

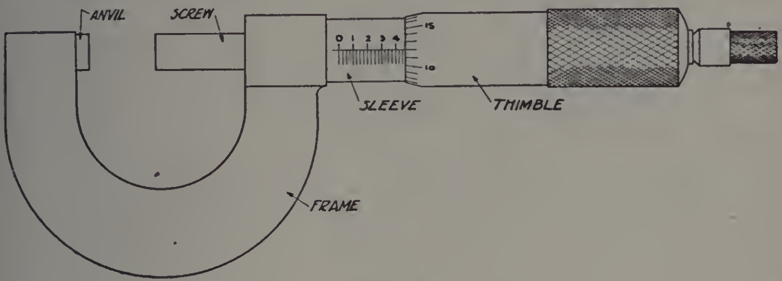


FIG. 72

The above is a representation of the micrometer for measuring in inches, the parts being indicated.

The principle of the instrument is as follows:

The screw is threaded inside of the sleeve with 40 threads to the inch. The thimble is attached to the end of the screw and the work to be measured is placed between the screw and anvil. The micrometer is then closed on the work by turning up the thimble.

Since the screw has 40 threads to the inch, one turn of the thimble closes the opening $\frac{1}{40}$th of an inch or $\cdot 025''$. Each mark on the sleeve represents one complete turn of the thimble, therefore, four turns equals $4 \times \frac{1}{40}''$ or $\frac{1}{10}''$. Figures are placed on the sleeve at every fourth mark, representing tenths of an inch. The thimble is divided into 25 equal divisions so that turning the thimble one division advances the screw $\frac{1}{25}$ of $\frac{1}{40} = \frac{1}{1000}''$.

To read the micrometer in the above figure:

The last number exposed on the sleeve is 4, therefore we set down $\frac{4}{10}''$ or $\cdot 4''$. Between the last number exposed and the edge of the thimble two small divisions are showing, therefore $2 \times \frac{1}{40}''$ or $2 \times \cdot 025'' = \cdot 05''$.

The thimble has evidently turned 12 spaces from the zero mark, therefore $\frac{12}{25}$ of $\frac{1}{40}'' = \frac{12}{1000}'' = \cdot 012''$.

$\therefore$ total reading $= \cdot 4'' + \cdot 05'' + \cdot 012'' = \cdot 462''$.

If the micrometer has a vernier, of the type described, the divisions on the thimble can be divided, making the micrometer read to 10,000ths.

Exercises LXIX.

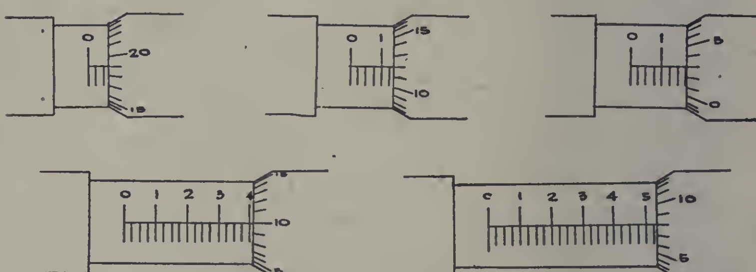


FIG. 73

1. Calculate the micrometer readings in the above figures.

2. If the thimble be turned backward through 6 complete revolutions, what decimal of an inch is the micrometer opened?

3. Through how many turns must the thimble be moved to open the micrometer $\cdot 7''$?

4. The sleeve reading is 4 and the thimble reading is 18. What is the opening of the micrometer?

5. How many turns must the micrometer be opened to read $\cdot 458''$?

6. A spindle is ground to $1 \cdot 345''$. What is the setting on the micrometer?

7. A ball measures $\cdot 864''$. What is the setting of the micrometer?

8. Calculate the setting of the micrometer for $\frac{7}{16}''$.

9. Explain how you would set a micrometer for $\frac{2}{1000}''$ over $\frac{3}{4}''$.

10. Calculate the setting of the micrometer for $\frac{5}{8}''$.

121. Vernier Caliper. The vernier caliper consists of a bar with a sliding jaw. The bar is divided the same as the

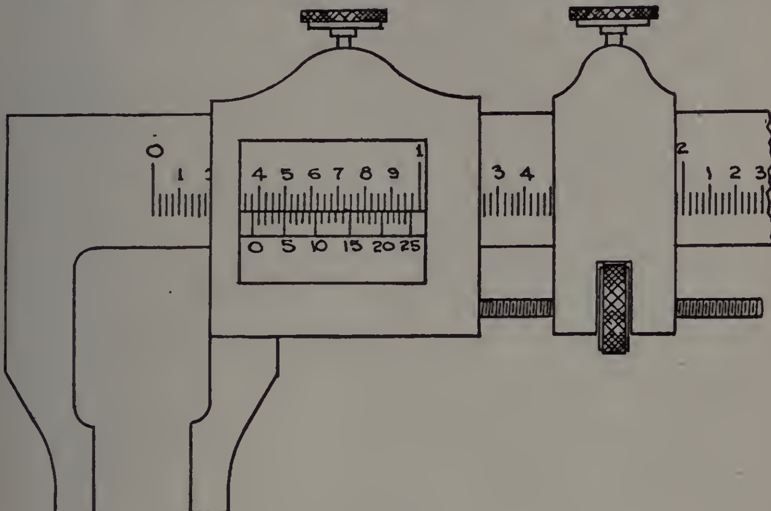


FIG. 74

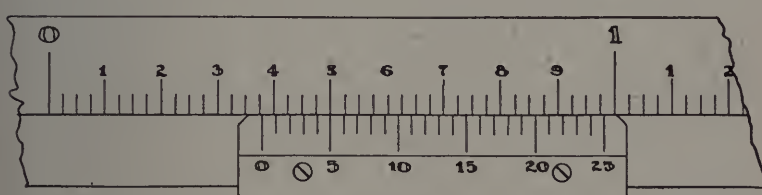


FIG. 74*a*

sleeve of the micrometer, i.e., the smallest division being $\frac{1}{40}$ of an inch.

On the sliding-jaw is a vernier. It is divided into 25 parts, the total length of these parts being equal to 24 divisions on the bar. As previously described in the case of the vernier, the distance between say the 4th mark on the vernier and the 4th mark on the bar will be $\frac{4}{25}$ of a division on the bar. Since a division on the bar is $\frac{1}{40}$ of an inch, this distance will be $\frac{4}{25}\times\frac{1}{40}=\frac{4}{1000}$ of an inch.

In the preceding figure our object is to obtain the bar reading opposite the 0 on the vernier. The last figure showing on the bar is 3, ∴ ·3″. From the 3 on the bar to the last division before the 0 on the vernier we have three small divisions. $\therefore 3 \times \frac{1}{40}'' = 3 \times \cdot 025'' = \cdot 075''$. To get the vernier reading we observe that the 5 line on the vernier is exactly opposite a line on the bar, $\therefore \frac{5}{1000}'' = \cdot 005''$

$\therefore$ total reading $= \cdot 3'' + \cdot 075'' + \cdot 005'' = \cdot 380''$.

Exercises LXX.

1. What would be the correct setting for a vernier caliper to read 1·642″?

2. A reading on the vernier caliper shows 1″, 3 tenths, 2 small divisions, while the 12th division on the vernier is in line with a beam division. What is the reading?

3. How would you set a vernier caliper to read $\frac{3}{8}''$?

4. What fraction of an inch is represented when the bar shows 2 tenths, 2 small divisions, and the 6th division on the vernier is in line with a beam division?

5. How would you set a vernier caliper to read ·7645″?

6. A reading on the vernier caliper shows 3″, 3 tenths, 3 small divisions, while the 8th division on the vernier is in line with a beam division. What is the reading?

122. Cutting and Surface Speed. In the running of machinery in the shop, the workman should know the speed at which to run the machine in order to give the best results. Lathes, milling machines, etc., are provided with attachments for changing the speed. This speed depends on the kind of material in the work, whether it is a roughing or finishing cut, etc.

In the lathe *the cutting speed is the rate at which the work passes the tool and is usually reckoned in feet per min.* The same definition would apply to the cutting speed of a planer. In the shaper, however, it is the tool that is moving and as a consequence *the cutting speed would be the rate at which the tool passes over the work.*

Cutting Speed of a Lathe. If a piece of work is being turned in a lathe, the tool will pass over the whole circumference of the work in a complete revolution. If the diameter of the work be 7″, then the circumference would be $\frac{22}{7}\times 7''=22''=\frac{22}{12}'$. If the lathe is making 40 revolutions per min., then the cutting speed of the lathe $=\frac{22}{12}\times 40=\frac{220}{3}=73\frac{1}{3}'$ per min.

From the above we have that *the cutting speed of the lathe in feet per minute = Circumference of work in ft. × R.P.M.* (R.P.M. being contraction for "revolutions per minute").

We have here three quantities involved—the cutting speed, the circumference of the work, and the revolutions per minute. If we know any two of these we can find the third.

Example 1:

A piece of work 5″ in diameter is to be turned in a lathe. How many revolutions per min. should the lathe make to give a cutting speed of 35 ft. per min.?

The circumference of the work $=5\times\frac{22}{7}=\frac{110}{7}''=\frac{110}{84}'$.

The tool travels $\frac{110}{84}'$ in 1 revolution.

It would travel 35′ in $\frac{84}{110}\times\frac{35}{1}$ revs. = 26+revs.

Example 2:

The surface speed of an emery-wheel is to be 200 ft. per min. It is belted to an arbor to run 50 R.P.M. Find the circumference of the wheel.

In 50 revolutions a point on the surface travels 200′.

In 1 revolution.......................... $\frac{200}{50}=4'$.

∴ circumference of wheel = 4′.

$$\frac{22}{7}D=48''.$$
$$D=\frac{48}{22}\times\frac{7}{1}=15\frac{3}{11}''.$$

Exercises LXXI.

$(\pi=\frac{22}{7})$.

1. A piece of steel $\frac{3}{4}''$ in diameter is turned in a lathe at 100 R.P.M. What is the cutting speed?

2. A brass rod $\frac{7}{8}''$ in diameter is revolving at the rate of 300 R.P.M.; find the cutting speed.

3. A piece of work with a diameter of 3″ is being turned at a cutting speed of 50 ft. per min. What are the R.P.M.?

4. A 30″ grinding wheel is run at 50 R.P.M. What is the surface speed?

5. At what R.P.M. should a 50″ wheel be run, for a surface speed of 300 ft. per min.?

6. What sized wheel should be ordered to go on a spindle running 1600 R.P.M., to give a surface speed of 4000 ft. per min.?

7. An 8″ shaft is being run to give a cutting speed of 50 ft. per min. What are the R.P.M.?

8. A cast-iron pulley is machined at a cutting speed of 30 ft. per min. If the R.P.M. is 10, what is the diameter of the pulley?

9. What would be the rim speed in ft. per min. of a fly-wheel 10′ in diameter, running 75 R.P.M.?

10. How many revolutions per min. will it take to turn a piece of tool steel 2″ in diameter with a cutting speed of 40 ft. per min.?

123. **Cutting Feed.** In turning a piece of work in the lathe, *the feed is the number of revolutions of the work to one inch travel of the carriage.*

In drilling, *the feed is the number of revolutions necessary to cause the drill to descend 1 in.*

Example 1:

How many revolutions are necessary to take one cut over a shaft 6′ in length with a feed of 30?

Length of shaft $=72''$.

$\therefore$ number of revolutions $=72\times30=2160$.

Example 2:

How long will be necessary to take one cut over a shaft 3′ long and 3″ in diameter, with a cutting speed of 30 ft. per min. and a feed of 34?

Circumference of work $=3\times\frac{22}{7}=\frac{66}{7}''=\frac{66}{84}'$

Since cutting speed is 30

$\therefore$ R.P.M. $=30\div\frac{66}{84}=30\times\frac{84}{66}$.

Revs. necessary to finish the work $=36\times34$

$\therefore$ time required $=(36\times34)\div(30\times\frac{84}{66})=32+$min.

On account of variations in the nature of materials used, especially of cast-iron, and also in the cutting capacity of tool steels, no fixed rule can be given for cutting speeds and feeds. Generally, for roughing—slow speed and heavy feed; for finishing—high speed and light feed.

Exercises LXXII.

1. How many revolutions will be necessary to take a cut over a steel rod 8′ in length with a feed of 24?

2. How long will be necessary to take a cut over a shaft 22″ long and $2\frac{1}{2}$″ in diameter with a feed of 20 and a speed of 30 ft. per min.?

3. A piece of work 5′ in length is being turned at the rate of 60 R.P.M. If the feed be 16, what time will be necessary to make one complete cut?

4. A cast-iron pulley is 18″ in diameter and has a 6″ face. If the cutting speed be 40 ft. per min. and the feed 16, how long will it take for one cut over the work?

5. A shaft 6′ long and 4″ in diameter is being turned at a cutting speed of 30 ft. per min. If the feed is 20, what fraction of the surface will be cut over in 15 min.?

6. A drill is being fed to the work at ·01″ per revolution. If it makes 40 revolutions per min., in what time will it cut through 2″ of metal?

7. A drill cuts $1\frac{1}{2}$″ into a piece of work in 15 minutes. If it makes 36 revolutions per min., what is the feed of the drill?

8. A drill with a feed of 100 is making 50 revolutions per min. In what time will it cut through $2\frac{1}{2}$″ of metal?

9. In 10 min. one cut is taken over a shaft 3′ long and 4″ in diameter. If the feed of the machine is 21, what is the cutting speed?

10. It takes 12 min. to take one cut over a shaft 18″ long and 3″ in diameter. If the cutting speed is 40 ft. per min., what is the feed?

124. The Trigonometrical Ratios. It is frequently necessary to make use of trigonometrical ratios in the machine shop. We will merely define these ratios without giving reasons for the names assigned.

1. The **Sine** of an angle $= \dfrac{\text{Side Opposite}}{\text{Hypotenuse}}$.

2. The **Cosine** of an angle $= \dfrac{\text{Side Adjacent}}{\text{Hypotenuse}}$.

3. The **Tangent** of an angle $= \dfrac{\text{Side Opposite}}{\text{Side Adjacent}}$.

4. The **Cosecant** of an angle $= \dfrac{\text{Hypotenuse}}{\text{Side Opposite}}$.

5. The **Secant** of an angle $= \dfrac{\text{Hypotenuse}}{\text{Side Adjacent}}$.

6. The **Cotangent** of an angle $= \dfrac{\text{Side Adjacent}}{\text{Side Opposite}}$.

The contractions Sin, Cos, Tan, Cosec, Sec, Cot, are used when writing the above.

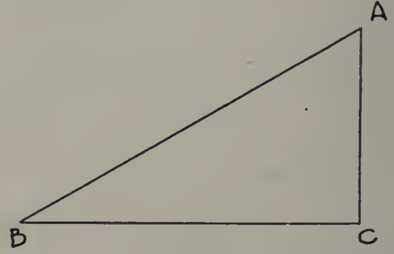


FIG. 75

In the above triangle:

$\text{Sin B} = \dfrac{AC}{AB}$. $\text{Cos B} = \dfrac{BC}{AB}$. $\text{Tan B} = \dfrac{AC}{BC}$. $\text{Cosec B} = \dfrac{AB}{AC}$.

$\text{Sec B} = \dfrac{AB}{BC}$. $\text{Cot B} = \dfrac{BC}{AC}$.

Tables giving the values of the trigonometrical ratios of all angles from 0° to 90° are available.

125. Taper. The taper on a piece of conical work is the difference in diameter for one foot of the work.

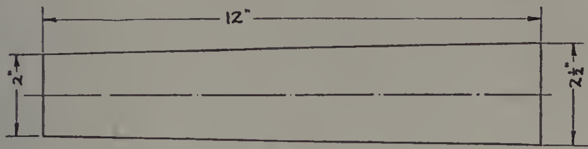


FIG. 76

If the work in the figure be 12″ in length and the diameters $2\frac{1}{2}''$ and 2″, the difference of $\frac{1}{2}''$ in the diameters is called the amount of taper, i.e., one-half inch per foot.

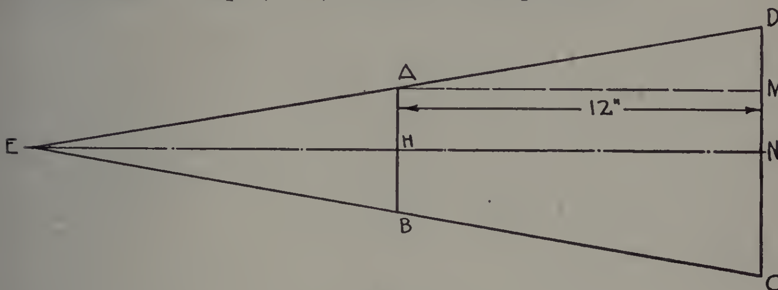


FIG. 77

Taper expressed as an Angle.

In the taper above the sides DA and CB when produced meet at E. The angle AEB is known as the angle of taper. Assume that the length HN of the piece is 12″, that the diameter AB of the small end is 6″, and the angle of taper $\angle AEB$ is 10°.

In $\triangle\ AMD : DM = 12 \tan 5°$

$= 1.04988''$

$\therefore DC = 6 + 2 \times 1 \cdot 04988 = 8 \cdot 1$ (Approx.)

The amount of taper is therefore 2·1 in. per foot.

Kinds of Tapers:

(1) Morse Taper.

Possibly the most common taper is the Morse. It is found chiefly on lathe spindles, drill spindles, and grinder spindles.

It is approximately $\frac{5}{8}''$ per ft., but varies somewhat according to the following table:

Number	Inches per Foot
0	·625
1	·600
2	·602
3	·602
4	·623
5	·630
6	·626

Of the above numbers 1, 2 and 3 are more commonly used.

(2) **Brown and Sharpe Taper (B. & S.).** The Brown and Sharpe taper is $\frac{1}{2}$ in. per ft. for all sizes except No. 10, which is ·516 in. per ft.

It is the taper used on milling machine arbors, the milling machine having been developed largely by Brown and Sharpe.

(3) **Jarno Taper.** The Jarno taper is ·6 in. per ft. for all sizes. It is frequently used on lathe centres.

126. Methods of Cutting Tapers on the Engine Lathe.

(1) By Means of the Compound Rest.

In cutting short tapers and bevels, this compound rest (Fig. 78) is used, the extent of the work being limited by the length of the compound rest screw. This attachment is used in turning head-stock centres. A graduated slide divided into degrees permits of adjustment to any required angle.

(2) By Offsetting the Tail Stock.

When the tail centre and head centre of the lathe are in alignment, the cutting tool moves in a line parallel to a line connecting the two centres. If a piece of work be turned in this position, a uniform cut will be taken throughout its

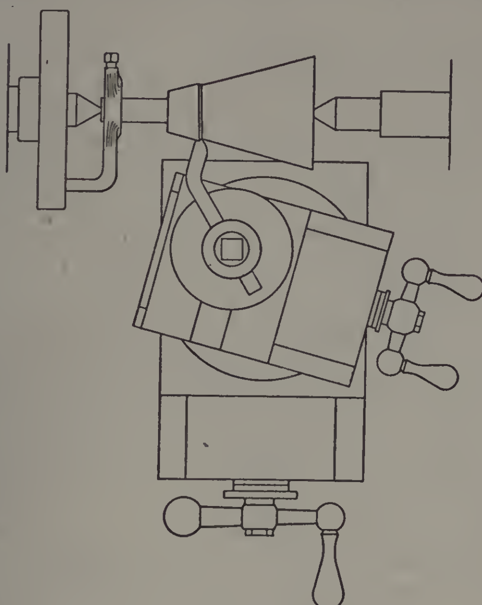


Fig. 78

length. If, however, the tail centre be moved out of alignment with the head centre, the cut will be deeper at one end than at the other.

The following diagram will help to make this clear:

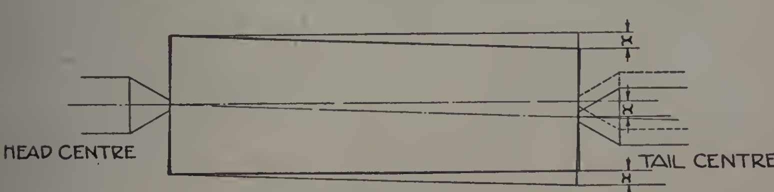


Fig. 79

In the above diagram the tail centre is represented as set over an amount x. If a piece of work be turned when the centres are related in this way, the radius of the work at the tail centre will be less by x than the radius of the work at the head centre. Since the taper is the difference in diameter between the centres, it follows that *the offset of the tail stock is one-half this difference in diameter.*

Example:

A piece of work 9″ long is to be turned with a taper of $\frac{1}{2}''$ per foot; find the amount of offset of the tail stock.

A taper of $\frac{1}{2}''$ in $12''$ = a taper of $\frac{9}{12} \times \frac{1}{2}''$ in 9″.

= a taper of $\frac{3}{8}''$ in 9″.

As the tail stock must be set over one-half of this amount, the required offset is $\frac{3}{16}''$.

In the above method it must be kept in mind that the amount of offset of the tail stock is one-half the difference of the end diameters **whether the taper extends the full length of the work or not.**

Example:

A steel pin 12″ long is to be tapered for 8″ and turned straight for the remaining 4″. The diameter at the large end is $1\frac{5}{8}''$ and the small end is to be 1″ in diameter; find the amount of offset.

The taper in $8'' = 1\frac{5}{8}'' - 1'' = \frac{5}{8}''$

$\therefore$ taper in $12'' = \frac{12}{8}$ of $\frac{5}{8}'' = \frac{15}{16}''$

$\therefore$ amount of offset $= \frac{15}{32}''$.

(3) By Means of a Taper Attachment.

Many lathes are now fitted with a taper attachment. This is attached to the back of the lathe and is connected to the cross-feed. A movable slide can be adjusted at various angles to the travel of the carriage, and the cross-feed screw having been released, the cross-feed slide will move backward and forward according to the alignment of the slide on the taper attachment. This method should always be used if a lathe with a taper attachment is available.

Exercises LXXIII.

1. A piece of steel 8″ long has end diameters of 1″ and $\frac{3}{4}$″. Find the amount of taper, i.e., taper per foot.

2. A piece of work 10″ long has a Jarno taper and has a diameter at the large end of $1\frac{3}{8}$″. What is the diameter at the small end?

3. A piece of work 6″ long has a No. 1 Morse taper and has a diameter at the small end of $\frac{9}{16}$″. What is the diameter at the large end?

4. A taper pin tapers $\frac{1}{4}$″ per foot and has end diameters of $\frac{1}{2}$″ and $\frac{3}{4}$″. What is the length of the pin?

5. A piece of work with a Jarno taper has end diameters of $\frac{7}{16}$″ and $\frac{11}{16}$″. What is the length of the work?

6. A piece of work with a B. & S. taper has end diameters of $1\frac{1}{4}$″ and $1\frac{9}{16}$″. What is the length of the work?

7. A piece of work 15″ long has end diameters of $1\frac{1}{2}$″ and $2\frac{9}{32}$″. What kind of taper was used in turning?

8. A piece of work 18″ long has end diameters of $1\frac{1}{4}$″ and 2″. What kind of taper was used in turning?

9. A piece of work $2\frac{1}{8}$″ long has end diameters of ·368″ and ·475″; find the amount of taper.

10. A piece of work 6″ long is to have end diameters of ·45″ and ·7625″. What kind of taper would the work have when finished?

11. A piece of work 9″ long is to have end diameters of ·3125″ and ·6875″. What is the amount of taper?

12. A piece of work 21″ long has end diameters of ·875″ and 1·925″. What kind of taper has it?

13. Explain why the offset of the tail stock is one half the difference of the required diameters.

14. A steel pin $7\frac{1}{2}$″ long is to be turned with a taper of $\frac{3}{4}$″ per foot. What is the offset of the tail stock?

15. The diameter at the large end of a piece of work is $1\frac{1}{2}$″ and the diameter at the small end 1″. What is the offset of the tail stock?

16. A taper pin is 1″ in diameter at the large end and $\frac{9}{16}$″ at the small end. What is the offset of the tail stock?

17. A taper gauge has end diameters of $2\frac{1}{2}$″ and $1\frac{7}{8}$″. If the length of the taper is 9″ and the total length 12″, find the offset of the tail stock.

18. Determine the distance that the tail stock should be set over to cut the following:

(*a*) A No. 0 Morse taper on a piece of work 9″ long.
(*b*) A No. 1 Morse taper on a piece of work $7\frac{1}{2}''$ long.
(*c*) A Jarno taper on a piece of work 10″ long.
(*d*) A Brown & Sharpe taper on a piece of work 18″ long.

19. A piece of work 18″ long is to be turned straight for 12″ and the remaining 6″ to be tapered. The diameter at the large end is to be 2″ and at the small end 1″; find the offset of the tail stock.

20. A tapered piece of work is 8″ long, and a Jarno taper was turned on the piece. What is the difference in end diameters?

21. A piece of work 20″ long is to be turned to a diameter of 3″ at the centre, and to be tapered from centre to each end with a taper of $\frac{1}{2}''$ per foot. Determine the end diameters and the offset of the tail stock.

22. A piece of work 12″ long having a diameter at the larger end of 6″, tapers to an angle of 10°. What is the amount of taper?

23. A piece of work 18″ long and a diameter at the smaller end of 8″, tapers to an angle of 8°. What is the amount of taper?

24. What is the angle of taper in a Morse No. 0, a Morse No. 2, a B. & S., a Jarno?

127. Threads. A thread is formed by cutting a uniform spiral groove around a piece of work.

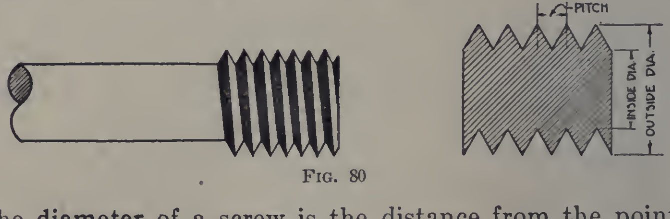


FIG. 80

The **diameter** of a screw is the distance from the point of a thread on one side to a point on the opposite side (outside diameter of diagram). The **inside diameter** is the diameter measured at the bottom of the groove (see diagram). The **pitch** of a thread on a screw is the distance from the middle point of one thread to the middle point of the next, measured in a line parallel to the axis. Pitch is usually stated as the number of threads per inch. Thus if there are 10 threads per inch, the pitch is $\frac{1}{10}$.

Stated generally : $Pitch = \dfrac{1}{No.\ of\ threads\ per\ inch.}$

To estimate the number of threads per inch, place a mark on the scale on the point of a thread and count the number of grooves within the inch line, or count the number of threads and subtract 1.

The **lead** of a screw is the distance the screw advances in one complete turn. In a **single threaded** screw the pitch is equal to the lead. Thus if the pitch is $\frac{1}{10}$, the screw will move forward $\frac{1}{10}''$ in one complete revolution. In a **double threaded** screw the pitch is $\frac{1}{2}$ the lead, in a **triple threaded** screw $\frac{1}{3}$ the lead, and so on.

If a screw has a **right-handed** thread it turns in the direction of the hands of a clock when screwed into the nut. If a **left-handed** thread it will turn in the opposite direction when screwed into the nut.

Exercises LXXIV.

1. Secure a number of different kinds of screws and find the number of threads per inch in each.

2. What is the lead of a single threaded screw if it has (*a*) 6 threads per inch, (*b*) 12 threads per inch, (*c*)15 threads per inch?

3. A single threaded screw advances 2″ in 12 turns, what is the pitch?

4. What is the pitch of a double threaded screw if it has 12 threads per inch?

5. A jack-screw has 4 threads per inch. How far does it move in $\frac{1}{2}$ a revolution?

6. What is the pitch of a triple threaded screw that advances 3″ in 6 revolutions?

7. What is the pitch of a double threaded screw which advances 1″ in 6 revolutions?

128. Kinds of Threads:

(1) Sharp "V" Thread.

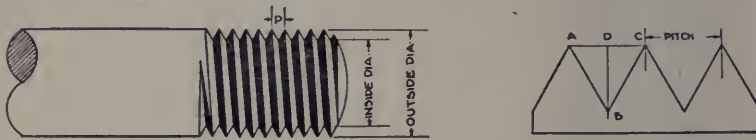


FIG. 81

The sharp "V" thread is a thread having its sides at an angle of 60° to each other and being perfectly sharp at both top and bottom. It is difficult to get a sharp "V" thread on account of the wear on the point of the tool in cutting.

Depth of "V" thread.

In figure above the thread has a pitch of 1″, then in the triangle ABC each side is 1″ in length. The depth of the thread will be equal to BD, the altitude of the triangle.

In the right-angled triangle BCD, $DB^2 = BC^2 - CD^2$

$$\therefore DB^2 = 1^2 - (\tfrac{1}{2})^2 \text{ or } DB = \sqrt{\tfrac{3}{4}} = \cdot 866''.$$

If the pitch be only $\frac{1}{2}''$, then since the triangle formed would be similar to the triangle ABC of the preceding, the depth would be $\frac{1}{2}$ of $\cdot 866'' = \cdot 433''$. If the pitch be $\frac{1}{12}''$ then for like reason the depth would be $\frac{1}{12}$ of $\cdot 866'' = \cdot 0721''$.

Calculations for threads are usually made on the **double depth.** In a thread of 1″ pitch the double depth would be $2 \times \cdot 866'' = 1 \cdot 732''$.

Since by the above the depth is proportional to the pitch, 1·732 is used as a constant for all "V" threads.

Example:

If the pitch of a "V" thread is $\frac{1}{10}''$ the double depth would be $\frac{1}{10}$ of $1 \cdot 732 = \cdot 1732''$.

$$\text{Since pitch} = \frac{1}{\text{Number of threads per inch}},$$

$$\therefore \textit{double depth} = \frac{1 \cdot 732}{\textit{Number of threads per inch}},$$

or, for brevity, $D = \dfrac{1 \cdot 732}{N}$, where D is the double depth, and N the number of threads per inch.

As the root diameter of the thread is the same size as the hole to be bored for tapping the thread, it is necessary to be able to find this double depth in selecting the size of drill.

Example:

What sized tap drill must be used for $\frac{1}{2}''$ screw, sharp "V" thread, having 12 threads per inch?

$$\textit{Root Diameter} = \textit{Outside Diameter} - \textit{Double Depth}$$

$$= \cdot 5'' - \frac{1\cdot 732''}{12}$$

$$= \cdot 5'' - \cdot 1443'' = \cdot 3557''.$$

From the table of decimal equivalents $\frac{23}{64}$ is the next above, therefore the correct size.

Exercises LXXV.

1. By means of the method used in the preceding find the double depth of sharp "V" threads having 8, 12, 14, 18, 20 threads per inch. Check by formula.

2. If the double depth of a sharp "V" thread is $\cdot 1443''$, find the number of threads per inch.

3. If the double depth of a sharp "V" thread is $\cdot 1732''$, find the number of threads per inch.

4. What size of tap drill would be necessary for a $\frac{7}{8}''$ screw with a sharp "V" thread having 9 threads per inch?

5. What size of tap drill would be necessary for a $1\frac{1}{8}''$ screw with a sharp "V" thread having 7 threads per inch?

6. What size of tap drill would be necessary for a $\frac{9}{16}''$ screw with a sharp "V" thread having 12 threads per inch?

7. If the single depth of a sharp "V" thread is $\cdot 1733''$, find the pitch.

8. The root diameter of a $3''$ bolt with a sharp "V" thread is $2\cdot 5052''$. What is the number of threads per inch?

9. The root diameter of a bolt with a sharp "V" thread is $\cdot 7835''$. If it has 8 threads per inch, what is the outside diameter?

10. What is the root diameter of a $2''$ bolt with a sharp "V" thread having $4\frac{1}{2}$ threads per inch?

(2) The United States' Standard Thread (U.S. Std.).

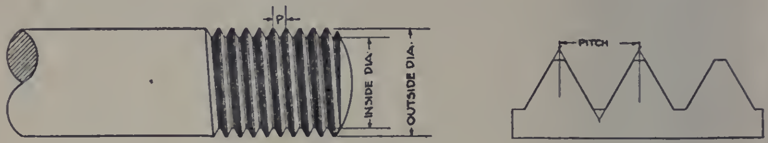


FIG. 82

This thread is commonly used in machine work as it gets over the difficulty of the sharp edges of the "V" thread.

This thread has the same triangular form as the sharp "V" thread but is flattened at the point and bottom. This flattened part is $\frac{1}{8}$ of the pitch in width. As $\frac{1}{8}$ of the height is taken from the top and bottom the depth of the thread is $\frac{3}{4}$ the depth of the "V" thread. $\therefore$ depth $= \frac{3}{4}$ of $\cdot 866'' = \cdot 649''$. Also double depth is $\frac{3}{4}$ of $1 \cdot 732'' = 1 \cdot 299''$.

As in the sharp "V" thread, the double depth of the U.S. Std. for different pitches may be found by dividing the constant by the number of threads per inch.

$$\therefore \textit{Double Depth of U.S. Std. thread} = \frac{1 \cdot 299}{\textit{Number of threads per inch}}.$$

$$\textit{Also Root Diameter} = \textit{Outside Diameter} - \frac{1 \cdot 299}{\textit{Number of threads per inch}}.$$

To find the size of tap drill for a U.S. Std. thread we would proceed as in the case of a sharp "V" thread.

Example:

What sized tap drill would be used for a $\frac{5}{8}''$ screw, U.S. Std. thread, 11 threads per inch?

Root Diameter = Outside Diameter − Double Depth

$$= \cdot 625'' - \frac{1 \cdot 299''}{11} = \cdot 5069''.$$

From table of decimal equivalents $\frac{33}{64}$ is the next above and consequently the correct size.

Exercises LXXVI.

1. What is the double depth of a U.S. Std. thread of $\frac{1}{8}$ pitch?

2. What is the root diameter of a $\frac{9}{16}''$ U.S. Std. threaded screw, 12 threads per inch?

3. The root diameter of a $\frac{3}{4}''$ U.S. Std. threaded screw is ·6201″. What is the pitch?

4. The root diameter of a U.S. Std. threaded bolt is 3·567″. If the pitch is $\frac{1}{3}$, what is the outside diameter of the screw?

5. If the single depth of a U.S. Std. thread is ·0491″, find the pitch?

6. If the double depth of a U.S. Std. thread is ·3248″, what is the number of threads per inch?

7. The single depth of a U.S. Std. thread is ·1998″, what is the number of threads per inch?

8. What sized tap drill would be used for a $1\frac{1}{8}''$ screw, U.S. Std. thread, 7 threads to the inch?

9. What sized tap drill would be used for a $1\frac{3}{4}''$ screw, U.S. Std. thread, 5 threads to the inch?

10. What sized tap drill would be used for a 1″ screw, U.S. Std. thread, 8 threads to the inch?

(3) Square Thread.

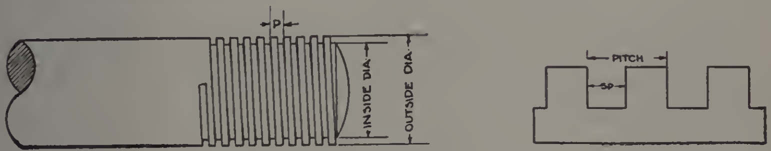


FIG. 83

The square thread is used in screws which are subjected to heavy loads, the jack-screw being an example.

In this thread the sides are parallel, the thickness of the tooth, the depth, and the width of the groove being all theoretically equal. In practice, however, the width of the groove is made slightly larger than the thickness of the thread to allow for clearance.

The pitch—or the distance from the middle point of one tooth to the middle point of the next—is in the square thread equivalent to one tooth and one space.

If, as in previous cases, we take a pitch of 1″, then the thickness of the thread will be $\frac{1}{2}$″, the depth $\frac{1}{2}$″, and the width of the groove $\frac{1}{2}$″.

Example:

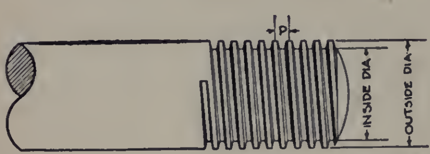
Find the root diameter of a square thread 3″ in diameter, with a pitch of $\frac{1}{4}$. If the pitch is $\frac{1}{4}$, then the depth is $\frac{1}{8}$″. $\therefore$ double depth $= \frac{1}{4}$″. $\therefore$ root diameter $= 2\frac{3}{4}$″.

We could obtain the same result by using a formula similar to that for preceding threads.

$$\textit{Root Diameter} = \textit{Outside Diameter} - \frac{1}{\textit{No. of threads per inch}}.$$

$\therefore$ Root Diameter $= 3'' - \frac{1}{4}'' = 2\frac{3}{4}''$.

(4) Acme 29° thread.



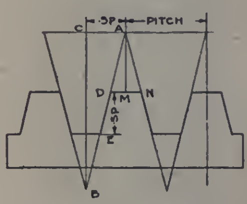


FIG. 84

This thread was designed to overcome the defects in the square thread. It is less difficult to make and does away with the sharp corners. Its principle use is in machine tool manufacture, where it is used for lead screws and other service where power is transmitted.

The angle between the threads is 29°, and theoretically the depth of the thread is one half the pitch.

If we consider the figure to the right above—pitch 1″—we have:

$$\text{In } \triangle\, ABC,\ BC = \cdot 5'' \text{ Cot } 14^\circ\, 30'$$
$$= \cdot 5'' \times 3 \cdot 86671$$
$$= 1 \cdot 93335''$$
$$\therefore\ 2\,AM = 1 \cdot 93335'' - \cdot 5'' = 1 \cdot 43335''$$
$$\therefore\ AM = \cdot 71667''.$$

In $\triangle$ *ADM*, $DM = AM \tan 14° \, 30'$

$$= \cdot 71667'' \times \cdot 25862$$

$$= \cdot 185345''$$

$\therefore$ 2 *DM* or $DN = \cdot 37069''$

$\therefore$ Width of flat at top $= \cdot 3707''$

Width of flat at bottom $= \cdot 3707''$

Width of space at top $= 1'' - \cdot 3707'' = \cdot 6293''$.

These are constants for all pitches. In practice to give clearance the following measurements are used (P = pitch):

Width of flat at top $= \cdot 3707 \; P$

Width of flat at bottom $= \cdot 3707 \; P - \cdot 0052''$

Width of space at top $= \cdot 6293 \; P$

Depth of thread $= \frac{1}{2} \; P + \cdot 010''$.

Exercises LXXVII.

1. Find the outside diameter of a screw with a square thread which has a root diameter of 1″ and a pitch of $\frac{3}{16}$.

2. Find the root diameter of a square thread which has an outside diameter of $2\frac{1}{2}''$ and a pitch of $\frac{1}{3}$.

3. A square thread has an outside diameter of 4″ and an inside diameter of $3\frac{1}{2}''$. What is the pitch?

4. Find the root diameter of a square thread which has an outside diameter of $3\frac{1}{2}''$ and a pitch of $\frac{1}{2}$.

5. By a method similar to that employed in the preceding for a 1″ pitch, find the width of flat at top, width at bottom, width of space at top, when the Acme 29° thread has a pitch of $\frac{1}{6}$. Check by means of data furnished for 1″ pitch.

6. If the depth of an Acme 29° thread is ·3850″, what is the number of threads per inch?

7. If the width of flat at the top of an Acme 29° thread is ·1853″, what is the pitch?

8. If the width of space at the top of an Acme 29° thread is ·1573″, what is the number of threads per inch?

9. If the width of space at the bottom of an Acme 29° is ·0566″, what is the number of threads per inch?

(5) Whitworth Thread.

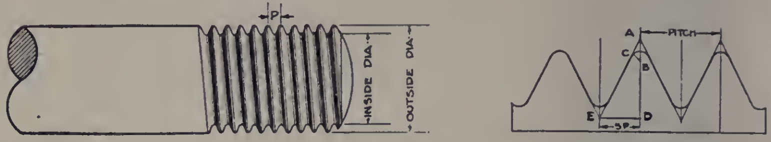


FIG. 85

This is a standard thread in England and on the Continent.

The sides form an angle of 55° with one another, while the top and bottom are rounded. The rounded part at both top and bottom is equal to one-sixth of the total depth of triangle above, leaving two-thirds for the depth of the thread.

In above figure if $AD = x$, $BC =$ radius of rounded part (r), then $AB = \frac{x}{6} + r$.

If the pitch be 1″ we have:

In $\triangle\, ADE$, $AD = \cdot 5 \text{ Cot } 27° \; 30'$

$$= \cdot 5'' \times 1 \cdot 92098$$
$$= \cdot 96049'' = x.$$

In $\triangle\, ABC$, Cosec $27° \; 30' = \dfrac{\frac{x}{6} + r.}{r}$ $\quad \therefore\; 2 \cdot 16568 = \dfrac{\frac{x}{6} + r}{r}$

$\therefore\;\; 2 \cdot 16568 \; r = \dfrac{x}{6} + r$

$\therefore\;\; 1 \cdot 16568 \; r = \dfrac{x}{6} = \dfrac{\cdot 96049}{6} = \cdot 16008''$

$\therefore\;\; r = \cdot 1373''$

depth of thread $= \frac{2}{3} \times \cdot 96049'' \doteq \cdot 64033''$.

The dimensions of this thread stated in terms of the pitch (P) are as a consequence of the above:

Depth $= \cdot 64033 \; P$.

Radius of rounded part $= \cdot 1373 \; P$.

Example:

Find the depth and radius of curvature of a Whitworth thread having a pitch of $\frac{1}{10}$.

Depth $= \cdot 64033 \times \frac{1}{10} = \cdot 064033''$.

Radius of Curvature $= \cdot 1373 \times \frac{1}{10} = \cdot 01373''$.

Exercises LXXVIII.

1. Find the depth and radius of curvature of a Whitworth thread having a pitch of $\frac{1}{12}$.

2. The depth of a Whitworth thread on a $\frac{3}{4}''$ screw is $\cdot 064033''$ What is the pitch and the diameter at the root?

3. A $1''$ screw has a Whitworth thread with a pitch of $\cdot 1250''$. What is the depth of the thread and the diameter at the root?

4. A $1\frac{1}{4}''$ screw with a Whitworth thread has a diameter at the root of $1\cdot 067''$. Find depth of thread, the pitch, and radius of curvature.

5. The depth of a Whitworth thread on a $2''$ screw is $\cdot 1423''$. Find the number of threads per inch, the diameter at the root, and the radius of curvature.

6. A $\frac{3}{8}''$ screw has a Whitworth thread with 16 threads per inch. Find the depth of the thread, the diameter at the root, and the radius of curvature.

129. Thread Cutting.

Gear Trains. One of the common ways of transmitting motion from one point to another is by means of gear trains.

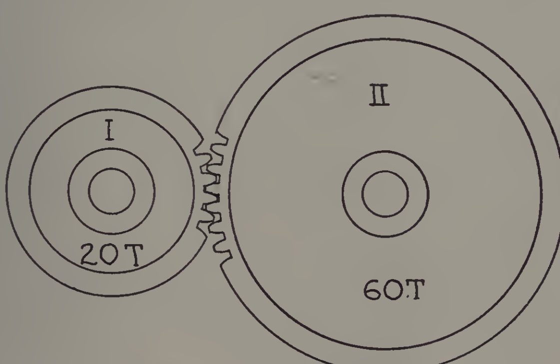


FIG. 86

The simplest form of gear train, having but two gears, is shown in Figure 86. Gears are usually known by their number of teeth. Thus, if *I* has 20 teeth it would be called a 20-toothed gear. Similarly *II* would be called a 60-toothed gear.

If two such gears are in **mesh**, as above, and the motion from *I* is transmitted to *II*, *I* would be known as the driver and *II* as the driven. As each tooth in *I* pushes along a corresponding tooth in *II*, it follows that one revolution of *I* will cause *II* to make only one-third of a revolution. Therefore the shaft to which *I* is keyed will make three revolutions while the shaft to which *II* is keyed is making one revolution. This principle is used extensively in gear trains.

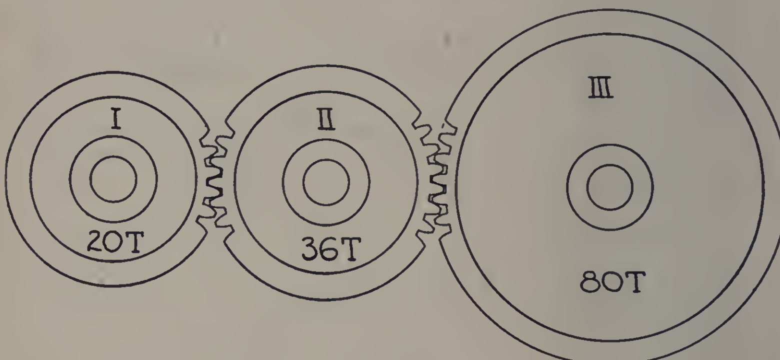


Fig. 87

In Figure 87 we have three gears in the train. It may be necessary to insert the **intermediate** gear *II*, either, that *I* and *III* may have the same direction, or to permit of *I* driving *III* without increasing the size of the gears. The gear *II* has no effect on the speed ratio of *I* and *III*, for when *I* moves one tooth the same amount of motion will be transmitted to *II*, which in turn will move *III* one tooth. Since each revolution of *I* will result in one-fourth of a revolution of *III*, therefore the speed ratio of *I* to *III* will be 4 to 1.

This may be stated as follows:

$$\frac{R.P.M.\ of\ driver}{R.P.M.\ of\ driven} = \frac{teeth\ on\ driven}{teeth\ on\ driver} = \frac{80}{20} = \frac{4}{1}.$$

Frequently it is necessary to make such a great increase or decrease in speed, that to accomplish it with a simple train of gears, would necessitate too great a difference in diameters. For this purpose a **compound gear train** is used.

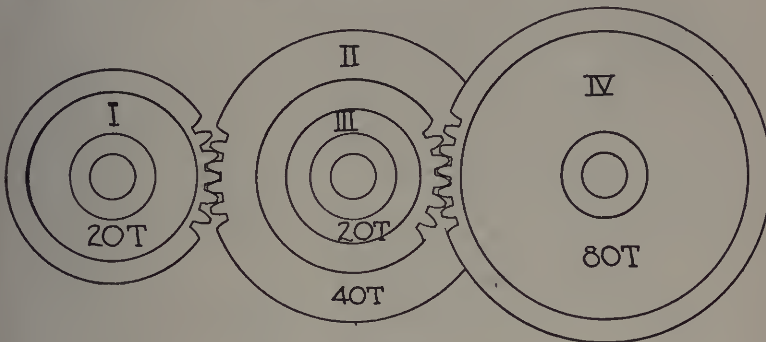


FIG. 88

Figure 88 represents a common form of a compound gear train, *I* drives *II* and causes a reduction of speed, *III* is keyed to the same shaft as *II* and therefore travels at the same speed, *III* meshes with *IV*, and on account of their relative number of teeth, a further reduction of speed is effected.

If we wish to find the speed ratio of *I* and *IV* we might proceed as follows:

Since *I* has 20 teeth and *II* 40 teeth, the speed of *I* is twice that of *II*. Since *III* has 20 teeth and *IV* 80 teeth, the speed of *III*, that is of *II*, is four times that of *IV*. Combining these statements we have that the speed of *I* is eight times that of *IV*.

The above is equivalent to the following:

$$\frac{\textit{R.P.M. of first driver}}{\textit{R.P.M. of last driven}} = \frac{\textit{Product of No. of teeth of all the driven}}{\textit{Product of No. of teeth of all the drivers}}.$$

In above figure *I* and *III* are the drivers and *II* and *IV* the driven.

Substituting the values from the figure in the above relation:

$$\frac{\text{R.P.M. of first driver}}{\text{R.P.M. of last driven}} = \frac{40 \times 80}{20 \times 20} = \frac{8}{1}.$$

Cutting a Thread. If a piece of work, on which a thread is to be cut, is placed in a lathe, it will revolve at the same rate as the spindle. If the spindle and lead screw turn at the same rate, then the number of threads per inch on the work will be the same as the number of threads per inch on the lead screw. If it is necessary that the number of threads per inch on the work differ from the number of threads per inch on the lead screw, then the principle of changing the speed by inserting gears of different sizes becomes necessary.

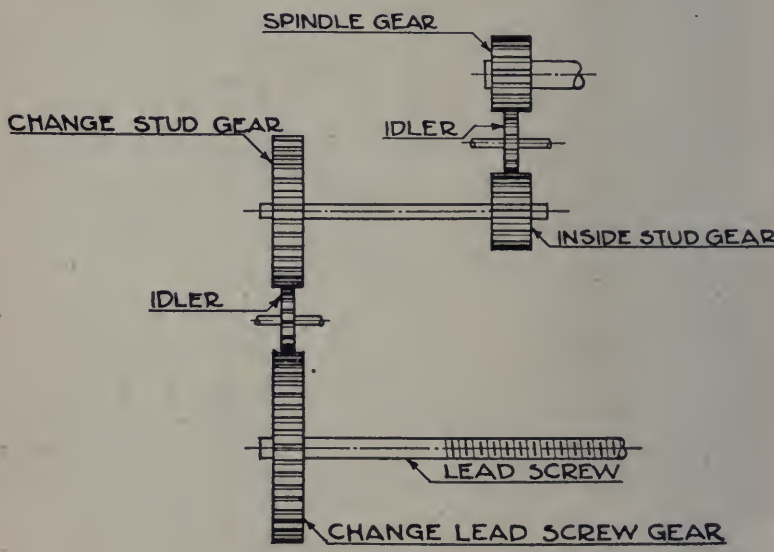


Fig. 89

Figure 89 shows the relation of gears in a **simple geared lathe.**

The spindle gear turns with the spindle, and drives the inside stud gear through the idler. The change stud gear, which is keyed to the inside stud, transfers the motion through another idler to the lead screw.

If the spindle gear in the above has 24 teeth, the inside gear on the stud 24 teeth, the change stud gear 40 teeth, and the lead screw 80 teeth, then:—Speed of spindle $\frac{24}{24}$ of speed of stud. Speed of stud $\frac{80}{40}$ of speed of lead screw. ∴ Speed of spindle $= \frac{24}{24} \times \frac{80}{40} = \frac{2}{1}$ of speed of lead screw.

The same result may be obtained by substituting in the formula, giving:

$$\frac{\textit{Revolutions of spindle}}{\textit{Revolutions of lead screw}} = \frac{\textit{Product of No. of teeth in driven}}{\textit{Product of No. of teeth in drivers}}.$$

$$= \frac{24 \times 80}{24 \times 40} = \frac{2}{1}.$$

In this case if the lead screw has 6 threads per inch, then the work would have 12 threads per inch.

Knowing the Lead of the Lathe we can find an arrangement of gears which will give the desired number of threads per inch on the work.

Example:

If the lead of the lathe is 8, find the necessary gears on stud and lead screw to cut a thread of $\frac{1}{10}$ pitch.

In this case the lead screw will advance $\frac{1}{8}''$ in one revolution and we want the work to advance $\frac{1}{10}''$ in one revolution.

This ratio of 8 to 10 would be obtained if we placed an 8-toothed gear on the stud and a 10-toothed gear on the lead screw. These gears are, however, not obtainable, but the same ratio may be maintained if we place a 48-toothed gear on the stud and a 60-toothed gear on the lead screw.

Gears furnished with a Lathe. Gears for a lathe usually vary in size by adding the same number of teeth each time to the gear just below. The two common sets are those obtained by adding 4 to the one below, giving 24, 28, 32 . . . . 120, and those obtained by adding 7, giving 21, 28, 35 . . . . 105. This is called gear progression.

Exercises LXXIX.

1. A lead screw has 6 threads per inch. What gears must be placed on stud and lead screw to cut 16 threads per inch?

2. Determine the change gears for cutting a $\frac{1}{12}$ pitch thread when the lead screw has a $\frac{1}{8}$ pitch.

3. A lathe with a lead screw of $\frac{1}{6}$ pitch has a 24-toothed gear on the stud and a 60-toothed gear on the lead screw. How many threads will be cut on a screw when the carriage has advanced 3 inches?

4. How many threads per inch will be cut by a lathe when the lead screw has a 64-toothed gear and the stud a 24-toothed gear, the lead screw having a $\frac{1}{6}$ pitch?

5. We wish to cut 24 threads per inch on a lathe with a lead of 8 and a gear progression of 4. What gears would be used?

6. The lead screw is $\frac{1}{8}$ pitch, the screw to be cut $\frac{1}{12}$ pitch. If there is a 24-toothed gear on the stud, what gear must be placed on the lead screw?

7. A lathe having a 72-toothed gear on the lead screw and a 24-toothed gear on the stud cuts 18 threads per inch. What is the pitch of the lead screw?

8. The lead screw has a 96-toothed gear and a $\frac{1}{6}$ pitch. What gear on the stud will cut 24 threads per inch?

9. What gear must be used on the lead screw in order to cut 12 threads per inch, when the lead screw has 6 threads per inch and a 36-toothed gear is used on the stud?

10. We wish to cut 14 threads per inch on a lathe with a lead of 6 and a gear progression of 7. What gears would be used?

Compound Gearing in the Lathe. Owing to the limited number of gears and also to give a wider range of speeds to those available, the principle of compound gears is used on the lathe.

Figure 90 represents the arrangement of gears on a lathe when compounding is necessary.

The only difference between this arrangement and that described in the simple geared lathe is that instead of the idler which meshes with the lead screw gear, there are two gears keyed to the same shaft. The inside one has usually twice as many teeth as the outside, and as a consequence causes an

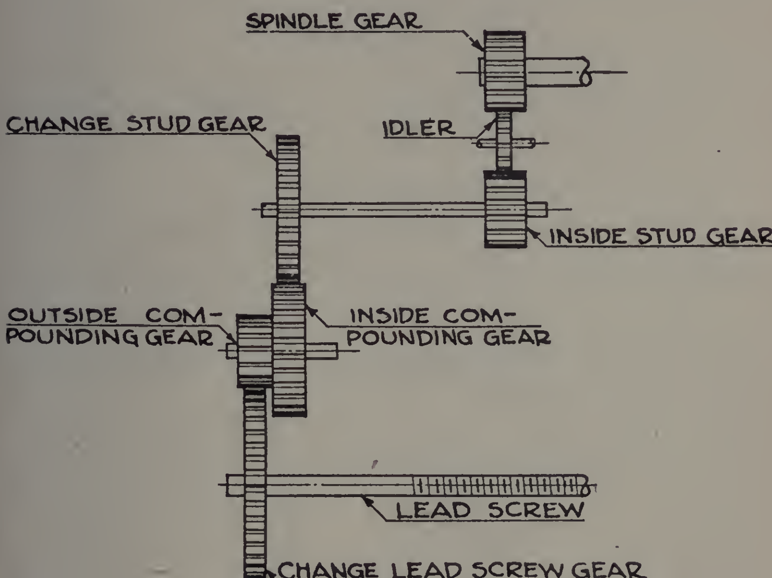


Fig. 90

additional reduction of speed in the ratio of 2 to 1. By the use of this compound the same gears which are used to cut 9, 10, 12, etc., threads on the simple geared lathe will cut 18, 20, 24, etc., threads.

Example 1:

We wish to cut 24 threads per inch on a lathe with a lead of 6.

We may use the same gears as in the example under the simple geared lathe, i.e., a 24 on both spindle and inside stud, a 40 on the change stud, and an 80 on the lead screw gear. If now we place a compound between the change stud and lead screw gear, consisting of a 72 and a 36 (see diagram), we would have the following speed ratio:

$$\frac{\text{Revolutions of spindle}}{\text{Revolutions of lead screw}} = \frac{\text{Product of No. of teeth in driven}}{\text{Product of No. of teeth in drivers}}.$$

$$= \frac{24 \times 72 \times 80}{24 \times 40 \times 36} = \frac{4}{1}.$$

In this arrangement the spindle will make four revolutions when the lead screw is making one, therefore 24 revolutions when the lead screw is making 6 revolutions.

Example 2:

We wish to cut 3 threads per inch on a lathe with a lead of 6. In this case it is necessary for the spindle to revolve only one-half as fast as the lead screw. For this purpose we might use the simple gear with say an 80 on the change stud and a 40 on the lead screw. We might also use the compound gear with equal gears on the change stud and lead screw, say 40 and 40, and interchange the gears on the compound, i.e., have the change stud mesh with the small gear on the compound and the lead screw mesh with the large gear on the compound. Then as in preceding cases:

$$\frac{\text{Revolutions of spindle}}{\text{Revolutions of lead screw}} = \frac{\text{Product of No. of teeth in driven}}{\text{Product of No. of teeth in drivers}}.$$

$$= \frac{24 \times 36 \times 40}{24 \times 72 \times 40} = \frac{1}{2} = \frac{3}{6}.$$

In practice the machinist reduces the method of finding the necessary gears when compounding to the following rule:

"*Write the ratio of the speed of the driving gear to the driven gear as a fraction, divide the numerator and denominator into two factors and multiply each pair of factors by the same number until gears with suitable number of teeth are found. The gears in the numerator are the driven and those in the denominator the driving gears.*"

Applying this rule to the two examples above, we have

In Example 1:

$$\frac{24}{6}=\frac{6\times4}{3\times2}=\left(\frac{6\times12}{3\times12}\right)\times\left(\frac{4\times20}{2\times20}\right)$$

$$=\frac{72}{36}\times\frac{80}{40}=\frac{4}{1}.$$

In Example 2:

$$\frac{3}{6}=\frac{3\times1}{6\times1}=\left(\frac{3\times12}{6\times12}\right)\times\left(\frac{1\times40}{1\times40}\right)$$

$$=\frac{36}{72}\times\frac{40}{40}=\frac{1}{2}.$$

Reduction Gears in the Head-stock. Some lathes, particularly those intended for cutting fine threads, have reduction gears in the head-stock. If in this case equal gears are placed on the change stud and lead screw, the spindle does not make the same number of revolutions as the lead screw. The ratio of this gearing in the head-stock is usually 2 to 1, so that with equal gears on the change stud and lead screw the spindle will turn twice as fast as the lead screw. In such lathes this must be taken into account in figuring the necessary gears.

Cutting of Double, Triple, etc., Threads. To cut a double thread on a screw, say 8 per inch, we would set the lathe for cutting half that number, in this case 4. Having cut this, turn the work **one-half** of a revolution and repeat the operation.

To cut a triple thread, set the lathe for cutting one-third the number. Having cut this, turn the work **one-third** of a revolution and repeat.

Exercises LXXX.

1. A lathe has a lead screw with a lead of 4, and has a 40-toothed gear on the stud and a 90-toothed gear on the lead screw. Using a 72 and 36 as compounding gears, how many threads are cut?

2. We wish to cut $11\frac{1}{2}$ threads per inch on a lathe with a lead of 6. If the gear progression is 4, what gears would do the work by compounding?

3. We wish to cut $1\frac{3}{4}$ threads per inch on a lathe, the lead screw having 6 threads per inch. If the gear progression is 4, what gears would do the work by compounding?

4. We wish to cut 64 threads per inch on a lathe with a lead screw having 8 threads per inch. If a 24-toothed gear is used on the stud, what gears placed on compound and lead screw would do the work?

5. A lathe has a lead of 6. If the gear progression be 7, calculate the change gears for cutting 14 threads per inch.

6. What gears must be used to cut 12 threads per inch on a lathe having a lead of 6, when 36 and 72 are used as compounding gears?

7. A special job requires $2\frac{1}{2}$ threads per inch. If the lathe has a lead of 4, what gears would do the work?

8. In a boat-lifting apparatus a 12-toothed gear meshes with a 48-toothed gear. Keyed to the latter is a 12-toothed gear, which meshes in turn with another 48-toothed gear on the revolving shaft. If the revolving shaft is 3 in. in diameter, how many turns of the handle will be necessary to raise the boat $5\frac{1}{2}$ feet?

9. A lathe has 6 threads per inch on the lead screw and a 40 and 80 on the inside and outside compound respectively. What gears must be used on stud and lead screw to cut 3 threads per inch?

10. It is desired to cut 4 threads per inch on a piece of work The lead screw gear has 6 threads per inch, while a 36-toothed gear is placed on the stud and a 48-toothed gear on the lead screw. What arrangement of compound gears would be suitable?

Quick Change Gears. To avoid the difficulty of having to calculate the necessary change gears, modern lathes are equipped with a mechanism for this purpose.

In Figures 91 and 92 this mechanism is shown.

The device is complete in one unit, and is contained in a box which is mounted on the front of the bed where its operating levers are convenient to the operator. The mechanism consists essentially of a cone of gears, an intermediate shaft, and a set of sliding gears. The tumbler gear is permanently in mesh with a long face pinion located inside the barrel about which the tumbler gear pivots. This gear may be tumbled into engagement with any of the nine gears in the cone, thus imparting

FIG. 91

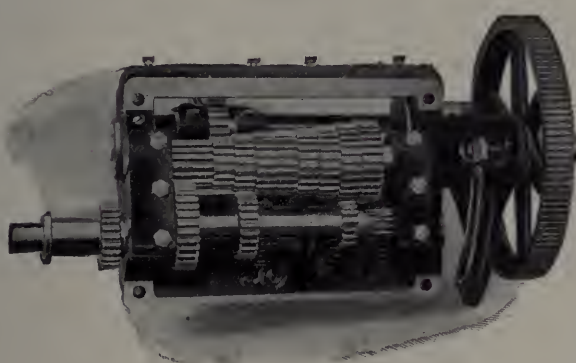


FIG. 92

nine changes of speed to the intermediate shaft which is permanently geared to the cone. It will thus be seen that thirty-six changes are obtained with two operating levers and without removing any of the gears.

130. **Gear Calculation.** In the last section the principle of change gears as applied to the lathe was dealt with. We will now consider some calculations pertaining to the gear itself.

In Figure 93 some of the more important terms with respect to a spur gear are indicated.

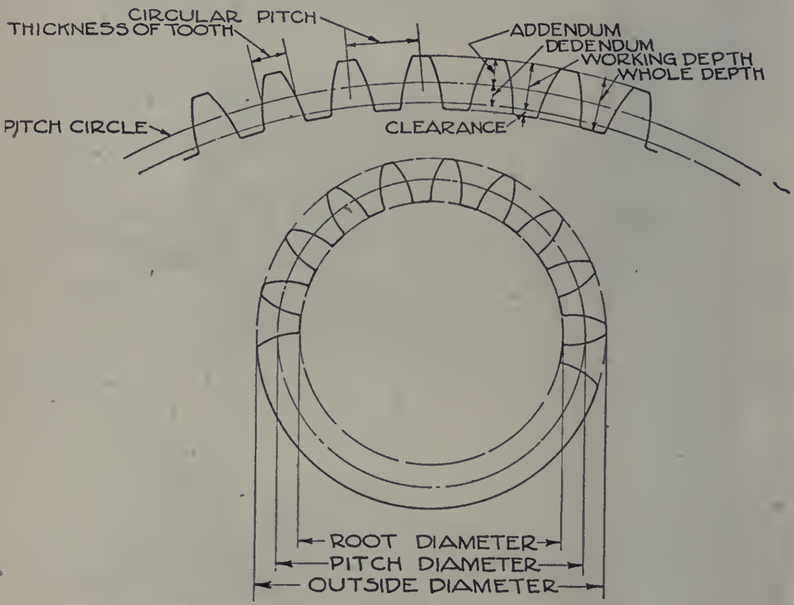


FIG. 93

The Pitch Circle is the line half-way between the top and bottom of the teeth. When two spur gears mesh, their pitch circles are regarded as being in contact.

The Pitch Diameter is the diameter of the pitch circle.

The Diametral Pitch is the number of teeth to every inch of pitch diameter of the gear. If the gear has 36 teeth and is 4 in. in diameter, it is said to be a 9 pitch gear.

The Circular Pitch is the distance from the centre of one tooth to the centre of the next, measured along the pitch circle.

The Thickness of the tooth should be slightly less than the space between the teeth to allow for clearance, but in practice they are calculated as being equal. As a result either the tooth or the space is one-half the circular pitch.

Clearance must be provided at the bottom of the space between the teeth (see diagram). It is usually $\frac{1}{10}$ of the thickness of the tooth measured on the pitch circle.

The Addendum is the part of the tooth projecting beyond the pitch circle. It is reckoned as a fraction of the size of the tooth. Thus in a 12 pitch gear the addendum would be $\frac{1}{12}''$.

The Dedendum is the part of the tooth between the pitch circle and the working depth.

The addendum plus the dedendum make the working depth of the tooth.

The Root Diameter is the diameter measured at the bottom of the space (see diagram).

The Outside Diameter is the diameter measured at the outside of the gear (see diagram).

Knowing the number of teeth in a gear and the diametral pitch, to find the size of gear blank, i.e., outside diameter.

Example:

What should be the outside diameter of a gear blank for a gear of 98 teeth and a diametral pitch of 14?

Diameter of pitch circle $= \frac{98}{14} = 7''$.

Addendum $= \frac{1}{14}''$ on one side.

$= \frac{1}{7}''$ on both sides.

$\therefore$ Outside diameter $= 7'' + \frac{1}{7}'' = 7 \cdot 1428''$.

To Find the Depth of Cut necessary in the Preceding Example.

Total depth = Addendum + Dedendum + Clearance.

Since the clearance depends on the thickness of the tooth it will first be necessary to determine the thickness.

Number of teeth = 98.

Since there are 14 teeth for 1″ of diameter there will be 14 teeth for 3·1416″ of circumference.

$$\therefore \text{Circular pitch} = \frac{3 \cdot 1416}{14} = \cdot 2244''.$$

Since the circular pitch is the distance which a space and tooth together occupy,

$\therefore$ thickness $= \frac{1}{2}$ of $\cdot 2244'' = 1122''$.

Since clearance $= \frac{1}{10}$ of thickness,

$\therefore$ clearance in above $= \cdot 01122''$.

$\therefore$ Total depth $= \frac{1}{14}'' + \frac{1}{14}'' + \cdot 01122'' = \cdot 15407''$.

Exercises LXXXI.

1. The circular pitch of a gear is $\cdot 3927''$. What is the diametral pitch?

2. The diametral pitch is 12. Find the circular pitch.

3. Find the thickness of tooth on a 14 pitch gear.

4. Find the total depth of tooth on a 14 pitch gear.

5. Find the thickness of tooth on a 16 pitch gear.

6. Find the total depth of tooth on a 16 pitch gear.

7. Find the outside diameter of a gear blank for a 60-toothed gear, 12 pitch.

8. Find the outside diameter of a gear blank for 20 teeth with a circular pitch of $\cdot 7854''$.

9. What is the number of teeth on a gear 6″ outside diameter, 12 pitch?

10. What is the number of teeth on a gear 8″ outside diameter, 6 pitch?

11. What is the pitch of a gear having 63 teeth and measuring $6 \cdot 5''$ outside diameter?

12. What is the distance between the centres of a pair of gears having 72 teeth and 54 teeth respectively, 9 pitch?

131. The Milling Machine. "Milling is the process of removing metal with rotary cutters. It is used extensively in machine shops to-day for forming parts of machinery, tools, etc., to required dimensions and shapes. A machine designed especially for this purpose was in existence as early as 1818, but little progress was made in the process until after the invention of the universal milling machine in 1861-62 by Joseph R. Brown of J. R. Brown and Sharpe."

132. Cutting Speed. In determining the cutting speed of a lathe we multiplied the circumference of the work in feet by the number of revolutions which the work made per minute. In the milling machine the diameter of the milling cutter corresponds to the diameter of the work in the lathe. The cutting speed of the milling cutter is therefore obtained by *multiplying the circumference of the cutter in feet by the number of revolutions which it makes per minute.*

Thus if a milling cutter 6″ in diameter makes 60 revolutions per min. the cutting speed = Circumference of cutter in ft. × Revolutions per min. = $\frac{22}{7} \times \frac{1}{2} \times \frac{60}{1} = 94+$ft. per min.

133. Feed. The feed on a milling machine is usually reckoned in inches per min. As in the case of the lathe only a general rule can be given. "In roughing, slow speed and heavy feed using a coarse-pitch cutter. In finishing, fast speed and light feed using a fine-pitch cutter." In Figure 94 following, a coarse-pitch cutter and a fine-pitch cutter are shown:

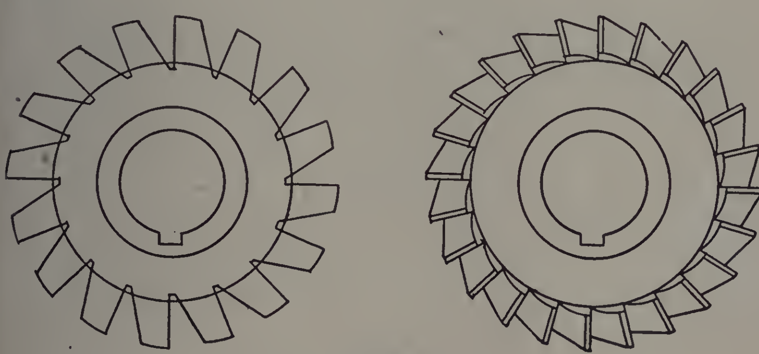


FIG. 94

R. H. Smith in "Advanced Machine Work" gives the following table for speeds and feeds:

Speeds.

With Carbon Steel Cutters. Cast iron—40 ft. per min. Machine steel—40 ft. per min. Annealed carbon steel—30 ft. per min. Brass or composition—80 ft. per min.

With High Speed Steel Cutters. Cast iron—80 ft. per min. Machine steel—80 ft. per min. Annealed carbon steel—60 ft. per min. Brass or composition—160 ft. per min.

Feeds.

Feeds for milling cutters are from ·002″ to ·250″ per cutter revolution, and depend on diameter of cutter, kind of material, width and depth of cut, size of work and whether light or heavy machine is used.

In order to calculate the feed it is necessary to know the lead of the feed screw and the number of revolutions per minute at which it is turning. Thus, if the lead of the feed screw is $\frac{1}{4}''$, and it is turning at the rate of 3 revolutions per min., then the feed $= \frac{1}{4}'' \times 3 = \frac{3}{4}''$ per min.

Exercises LXXXII.

1. A milling cutter 4″ in diameter is turning at a rate of 40 R.P.M. What is the cutting speed?

2. A milling cutter $3\frac{1}{2}''$ in diameter is cutting at a speed of $36\frac{2}{3}$ ft. per min. What is the R.P.M.?

3. A milling cutter turning at a rate of 56 R.P.M. has a cutting speed of 60 ft. per min. What is the diameter of the cutter?

4. A milling cutter 6″ in diameter is cutting at a speed of 66 ft. per min. What is the R.P.M.?

5. A milling cutter 2″ in diameter is running at 58 R.P.M. What is the cutting speed?

6. A milling cutter turning at a rate of 30 R.P.M. has a cutting speed of 40 ft. per min. What is the diameter of the cutter?

7. The feed screw in a milling machine is single threaded and has a pitch of $\frac{1}{8}$. If it is turned at a rate of 6 R.P.M., what is the feed?

8. The feed screw in a milling machine has a double thread with a pitch of $\frac{1}{4}$. If it is turned at a rate of 4 R.P.M., find the feed.

9. The feed screw on a milling machine has a lead of $\frac{1}{4}''$. How many R.P.M. does it make if the feed is $1\frac{1}{4}''$ per min.?

10. The feed screw on a milling machine has a feed of $1\frac{1}{2}''$ per min., and is being turned at 6 R.P.M. What is the lead of the screw?

134. **Indexing.** One of the purposes of the milling machine is to cut slots or grooves in a circular piece of work at regular intervals. It is, therefore, necessary that it should have an attachment for dividing the circumference of the work into equal parts. This attachment is called the dividing head. The process of dividing the work into equal parts is called **indexing.**

The methods of indexing may be classified as—**Rapid Indexing, Plain Indexing, Differential Indexing.**

Rapid Indexing permits of only a limited number of divisions of the circumference of the work, **plain indexing** extends the number of divisions, while **differential indexing** permits of a still wider range.

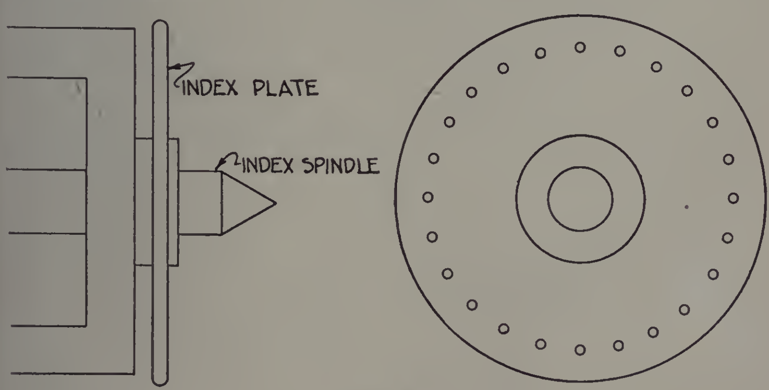


FIG. 95

135. **In Rapid Indexing** the index plate is fastened directly to the nose of the spindle as shown in Figure 95. This plate usually has 24 holes and is rotated by hand to any desired position, being held in place by a stop-pin.

Assume that in Figure 96 we have a round-headed bolt which is required to be milled so that the head becomes

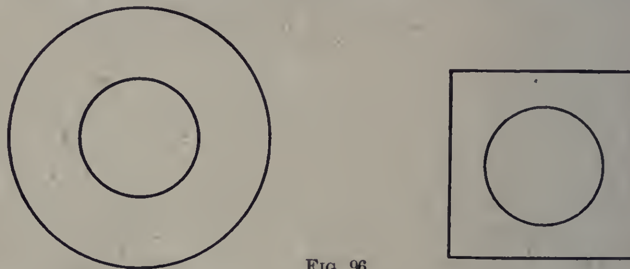


FIG. 96

square. In this case it is evident that the work must be turned through $\frac{1}{4}$ of a revolution when one side of the work has been milled and we are ready to mill the next. We would therefore turn the index plate $\frac{1}{4}$ of a revolution, i.e., 6 holes.

Using this kind of indexing we may obtain any number of divisions which will divide evenly into 24, as two, three, four, six, etc.

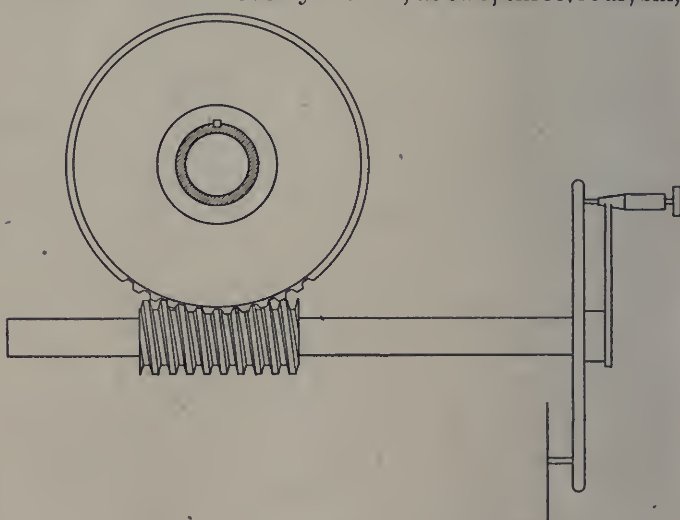


FIG. 97

136. **Plain Indexing.** In the figure above the index spindle is shown with a worm and worm-wheel mechanism, the worm

being attached to the crank turned when indexing. The worm-wheel is keyed to the index spindle to which the work is attached.

The principle of **Plain Indexing** may be seen from the diagram above. In the majority of index heads the worm is single threaded and the worm-wheel has 40 teeth. If, therefore, the index crank is turned one complete revolution, the worm will make one revolution, which moves the worm-wheel one tooth or $\frac{1}{40}$ of its circumference. If, therefore, we want to turn the worm-wheel, and hence the spindle to which it is attached, one full revolution, we must turn the index crank 40 revolutions. If we want to turn the spindle $\frac{1}{5}$ of a revolution, we will turn the index crank 8 revolutions, and so on.

If now we assume that it is required to cut seven flutes equally spaced in a reamer, we would first insert the stop-pin

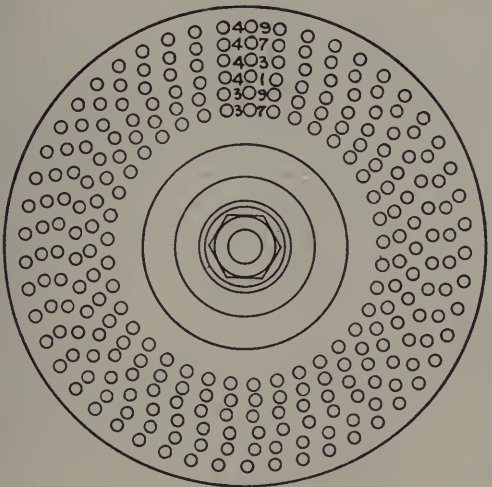


FIG. 98

(see diagram) and then estimate the required number of revolutions of the index crank. In order to index for each flute the index crank must be turned $\frac{40}{7}=5\frac{5}{7}$ revolutions.

To accurately fix this $\frac{5}{7}$ of a revolution the index plate is made with a series of holes arranged in concentric circles. A sample plate is shown (Fig. 98).

It is required to choose some circle where the total number of holes can be divided by 7. In the plate shown the outside row having 49 holes will suffice.

We will, therefore, turn the index crank five complete revolutions, afterwards turn to the 35th hole in the outside circle of holes and insert crank-pin.

Most milling machines are furnished with three index plates, each having six index circles. The following numbers of holes in the index circles of the three plates are used:

15	16	17	18	19	20
21	23	27	29	31	33
37	39	41	43	47	49

Exercises LXXXIII.

1. By the rapid method show how you would index for milling a hexagonal head on a round bolt, if the index plate has 24 holes.

2. A piece of work is to have eight sides regularly spaced. How would you index by the rapid method, if the index plate has 24 holes?

3. What diameter must a piece be to mill square $1\frac{1}{4}''$ across the flats?

4. What diameter must a piece be to mill hexagonal $1\frac{1}{4}''$ across the flats?

5. Twelve flutes are to be milled in a tap. How would you index, using plain indexing, assuming that 40 turns of the index crank are required for one turn of the spindle?

6. It is required to cut nine regularly spaced flutes in a reamer. How would you index, assuming a ratio of 40 to 1 between index crank and spindle?

7. Assuming a ratio of 40 to 1 between index crank and spindle, find the number of complete turns, the proper plate, and the number of holes for indexing 15 divisions.

8. If the ratio between index crank and spindle be 60 to 1, what indexing would be used for 21 divisions?

9. If the ratio between index crank and spindle be 40 to 1, what indexing would be used to cut 84 teeth in a spur gear?

10. If the ratio between index crank and spindle be 40 to 1, what indexing would be used to cut 105 teeth in a spur gear?

11. It is required to cut 85 teeth in a spur gear. How would you index assuming a ratio of 40 to 1 between index crank and spindle?

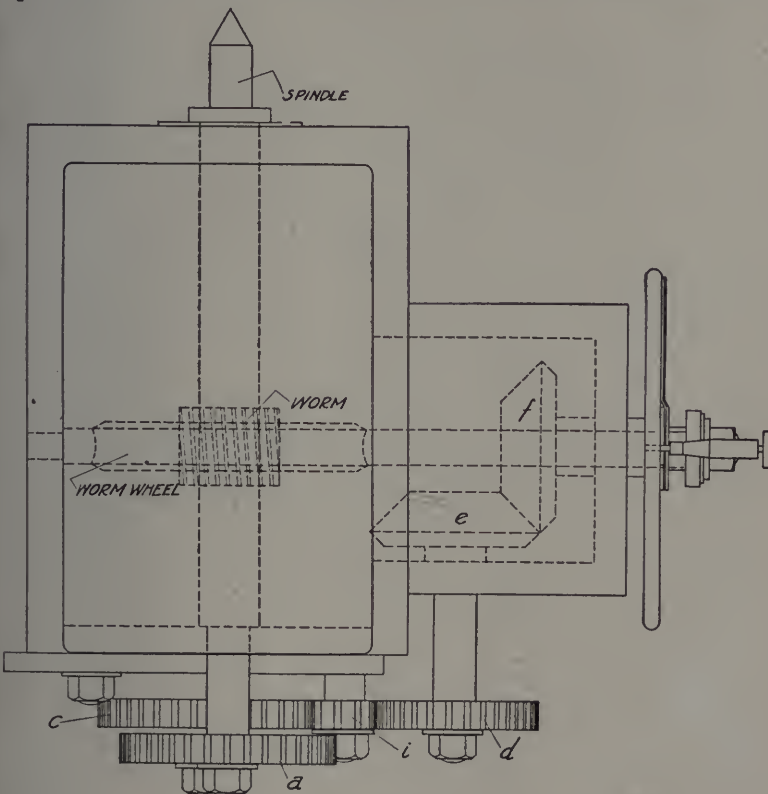


FIG. 99

137. **Differential Indexing.** Assume that we require to cut 101 teeth in a spur gear. If we proceed as in plain indexing we would conclude that the index crank must turn $\frac{40}{101}$ revolutions for milling each slot. As this fraction will not reduce

to lower terms it would be necessary to have a plate with a circle containing 101 holes. As such a plate is not available for all machines a different mechanism is necessary. The method employed in such cases is called **differential indexing.**

The diagrams 99 and 99*a* will help to explain the method.

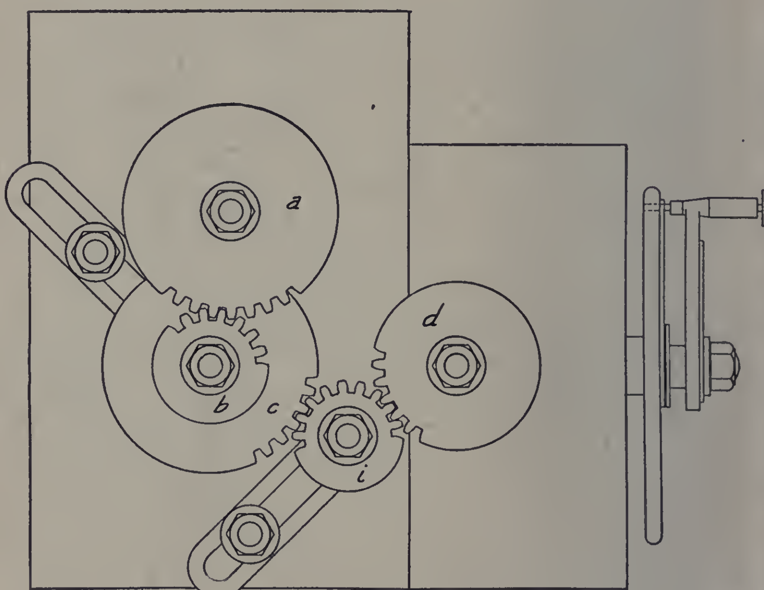


FIG. 99*a*

The index crank and index plate are shown connected with the worm and worm-wheel as in plain indexing. On the outer end of the spindle the gear **a** is fastened. To an adjustable bracket are keyed the two gears **b** and **c**. The gear **b** meshes with **a** and the gear **c** with an idler **i**, which in turn meshes with the gear **d**. Keyed to the same shaft with **d** is a bevel gear **e**, which meshes with another bevel gear **f**.

If the index plate in the accompanying figure be held stationary then for one complete turn of the spindle it will

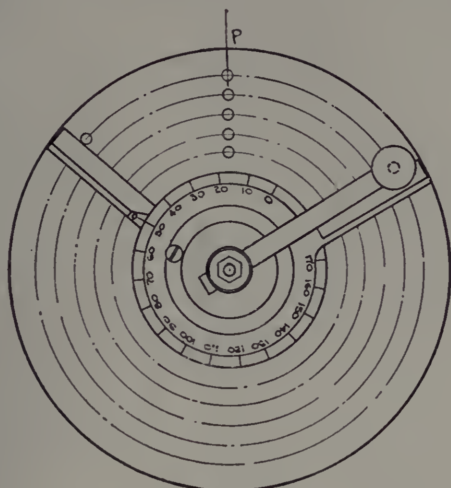


FIG. 100

be necessary for the crank-pin to pass the point marked **P** 40 times. Suppose, however, that the index plate is left free to rotate and gears in the ratio of 1 to 1 be placed on the spindle and on the worm, then when the spindle is making one revolution the index plate makes one revolution. If the gears be so arranged that the index plate turns in the same direction as the spindle, it would only be necessary for the crank-pin to pass **P** 39 times in order to turn the spindle once. If however the index plate turns in the opposite direction to the spindle, it is evident that the crank-pin must pass **P** 41 times in order to turn the spindle once. *This principle of having the index plate connected with gearing so that the crank makes other than 40 turns for one complete turn of the spindle is the special feature of differential indexing.*

We will now return to the difficulty of cutting 101 teeth in a spur gear. Since our object is to turn the spindle $\frac{1}{101}$ of a revolution, and 101 is not a multiple of any of the numbers on the index plates, we will select a number on either side of 101 which is a multiple of some of the numbers. It is readily seen that 20 is a multiple of 100 and also that $\frac{8}{20} \times 100 = 40$. If, therefore, we make 100 moves of 8 holes each on the 20 hole circle we will turn the worm 40 revolutions.

If 101 such moves be made we would have $\frac{8}{20} \times 101 = 40\frac{2}{5}$ rev. of worm. This is $\frac{2}{5}$ of a revolution **too many**, which may be offset by moving the index plate in the opposite direction to the spindle by suitable gears. Splitting this ratio into two parts we have $\frac{2}{5} = \frac{2}{1} \times \frac{1}{5}$.

Since we cannot multiply these fractions by any numbers which will give gears in stock, we write—

$$\frac{2}{5} = \frac{2}{3} \times \frac{3}{5}$$

$$= \left(\frac{2}{3} \times \frac{24}{24}\right) \times \left(\frac{3}{5} \times \frac{8}{8}\right)$$

$$= \frac{48}{72} \times \frac{24}{40}.$$

The 48 and 24 will be placed on the drivers, i.e., **a** and **c**, the 40 and 70 on the driven, i.e., **b** and **d**. It will be necessary to place one idler in the train, as in diagram, in order that the index plate may turn in the opposite direction to the spindle.

Example 2:

Required to cut 83 teeth in a spur gear.

Our object here is to turn the spindle $\frac{1}{83}$ of a revolution. Since 83 is not a multiple of any of the numbers on the index plates, we will select a number on either side of 83 which is a multiple of some of the numbers. Thus we observe that 16 is a multiple of 80 and also that $\frac{8}{16} \times 80 = 40$. If, therefore, we make 80 moves of 8 holes each on the 16 hole circle, we will turn the worm 40 revolutions.

If 83 such moves be made we would have $\frac{8}{16} \times 83 = 41\frac{1}{2}$ rev. of the worm. As this is $1\frac{1}{2}$ revolutions **too many**, the index plate must move opposite to the spindle.

As gears in the ratio of 3 to 2 are in stock, it is not necessary to split the ratio $\frac{3}{2}$, but write $\frac{3}{2}=\frac{48}{32}$.

Here the compound would be removed and the gears on worm and spindle connected by means of two idlers. The 48 gear would go on the driver, i.e., on **a** and the 32 gear on the driven, i.e., on **d**.

Example 3:

Required to cut 137 teeth in a spur gear.

By trying different combinations as in the preceding cases we find that $\frac{6}{21}\times 140=40$.

If, therefore, we make 140 moves of 6 holes each on the 21 hole circle, we will turn the worm 40 revolutions.

If 137 such moves be made we would have $\frac{6}{21}\times 137=39\frac{3}{21}$ rev. of worm.

This lacks $\frac{18}{21}$ of a revolution, which may be offset by moving the index plate in the same direction as the spindle by suitable gears. As in the preceding case it is not necessary to split the ratio but use gears in the ratio of 6 to 7, i.e., 24 and 28. The 24 will go on the driver, i.e., on **a**, and the 28 on the driven i.e., on **d**. One idler would be inserted to connect the worm and spindle.

Note.—When a simple gearing is used the number of idlers depends on whether the index plate is to turn in the same or opposite direction to that of the spindle. If in the same direction one idler will be inserted, if in the opposite direction two idlers.

To obviate the necessity of working out these gears, tables are available giving the necessary gears for all required divisions of the work.

138. **Cutting Spirals.** If the gears that drive the shaft carrying the worm gear be connected with the feed screw, then as the table advances the spindle will rotate. This will produce a spiral cut in the work, such as may be seen in a spiral reamer or a twist drill.

The gears for this purpose are shown in the following diagram:

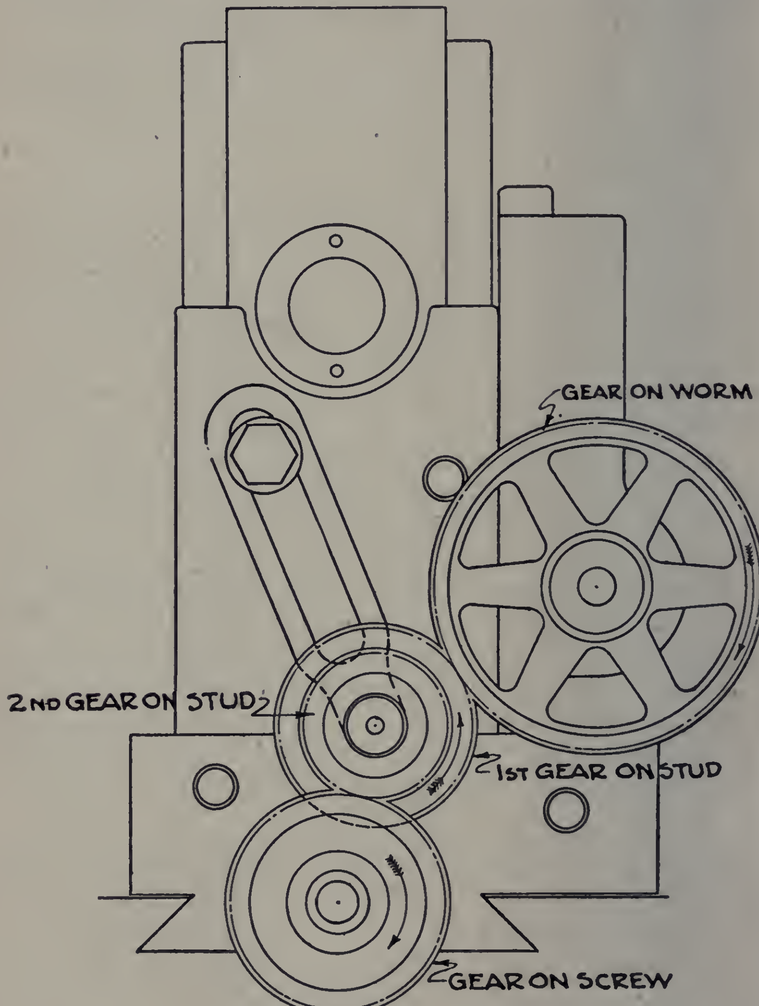


FIG. 101

The four change gears are indicated in the above figure. The screw gear and the first stud gear are the drivers, the others being the driven. By using different combinations of change gears the ratio of the lengthwise motion of the table to the rotary motion of the spindle can be varied.

139. **Lead of the Machine.** If the feed screw of the table has 4 threads to the inch, and 40 revolutions of the worm are necessary for one revolution of the spindle, then, if change gears of equal diameters are used, the work will make one revolution while the table advances 10 in. **The lead of the machine is therefore 10 in.**

Some machines have feed screws with other than 4 threads to the inch, but the same principle as the above will give the lead of the machine.

140. **Calculating Change Gears.** In calculating gears for screw cutting we had the following proportion:

$$\frac{\text{Product of teeth in driven}}{\text{Product of teeth in drivers}} = \frac{\text{Revolutions of spindle}}{\text{Revolutions of feed screw}}.$$

In a similar way we will now have:

$$\frac{\textit{Product of teeth in driven}}{\textit{Product of teeth in drivers}} = \frac{\textit{Lead of spiral}}{\textit{Lead of feed screw}}.$$

Example 1:

Find the change gears for cutting a spiral with a lead of 20″, when the lead of the machine is 10″.

Here, $\frac{\text{Lead of spiral}}{\text{Lead of machine}} = \frac{20}{10} = \frac{2}{1}.$

Since four gears are used we split the ratio thus:

$$\frac{2}{1} = \frac{2}{1} \times \frac{1}{1} = \left(\frac{2}{1} \times \frac{32}{32}\right) \times \left(\frac{1}{1} \times \frac{24}{24}\right) = \frac{64}{32} \times \frac{24}{24}.$$

We will, therefore, place 64 and 24 on the worm and 2nd stud respectively, and 24 and 32 on the 1st stud and feed screw respectively.

Example 2:

Find the change gears for cutting a spiral with a lead of 8·333″, when the lead of the machine is 10″.

$$\frac{\text{Lead of spiral}}{\text{Lead of machine}}=\frac{8\frac{1}{3}}{10}=\frac{5}{6}.$$

Splitting the ratio $\frac{5}{6}=\frac{5}{2}\times\frac{1}{3}.$

$$=\left(\frac{5\times 20}{2\times 20}\right)\times\left(\frac{1\times 24}{3\times 24}\right)$$

$$=\frac{100}{40}\times\frac{24}{72}.$$

We will, therefore, place 100 and 24 on the worm and 2nd stud respectively, and 72 and 40 on the feed screw and 1st stud respectively.

141. Position of Table in Cutting Spirals. In order that the cutter may have clearance in cutting the groove, it is necessary that the table of the machine should be set at an angle. This angle depends on two things:—**The lead of the spiral and the diameter of the work to be milled.** This angle may be determined either **graphically** or by **calculation.**

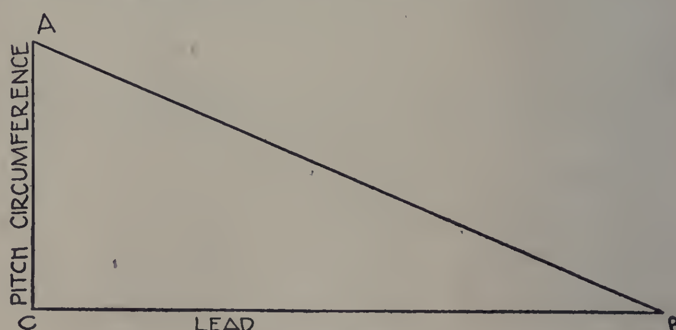


Fig. 102

In the figure ABC is a right-angled triangle in which BC is equal to the lead of the spiral and AC the circumference of the work. The angle ABC will then be the required angle.

Finding the angle by calculation is however a more accurate method, thus:

$$\text{Tangent of required angle} = \frac{\text{Circumference of work}}{\text{Lead}}.$$

From trigonometrical tables this angle can readily be found.

Example:

Find the angle at which the table must be set in milling a twist drill 1″ in diameter, lead 8·68″.

If θ be the required angle, then $\tan \theta = \dfrac{3 \cdot 1416 \times 1}{8 \cdot 68} = \cdot 36193$

$$\therefore \theta = 19° \ 54' = 20° \text{ (approx.)}$$

Tables are available giving the proper gears and angle of the table for all necessary cases.

Exercises LXXXIV.

1. If the ratio between the worm and spindle of a dividing head is 40 to 1, find the differential indexing for the following divisions:—83, 99, 111, 139, 159, 161, 171, 238, 269, 351. Verify from table.

2. What is the lead of a milling machine if the feed screw has a lead of $\frac{1}{4}$″ and the ratio of worm to spindle is 60 to 1?

3. If the lead of a milling machine is 10″, calculate the change gears for cutting spirals with the following leads:—·9″, 1·067″, 1·200″, 1·667″, 2·200″, 3·056″, 4·000″, 5·500″, 6·482″, 8″. Verify from table.

4. The following change gears were used in cutting a spiral, on worm 72, on 1st stud 24, on 2nd stud 24, on screw gear 48. If the lead of the machine was 10″, what was the lead of the spiral?

5. The following change gears were used in cutting a spiral, on worm 64, on 1st stud 24, on 2nd stud 32, on screw gear 40. If the lead of the machine was 10″, what was the lead of the spiral?

6. The following change gears were used in cutting a spiral, on worm 86, on 1st stud 24, on 2nd stud 24, on screw gear 40. If the lead of the machine was 10″, what was the lead of the spiral?

7. A spiral with a lead of 7·92″ is to be cut on a gear blank with a pitch diameter of 3″; find the angle for setting the table.

8. A spiral with a lead of 9·34″ is to be cut on a twist drill with a diameter of $1\frac{1}{4}$″; find the angle for setting the table.

9. In milling a twist drill the table is set at an angle of 15°, and the lead of the spiral is 11·724″. Find the diameter of the drill.

10. In milling a twist drill the table is set at an angle of 17° 30′, and the diameter of the drill is $1\frac{1}{2}$″. Find the lead of the spiral.

Review Exercises LXXXV

1. What is the lead on a double-threaded screw of $\frac{1}{8}$ pitch?

2. A screw with a triple thread has a lead of 1″. What is the pitch?

3. How many revolutions must be made with a double-threaded screw, with a pitch of $\frac{1}{10}$, so that it may advance 2″?

4. A sharp "V" thread with a pitch of $\frac{1}{12}$, makes 6 turns to the inch. How is it threaded? What is the double depth of the thread?

5. If the double depth of a sharp "V" thread is ·1924″, what is the number of threads per inch?

6. A $1\frac{5}{8}$″ bolt with a sharp "V" thread has a diameter at the root of 1·2784″. What is the depth of the thread? What is the pitch?

7. How long will be necessary to take a cut over a shaft 3′ long and 2″ in diameter with a feed of 18 and a speed of 36 ft. per min?

8. A drill cuts $\frac{7}{8}$″ into a piece of work in 10 min. If it makes 40 revolutions per min., what is the feed of the drill?

9. A piece of work 6″ long is to have end diameters of ·7432″ and ·6182″; find the amount of taper and also the angle of taper.

10. A screw with a U.S. Std. thread has 16 threads to the inch. It has a diameter at the root of ·2936″. What is the diameter of the screw?

11. A 6″ screw with a Whitworth thread has a pitch of $\frac{2}{5}$. What is the root diameter of the thread?

12. The width of the flat at the top of an Acme 29° thread is ·0371″. What is the pitch?

13. A lathe has an 84-toothed gear on the lead screw and the pitch of the lead screw is $\frac{1}{6}$. What gear on the stud will cut 18 threads per inch, simple gearing?

14. A simple geared lathe having a 96-toothed gear on the lead screw and a 32 on the change stud cuts 24 threads per inch. What is the pitch of the lead screw?

15. The lead screw on a lathe is $\frac{1}{4}$ pitch, the screw to be cut $\frac{1}{12}$ pitch, and the change stud gear has 24 teeth. If the lathe be simple geared what gear must be placed on the lead screw?

16. If the lathe in the preceding question had reduction gears in the head-stock in the ratio of 1 to 2, what gear would be necessary on the lead screw?

17. In a lathe with a lead screw of $\frac{1}{6}$ pitch a 40-toothed gear is placed on the change stud and a 75 on the lead screw. If the lathe be simple geared how many threads per inch will be cut on the screw?

18. How many threads per inch will be cut when the lead screw gear is 100, the change stud 60, the outside compound 24, the inside compound 48, and the pitch of the lead screw $\frac{1}{6}$?

19. A lathe with a 60-toothed gear on the change stud and a 40 on the lead screw, has 6 threads per inch on the lead screw. What compound gears are used in cutting 2 threads per inch?

20. What should be the outside diameter of a gear blank for a gear of 24 teeth and a pitch diameter of 4?

21. Two gears in mesh have 66 teeth and 88 teeth respectively. If they are 14 pitch gears, what is the distance between their centres?

22. A milling cutter turning at the rate of 40 R.P.M. has a cutting speed of 40 ft. per min. What is the diameter of the cutter?

23. What is the lead of a milling machine if the feed screw has a lead of $\frac{1}{6}$ and the ratio of worm to spindle is 40 to 1?

24. Find the change gears for cutting a spiral with a lead of 1·64″, when the lead of the machine is 10″.

25. A spiral with a lead of 7·5″ is to be cut on a twist drill with a diameter of $1\frac{1}{2}$″; find the angle for setting the table.

CHAPTER XV.

LOGARITHMS.

142. If we wish to multiply 100 by 1000 we may do so in either of two ways:

(1) $100 \times 1000 = 100000$

(2) $100 = 10^2$ and $1000 = 10^3$

$\therefore\ 100 \times 1000 = 10^2 \times 10^3 = 10^5 = 100000.$

In the second method we observe that the product is obtained by adding the exponents of the powers of 10 which equal 100 and 1000.

If then we had numbers expressed as powers of 10, it would be possible to multiply them together by adding their exponents.

Thus, if we wished to multiply 23 by 432 we might do so by addition of the exponents of the powers of 10 which equal 23 and 432.

We are here met by two difficulties.

(1) What powers of 10 equal 23 and 432?

(2) What number is represented by 10 when raised to the sum of these two powers?

Let us consider the following set of numbers:

(1) 10, (2) 25, (3) 100, (4) 365, (5) 1000, (6) 7628, (7) 10000.

We know that in (1) $10 = 10^1$, and that in (3) $100 = 10^2$.

Now in (2) 25 is greater than 10, or 10^1, and less than 100, or 10^2, therefore $25 = 10^{1 + \text{a decimal.}}$

Again in (4) 365 is greater than 100, or 10^2, and less than 1000, or 10^3, therefore $365 = 10^{2 + \text{a decimal.}}$

Further in (6) 7628 is greater than 1000, or 10^3, and less than 10000, or 10^4, therefore $7628 = 10^{3 + \text{a decimal.}}$

Tables have been worked out giving the decimal parts of the powers of 10 in the above.

Thus, from the tables $25 = 10^{1.39794.}$

$365 = 10^{2.56229.}$

$7628 = 10^{3.88241.}$

This exponent of the power to which we must raise 10 to give the number is called the logarithm of the number.

Thus, logarithm of $25 = 1 \cdot 39794.$

logarithm of $365 = 2 \cdot 56229.$

logarithm of $7628 = 3 \cdot 88241.$

In this system—called the Briggs' System—the **base** is 10, and all numbers are considered as powers of 10.

The contraction "log" is used instead of logarithm.

143. Characteristic and Mantissa.

In $25 = 10^{1 \cdot 39794.}$

the 1 in the exponent is called the **Characteristic** and the ·39794 the **Mantissa. The Mantissa is always positive.**

Characteristic written at sight.

In the above set of numbers we observe that 25 which is greater than 10 and less than 100, has 1 for its characteristic; that 365 which is greater than 100 and less than 1000 has 2 for its characteristic; that 7628 which is greater than 1000 and less than 10,000, has 3 for its characteristic. We, therefore, infer that *the characteristic of the logarithm of any number greater than 1 is one less than the number of integral figures in the number.*

144. How to find the Logarithm of a number from the Tables.

Find the log of 36.

As previously explained we at once write down the characteristic, 1.

To get the decimal part we go down the left-hand column to 36, then along the horizontal row to the right, and under the vertical column headed 0, we read ·55630,

$$\therefore \log 36 = 1 \cdot 55630.$$

Find the log of 365.

The characteristic here is 2.

To get the decimal part we go down the left-hand column to 36 as before, then along the horizontal row to the right, and under the vertical column headed 5, we read ·56229.

$$\therefore \log 365 = 2 \cdot 56229.$$

Find the log of 3658.

The characteristic here is 3.

To get the decimal part we proceed as in the last case, giving 3·56229. To make the adjustment for the 8, we follow the same horizontal row out to the mean differences. In the vertical column headed 8, we read 95. This we add to 3·56229, giving the log of 3658 = 3·56229

$$\begin{array}{r} 3 \cdot 56229 \\ 95 \\ \hline 3 \cdot 56324 \end{array}$$

Find the log of 36587.

The characteristic here is 4.

To get the decimal part we proceed as in the last example, giving 4·56324. To make the adjustment for the 7, we observe that in the same horizontal row, under 7 in mean differences, we have 83. Since the 7 is in the fifth place, it has only one-tenth the value that it would have in the fourth place, therefore we move the 83 one place to the right before adding, thus 4·56324

$$\begin{array}{r} 4 \cdot 56324 \\ 83 \\ \hline 4 \cdot 563323 \end{array}$$

$$\therefore \log 36587 = 4 \cdot 563323 = 4 \cdot 56332 \text{ to 5 places.}$$

145. Position of Decimal Point. Since the division of a number by 10 or 100 is made by moving the decimal place to the left, the position of the decimal affects the characteristic only.

$$\begin{aligned} \text{Thus,}\quad \log 3658 &= 3\cdot 56324. \\ \log 365\cdot 8 &= 2\cdot 56324. \\ \log 36\cdot 58 &= 1\cdot 56324. \\ \log 3\cdot 658 &= 0\cdot 56324. \end{aligned}$$

146. Knowing the Logarithm of a Number to find the Number.

Tables, called antilogarithms, have been worked out which enable us to find a number if we know its logarithm.

If $\log x = 2\cdot 34563$, find x.

Since only mantissas are recorded in the tables, the characteristic 2 has no bearing on what we look up, but only serves to fix the decimal place in the result.

Proceeding with $\cdot 34563$ in antilogarithms, just as outlined in finding a logarithm, we have 22131

$$\begin{array}{r} 22131 \\ 31 \\ 15 \\ \hline 221635 \end{array}$$

Since the characteristic of the logarithm of the required number is 2, it must have three figures in the integral part,

$\therefore x = 221\cdot 635 = 221\cdot 64$ to 5 figures.

Returning to the difficulty raised when we wished to multiply 23 by 432 we have:

$$23 = 10^{1.36173.}$$

$$432 = 10^{2.63548.}$$

$\therefore 23 \times 432 = 10^{1.36173+2.63548} = 10^{3.99721} = 9936\cdot 03 = 9936$ to 5 figures.

In practice the base 10 is not written down, but only the exponents.

Example:

Find the value of $45 \cdot 236 \times 31 \cdot 341$.

$\log 45 \cdot 236 = 1 \cdot 65514$	$\log 31 \cdot 341 = 1 \cdot 49554$
29	55
57	14
$1 \cdot 655487$	$1 \cdot 496104$

$$\begin{aligned} \text{Sum of logs} &= 1 \cdot 655487 \\ & 1 \cdot 496104 \\ \hline & 3 \cdot 151591 = 3 \cdot 15159 \text{ to 5 places.} \end{aligned}$$

$$\begin{aligned} \text{Antilog } 3 \cdot 15159 = \ & 14158 \\ & 16 \\ & 30 \\ \hline & 1417 \cdot 70 \end{aligned}$$

$\therefore 45 \cdot 236 \times 31 \cdot 341 = 1417 \cdot 7$ to 5 figures.

Exercises LXXXVI.

Employ logarithms to find the value of:

1. 53×82.
2. $10 \cdot 64 \times 150$.
3. $483 \cdot 26 \times 108$.
4. $381 \cdot 56 \times 17 \cdot 928$.
5. $493 \cdot 75 \times 4 \cdot 73$.
6. $7 \cdot 53 \times 20 \cdot 08 \times 14 \cdot 93$.
7. $146 \cdot 32 \times 78 \cdot 49 \times 10 \cdot 09$.
8. $9 \cdot 36 \times 4 \cdot 592 \times 3 \cdot 61 \times 1 \cdot 08$.
9. $8 \cdot 99 \times 61 \cdot 3 \times 7 \cdot 6297 \times 3 \cdot 92$
10. $5 \cdot 037 \times 236 \cdot 84 \times 1 \cdot 009$.

147. Logarithms Applied to Division.

We have learned by the foregoing that *to multiply two numbers together, we add their logarithms and find the antilogarithm of the result.*

Since division is the reverse of multiplication, we could without further detail infer that, *to divide one number by another, we subtract their logarithms and find the antilogarithm of the result.*

Thus, divide 365 by 73.

log of 365 = 2·56229

log of 73 = 1·86332

difference = ·69897

antilog of ·69897 = 49888

103

80

4·99990 ∴ 365 ÷ 73 = 4·9999.

Example:

Find the value of $\frac{43 \cdot 21 \times 148 \cdot 92}{149 \cdot 7 \times 37 \cdot 42}$

log 43·21 = 1·63548
10
1·63558

log 148·92 = 2·17026
265
59
2·172969

Sum of logs of numbers in numerator = 3·808549 (*a*).

log 149·7 = 2·17319
206
2·17525

log 37·42 = 1·57287
23
1·57310

Sum of logs of numbers in denominator = 3·74835 (*b*).

(*a*) − (*b*) = 3·808549
3·74835
·060199 = ·06020 to 5 places.

Antilog ·06020 = 11482
5
1·1487

∴ result = 1·1487 to 5 figures.

An abbreviated arrangement of the work is as follows:

$$
\begin{array}{rr}
\log\ 43\cdot21 = 1\cdot63548 & \\
10 & \\
\log 148\cdot92 = 2\cdot17026 & \\
265 & \\
59 & \\
\hline
3\cdot808549 & \\
\text{subtract } 3\cdot74835 & \\
\hline
0\cdot060199 &
\end{array}
\qquad
\begin{array}{rr}
\log 149\cdot7 = 2\cdot17319 \\
206 \\
\log 37\cdot42 = 1\cdot57287 \\
23 \\
\hline
3\cdot74835
\end{array}
$$

$$
\text{anti } 0\cdot06020 = \dot{1}1482 \qquad
\begin{array}{r}
5 \\
\hline
1\cdot1487
\end{array}
$$

148. Logarithm of a Number Less than Unity.

We have the following:

$$\cdot1 = \frac{1}{10^1} = 10^{-1}$$

$$\cdot01 = \frac{1}{100} = \frac{1}{10^2} = 10^{-2}$$

$$\cdot001 = \frac{1}{1000} = \frac{1}{10^3} = 10^{-3}$$

$$\cdot0001 = \frac{1}{10000} = \frac{1}{10^4} = 10^{-4}$$

By our definition of logarithms we have from the above

$$\log\ \cdot1 = -1.$$
$$\log\ \cdot01 = -2.$$
$$\log\ \cdot001 = -3.$$
$$\log\ \cdot0001 = -4.$$

Consider the following set of numbers:

(1) ·1. (3) ·01. (5) ·001. (7) ·0001.
(2) ·06. (4) ·008. (6) ·0007.

We know from the above that in (1) $\cdot1 = 10^{-1}$ and that in (3) $\cdot01 = 10^{-2}$.

Now in (2) $\cdot 06$ is greater than $\cdot 01$, or 10^{-2}, and less than $\cdot 1$ or 10^{-1}, therefore $\cdot 06 = 10^{-2+\text{a decimal.}}$

Again in (4) $\cdot 008$ is greater than $\cdot 001$, or 10^{-3}, and less than $\cdot 01$, or 10^{-2}, therefore $\cdot 008 = 10^{-3+\text{a decimal.}}$

Further in (6) $\cdot 0007$ is greater than $\cdot 0001$, or 10^{-4} and less than $\cdot 001$, or 10^{-3}, therefore $\cdot 0007 = 10^{-4+\text{a decimal.}}$

From observing the above results we infer:

(1) *That the characteristic of the logarithm of a number less than unity is negative.*

(2) *That the characteristic of the logarithm of a number less than unity is one more than the number of zeros between the decimal point and the first significant figure.*

149. **How to write the Logarithm of a Number less than Unity.**

Find log $\cdot 067$.

By the above the characteristic is -2, and from the tables the mantissa is $\cdot 82607$.

$\therefore \log \cdot 067 = -2 + \cdot 82607$.

Since the mantissa is always positive we could not correctly write this as $-2\cdot 82607$, for that would imply that the whole quantity $2\cdot 82607$ is negative. To avoid this difficulty the minus sign is placed immediately above the characteristic.

$\therefore \log \cdot 067 = \bar{2}\cdot 82607$, the characteristic being read "bar" 2.

Example 1: Divide $\cdot 0432$ by $82\cdot 624$.

$$\log \cdot 0432 = \bar{2}\cdot 63548$$

$$\log 82\cdot 624 = 1\cdot 917111$$

$$\text{Diff. of logs} = \bar{2}\cdot 63548$$

$$1\cdot 917111$$

$$\bar{4}\cdot 718369 = \bar{4}\cdot 71837 \text{ to 5 places.}$$

Here note that we are subtracting the greater quantity from the less, therefore in obtaining the $\bar{4}$ we use the law for algebraic subtraction, i.e., change the sign of the lower line and add.

Antilog of $\bar{4}\cdot 71837 = \cdot 000522844 = \cdot 00052284$ approx.

Example 2:

Find the value of $\frac{36 \cdot 215 \times \cdot 0724}{\cdot 0027 \times 936}$.

$\log 36 \cdot 215 = 1 \cdot 55889$

$\log \cdot 0724 = \bar{2} \cdot 85974$

$\cdot 41863$ (*a*)

$\log \cdot 0027 = \bar{3} \cdot 43136$

$\log 936 = 2 \cdot 97128$

$\cdot 40264$ (*b*)

$(a) - (b) = \cdot 41863$

$\cdot 40264$

$\cdot 01599$

Antilog of $\cdot 01599 = 1 \cdot 03741 = 1 \cdot 0374$ approx.

Exercises LXXXVII.

Employ logarithms to find the value of :

1. $43 \cdot 752 \div 8 \cdot 75$.
2. $\cdot 0752 \div \cdot 648$.
3. $\frac{26 \cdot 584 \times \cdot 075}{8 \cdot 359}$.
4. $408 \cdot 039 \div 3423 \cdot 08$.
5. $\frac{472 \cdot 86 \times 15 \cdot 8 \times 10^{-3}}{\cdot 0728 \times \cdot 63 \times 10^{2}}$.
6. $\frac{728 \cdot 43 \times \cdot 00625 \times 19}{\cdot 0946 \times 1 \cdot 0009}$.

150. Logarithm of a Power.

From tables $\log 2 = \cdot 30103$.

$$\therefore\ 2 = 10^{\cdot 30103}.$$

$$\therefore\ (2)^2 = (10^{\cdot 30103})^2 = 10^{\cdot 60206}.$$

In the above we observe that the log of 2^2 is twice the log of 2, therefore to find the value of 2^2, we would find the log of 2, double it and find the antilog of the result.

Thus, $\log 2 = \cdot 30103$,

twice $\log 2 = \cdot 60206$.

Antilog $\cdot 60206 = 3 \cdot 99996 = 4$ (nearly).

Again, $\log 3 = \cdot 47712$.

$$\therefore\ 3 = 10^{\cdot 47712}.$$

$$\therefore\ 3^4 = (10^{\cdot 47712})^4 = 10^{1.90848}.$$

Here we observe that the log of 3^4 is four times the log of 3, therefore to find the value of 3^4, we would find log 3, take four times it, and find the antilog of the result.

Thus, log 3 $= \cdot 47712$,

four times log $3 = 1 \cdot 90848$.

Antilog $1 \cdot 90848 = 80 \cdot 9988 = 81$ (nearly).

Further log $9 = \cdot 95424$.

$$\therefore 9 = 10^{\cdot 95424.}$$

$$\therefore 9^{\frac{1}{2}} = (10^{\cdot 95424})^{\frac{1}{2}} = 10^{\cdot 47712.}$$

Here we observe that log $9^{\frac{1}{2}}$ is one-half the log 9, therefore to find the value of $9^{\frac{1}{2}}$, we would find log 9, take one-half of it, and find the antilog of the result.

Thus, log $9 = \cdot 95424$.

$\frac{1}{2}$ log $9 = \cdot 47712$.

Antilog $\cdot 47712 = 3 \cdot 00004 = 3$ (nearly).

From these examples we infer:—*To obtain any power of a number multiply its logarithm by the exponent of the power and find the antilogarithm of the result.*

Example 1:

Find value of $(\cdot 026)^3$.

Let $x = (\cdot 026)^3$.

Then by above log $x = 3$ log $\cdot 026$.

$$= 3\ (\bar{2} \cdot 41497).$$

Here we have to multiply a logarithm by 3, the mantissa being positive and the characteristic negative. We should first multiply them separately, giving $\bar{6} + 1 \cdot 24491$, and afterwards combine giving $\bar{5} \cdot 24491$.

$\therefore$ log $x = \bar{5} \cdot 24491$.

$\therefore \quad x = \cdot 0000175754 = \cdot 000017575$ approx.

Example 2:

Find value of $(\cdot 026)^{\frac{1}{3}}$.

Let $\log x = (\cdot 026)^{\frac{1}{3}}$.

then $\log x = \frac{1}{3} \log \cdot 026$.

$= \frac{1}{3} (\bar{2} \cdot 41497)$.

The same difficulty is presented here as in the preceding example, only we have to divide by 3 instead of multiplying. We, therefore, write $\frac{1}{3}(\bar{2} \cdot 41497)$ as $\frac{1}{3}(\bar{3}+1 \cdot 41497)$, **the object being to make the negative part so that the 3 will divide it evenly.**

$\frac{1}{3}(\bar{3}+1 \cdot 41497) = \bar{1} \cdot 471656$.

$\therefore \log x = \bar{1} \cdot 471656 = \bar{1} \cdot 47166$ to 5 places.

$\therefore \quad x = \cdot 296251 = \cdot 29625$ approx.

Example 3:

Find the value of $\dfrac{1}{(1 \cdot 05)^6}$.

Let $x = \dfrac{1}{(1 \cdot 05)^6}$.

then $\log x = \log 1 - 6 \log 1 \cdot 05$.

$= 0 - 6(\cdot 02119)$.

$= - \cdot 12714$.

Since the mantissa must always be positive we must now change $- \cdot 12714$ to a number having a positive mantissa.

Thus, $- \cdot 12714 = \bar{1} + 1 - \cdot 12714$.

$= \bar{1} \cdot 87286$.

$\therefore \log x \quad = \bar{1} \cdot 87286$.

$\therefore \quad x \quad = \cdot 746214 = \cdot 74621$ approx.

151. Solution of an Exponential Equation.

If $3^x = 148$, find x.

This is an exponential equation, the unknown quantity being the exponent.

Here $x \log 3 = \log 148$

$$\therefore \qquad x = \frac{\log 148}{\log 3}$$

$$\therefore \qquad x = \frac{2 \cdot 17026}{\cdot 47712}$$

$$\therefore \log x = \log 2 \cdot 17026 - \log \cdot 47712.$$

$$= \cdot 336512 - \bar{1} \cdot 678628.$$

$$= \cdot 657884 = \cdot 65788 \text{ to 5 places.}$$

$$\therefore \qquad x = 4 \cdot 54853 = 4 \cdot 5485 \text{ approx.}$$

Exercises LXXXVIII.

Find the cube root of the following numbers:

1. $27 \cdot 27$.
2. $\cdot 08765$.
3. $\cdot 00069$.
4. $\cdot 7248$.
5. $437 \cdot 72$.
6. $9281 \cdot 4$.

Find the numerical value of:

7. $\left\{\dfrac{3128}{25 \cdot 34}\right\}^{\frac{1}{2}}$.

8. $\left\{\dfrac{\cdot 0275}{\sqrt{\cdot 0183}}\right\}^{\frac{1}{3}}$.

9. $\dfrac{41 \times (\cdot 015)^{\frac{1}{4}}}{\sqrt[3]{105}}$.

10. $\dfrac{(46 \cdot 43)^{10} \times (\cdot 0348)^{12}}{1 \cdot 245 \times 163}$.

11. $15^{3 \cdot 2}$.

12. $\dfrac{(\cdot 19)^{\frac{1}{2}} \times (\cdot 19)^{\frac{1}{3}} \times (\cdot 19)^{\frac{3}{4}} \times 264^2}{(\cdot 0418)^2 \times (\cdot 4365)^3 \times \sqrt[3]{472}} \times \dfrac{1}{10^6}$.

13. $\dfrac{(34 \cdot 96)^{1.4} \times (165 \cdot 3)^{.34}}{(\cdot 258)^{.7} \times (\cdot 045)^{6.5}} \times \dfrac{1}{10^{12}}$.

14. $\sqrt{5^{\frac{1}{3}} + 6^{\frac{1}{3}}}$.

15. $\dfrac{1}{(1 \cdot 05)^8} - \dfrac{1}{(1 \cdot 05)^{20}}$.

16. $\dfrac{1}{(1 \cdot 06)^8} \times \dfrac{1}{1 \cdot 05)^{15}}$.

Find the value of x in the following:

17. $13^x = 432$. 18. $6^x = 25 \cdot 2$. 19. $15^{\frac{1}{x}} = 5$.

Employ the formula for the area of a triangle in terms of its sides to find the area of the following traingles:

20. 36·4 yd., 21·3 yd., 26·5 yd.

21. 16·48″, 23·39″, 31·18″.

22. 2500 links, 3500 links, 4000 links (area in acres).

23. 27·6 chains, 19·5 chains, 14·3 chains (area in acres).

24. Find the length of the perpendicular drawn from A on BC in the triangle ABC, if $a = 700'$, $b = 670'$, $c = 527 \cdot 2'$.

25. The sides of a triangle are 43·6″, 51·8″, and 62·4″. Find the side of an equilateral triangle of equal area.

26. Find the area of a circle whose radius is 72·46″.

27. A circle has a radius of 43·46″. Find the radius of the concentric circle which divides the first circle into two equal areas.

28. Find the diameter of a circle whose area is equal to that of an equilateral triangle on a side of 18″.

29. Find the number of gallons in a cubical cistern, each side of which measures 18·6′ (1 gal. = 277·274 cu. in.).

30. The water contained in a cubical cistern, each edge of which measures 5′, is found to lose by evaporation ·03 of its volume in a day. If the total loss be due entirely to evaporation, find how many gallons will be left in the cistern at the end of 9 days, assuming it to be full at the outset.

CHAPTER XVI.

MENSURATION OF SOLIDS.

152. **We have already found the surfaces and volumes** of various rectangular solids. We will now proceed to deal with some of the more specialized forms of solids.

If the block in Figure 103 has the dimensions indicated, we can find the area of the sides, i.e., the lateral surface by finding the area of each lateral face and adding the results. Thus the area of the front and back faces $=6'' \times 18'' \times 2 = 216$ sq. in., the area of the two side faces $=4'' \times 18'' \times 2 = 144$ sq. in., giving a total lateral area of 360 sq. in.

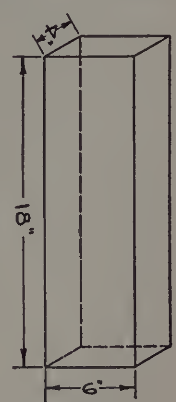


FIG. 103

The same result might have been obtained by first finding *the perimeter of the base and multiplying this result by the height.* Thus perimeter of base $= 6'' + 6'' + 4'' + 4'' = 20''$. $\therefore$ lateral surface $= 20'' \times 18'' = 360$ sq. in. Further in finding the volume of this solid we multiplied together the three dimensions—length, breadth and thickness. Thus volume $= 18'' \times 6'' \times 4'' = 432$ cu. in. The same result might have been obtained by first finding *the area of the base and then multiplying this area by the height.* Thus area of end $= 6'' \times 4'' = 24$ sq. in. and volume $= 24 \times 18'' = 432$ cu. in.

153. **The Prism.** A prism is a solid whose sides are parallelograms and whose top and bottom are parallel to each other.

In Figure 104 we have represented a number of prisms each complying with the conditions in the definition.

A prism is called triangular, rectangular, pentagonal, etc., according as the base is one or other of these polygons.

To find the lateral surface of any of the prisms below we would proceed as in Figure 103, i.e., *multiply the perimeter of the base by the height.* Thus if p be the perimeter of the base and h the height, the area of the lateral surface of the prism $= ph$.

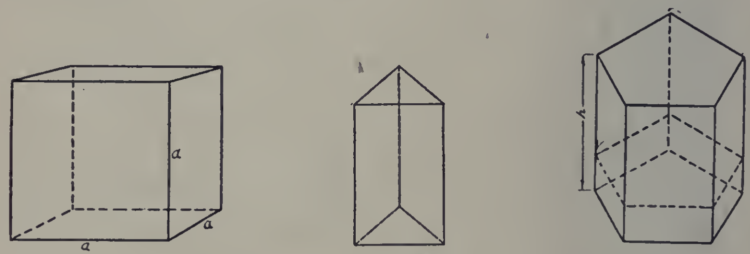


FIG. 104

To find the volume of any one of the above prisms we would as in Figure 103 *multiply the area of the base by the height.* Thus if b be the area of the base and h the height, the volume of the prism $= bh$.

If we wish to find the area of the total surface of a prism, *we would add the areas of the two ends to the area of the lateral surface.*

Exercises LXXXIX.

1. Measure the various prisms in the laboratory. Make drawings in your laboratory book and find total area and volume.

2. The internal dimensions of a box, without a lid, are length 8′, breadth 3′, depth 2′. Find the cost of lining it with zinc at 40c. a sq. ft.

3. A rectangular tank, 13′ 6″ in length by 9′ 9″ in breadth, is full of water. How many gallons of water must be drawn off to lower the surface 1″?

4. How many sq. ft. of metal are there in a rectangular tank, open at the top, 12′ in length 10′ in breadth and 8′ deep?

5. A prism whose base is a regular pentagon with a side of $9\frac{1}{2}$″ is $25\frac{1}{2}$″ in height. Find its total area and volume.

6. A rectangular tank is $11\frac{1}{2}$″ long, $14\frac{1}{2}$″ wide, and 10″ deep. Find the number of gallons it contains when filled with water within an inch of the top.

154. The Cylinder. A cylinder is a solid whose lateral surface is curved and whose bases are parallel to each other.

To find the lateral surface of a cylinder. If we roll a cylinder on a sheet of paper until it has made one complete revolution, we observe that the area of the paper touched by the cylinder is a rectangle whose length is equal to the circumference of the cylinder, and whose breadth is equal to the height of the cylinder. From this experiment we infer that *the lateral surface of a cylinder ≐ the circumference of base multiplied by the height.*

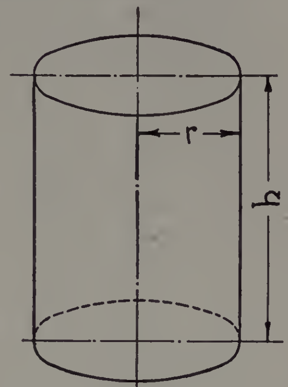


FIG. 105

Therefore with the notation in the figure the area of the lateral surface $= 2\pi rh$

$= \pi dh$ ($d =$ diameter).


FIG. 106

To find the volume of the cylinder. If the cylinder in Figure 106 be cut into a number of triangular prisms as indicated, we can find its volume by adding together the volumes of the prisms. Since the volume of a triangular prism is found by multiplying the area of the base by the height, the volume of all the prisms, i.e., of the cylinder, may be found by multiplying the sum of the areas of the bases by the height. Therefore *the volume of a cylinder = area of base multiplied by the height,* or $V = \pi r^2 h = \cdot 7854 d^2 h$.

Example:

A cylindrical tank open at top is 6′ high and has a diameter of 3′. Find (1) the cost of lining with galvanized iron at 20c. a sq. ft., (2) its capacity in gallons.

Area of lateral surface $=\pi\times3\times6=18\pi$ sq. ft.
Area of bottom $=\pi(\frac{3}{2})^2$ sq. ft. $=2\cdot25\ \pi$ sq. ft.
$\therefore$ total area $=\pi(18+2\cdot25)$ sq. ft.
$=\pi(20\cdot25)$ sq. ft.
$\therefore$ cost $=\pi(20\cdot25)\times20=\$12\cdot72$
Volume $=\pi(\frac{3}{2})^2\times6$ cu. ft.
Capacity $=\pi(\frac{3}{2})^2\times6\times6\cdot232$ gal.
$=264\cdot31$ gal

Exercises XC.

1. Measure the various cylindrical models in the laboratory. Make drawings in your laboratory book and obtain the lateral area and volume in each case. In the case of the iron and steel models find their weights from knowing their volumes. Check by weighing.

2. Fill in the omitted entries in the following cylinders:

No.	Diameter	Height	Circ. at Base	Area of Base	Lateral Area	Volume
1	5″	$3\frac{1}{2}$″				
2	8″	7″				
3	1″				28 sq. ft.	
4		3′		154 sq. ft.		
5		7′				616 cu. ft.
6		8′	44′			

3. Find the weight of a steel shaft 2″ in diameter and 12′ long.

4. A tank car is $33\frac{1}{2}$′ long and $8\frac{1}{2}$′ in diameter. How many gallons of oil will it contain?

5. Find the cost of painting the inside of an open cylindrical tank 10′ in diameter and 15′ high at 20c. a sq. yd.

6. A cylindrical vessel partly filled with water is 8″ in diameter. A steel crane hook is immersed in the vessel and the surface of the water is raised 2″. Find the weight of the crane hook.

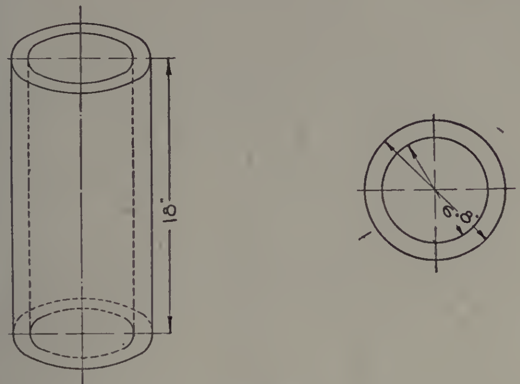


FIG. 107

155. **The Hollow Cylinder.** The total surface of the hollow cylinder in Figure 107 would consist of the outside lateral surface, the inside lateral surface, and the two rims.

The outside lateral surface $= \pi \times 8 \times 18 = 144\pi$ sq. in.
The inside lateral surface $= \pi \times 6 \times 18 = 108\pi$ sq. in.
The area of the rims $= \pi \times 7 \times 1 \times 2 = 14\pi$ sq. in.
The total lateral surface $= 266\,\pi$ sq. in. $= 835 \cdot 68$ sq. in.

The volume of the hollow cylinder would be the area of the base multiplied by the height.

Area of the base, i.e., the area of the ring in Figure 107
$= \pi \times 7 \times 1 = 7\pi$ sq. in.
$\therefore$ the volume $= 7\pi \times 18 = 395 \cdot 84$ cu. in.

Exercises XCI.

1. Measure the hollow cylindrical models in the laboratory. Make drawings in your laboratory book and calculate the total surfaces and volumes.

2. Find the whole surface of a hollow cylindrical pipe, open at the ends, if the length is 8″, the external diameter 10″ and the thickness 2″.

3. An iron roller is in the shape of a hollow cylinder whose length is 4′, external diameter 2′ 8″ and thickness $\frac{1}{2}$″. Find its weight if a cu. ft. of iron weighs 486 lb.

4. A portion of a cylindrical steel shaft casing is $12\frac{1}{2}$′ in length, $1\frac{1}{4}$″ thick, and its external diameter is 14″. Find its weight.

5. Find the weight of a lead pipe 8′ long, external diameter 8″, internal diameter 7″, assuming that the weight of the two flanges is equivalent to one foot length of pipe.

6. Find the weight of a hexagonal cast-iron nut 1″ to the side, $\frac{1}{2}$″ thick, inside diameter $\frac{3}{4}$″.

156. The Right Cone. A cone is a solid whose base is a circle and whose sides taper uniformly to a point directly over the base.

Fig. 108

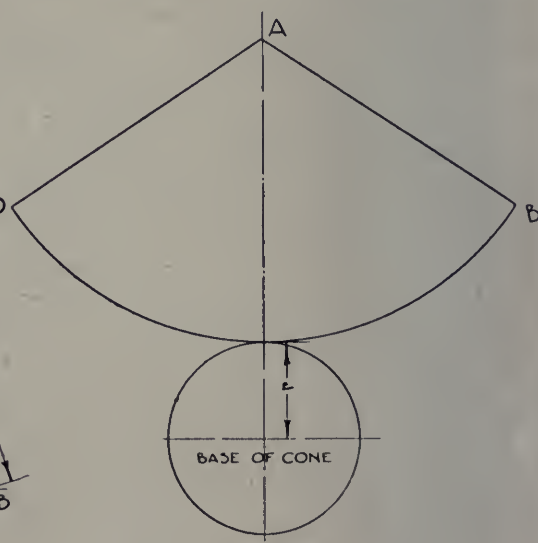


Fig. 109

Lateral Surface of a Cone. If a piece of paper be wrapped, without crumpling or tearing, around the lateral surface of a cone (Figure 108) and cut along the edge of the base and the line **AB**, and then folded out, the paper will be a sector of a circle (Figure 109).

The radius **AB** of this sector is equal to the slant height of the cone and the length of the arc **BD** is equal to the circumference of the base of the cone. The area of a sector of a circle has previously been found to be equal to $\frac{1}{2}$arc $\times$ radius. Therefore *the lateral surface of a cone* $= \frac{1}{2}$*circumference of the base multiplied by the slant height,* or with the notation of the figure, the lateral surface of a cone $= 2\pi r \times \frac{1}{2}s = \pi d \times \frac{1}{2}s$.

Perform the experiment suggested above. Make drawings and write conclusions in your laboratory book.

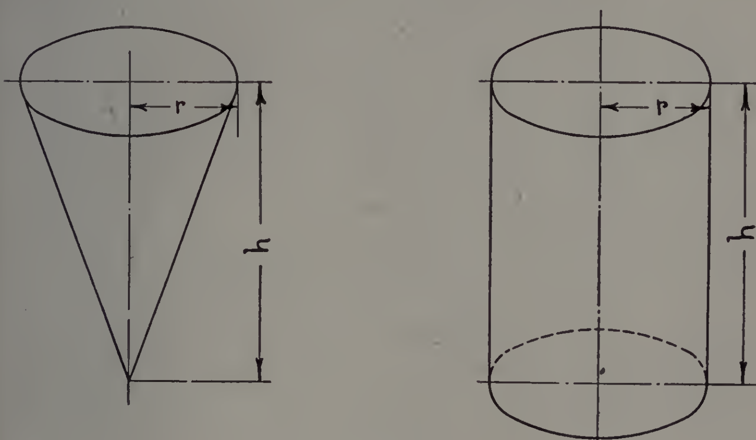


FIG. 110

The Volume of a Cone. Take two vessels, one conical and the other cylindrical, having the same height and radius of end. (Figure 110).

If we fill the conical vessel with water and empty it into the cylindrical one, we find that it will take three fillings of the conical vessel to fill the cylindrical one.

We, therefore, infer that when the vessels are related as in the above illustration, the volume of the cone is one-third that of the cylinder.

But the volume of the cylinder = area of base multiplied by the height. Therefore *the volume of a cone = area of base* $\times \frac{1}{3}$ *perpendicular height*, or $V = \pi r^2 \times \frac{1}{3} h = \frac{1}{3} \pi r^2 h$.

Note.—Height means perpendicular height, unless otherwise stated, but in the formula for the volume of a cone we should state "perpendicular" height to distinguish from "slant" height in the formula for the lateral surface.

Perform the experiment suggested above. Make drawings and write conclusions in your laboratory book.

Example :

A conical tent has a diameter at the base of 14′ and a height of 7′.

Find (1) the number of sq. yd. of canvas in the tent.

(2) the number of cu. ft. of air space.

Slant height of cone $= \sqrt{7^2 + 7^2} = 9 \cdot 89'$

Number of sq. yd. $= \pi \times 14 \times \frac{9 \cdot 89}{2} \times \frac{1}{9}$

$= 24 \cdot 17$

Air space $= \pi \times 7 \times 7 \times \frac{7}{3} = 359 \cdot 19$ cu. ft.

Exercises XCII.

1. Measure the various conical models in the laboratory. Make drawings in your laboratory book and calculate lateral surfaces and volumes. Find the weights of iron models from knowing their volumes. Check by weighing.

2. A piece of paper in the form of a circular sector, of which the radius is 8″ and the length of the arc 12″, is formed into a conical cap. Find the area of the conical surface and the base of the cone.

3. Find the weight of a cast-iron cone, diameter of base 7″ and height 15″.

4. Find the weight of petroleum in a conical vessel, diameter of the base 14″, height 10″, specific gravity of petroleum ·87.

5. The interior of a building is in the form of a cylinder of 20′ radius and 15′ in height. A cone surmounts it, radius of base 20′ and height 8′. Find (*a*) the cost of painting the interior at 20c. a sq. yd., making no allowance for openings, (*b*) cubic feet of air space in the building.

6. How many yards of canvas 27″ wide will be required to make a conical tent 7 yd. in diameter and 10′ high?

157. The Pyramid. A pyramid is a solid whose sides are triangles and whose base is any figure bounded by straight lines.

In Figure 111 we have the simplest type of a right pyramid, the base being a square.

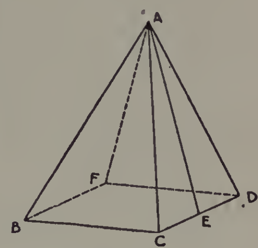


FIG. 111

Lateral Surface of a Pyramid. In Figure 111 the lateral surface consists of four equal isosceles triangles. Area of $ACD = CD \times \frac{1}{2} AE$.

$\therefore$ area of four faces = 4 times $CD \times \frac{1}{2} AE$.

But 4 times CD = perimeter of base, and AE = slant height of pyramid.

$\therefore$ *lateral surface of pyramid = perimeter of base* $\times \frac{1}{2}$ *slant height.*

$= \frac{1}{2}\, ps$ (p = perimeter, s = slant ht.).

It may readily be shown that this formula holds where the base is any regular polygon.

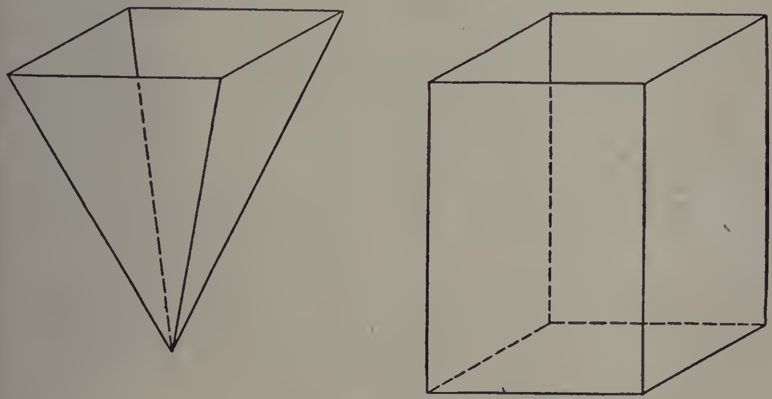


FIG. 112

Volume of a Pyramid. Take two vessels one a square pyramid and the other a rectangular prism of the same height and area of end as in Figure 112. If we fill the pyramidal vessel with sand and empty it into the prism, we find that it takes three fillings of the pyramid to fill the prism. We therefore infer that, when the vessels are related as above, the volume of the pyramid is one-third that of the prism.

The volume of prism = Area of base multiplied by the height.

∴ *volume of pyramid = area of base* $\times \frac{1}{3}$ *perp. height.*
or $V = \frac{1}{3}Ah$ (A = area of base, h = height).

Example:

A granite pyramid 12′ high stands on a square base 10′ to the side. Find (1) cost of polishing the lateral surface at 10c. a sq. ft. (2) weight, if 1 cu. ft. weighs 165 lb.

Slant height $= \sqrt{12^2+5^2} = 13'$

Lateral surface $= 4 \times 10 \times \frac{13}{2}$ sq. ft.

Cost of polishing $= 4 \times 10 \times \frac{13}{2} \times \frac{10}{1} = \$26.00$

Volume $= \frac{1}{3} \times 10 \times 10 \times 12$
$= 400$ cu. ft.

Weight $= 165 \times 400$
$= 66{,}000$ lb.

Exercises XCIII.

1. Measure the various pyramidal models in the laboratory. Make drawings and calculate lateral surfaces and volumes.
2. What is the weight of a cast-iron pyramid with a square base 6″ to a side and a height of 10″?
3. Find the total surface of a hexagonal pyramid with a base 3″ to the side and a slant height of 12″. Find its weight if made of cast-iron.
4. Find the number of cu. ft. of air space in a hexagonal room, each side of which is 12′, and its height 18′, which is furnished above with a pyramidal roof 9′ high. Find also the cost of painting the interior at 25c. a sq. yd., making no allowance for openings.
5. A pyramid has a square base each side of which is 2·48″, and the pyramid has equilateral triangles for sides. Find its volume.

Frustum of Cone or Pyramid. A frustum of a cone or pyramid is the part contained between the base and a plane drawn parallel to it.

Lateral Surface of Frustum of Cone. **aABb** in Figure 113 may be considered as a trapezium, **ab** and **AB** being the parallel sides and either **aA** or **bB** representing the perpendicular distance between the parallel sides.

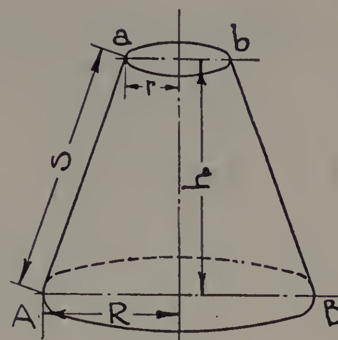


FIG. 113

If we consider this figure as being bent around until **a** coincides with **b** and **A** with **B**, it would take the form of a frustum of a cone, the parallel sides of the trapezium becoming the circumferences of the ends and the perpendicular distance between the parallels becoming the slant height of the frustum.

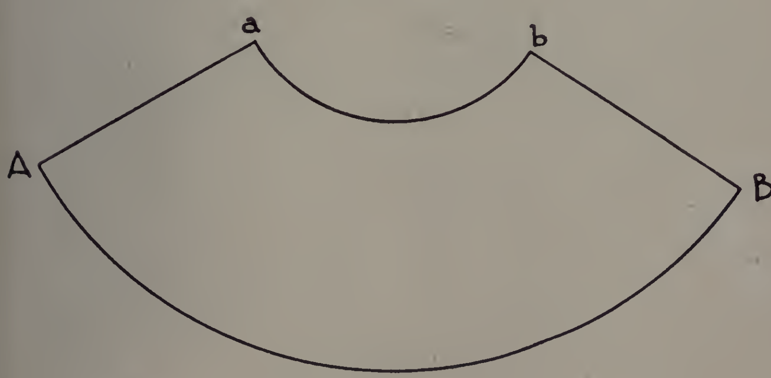


FIG. 114

Since area of trapezium = Sum of parallel sides multiplied by $\frac{1}{2}$ perp. distance between them.

∴ *lateral surface of frustum of cone = sum of circumferences of ends* $\times \frac{1}{2}$ *slant ht.*

or, Lateral surface $= \frac{1}{2}(C+c)S.$

$= \pi(R+r)S.$

Lateral Surface of Frustum of Pyramid. If we consider the frustum of a pyramid in Figure 115, we observe that its lateral surface is made up of four equal trapeziums.

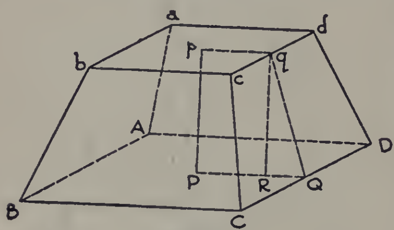


Fig. 115

Area of the face $cCDd = (cd+CD)\frac{1}{2}qQ.$

∴ area of the four faces $= 4(cd+CD)\frac{1}{2}qQ.$

But $4(cd+CD) =$ Sum of perimeters of ends and $qQ =$ slant height of frustum.

∴ *lateral surfaces of frustum of pyramid = sum of perimeters of ends.* $\times$ $\frac{1}{2}$ *slant ht.* $= \frac{1}{2}(P_1+P_2)\,S$ *(P_1 and P_2 being perimeters and S slant height).*

Volume of Frustum of Cone or Pyramid. In Figure 116 from similar triangles **Ocb** and **OCB** we have $\frac{x}{x+h} = \frac{r}{R}.$

$\therefore x = \frac{hr}{R-r} \quad \therefore x+h = \frac{hR}{R-r}.$

Volume of whole cone

$= \frac{1}{3}\pi R^2 \frac{hR}{R-r}.$

Volume of small cone

$= \frac{1}{3}\pi r^2 \frac{hr}{R-r}.$

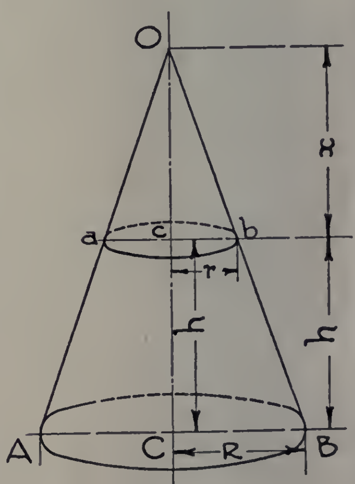


Fig. 116

$$\therefore \text{ volume of frustum} = \tfrac{1}{3}\pi h\left\{\frac{R^3-r^3}{R-r}\right\}.$$

$$= \tfrac{1}{3}\pi h\left\{R^2+Rr+r^2\right\}.$$

If A represents area of large end and a area of small end, then $A = R^2$ and $a = \pi r^2$.

$$\therefore \text{volume of frustum} = \frac{h}{3}\{A + a + \sqrt{Aa}\}.$$

Work through a similar proof to show that the volume of a frustum of a pyramid is the same as the above.

Example:

A vessel in the form of a frustum of a cone has the following dimensions: Depth 16″, diameter of large end 12″, diameter of small end 8″. Find (*a*) its lateral surface (*b*) its capacity in gallons.

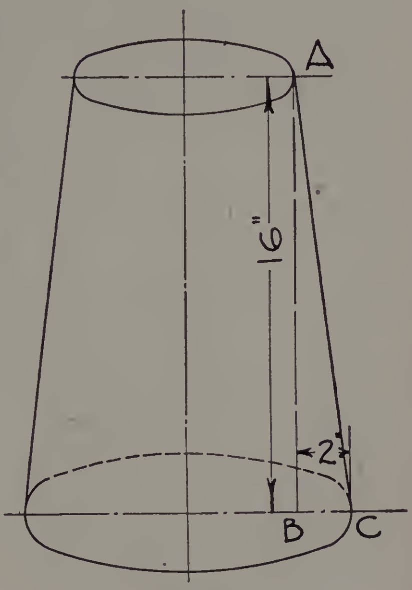


FIG. 117

In the rt.-angled triangle ABC, $AC = \sqrt{16^2 + 2^2}$
$= 16 \cdot 12''$.

Lateral surface

$$= (\pi 12 + \pi 8)\frac{16 \cdot 12}{2}$$

$= 20\pi \times 8 \cdot 06 = 506 \cdot 43$ sq. in.

Volume

$= \frac{16}{3}\{\pi 6^2 + \pi 4^2 + \sqrt{\pi 6^2 \pi 4^2}\}$

$= \frac{16}{3}\{\pi 6^2 + \pi 4^2 + \pi 6 \times 4\}$

$= \frac{16}{3}\pi\{6^2 + 4^2 + 24\}$

$= \frac{16}{3}\pi\, 76$ cu. in.

$$\text{Capacity in gallons} = \frac{16\pi}{3} \times \frac{76}{277 \cdot 274} = 4 \cdot 59.$$

Exercises XCIV.

1. Measure the frustum models in the laboratory. Make drawings in your laboratory book and calculate lateral surfaces and volumes.

In the case of the iron and steel models find weights from knowing their volumes. Check by weighing.

2. Find the lateral surface of the frustum of a pyramid, perpendicular height 6″, and a square base, side 6″, the side of the upper square being 1″.

3. A tapered piece of cast-iron 2′ long is 8″ in diameter at one end and 12″ in diameter at the other; find its weight.

4. A piece of steel 16″ long is 4″ in diameter at the large end. The taper is a Brown and Sharpe—$\frac{1}{2}$″ to 1′; find its weight.

5. Find the volume of a steel pin 8″ long, diameter of small end 2″, the taper being a No. 0 Morse—$\frac{5}{8}$″ to 1′.

6. Two buckets, one cylindrical of 7″ diameter, the other a frustum of a cone with the diameters of its ends 6″ and 8″ are of the same depth, 9″. Find the difference in their volume.

158. **The Sphere.** A sphere is the geometrical name for a round or ball-shaped solid.

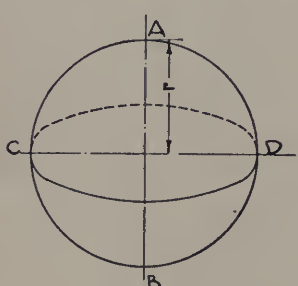


FIG. 118

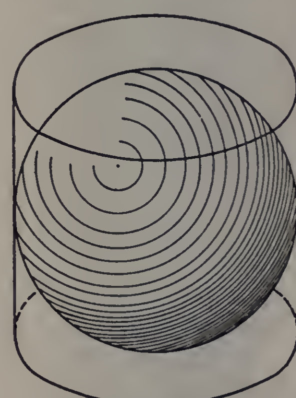


FIG. 119

Area of Surface of Sphere. It has been found by measurement that the surface of a sphere is equal to the lateral surface of a cylinder of the same diameter and height, as illustrated in Figure 119.

The circumference of the cylinder is $2\pi r$ and its height $2r$, hence *area of surface of sphere* $=2\pi r \times 2r = 4\pi r^2$.

An approximate idea of this relation may be obtained by the following experiment:

Insert a nail in the centre of the curved surface of a hemisphere. Fasten the end of a cord to this nail and wrap it with the object of completely covering the curved surface.

Next insert a nail in the centre of the flat face and wrap the cord with the object of completely covering the flat surface. It may then be observed that the length of cord required to cover the flat surface is only half that required to cover the curved surface.

But area of flat surface $=\pi r^2$.

$\therefore$ area of curved surface $=2\pi r^2$.

$\therefore$ total surface of sphere $=4\pi r^2$.

Since $4\pi r^2=\pi(2r)^2$ $\therefore$ area of surface of sphere in terms of diameter $=\pi D^2$.

Volume of Sphere. The sphere may be considered as made up of a number of pyramids, as indicated in Figure 120, whose bases together form the surface of the sphere and whose apexes meet at the centre.

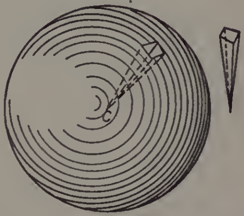


FIG. 120

Since the volume of a pyramid equals area of base multiplied by $\frac{1}{3}$ perp. ht. (= radius of sphere), $\therefore$ volume of sphere

$= \text{Surface} \times \frac{1}{3}$ radius.

$=4\pi r^2 \times \frac{1}{3} r$.

$=\frac{4}{3}\pi r^3$.

Volume in terms of diameter $= \cdot 5236 D^3$.

Example:

Find the surface and weight of a cast-iron ball, radius 5″.

Surface $=4\pi 5^2 = 314\cdot 16$ sq. in.

Volume $=\frac{4}{3}\pi 5^3 = \frac{4}{3}\pi 125$ cu. in.

Weight $=\frac{4}{3}\pi 125 \times \cdot 26 = 136\cdot 13$ lb.

Exercises XCV.

1. Measure the spherical models in the laboratory. Calculate areas and volumes.

2. Secure cylinder and sphere related as in Figure 119. After placing sphere in cylinder, fill the remaining space with sand. Remove sphere and replace the sand. By estimating the part of the cylinder now occupied by the sand derive the formula for the volume of the sphere.

3. Find the number of yards of material, 27″ wide, necessary to make a spherical balloon 12′ in diameter.

4. Find the weight of a ball composed of a cast-iron sphere 4″ in diameter, covered with a layer of lead 1″ thick.

5. Find the weight of a hollow cast-iron sphere, internal diameter $2\frac{1}{2}$″, thickness $\frac{1}{4}$″.

6. How many ounces of nickel would be used in plating a ball 3″ in diameter, to a depth of $\frac{1}{64}$″? (1 cu. in. nickel weighs 5·14 oz.).

Segment of a Sphere. A segment of a sphere is the part cut off from a sphere by a plane.

Lateral Surface of a Segment. If we roll a sphere on a sheet of paper, and keep in mind that the area of the surface is equal to that of a cylinder with radius of base equal to the radius of the sphere and height equal to the diameter of the sphere, we could infer that the surface traced out by any segment is equal in area to a rectangle having the circumference of the sphere for length and the height of the segment for width.

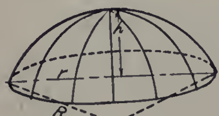


Fig. 121

∴ *lateral surface of segment*

$$= 2\pi R \times h.$$
$$= 2\pi Rh.$$

The volume of the segment in Figure 121 (less than a hemisphere) is given by the formula: $V = \frac{\pi h r^2}{2} + \frac{\pi h^3}{6}$.

Zone of Sphere. A zone of a sphere is the part cut off from the sphere between two parallel planes.

Lateral Surface of Zone. As in the segment of a sphere the area traced out when rolled on the paper would be equal in area to a rectangle having the circumference of the sphere for length, and the thickness of the zone for breadth.

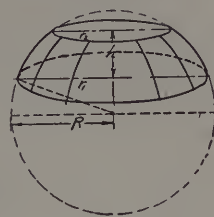


FIG. 122

∴ *lateral surface of zone* $= 2\pi Rh$.

The volume of the zone in Figure 122 is given by the formula:

$$V = \frac{\pi h}{2}\,(r_1{}^2 + r_2{}^2) + \frac{\pi h^3}{6}.$$

Sector of Sphere. A sector of a sphere consists of a segment and a cone whose bases are coincident, the apex of the cone being at the centre of the sphere.

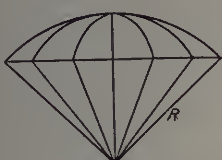


FIG. 123

The surface of the sector would be equal to the surface of the segment plus the surface of the cone.

The volume of the sector is given by the formula:

$$V = \frac{\pi}{6}\left\{r^2(h+2R) + h^3\right\}.$$

159. Bead. Volume remaining when sphere is pierced by a cylindrical solid.

FIG. 124

Volume of bead as shown $= \dfrac{\pi h^3}{6}$ (h = ht. of bead).

Exercises XCVI.

1. The silk covering of an umbrella forms a portion of a sphere of $3\frac{1}{2}'$ in diameter, the area of the silk being $14\frac{2}{3}$ sq. ft. Find the area sheltered from vertical rain when the handle is held upright.

2. A sphere of diameter 24′ is placed so that its centre is 37′ distant from the observer's eye. Find the area of that part of the sphere's surface that is visible to the observer.

3. A cylindrical tank is 8′ long and $2\frac{1}{2}'$ in diameter. The ends are spherical segments whose centre of curvature projects 6″ beyond the base of the segment. Find the total surface and volume of the tank.

4. If the diameters of two circles of a spherical zone are 12″ and 4″, and the thickness of the zone 6″, find its total surface and volume.

5. In the sector of a sphere of radius 10″, the height of the segment is 4″; find the volume of the sector.

160. Solid Ring. Examples of solid rings are found in anchor rings, curtain rings, etc. It will be observed that any cross-section of such a ring will be a circle, so it may be considered as a cylinder bent around in a circular arc until the ends meet. The mean length of the cylinder will be $2\pi R$.

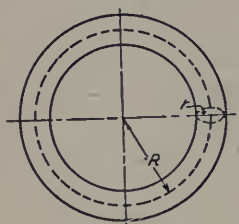


Fig. 125

$\therefore$ with notation of figure:

$$\text{Surface} = 2\pi r \times 2\pi R = 4\pi^2 r R.$$

$$\text{Volume} = \pi r^2 \times 2\pi R = 2\pi^2 r^2 R.$$

161. Wedge. A wedge, as shown, is a solid contained by five plane faces; the base is a rectangle, the two ends are triangles, and the two remaining faces are trapeziums having a common side, called the edge, which is parallel to the base.

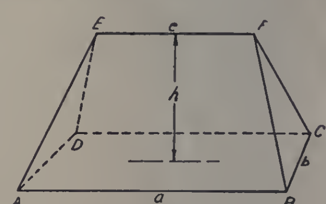


Fig. 126

The surface of the wedge is found by calculating separately the area of each of the faces. To do this, the slant heights of the faces, or means of finding them, must be given.

The volume of the wedge is given by the formula:

$$V = \frac{hb}{6}\{2a + e\}.$$

162. **Prismoid**—An irregular-shaped solid having five or more flat or plane faces, two of which are parallel.

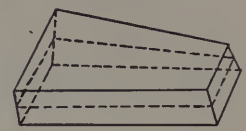


FIG. 127

Volume of prismoid given by the formula: $V=\frac{h}{6}\{A+B+4M\}$,

where A and B are the areas of the parallel faces, M the area of a section half-way between them, and h the height.

Exercises XCVII.

1. Find the weight of a brass wedge, whose height is 5″ and edge 4″, the base being a rectangle which measures 8″ by 6″ (1 cu. in. = ·3 lb.).

2. How many tons of earth are removed in excavating a trench of which the top and bottom are rectangles? At the top it is 400′ long by 18′ wide, and at the bottom it is 350′ long by 15′ wide. The bottom is horizontal and the depth 12′, (given 1000 cu. ft. earth weighs 40 tons).

3. In a cast-iron wheel the inner diameter of the rim is 2′ and the cross-section of the rim is a circle of 6″ radius; find the weight of the rim.

4. The cross-section of the rim of a cast-iron fly-wheel is a rectangle 8″ by 10″. If the mean diameter is 10′, find the weight of the rim.

5. A wedge-shaped trench is 40 yards long at the top and 8′ wide; the length of the bottom edge is 32 yards and the depth is 10′. How many cu. yd. of earth have been excavated?

6. The cross-section of the rim of a fly-wheel is a rectangle 6″ by 8″, the shorter dimension being in the diameter of the wheel. The wheel is 22′ in outer diameter; find the weight if the specific gravity of the material is 7·2.

Miscellaneous Exercises XCVIII.

(1 cu. in. cast-iron = ·26 lb. 1 cu. in. steel = ·2834 lb.)

1. A square bar of wrought-iron 12′ long, weighs 80 lb. What is the size of the end?

2. A closed cast-iron tank is 3′ long, $2\frac{3}{4}$′ wide, and $2\frac{1}{2}$′ deep, outside measurements. If the material is $\frac{1}{2}$″ thick, find the weight of the tank.

3. The rain which falls on a roof 22′ by 36′ is conducted to a cylindrical cistern 8′ in diameter. How great a rainfall would it take to fill the cistern to a depth of $7\frac{1}{2}'$?

4. Water is poured into a cylindrical reservoir 20′ in diameter, at the rate of 300 gallons per minute. Find the rate, in feet per minute, at which the water rises in the reservoir.

5. The internal diameter of a cylinder, open at the top, is $1\frac{1}{2}'$, and its weight is 180 lb.; when filled with water it weighs 2000 lb.; find the depth of the cylinder.

6. Find the weight of a copper tube $\frac{5}{8}''$ outside diameter, ·05″ thick, and 5′ 10″ long.

7. A steel bar whose cross-section is a regular hexagon 1″ to the side, is 8′ in length. Find its weight.

8. A trough whose cross-section is an equilateral triangle 8″ to the side contains 30 gallons of water; how long is it?

9. A boiler has 275 tubes, each 19′ 3″ long and $2\frac{3}{8}''$ in diameter. What is the total heating surface of the tubes?

10. Find the capacity in gallons of a conical vessel 15″ in diameter and 2′ in slant height.

11. A conical tent covers an area of 154 sq. ft. and is 6′ in height. How many sq. yd. of canvas does it contain?

12. What is the volume of a cylindrical ring having an outside diameter of $6\frac{1}{4}''$, an inside diameter of $5\frac{3}{16}''$, and a height of $5\frac{3}{8}''$?

13. Water flows at the rate of 20′ per min. from a cylindrical pipe ·25″ in diameter. How long would it take to fill a conical vessel, whose diameter at the surface is 10″ and depth 9″?

14. The external diameter of a hollow steel shaft is 20″, and the internal diameter 12″. Find the weight of 20′ of this shafting.

15. From a cylinder whose height is 8″, and diameter 12″, a conical cavity of the same height and base is hollowed out. Find the whole surface of the remaining solid.

16. Find the cost of polishing the lateral surface of a pyramid 6′ 5″ high, standing on a square base 6′ to the side, at the rate of 20 c. a sq. ft.

17. How many gallons of water will be discharged per min. from a 4″ pipe if it flows at the rate of 300′ per minute?

18. The cross-section of a water pipe is a regular hexagon whose side is 1″. At what rate, in feet per min., must the water flow through the pipe in order to fill in one hour a cylindrical tank the radius of whose base is 16″ and whose depth is 5′?

19. The base of a prism whose altitude is 15″ is a quadrilateral whose sides are 10″, 18″, 12″, 16″, the last two forming a rt. angle. Find its volume.

20. A tower whose ground plan is a square on a side of 30′, is furnished with a pyramidal roof 8′ high. Find the cost of covering the roof with sheet-iron at 25c. a sq. ft.

21. A steel bar whose cross-section is an equilateral triangle $1\frac{1}{2}''$ to the side is 8′ long; find its weight.

22. A cylindrical granite pillar 10′ high and 30″ in diameter, is surmounted by a cone $2\frac{1}{2}'$ high. Find the weight of the whole if a cu. ft. of granite weighs 165 lb.

23. How many cu. in. are there in a hexagonal blank nut ·5″ to a side and $\frac{3}{8}''$ thick?

24. It is desired to make a conical oil can with a base 5″ in diameter to contain $\frac{1}{2}$ pint; what must be the height?

25. A piece of cast-iron has a B. & S. taper—$\frac{1}{2}''$ to 1′. It is 10″ long and the diameter at the large end is 3·5″; find its weight.

26. Find the height of a pyramid, of which the volume is 625 cu. in. and the base a regular hexagon 12″ to the side.

27. The perpendicular height of a square chimney is 150′ 3″. The side of the base measures 12′ 6″ and the side at the top 6′ 3″, the cavity is a square prism whose side measures 3′ 9″. How many cu. ft. of masonry in the chimney?

28. A circular disc of lead, 3″ in thickness and 12″ diameter, is converted into shot, each ·05″ in radius. How many shot does it make?

29. The interior of a building, in the form of a cylinder of 15′ 0″ radius and 10′ 0″ high, is surmounted by a cone whose vertical angle is a rt. angle. Find the area of the surface and the cubical contents of the building.

30. A square building 20′ 0″ to the side has a hip roof in the form of a pyramid. The peak of the roof is 10′ above the plate level and the rafter heel is 2′; find the cost of roofing with shingles, laid $4\frac{1}{2}''$ to the weather, material and labour costing \$12 a square of shingles.

31. The base of a cone is an ellipse, major axis 4″, minor axis 2″, height 6″. Find the volume.

32. A quart measure is 8″ in height. Find the diameter of its base.

33. Find the weight of a log 40′ long, 4′ 6″ in diameter at one end and 30″ in diameter at the other, the specific gravity of the wood being ·78.

34. A piece of copper 6″ long, 2″ wide, and $\frac{1}{2}$″ thick, is drawn out into a wire of uniform thickness and 100′ long. Find the diameter of the wire in mils.

35. A conical vessel $7\frac{1}{2}$″ deep and 20″ across the top is completely filled with water. If sufficient water is now drawn off to lower the surface 6″, find the area of the surface of the vessel thus exposed.

36. A cylinder 2″ in diameter and 8″ in height contains equal volumes of mercury, oil and water. If the specific gravity of the mercury be 13·6, of oil ·92, find the total weight of contents.

37. The radii of the internal and external surfaces of a hollow spherical shell of metal are 10″ and 12″ respectively. If it is melted down and the material formed into a cube, find the edge of the cube.

38. An automobile gasoline tank has an elliptical cross-section 9″ by 15″ and is 3′ long. How many gallons of gasoline will it hold?

39. A hemispherical basin holds 2 gallons. Find its internal diameter.

40. If 30 cu. in. of gunpowder weigh 1 lb., find the internal diameter of a spherical shell that holds 15·4 lb.

CHAPTER XVII.

RESOLUTION INTO FACTORS.

163. **When a quantity is the product of two or more quantities, each of these is called a factor of the quantity, and the finding of these quantities is called factoring the quantity.**

Thus, 6 is the product of 3 and 2, therefore 3 and 2 are called the factors of 6.

Further $2xy$ is the product of 2, x and y, therefore 2, x and y are called the factors of $2xy$.

164. **First Type.** If we wish to find the value of $6\times4+6\times3$, we could find the value of $6\times4=24$, then $6\times3=18$, and then add the results giving 42.

We might also write as follows:—$6\times4+6\times3=6(4+3)=6\times7=42$.

The following examples will illustrate this algebraically:

1. $bx+by+bz=b(x+y+z)$.
2. $2y+4=2(y+2)$.
3. $4x^2-16xy=4x(x-4y)$.
4. $ab+ac+a=a(b+c+1)$.

Frequently an expression may be resolved into factors under this type by arranging in groups which have a compound factor common.

Thus, factor $x^2-ax+bx-ab$.

By taking x out of the first and second and b out of the third and fourth we may write as follows:—$x\ (x-a)+b\ (x-a)$.

We have now the sum of two products with $x-a$ in each, therefore we may write as $(x-a)(x+b)$.

Examples:

1. Factor $a^2+ab+ac+bc$.
$=a(a+b)+c(a+b)$.
$=(a+b)(a+c)$.

2. Factor $12a^2-4ab-3ax^2+bx^2$.
$=3a\ (4a-x^2)-b(4a-x^2)$.
$=(4a-x^2)(3a-b)$.

Exercises XCIX.

Factor:

1. $ax-a^2$.
2. x^2-3ax.
3. $5x^3-15x^2y$.
4. $8a^3-16ab$.
5. $21-56x$.
6. $-ay+by+cy$.
7. $ax-bx-cx$.
8. $3a^2b^2-9ab+12$.
9. $14x^3-7x^2y+56xy^2$.
10. $5a^2+15ax+20ab$.
11. $ax-bx-ay+by$.
12. $x^2-xy+xz-yz$.
13. $3x-3y+ax-ay$.
14. $x^3-xy-2x^2+2y$.
15. $ab\,(x^2+1)-x\,(a^2+b^2)$.
16. a^5+a^4+a+1.
17. $a^2-bc-b+a^2c$.
18. $2a^3+6a^2-ca-3c$.
19. $x^2+mx\,(m+1)+m^3$.
20. $ax+bx+ay+by-az-bz$.

165. **Second Type.** In the treatment of multiplication we found the product of two binomials as $x+2$ and $x+5$ as follows:

$$\begin{array}{r} x+2 \\ x+5 \\ \hline x^2+2x \\ +5x+10 \\ \hline x^2+7x+10 \end{array}.$$

While the result could always be obtained by this method, it is important that the student should be able to write down the product of two binomials by inspection. In the result above we observe that the first term is the product of the first terms of the two expressions; the third term is the product of the second terms of the two expressions; the middle term has for its coefficient the sum of the numerical quantities (with proper sign) in the second terms of the two expressions.

Write down the values of the following products:

1. $(x+4)(x+5)$.
2. $(x-6)(x+2)$.
3. $(p+3)(p-6)$.
4. $(r+4)(r-6)$.
5. $(x+6)(x+3)$.
6. $(p-9)(p+1)$.
7. $(x-3a)(x+2a)$.
8. $(x+7y)(x-3y)$.
9. $(2x-5)(2x+6)$.
10. $(3x-1)(3x+1)$.
11. $(2x+7y)(2x-5y)$.
12. $(2x+a)(2x+b)$.

The converse problem gives us our second type and consists in finding the two factors if we know the product.

Thus, factor $x^2+7x+12$.

The second terms of the factors must be such that their product is $+12$ and their sum $+7$. Hence they must both be positive, and it is readily seen that they must be $+4$ and $+3$.

$\therefore\ x^2+7x+12=(x+4)(x+3)$.

Factor $x^2-10ax+9a^2$.

The second terms of the factors must be such that their product is $9a^2$ and their sum $-10a$. Hence they must be $-9a$ and $-a$.

$\therefore\ x^2-10a^2+9a^2=(x-9a)(x-a)$.

If we multiply $3x+4$ by $2x+1$ we get $3x(2x+1)+4(2x+1)$ $=6x^2+3x+8x+4=6x^2+11x+4$.

The converse problem is now to be considered.

Factor $4x^2+11x-3$.

Here the numerical coefficients of the first terms of the factors must be 4 and 1, or 2 and 2, and the last terms must be 3 and 1.

The possible sets (omitting the signs) are:

$4x$	3,	x	3,	$2x$	1.
x	1,	$4x$	1,	$2x$	3.

Since the sign of the last term in $4x^2+11x-3$ is minus, we at once decide that the signs of the last terms in the factors must be different, and therefore that the partial products must be subtracted. The second arrangement is the only one from which we can obtain $11x$, and also since the middle term is positive, the larger of the cross products must be positive.

$\therefore\ 4x^2+11x-3=(x+3)(4x-1)$.

Example 1:

$12x^2-x-20=(3x-4)(4x+5)$.

Example 2:

Factor $3x^2-7x+2=(3x-1)(x-2)$.

Exercises C.

Factor and verify:

1. $x^2+10x+21.$
2. $x^2-10x+24.$
3. $x^2-4x+4.$
4. $x^2-x-2.$
5. $x^2-11x+10.$
6. $x^2-x-42.$
7. $x^2-3x-130.$
8. $1-3x+2x^2.$
9. $x^2+x-72.$
10. $x^2+4x-5.$
11. $5-4x-x^2$
12. $40-13x+x^2.$
13. $1-5x+6x^2.$
14. $40-3x-x^2.$
15. $1-3x-130x^2.$
16. $5x^2+42x-27.$
17. $4x^2-16x+15.$
18. $3x^2-22x+7.$
19. $6x^2-11x+3.$
20. $9x^2-9x-28.$
21. $26x^2-41x+3.$
22. $12x^2-17x+5.$
23. $5x^4-10x^2y^2-400y^4.$
24. $2x^2+5xy+3y^2.$
25. $12x^2-2xy-30y^2.$
26. $12x^2-5xy-3y^2.$
27. $8x^2+22x+9.$
28. $6x^2-13xy+6y^2.$
29. $13x^2y^2-9x^4-4y^4.$
30. $14x^2+83xy-6y^2.$

166. Third Type. If we multiply $x+y$ by $x-y$ we have:

$$\begin{array}{l} x+y \\ \underline{x-y} \\ x^2+xy \\ \underline{\quad\; -xy-y^2} \\ x^2 \qquad\;\; -y^2. \end{array}$$

Observing the above we find that, when we multiply the sum of x and y by the difference of x and y, the result is the difference of the squares of x and y. Therefore we may say that *the difference of the squares of two quantities is equal to the sum of the quantities multiplied by the difference of the quantities.*

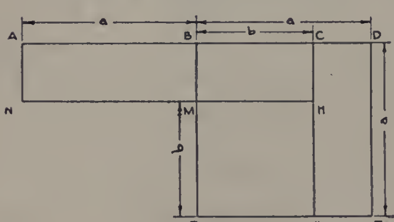


FIG. 128

Geometrical Illustration:

$$\begin{aligned}(a+b)\,(a-b) &= ANHC \\ &= BMHC+CKFD \\ &= BEFD-MEKH \\ &= a^2-b^2.\end{aligned}$$

Examples:

1. $p^2-q^2=(p+q)(p-q)$.
2. $9a^2-25b^2=(3a)^2-(5b)^2=(3a+5b)(3a-5b)$.
3. $\pi R^2-\pi r^2=\pi(R^2-r^2)=\pi(R+r)(R-r)$.
4. $(a+b)^2-c^2=(a+b+c)(a+b-c)$.

167. Incomplete Squares. Sometimes an expression comes under this type, but it is not stated directly as the difference of two squares.

Thus, factor $a^4+a^2b^2+b^4$.

This expression would be the square of a^2+b^2 if the middle term were $2a^2b^2$ instead of a^2b^2.

We will then add a^2b^2 to complete the square and subtract it again to maintain the value of the expression.

Thus, $a^4+a^2b^2+b^4=(a^4+2a^2b^2+b^4)-a^2b^2=(a^2+b^2)^2-(ab)^2$.

$$=(a^2+b^2+ab)(a^2+b^2-ab).$$

Examples:

1. $x^4+x^2+1 \quad =(x^4+2x^2+1)-x^2$.
$$=(x^2+1)^2-x^2.$$
$$=(x^2+1+x)(x^2+1-x).$$
2. $x^4+9x^2+25=(x^4+10x^2+25)-x^2$.
$$=(x^2+5)^2-x^2.$$
$$=(x^2+5+x)(x^2+5-x).$$
3. $4x^4+1=(4x^4+4x^2+1)-4x^2$.
$$=(2x^2+1)^2-(2x)^2.$$
$$=(2x^2+1+2x)(2x^2+1-2x).$$

Exercises CI.

Factor:

1. $16x^2-25y^2$.
2. x^2-9y^2.
3. $25x^2y^2-16a^2b^2$.
4. $9x^2y^2-4p^2q^2$.
5. $a^2-(b+c)^2$.
6. x^4-y^4.
7. x^8-y^8.
8. $a^2-b^2-2bc-c^2$.
9. $(x+y)^2-(a+b)^2$.
10. $(x^2+y^2)^2-4x^2y^2$.

11. $x^2-y^2+2yz-z^2$.
12. $1-a^2-2ab-b^2$.
13. $a^{16}-1$.
14. $x^2-2xy+y^2-a^2-2ab-b^2$.
15. $\pi 10^2-\pi 7^2$.
16. $\pi 7\cdot 5^2-\pi 2\cdot 5^2$.
17. $5a^2-10ab+5b^2-20c^2$.
18. $16-a^2-b^2+2ab$.
19. $4x^4+11x^2y^2+9y^4$.
20. x^4+9x^2+81.
21. x^4+4y^4.
22. x^8-7x^4+1.
23. $4x^4-37x^2y^2+9y^4$.
24. x^4+4x^2+16.
25. $9x^4-10x^2y^2+y^4$.
26. $4x^4-13x^2y^2+9y^4$.
27. $x^4+5x^2y^2+9y^4$.
28. x^4+x^2+25.

168. Fourth Type. Divide x^3+y^3 by $x+y$.

$$
\begin{array}{l}
x+y)\,x^3+y^3\,/\,\underline{x^2-xy+y^2} \\
\qquad \underline{x^3+x^2y} \\
\qquad\quad -x^2y+y^3 \\
\qquad\quad \underline{-x^2y-xy^2} \\
\qquad\qquad\quad xy^2+y^3 \\
\qquad\qquad\quad \underline{xy^2+y^3}
\end{array}
$$

Divide x^3-y^3 by $x-y$.

$$
\begin{array}{l}
x-y)\,x^3-y^3\,/\,\underline{x^2+xy+y^2} \\
\qquad \underline{x^3-x^2y} \\
\qquad\quad x^2y-y^3 \\
\qquad\quad \underline{x^2y-xy^2} \\
\qquad\qquad\quad xy^2-y^3 \\
\qquad\qquad\quad \underline{xy^2-y^3}
\end{array}
$$

As a result of the above we may write:

$x^3+y^3=(x+y)(x^2-xy+y^2)$ and $x^3-y^3=(x-y)(x^2-xy+y^2)$.

The above results might be stated as follows:

The sum of the cubes of two quantities is divisible by the sum of the quantities, and the difference of the cubes of two quantities divisible by the difference of the quantities. The other factor consists of the sum of the squares of the quantities, minus their product, if the sum of two cubes, and plus their product if the difference of two cubes.

Examples:

1. $p^3+q^3=(p+q)(p^2-pq+q^2)$.
2. $8x^3-27y^3=(2x)^3-(3y)^3=(2x-3y)(4x^2+6xy+9y^2)$.
3. $5a^3-40=5(a^3-8)=5(a-2)(a^2+2a+4)$.

Exercises CII.

Factor:

1. y^3+27.
2. a^3-125.
3. x^6+1.
4. a^6-b^6.
5. x^6-64.
6. a^3-216.
7. $3-81x^3$.
8. x^4-27x.
9. $2x^3+250$.
10. $x^{12}-y^{12}$.
11. $(a+b)^3-c^3$.
12. $(a+b)^3-(a-b)^3$.

CHAPTER XVIII.

INDICES AND SURDS.

169. **Indices.** In the introductory chapter in Algebra we inferred the laws with respect to indices from particular cases.

Thus, (1) $x^3 \times x^2 = x^{3+2} = x^5$. (3) $(x^3)^2 = x^3 \times x^3 = x^6$.

(2) $x^5 \div x^2 = x^{5-2} = x^3$. (4) $(xy)^2 = xy \times xy = x^2y^2$.

In the following discussion general proofs will be given for these laws and also their application when the indices are fractional, zero, or negative.

Definition. If x is any number and m any positive integer x^m means the product of m factors each equal to x.

1. To prove $x^m \times x^n = x^{m+n}$.

By definition:

$$x^m \times x^n = (x \times x \times x \ldots \text{to } m \text{ factors}) \times (x \times x \times x \text{ to } n \text{ factors}).$$
$$= x \times x \times x \ldots \text{to } (m+n) \text{ factors}.$$
$$= x^{m+n} \text{ by definition.}$$

From the above it follows that:

$$x^m \times x^n \times x^p = x^{m+n} \times x^p = x^{m+n+p}.$$

2. To prove $\dfrac{x^m}{x^n} = x^{m-n}$ $\quad m > n$.

By definition:

$$\frac{x^m}{x^n} = (x \times x \times x \ldots \text{to } m \text{ factors}) \div (x \times x \times x \ldots \text{to } n \text{ factors}).$$
$$= x \times x \times x \ldots \text{to } (m-n) \text{ factors}.$$
$$= x^{m-n} \text{ by definition.}$$

3. To prove $(x^m)^n = x^{mn}$.

$$(x^m)^n = (x^m) \times (x^m) \times (x^m) \text{ to } n \text{ factors}.$$
$$= x^{m+m+m \ldots \text{to } n \text{ terms}} \text{ by } \mathbf{1}.$$
$$= x^{mn}.$$

4. To prove $(xy)^m = x^m y^m$.

$$(xy)^m = (xy) \times (xy) \times (xy) \ldots \text{to } m \text{ factors.}$$
$$= (x \times x \times x \ldots \text{to } m \text{ factors}) \times (y \times y \times y \ldots \text{to } m \text{ factors}).$$
$$= x^m \times y^m = x^m y^m.$$

The above are known as the fundamental laws of indices.

In assigning a value to x^m, the definition requires that m be a positive integer, so that $x^{\frac{1}{3}}$, x^{-3}, x^0 have as yet no meaning. However, one of the advantages of Algebra over Arithmetic is that it extends the principles of Arithmetic to negative numbers, so in harmony with this principle we will assume that the laws proved for positive integral indices holds for negative and fractional indices.

(*a*) Meaning of x^0.

Since $x^m \times x^n = x^{m+n}$ for all values of m and n, if we replace m by 0, we have $x^0 \times x^n = x^{n+0} = x^n$.

$$\therefore\ x^0 = \frac{x^n}{x^n} = 1.$$

This relation was assumed when we said that $\log 1 = 0$, for by the above $10^0 = 1$, therefore by definition of logarithm, $\log 1 = 0$.

(*b*) Meaning of x^{-n}.

Since $x^m \times x^n = x^{m+n}$ for all values of m and n, if we replace m by $-n$, we have:

$$x^{-n} \times x^n = x^{-n+n} = x^0.$$
$$\text{But } x^0 = 1.$$
$$\therefore\ x^{-n} = \frac{1}{x^n}.$$

From the above it follows *that any factor may be transferred from the numerator to the denominator of an expression, or vice versa, by changing the sign of the index.*

Exercises CIII.

Express with positive indices:

1. x^{-2}.
2. p^{-6}.
3. $\frac{1}{x^{-3}}$.
4. $\frac{2}{3^{-2}}$.
5. $\frac{6^{-1}}{3^{-2}}$.
6. $\frac{4^{-1}}{2^{-2}}$.
7. $\frac{5^{-2}}{25^{-1}}$.
8. $\frac{a^{-2}}{a^{-4}}$.
9. $a^{-6}\times\frac{1}{a^2}\times\frac{1}{a^{-5}}$.
10. $\frac{1}{2^2}\times8^{-1}\times\frac{1}{16}$.
11. $3^{-2}\times\frac{1}{3}\times3^3$.
12. $\frac{1}{a^2}\times a^3\times\frac{1}{a^{-2}}$.

(*c*) Meaning of $x^{\frac{p}{q}}$, p and q being positive integers.

Since $x^m\times x^n=x^{m+n}$ for all values of m and n, if we replace both m and n by $\frac{1}{2}$, we have $x^{\frac{1}{2}}\times x^{\frac{1}{2}}=x^{\frac{1}{2}+\frac{1}{2}}=x^1=x$.

Thus if $x^{\frac{1}{2}}$ be multiplied by $x^{\frac{1}{2}}$ we get the product x, or otherwise stated the square of $x^{\frac{1}{2}}=x$.

We have, however, previously represented the quantity whose square is x by $\sqrt{x}$.

$$\therefore\ x^{\frac{1}{2}}=\sqrt{x}.$$

$$\text{Similarly } x^{\frac{1}{3}}\times x^{\frac{1}{3}}\times x^{\frac{1}{3}}=x^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}=x,$$

$$\therefore\ x^{\frac{1}{3}}=\sqrt[3]{x} \text{ (cube root of } x).$$

$$\text{Generally } x^{\frac{1}{n}}=\sqrt[n]{x} (n\text{th root of } x).$$

$$\text{Again, since } (x^m)^n=x^{mn},$$

$$\text{then } (x^{\frac{3}{4}})^4=x^3,$$

$$\therefore\ x^{\frac{3}{4}}=\sqrt[4]{x^3}.$$

$$\text{Similarly, } (x^{\frac{p}{q}})^q=x^p,$$

$$\therefore\ x^{\frac{p}{q}}=\sqrt[q]{x^p}, \ p \text{ and } q \text{ being positive integers.}$$

Example 1: $16^{\frac{1}{4}}=\sqrt[4]{16}=\sqrt[4]{2^4}=2$.

Example 2: $27^{\frac{2}{3}}=(\sqrt[3]{27})^2=3^2=9$.

Example 3: $64^{\frac{5}{6}}=(\sqrt[6]{64})^5=2^5=32$.

Exercises CIV.

Write with positive indices:

1. a^3b^{-2}.
2. $\dfrac{a^{-2}}{b^{-2}} \times \dfrac{a^3}{b^2}$.
3. $\dfrac{a^3}{a^2} \times a^{-4}$.
4. $\dfrac{a^{-2}b^{-3}}{c^{-4}d^{-5}}$.
5. $\dfrac{2x^{-1}}{4y^{-3}}$.
6. $2x^{\frac{1}{2}} \times 3x^{-1}$.
7. $2a^{-\frac{1}{x}}$.
8. $\dfrac{2}{x^{-\frac{3}{4}}}$.
9. $\dfrac{2a^{-2}}{a^{-\frac{3}{2}}}$.
10. $\dfrac{1}{\sqrt{x^3}}$.
11. $\dfrac{1}{\sqrt[3]{x^{-3}}}$.
12. $\dfrac{2}{\sqrt{y^{-3}}}$.
13. $\dfrac{4x^{-1}}{x^{-\frac{1}{2}}}$.
14. $7a^{-\frac{1}{2}} \times 3a^{-1}$.
15. $\dfrac{a^{-\frac{1}{2}}}{3a}$.

If $a=1$, $b=2$, $n=3$, find the value of:

16. $(ab)^n$.
17. $(\frac{a}{b})^n$.
18. $(a^2b^{-1})^n$.
19. $(a^nb^n)^2$.
20. $(a^{-1}b^{-1})^{-n}$.
21. $(a^{-2}b^2)^{-n}$.
22. $(a^3b^3)^{-n}$.
23. $(a^{-4}b)^n$.

Find the value of:

24. $\dfrac{2 \times 6^{-2}}{3^{-2}}$.
25. $16^{\frac{3}{4}}$.
26. $\dfrac{1}{25^{-\frac{1}{2}}}$.
27. $\dfrac{1}{8^{-\frac{1}{3}}}$.
28. $(\frac{24}{81})^{-\frac{1}{3}}$.
29. $16^{1\cdot5}$.
30. $(\frac{32}{243})^{-\frac{7}{5}}$.
31. $(\frac{162}{32})^{\frac{3}{4}}$.
32. $36^{-\frac{3}{2}}$.

Show that:

33. $12^{\frac{1}{2}} = 2 \times 3^{\frac{1}{2}}$.
34. $108^{\frac{1}{3}} = 3 \times 2^{\frac{2}{3}}$.
35. $80^{\frac{1}{4}} = 2 \times 5^{\frac{1}{4}}$.

Express as the root of an integer:

36. $3^{\frac{1}{4}} \times 3^{\frac{1}{5}}$.
37. $3^{\frac{2}{3}} \times 9^{\frac{1}{4}}$.
38. $3^{\frac{1}{2}} \times 9^{\frac{1}{4}} \div 27^{\frac{1}{5}}$.
39. Multiply $x^{\frac{1}{2}} + y^{\frac{1}{2}}$ by $x^{\frac{1}{2}} - y^{\frac{1}{2}}$.
40. Multiply $x^{\frac{1}{2}} + y^{\frac{1}{2}}$ by $x - y$.

Solve:

41. $x^{\frac{1}{3}} = 2$.
42. $x^{-\frac{1}{2}} = 4$.
43. $\dfrac{1}{x^{\frac{1}{3}}} = 4$.
44. $x^{\frac{3}{4}} = 27$.

170. Surds.

Definition. If the root of a number cannot be exactly determined, the root is called a **surd**.

Thus, $\sqrt{2}$ is a surd because we cannot find a number whose square is exactly equal to 2. We can find its value to a number of decimal places (1·4142), but this is only an approximate value.

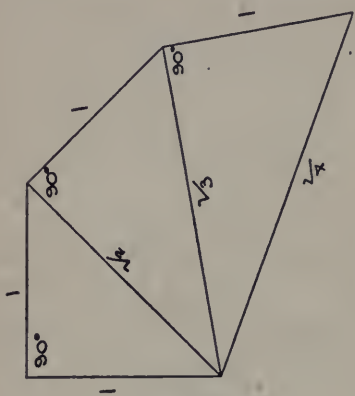


FIG. 129

We can, by a geometrical process (Fig. 129), find a line which is the $\sqrt{2}$ units in length. If we draw two lines at right angles to each other and each 1 unit in length, then the hypotenuse of the right-angled triangle so formed would be the $\sqrt{2}$ units in length. By continuing as in diagram, lines $\sqrt{3}$, $\sqrt{4}$, etc., may be found.

171. Quadratic Surds. We are chiefly concerned with surds in which the square root is to be found. These are called **quadratic surds.**

Thus, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{6}$, $\sqrt{8}$ are quadratic surds.

172. Surds other than Quadratic. These are indicated by the root symbol.

Thus, $\sqrt[3]{6}$, $\sqrt[4]{9}$, $\sqrt[5]{10}$, the first being called a surd of the third order, the second a surd of the fourth order, the third a surd of the fifth order.

A surd is sometimes called an **irrational quantity,** and for the sake of distinction, quantities which are not surds, are called **rational quantities.**

173. Like and Unlike Surds. When surds in their simplest form have the same surd factor they are called **like surds,** otherwise they are **unlike surds.**

Thus, $2\sqrt{3}$, $3\sqrt{3}$, $5\sqrt{3}$ are like surds, and $2\sqrt{3}$, $3\sqrt{2}$, $5\sqrt{6}$ are unlike surds. Just as we add and subtract like terms in Algebra, so we may add and subtract like surds.

Thus, $2\sqrt{3}+3\sqrt{3}=5\sqrt{3}$.

$$5\sqrt{2}-3\sqrt{2}=2\sqrt{2}. \quad 5\sqrt{6}+2\sqrt{6}-3\sqrt{6}=4\sqrt{6}.$$

174. **Multiplication of Surds.** Since $\sqrt{3}$ represents a quantity whose square is 3, $\therefore \sqrt{3}\times\sqrt{3}=3$.

Again, since $(\sqrt{3}\times\sqrt{2})^2=\sqrt{3}\times\sqrt{3}\times\sqrt{2}\times\sqrt{2}$.

$$=3\times2=6.$$

$$\therefore \sqrt{3}\times\sqrt{2}=\sqrt{6}.$$

Similarly, $\sqrt{3}\times\sqrt{5}=\sqrt{15}$.

Generally, $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$.

In the above the surds multiplied together are of the same order. If, however, we wished to multiply $\sqrt{2}$ and $\sqrt[3]{3}$, it would first be necessary to change them to surds of the same order.

By the previous section on indices:

$$\sqrt{2}=2^{\frac{1}{2}}=2^{\frac{3}{6}},\ \sqrt[3]{3}=3^{\frac{1}{3}}=3^{\frac{2}{6}}.$$

$$\therefore \quad \sqrt{2}\times\sqrt[3]{3}=2^{\frac{3}{6}}\times3^{\frac{2}{6}}=\sqrt[6]{2^3}\times\sqrt[6]{3^2}.$$

$$=\sqrt[6]{8}\times\sqrt[6]{9}=\sqrt[6]{72}.$$

Exercises CV.

Express as surds of the same lowest order:

1. $\sqrt{3}$, $\sqrt[3]{4}$, $\sqrt[4]{6}$.
2. $\sqrt{5}$, $\sqrt[3]{11}$, $\sqrt[6]{13}$.
3. $\sqrt[3]{2}$, $\sqrt{8}$, $\sqrt[6]{4}$.
4. $\sqrt[4]{8}$, $\sqrt[3]{3}$, $\sqrt{6}$.

Find the product of:

5. $\sqrt{2}$, $\sqrt{3}$.
6. $\sqrt{3}$, $\sqrt{5}$, $\sqrt{2}$.
7. $\sqrt{\frac{3}{2}}$, $\sqrt{\frac{4}{3}}$, $\sqrt{\frac{8}{5}}$.
8. $\sqrt{5}$, $\sqrt[3]{6}$.
9. $\sqrt{2}$, $\sqrt[3]{3}$, $\sqrt[4]{4}$.
10. $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$.

175. Mixed and Entire Surds. When a surd quantity is the product of a rational quantity and a surd, it is called a **mixed surd.** If there is no rational factor it is called an **entire surd.**

Thus, $6\sqrt{3}$ is a mixed surd, and $\sqrt{7}$ is an entire surd.

The expressing of a mixed surd as an entire surd would be of little value practically, but the reverse process is of frequent application.

$$\text{Thus,}\quad \sqrt{27}=\sqrt{9\times 3}=3\sqrt{3}.$$

$$\text{Again,}\quad \sqrt{72}=\sqrt{36\times 2}=6\sqrt{2}.$$

Exercises CVI.

Express as a single surd:

1. $2\sqrt{63}+5\sqrt{28}-\sqrt{7}$.
2. $10\sqrt{44}-4\sqrt{99}$.
3. $\sqrt{72}+\sqrt{98}-\sqrt{128}+\sqrt{32}+\sqrt{50}$.
4. $\sqrt{45}-\sqrt{20}+\sqrt{80}$.

Find the value correct to two places of decimals:

5. $\sqrt{288}$.
6. $\sqrt{147}$.
7. $\sqrt{250}$.
8. $3\sqrt{150}$.
9. $5\sqrt{245}$.
10. $4\sqrt{63}$.
11. $\sqrt{36}-\sqrt{72}+\sqrt{90}$.
12. $4\sqrt{63}+5\sqrt{7}-8\sqrt{28}$.
13. $2\sqrt{363}-5\sqrt{243}+\sqrt{192}$.
14. $5\sqrt{24}-2\sqrt{54}-\sqrt{6}$.
15. $4\sqrt{128}+4\sqrt{75}-5\sqrt{162}$.

Express in simplest form:

16. $\sqrt[3]{256}$.
17. $\sqrt[3]{432}$.
18. $\sqrt[4]{3125}$.
19. $\sqrt[3]{-2187}$.

Find the value to two decimal places:

20. $2\sqrt{14}\times\sqrt{21}$.
21. $3\sqrt{8}\times\sqrt{128}$.
22. $\sqrt{50}\times\sqrt{75}$.
23. $3\sqrt{6}\times 4\sqrt{2}$.
24. $2\sqrt{14}\times 3\sqrt{28}$.
25. $2\sqrt{15}\times 3\sqrt{5}$.
26. $8\sqrt{12}\times 3\sqrt{24}$.

176. Division of Surds.

Since $\sqrt{x}\times\sqrt{y}=\sqrt{xy}$,

$\therefore\ \sqrt{xy}\div\sqrt{x}=\sqrt{\frac{xy}{x}}=\sqrt{y}$.

Similarly, $\sqrt{x}\div\sqrt{y}=\sqrt{\frac{x}{y}}$,

and $2\sqrt{30}\div3\sqrt{6}=\frac{2}{3}\sqrt{\frac{30}{6}}=\frac{2}{3}\sqrt{5}$.

Example:—Find the numerical value of $\frac{1}{\sqrt{3}}$ (tan 30°).

We might find the square root of 3 and perform the division. This, however, would not be the best method,

For $\frac{1}{\sqrt{3}}=\frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{3}}{3}=\frac{1\cdot7321}{3}=\cdot5774$.

Here we changed $\frac{1}{\sqrt{3}}$ into $\frac{\sqrt{3}}{3}$ by multiplying both numerator and denominator by $\sqrt{3}$.

This operation of making the denominator a rational quantity is called **rationalizing the denominator.**

Example:—Find the value of $\frac{1}{\sqrt{2}}$ (sin 45°).

$$\frac{1}{\sqrt{2}}=\frac{1}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2}}{2}=\frac{1\cdot4142}{2}=\cdot7071.$$

Example:—To rationalize the denominator of an expression of the form $\frac{1+\sqrt{2}.}{2-\sqrt{2}.}$

Here we wish to convert $\frac{1+\sqrt{2}}{2-\sqrt{2}}$ into an equivalent expression but with a rational denominator.

Since the product of the sum and difference of two quantities is equal to the difference of their squares, then $(2-\sqrt{2})(2+\sqrt{2})=4-2=2$.

$$\therefore\ \frac{1+\sqrt{2}}{2-\sqrt{2}}\times\frac{2+\sqrt{2}}{2+\sqrt{2}}=\frac{(1+\sqrt{2})(2+\sqrt{2})}{2}=\frac{4+3\sqrt{2}}{2}.$$

The expression $2+\sqrt{2}$ is known as the conjugate expression to $2-\sqrt{2}$. If the denominator of the fraction had been $2+\sqrt{2}$ we would then have multiplied by $2-\sqrt{2}$.

Exercises CVII.

Calculate the value of the following to 3 places of decimals:

1. $\frac{15}{\sqrt{3}}$.
2. $\frac{2}{\sqrt{3}}$.
3. $\frac{12\sqrt{2}}{\sqrt{3}}$.
4. $\frac{6}{\sqrt{5}}$.
5. $\frac{1}{\sqrt{24}}$.
6. $\frac{48}{\sqrt{6}}$.
7. $\sqrt{\frac{256}{1575}}$.
8. $\frac{1}{\sqrt{500}}$.
9. $\frac{4}{\sqrt{243}}$.
10. $\sqrt{\frac{25}{252}}$.
11. $\frac{1}{2-\sqrt{2}}$.
12. $\frac{3}{\sqrt{5}+\sqrt{2}}$.
13. $\frac{5+2\sqrt{6}}{6-2\sqrt{6}}$.
14. $\frac{1}{\sqrt{5}-1}$.
15. $\frac{7\sqrt{2}+3}{7\sqrt{2}-3}$.
16. $3-\frac{2}{\sqrt{6}}$.
17. $(\sqrt{3}-\sqrt{2})^2$.
18. $\frac{\sqrt{3}-1}{\sqrt{2}-1}$.
19. $\frac{3\sqrt{3}-1}{3\sqrt{2}-1}$.
20. $\frac{4\sqrt{7}+3\sqrt{2}}{\sqrt{3}-\sqrt{2}}$.

CHAPTER XIX.

QUADRATIC EQUATIONS.

177. **Quadratic Equations.** The following problems will lead to equations which differ somewhat from those previously solved.

Problem 1:

The area of a square is 64 sq. in. What is the length of a side?

If x represents a side of the square, then $x^2 = 64$.

In the equations previously met the unknown x occurred to only one power, and that the first. Here, however, the unknown occurs to the second power. When an equation contains the square of the unknown quantity, but no higher power, it is called a **quadratic equation.**

In the equation $x^2 = 64$ we have the simplest form of the quadratic equation:

$$\text{If} \quad x^2 = 64.$$
$$\text{then } x = \pm 8.$$

We have here two values of x, i.e., $+8$ and -8, which will satisfy the equation. If we regard $+$ and $-$ as opposite directions in the same straight line, the minus value has no significance in determining the side of the square.

Problem 2:

The length of a number-plate on a machine is 6″ more than its width. If its area is 72 sq. in., find its dimensions.

If $x =$ No. of in. in width.

then $x + 6 =$ No. of in. in length.

then $x(x+6) = 72$.

or $x^2 + 6x - 72 = 0$.

By our previous principles in factoring $x^2+6x-72=(x+12)(x-6)$.

Now, if $x^2+6x-72=0$, then $(x+12)(x-6)=0$.

In order that the product of these two factors may be equal to zero, it is necessary that one factor should be equal to zero.

Thus the equation will be satisfied if $x+12=0$ or $x-6=0$ or if $x=-12$ or 6.

As $+6$ is the only admissible value, therefore the width $=6''$ and the length $6+6=12''$. The equation $x^2+6x-72=0$ is known as a complete quadratic equation, containing as it does both the square and the first power of the unknown quantity.

Exercises CVIII.

Solve the following equations and verify:

1. $x^2=49$.
2. $x^2+3x=0$.
3. $x^2=7x$.
4. $x^2-4=0$.
5. $6x^2=54$.
6. $x^2-3=1$.
7. $x^2-10x+21=0$.
8. $x^2-14x+48=0$.
9. $x^2-x-20=0$.
10. $x^2+10=11x$.
11. $x^2-2=x$.
12. $4x=45-x^2$.
13. $5x^2-12x+4=0$.
14. $3x^2+14x-15=0$.
15. $20x^2+41x+20=0$.
16. $5+9x-2x^2=0$.
17. $18x^2-9x-2=0$.
18. $13x^2+41x+6=0$.
19. $12x^2-x-20=0$.
20. $6x^2-x-2=0$.

178. **Solving by Completing Squares.** In connection with the squaring of a binomial we recall that $(a+b)^2=a^2+b^2+2ab$, or that the square of a binomial equals the square of each term, plus twice their product. If then we have x^2+6x and we wish to add a sufficient quantity to make a complete square, we could reason as follows: x^2 is the square of x, $6x$ is twice the product of x and 3, therefore it is necessary to add 3^2 or 9.

$\therefore x^2+6x+9$ is a complete square $=(x+3)^2$.

Similarly, to x^2-8x or $x^2-2\times x\times 4$ we must add 4^2 or 16, giving $x^2-8x+16=(x-4)^2$.

Again, to x^2+9x, or $x^2+2\times x\times\frac{9}{2}$ we must add $(\frac{9}{2})^2$ or $\frac{81}{4}$, giving $x^2+9x+\frac{81}{4}=(x+\frac{9}{2})^2$.

An analysis of the three cases above would lead us to infer *that we completed the square in each case by adding the square of half the coefficient of x.*

This method is necessary where the quadratic equation cannot readily be resolved into factors.

Thus, Example 1:—Solve $x^2-6x-13=0$.

$$\text{or,}\quad x^2-6x=13.$$

Completing the square on the left-hand side we have:

$$x^2-6x+9=13+9.$$

$$\text{or,}\quad (x-3)^2=22.$$

Extracting square root, $x-3=\pm 22$.

$$\therefore\quad x=3+\sqrt{22} \text{ or } 3-\sqrt{22}.$$

$$=7\cdot 69 \text{ or } -1\cdot 69.$$

Example 2:—Solve $-3x^2+8x+12=0$.

In the examples above on completing the square, we observe that the coefficient of x^2 in each case is unity and further that it is positive. **Before attempting then to solve this equation we must make these two changes.**

$$-3x^2+8x+12=0.$$

$$=3x^2-8x-12=0.$$

$$=\ x^2-\tfrac{8}{3}x-\ 4=0.$$

Complete the square, giving:

$$x^2-\tfrac{8}{3}x+(\tfrac{4}{3})^2=4+(\tfrac{4}{3})^2.$$

$$\text{or,}\ (x-\tfrac{4}{3})^2=\tfrac{52}{9}.$$

$$\text{or,}\quad x-\tfrac{4}{3}=\pm\sqrt{\tfrac{52}{9}}.$$

$$\text{or,}\quad x=\tfrac{4}{3}\pm\sqrt{\tfrac{52}{9}}.$$

$$=\tfrac{4}{3}+\sqrt{\tfrac{52}{9}} \text{ or } \tfrac{4}{3}-\sqrt{\tfrac{52}{9}}.$$

$$=3\cdot 74 \text{ or } -1\cdot 07.$$

Exercises CIX.

Solve by completing the square:

1. $5x^2+14x-55=0.$
2. $9x^2-143-6x=0.$
3. $19x=15-8x^2.$
4. $6x^2-9x-15=0.$
5. $5x^2+11x-12=0.$
6. $2x^2+7-9x=0.$
7. $5x^2-15x+11=0.$
8. $x^2-7x+5=0.$
9. $x^2+11=7x.$
10. $4x^2=\frac{4}{15}x+3.$
11. $x^2-2=\frac{23}{12}x.$
12. $\frac{5x+7}{x-1}=3x+2.$
13. $\frac{5x-1}{x+1}=\frac{3x}{2}.$
14. $\frac{1}{1+x}-\frac{1}{3-x}=\frac{6}{35}.$
15. $\frac{5}{x-2}-\frac{4}{x}=\frac{3}{x+6}.$
16. $\frac{4}{x-1}-\frac{5}{x+2}=\frac{3}{x}.$
17. $\frac{x+3}{2x-7}-\frac{2x-1}{x-3}=0.$
18. $21x^2-2ax-3a^2=0.$
19. $12x^2+23kx+10k^2=0.$
20. $\frac{1}{2x-5a}+\frac{5}{2x-a}=\frac{2}{a}.$

179. **The General Quadratic Equation.** From the preceding examples it is apparent that every quadratic equation can be reduced to the form

$$ax^2+bx+c=0,$$

where a, b, c may have any numerical values whatever. If then we would solve this general quadratic equation we could use the result as a formula to solve particular cases and consequently save the labour entailed.

$$ax^2+bx+c=0.$$

Transposing, $ax^2+bx=-c.$

dividing by a, $x^2+\frac{b}{a}x=-\frac{c}{a}.$

Completing the square by adding to each side the square of half the coefficient of x, i.e., $\left(\frac{b}{2a}\right)^2.$

giving $x^2+\frac{bx}{a}+\left(\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}$

or, $\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}.$

Extracting the square root

$$x+\frac{b}{2a}=\frac{\pm\sqrt{b^2-4ac}}{2a}.$$

Hence, $$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

We might here restate the steps required in solving a quadratic of the above form.

(1) *Simplify the equation so that the terms in* x^2 *and* x *are on one side of the equation, and the term without* x *on the other.*

(2) *Make the coefficient of* x^2 *unity and positive by dividing throughout by the coefficient of* x^2.

(3) *Add to each side the square of half the coefficient of* x *thus completing the square.*

(4) *Take the square root of each side.*

(5) *Solve the resulting simple equations.*

Example 1:—Solve $x^2-5x-3=0$.

Here, $a=1, b=-5, c=-3$.

$$\therefore x=\frac{5\pm\sqrt{(-5)^2-4\times1\times(-3)}}{2}.$$

$$=\frac{5\pm\sqrt{25+12}}{2}.$$

$$=\frac{5\pm\sqrt{37}}{2}=\frac{5\pm6\cdot08}{2}.$$

$$=5\cdot54 \text{ or } -\cdot54.$$

Example 2:—Solve $x^2-2x+5=0$.

Here, $a=1,\ b=-2,\ c=5$.

$$\therefore x=\frac{2\pm\sqrt{(-2)-4\times1\times5}}{2}.$$

$$=\frac{2\pm\sqrt{4-20}}{2}.$$

$$=\frac{2\pm\sqrt{-16}}{2}.$$

In the preceding result the numerical value of the roots cannot be found, as there is no number whose square is negative.

Such a quantity as $\sqrt{-16}$ is called an **imaginary quantity,** and the roots are said to be **imaginary.** This is equivalent to saying that there is no real number which will satisfy the equation $x^2-2x+5=0$.

180. **There are some Equations** that are not really quadratics but may be solved by the methods of this chapter.

Example 1:—Solve $x^4-5x^2+4=0$.

Factoring $(x^2-4)(x^2-1)=0$.

$$\therefore x^2=4 \text{ or } x^2=1.$$

$$\therefore x=\pm 2 \text{ and } x=\pm 1.$$

Example 2:—Solve $x^2-x+\dfrac{72}{x^2-x}=18$.

Write y for x^2-x, then we have:

$$y+\frac{72}{y}=18.$$

or, $y^2-18y+72=0$.

Factoring, $(y-12)(y-6)=0$.

giving $y=12$ or 6.

$\therefore\ x^2-x=12$ or 6.

If $x^2-x=12$ then $x^2-x-12=0$.

then $(x-4)(x+3)=0$.

giving $x=4$, or -3.

If $x^2-x=6$, then $x^2-x-6=0$.

then $(x-3)(x+2)=0$.

giving $x=3$ or -2.

Exercises CX.

Solve the following examples:

1. $3x^2-17x+10=0$.
2. $2x^2+19x+9=0$.
3. $2(x^2+1)-5x=0$.
4. $25x^2-7x-86=0$.
5. $7x^2+32x-15=0$.
6. $(2x-1)^2=25$.
7. $10x^2=13x+9$.
8. $x^4-13x^2+36=0$.
9. $4x^2=\frac{4x}{15}+3$.
10. $x^2+3x-\frac{20}{x^2+3x}=8$.
11. $(x^2+2)^2+198=29(x^2+2)$.
12. $x^6-19x^3-216=0$.

13. A rectangular name-plate for a machine is to be $1\frac{1}{2}''$ longer than it is wide and to have an area of 10 sq. in. What will be its dimensions?

14. Three holes are to be drilled so that they will lie at the three corners of a triangle ABC, right angled at B. The distance from A to C is to be $10''$ and the distance from B to C is to be $2''$ more than from A to B. Find AB and BC.

15. The sides AB, BC, CA of a triangle measure 13, 14, 15 respectively. From A a perpendicular AD is drawn to BC. If BD measures x, express the length of AD in two ways. Equate the results and find x.

16. The owner of a rectangular lot 15 rods by 5 rods, wishes to double the size of the lot by increasing the length and the width by the same amount. What should be the increase?

17. A straight line is $10''$ long. Divide it into two parts so that the rectangle contained by the whole line and one of the parts is equal to the square on the other part.

18. $S=\frac{1}{2}gt^2$ is the law governing a body falling from rest, s = space, g = acceleration due to gravity (32 ft.), t = time in seconds. How long will it take a stone to fall from the top of the City Hall tower, Toronto, if it be 305 ft. high?

19. $S=ut+\frac{1}{2}gt^2$ is the law for a falling body when it has an initial velocity, u representing this initial velocity. If a stone be thrown with an initial velocity of 8 ft. per sec. from the top of the Eiffel tower, 984 ft. high, in what time will it reach the ground?

181. Simultaneous Quadratic Equations. The following problems will lead to simultaneous equations where one at least is of higher degree than the first.

Problem:

The perimeter of a rectangle is 18″, and its area is 20 sq. in.; find its length and breadth.

If x represent the length and y the breadth, then:

$$2x+2y=18.$$

$$\text{or,}\quad x+y=9 \ldots\ldots\ldots\ldots (a).$$

$$\text{also,}\quad xy=20 \ldots\ldots\ldots\ldots (b).$$

Solution—1st method.

$$\text{from } (a),\ y=9-x.$$

$$\text{Substitute in } (b),\ x(9-x)=20.$$

$$\text{or,}\quad 9x-x^2=20.$$

$$\text{or,}\quad x^2-9x+20=0.$$

$$\text{or,}\quad (x-5)(x-4)=0.$$

$$\therefore\quad x=5 \text{ or } 4.$$

$$\therefore\quad y=\frac{20}{5}=4 \text{ or } \frac{20}{4}=5.$$

$$\therefore\quad x=5 \text{ or } 4.$$

$$y=4 \text{ or } 5.$$

Solution—2nd method.

$$(a)^2=x^2+2xy+y^2=81.$$

$$4\times(b)=\quad 4xy \quad =80.$$

$$\text{Subtracting,}\quad x^2-2xy+y^2=1.$$

$$\therefore\quad (x-y)^2=1.$$

$$\text{or,}\quad x-y=\pm 1.$$

$$\begin{array}{ll} x+y=9 & x+y=9 \\ x-y=1 & x-y=-1 \\ \hline 2x \quad =10,\ x=5. & 2x \quad =8,\quad x=4. \\ 2y \quad =\ 8,\ y=4. & 2y \quad =10,\ x=5. \end{array}$$

$$\therefore\ x=5 \text{ or } 4,\ y=4 \text{ or } 5.$$

Exercises CXI.

Solve the following equations:

1. $x+y=28,$
   $xy=187.$

2. $x-y=5,$
   $xy=126.$

3. $x-y=8,$
   $xy=513.$

4. $x+y=84,$
   $xy=923.$

5. $x^2+y^2=74,$
   $xy=35.$

6. $x^2+y^2=89,$
   $xy=40.$

7. $x^2+y^2=178,$
   $x+y=16.$

8. $x^2+y^2=185,$
   $x-y=3.$

9. $\frac{1}{x}+\frac{1}{y}=2,$
   $x+y=2.$

10. Divide a straight line 7″ long into two parts so that the rectangle contained by the parts may be equal to 12 sq. in.

11. Divide a straight line 12″ long into two parts so that the sum of the squares on the parts may be equal to 74 sq. in.

12. The hypotenuse of a right-angled triangle is 25″, and the perimeter is 56″, find the sides.

CHAPTER XX.

VARIATION.

182. **Quantities are often related** to each other in such a way that any change in one quantity produces a corresponding change in the other.

For example, consider a train travelling with a uniform speed. If in one hour the train travels 30 miles, then in two hours it will travel 60 miles, and so on.

We may state this by saying that the distance travelled is proportional to the time, or that the distance varies directly as the time.

If we represent distance by d and time by t, the relation may be expressed by d varies as t.

The symbol $\propto$ represents "varies as", therefore we have $d \propto t$.

If we let $d_1, d_2, d_3 \ldots\ldots$ be successive values of d and $t_1, t_2, t_3 \ldots$ corresponding successive values of t, then, we have:

$$\frac{d}{d_1}=\frac{t}{t_1} \text{ or } d=\frac{d_1}{t_1}t.$$

$$\text{also, } \frac{d}{d_2}=\frac{t}{t_2} \text{ or } d=\frac{d_2}{t_2}t.$$

$$\text{also, } \frac{d}{d_3}=\frac{t}{t_3} \text{ or } d=\frac{d_3}{t_3}t, \text{ etc.}$$

Let us consider the expressions $\frac{d_1}{t_1}, \frac{d_2}{t_2}, \frac{d_3}{t_3} \ldots\ldots$ in the light of the above illustration.

$$\text{If } d_1=60,\ t_1=2, \text{ then } \frac{d_1}{t_1}=\frac{60}{2}=30.$$

$$\text{If } d_2=90,\ t_2=3, \text{ then } \frac{d_2}{t_2}=\frac{90}{3}=30.$$

$$\text{If } d_3=120,\ t_3=4, \text{ then } \frac{d_3}{t_3}=\frac{120}{4}=30.$$

From the above we see that the ratios $\frac{d_1}{t_1}, \frac{d_2}{t_2}, \frac{d_3}{t_3}$ are each equal to 30, therefore we infer that $d \propto t$ becomes $d=30t$ or generally $d=\text{Constant} \times t$ or $d=kt$ (k being a constant).

Example 1:

The circumference of a circle varies directly as its diameter. A circle 7″ in diameter has a circumference of 22″, find the circumference of a circle of 64″ diameter.

From the above $C=kd$,

then, $22=k7$,

or, $k=\frac{22}{7}$,

$$\therefore\ C=\frac{22}{7}D.$$

Substituting for D the value 64,

$$C=\frac{22}{7}\times 64=201\cdot 14''.$$

In the above we observe that the first set of conditions enable us to find the constant k. The equation is then one between C and D, and from any value of one of these we can find the other.

Example 2:

The areas of circles vary directly as the squares of their radii. If a circle with a radius of 7″ has an area of 154 sq. in., find the radius of a circle with an area of 1386 sq. in.

From the above $A \propto r^2$. $\therefore A=kr^2$, then $154=k\ 7^2$, giving $k=\frac{22}{7}$.

Then substituting for A the value 1386 we have:

$$1386 = \frac{22}{7} r^2.$$

$$\therefore\ r = 21.$$

Example 3:

The volume of a gas varies inversely as the height of the mercury in the barometer. If the volume is 22 cu. in. when the barometer registers 30″, what is the volume when the barometer registers 32″?

Here we have a case of varying "inversely." This means that an **increase** in one quantity gives a proportionate **decrease** in the other. *Hence, when one quantity varies inversely as another it varies as the reciprocal of the other.*

In the above $V \propto \frac{1}{H}$ or $V = k\frac{1}{H}$.

From the conditions given, $22 = \frac{k}{30}$, giving $k = 660$,

then, $V = \frac{660}{32} = 20 \cdot 6$ cu. in.

Exercises CXII.

1. The strength of a beam varies directly as the square of its thickness.

A beam of given length and width and 6″ thick carries a maximum load of 5 tons. What load will a beam of the same width and length, but 12″ in thickness carry?

2. The weight of a substance varies directly as its volume. A steel bar containing 100 cu. in. weighs 28·3 lb. What is the weight of a bar of the same material containing 642 cu. in.?

3. The velocity of a falling body varies directly as the time during which it is falling. When a body falls from rest, its velocity at the end of 1 sec. is approximately 32 ft. per second. Compute its velocity at the end of 15 seconds.

4. The velocity of the rim of a pulley varies directly as its diameter. A 12″ pulley has a rim velocity, at a certain moment, of 160′ per min. What is the rim velocity, at the same moment, of a $9\frac{1}{2}$″ pulley which is keyed to the same shaft?

5. The deflection of a beam under a given load varies inversely as the square of the thickness. If a given beam carrying a certain load is 5″ in thickness and has a deflection (due to the load) of 2″, what deflection will be produced by the same load if the thickness be $7\frac{1}{2}$″?

6. The weight of a body varies inversely as the square of its distance from the centre of the earth. If a substance weighs 10 lb. at sea level (3960 miles from the centre), compute its weight on the top of a mountain 29,000 ft. above sea level.

7. The areas of circles are to one another as the squares of their diameters. If a circle with a diameter of 14″ has an area of 154 sq. in., find the diameter of a circle with an area of 320 sq. in.

8. The pressure per sq. in. on a hydraulic ram varies inversely as the square of the diameter, if the total load on the ram is constant. If a load supported by a hydraulic ram of 8″ diameter gives a pressure per sq. in. of 40 lb., find the pressure per sq. in. on a $3\frac{1}{2}$″ ram which supports an equal load.

9. The horse-power of the engines of a ship varies directly as the cube of the speed. If the horse-power is 1800 at a speed of 10 knots, what is the power when the speed is 23·5 knots?

10. The volumes of spheres vary directly as the cubes of their diameters. If a sphere with a diameter of 6″ has a volume of $113\frac{1}{7}$ cu. in., find the diameter of a sphere whose volume is 616 cu. in.

11. The number of rivets required for a boiler seam varies inversely as the pitch (the distance between rivet centres).

If 35 rivets are required when the pitch is $2\frac{3}{4}$″, determine the pitch when 40 rivets are required for a boiler of the same size.

12. The resistance of a wire varies inversely as the square of its diameter.

The resistance of a coil of copper wire $\frac{1}{4}$″ in diameter was 3 ohms. What is the diameter of a wire of the same length with a resistance of 2·3 ohms?

183. **Problems involving more than Two Variables.** If we take two rectangles of the same width, it is readily seen that their areas vary as their lengths. If again we take two rectangles of different widths but the same length, it is further agreed that their areas vary as their widths.

We now wish to consider the variation in the area when both the length and width vary.

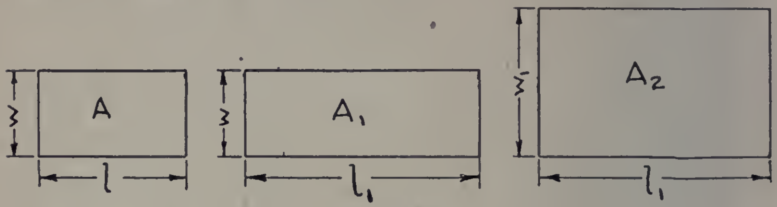


FIG. 130

In Figure 130 above the rectangles (1) and (2) have the same width but different lengths, while the rectangles (2) and (3) have the same length but different widths.

If A, A_1, and A_2 represent the respective areas, then with the above notation:

$$\frac{A}{A_1}=\frac{lw}{l_1w}=\frac{l}{l_1} \qquad (a),$$

also, $$\frac{A_1}{A_2}=\frac{l_1w}{l_1w_1}=\frac{w}{w_1} \qquad (b).$$

$$(a)\times(b) \quad \frac{A}{A_1}\times\frac{A_1}{A_2}=\frac{l}{l_1}\times\frac{w}{w_1}.$$

or, $$\frac{A}{A_2}=\frac{lw}{l_1w_1} \text{ or } A=\frac{A_2}{l_1w_1}\,lw.$$

Since, $A_2=l_1w_1 \; \therefore \; \frac{A_2}{l_1w_1}=1$ (constant).

$\therefore \; A=\text{Constant}\times lw.$

$\therefore \; A \propto lw.$

From this we state that the area of a rectangle varies as the product of its length and width, when both the length and width vary.

Further we know that triangles of the same altitude are to one another as their bases, and also that triangles of equal bases are to one another as their altitudes. Hence we might, as in the case of the rectangle, prove that the area of a triangle varies as the product of the base and altitude when both base and altitude vary.

Again, the volumes of cylinders of the same height are to one another as their bases, and also the volumes of cylinders with equal bases are to one another as their heights. Hence it might be proved that the volume of a cylinder varies as the product of the base and height, when both base and height vary.

From these illustrations we infer the general theorem:—*If A varies as B when C is constant, and A varies as C when B is constant, then A varies as BC when both B and C vary.*

Definition—One quantity is said to vary **jointly** as a number of others when it varies directly as their product.

Example 1:

The volume of a cone varies jointly as its altitude and the area of its base. The volume is 392·7 cu. in. when the altitude is 15″ and the diameter of the base 10″. Find the diameter when the altitude is 22″ and the volume 436 cu. in.

$$\text{Here } V \propto AB \text{ or } V = kAB.$$

Substituting the first conditions:

$$392{\cdot}7 = k\ 15 \times {\cdot}7854 \times 10^2.$$

$$\text{giving } k = \frac{392{\cdot}7}{15 \times {\cdot}7854 \times 10^2}.$$

From the second conditions:

$$436 = \frac{392{\cdot}7}{15 \times {\cdot}7854 \times 10^2} \times 22B.$$

$$\therefore\ B = \frac{436 \times 15 \times {\cdot}7854 \times 10^2}{22 \times 392{\cdot}7}.$$

If d be required diameter,

$$\text{then}\quad {\cdot}7854\ d^2 = \frac{436 \times 15 \times {\cdot}7854 \times 10^2}{22 \times 392{\cdot}7}.$$

$$\therefore\ d^2 = \frac{436 \times 15 \times 10^2}{22 \times 392{\cdot}7}.$$

$$\therefore\ d = 8{\cdot}7''.$$

Example 2:

The volume of a gas varies inversely as the pressure and directly as the absolute temperature (the absolute temperature is obtained by adding 273 to the temperature on the Centigrade scale).

If a quantity of nitrogen under 900mm. pressure at 20° C. occupies a volume of 300cc., what volume will it occupy at 100° C. under 600mm. pressure?

$$\text{Here, } V \propto \frac{T}{P} \text{ or } V = k\,\frac{T}{P}.$$

From first conditions:

$$300 = k\,\frac{293}{900}, \text{ giving } k = \frac{300 \times 900}{293}.$$

From second conditions:

$$V = \frac{300 \times 900}{293} \times \frac{373}{600} = 572{\cdot}87\text{cc.}$$

Exercises CXIII.

1. The area of a triangle varies jointly as its base and altitude. The area of a triangle whose base is 19′ and whose altitude is 10′ is 95 sq. ft. Find the altitude when the base is 22·5′ and the area 134 sq. ft.

2. The volume of a pyramid varies jointly as its height and the area of its base. When the height is 18′ and the base a square 8′ to the side, the volume is 384 cu. ft. What is the side of the base if a pyramid of the same form, 10′ high, has a volume of 432 cu. ft.?

3. The pressure of the wind perpendicular to a plane surface varies jointly as the area of the surface and the square of the velocity of the wind. Under a velocity of 16 miles per hour the pressure on 1 sq. ft. is 1 lb., what is the velocity when the pressure on 3 sq. yd. is 68 lb.?

4. The amount of illumination received by a body varies directly as the intensity of the light and inversely as the square of the distance from the light.

From a light of 14 candle-power, the illumination is 5 at a distance of 8′. Find the illumination at a distance of 10′ from a light of 40 candle-power.

5. The intensity of a magnetic field varies directly as the number of vibrations and inversely as the square of the distance of the magnet. When the distance is ·5″, and the number of vibrations 15, the intensity is ·14. Find the intensity when the distance is ·075″, and the number of vibrations 90.

6. The heat developed in a conductor varies jointly as the resistance of the conductor, the time the current flows, and the square of the current. In 2 minutes a current of 4 amperes developed 1400 units of heat in a wire having a resistance of 11 ohms. Find the resistance of a wire of the same size in which 20,000 units of heat were developed by a current of 6·84 amperes in $3\frac{1}{2}$ minutes.

7. The stiffness of a rectangular beam varies jointly as the breadth and as the cube of the depth. Show that two beams of the same material of breadth and depth (1) 2″, 3″, (2) 1″ 3·78″, are of nearly the same stiffness.

8. If the attraction between two masses varies directly as their product and inversely as the square of the distance between their centres, what would 1 lb. weigh on the surface of a planet of the same density as the earth, but 1·5 times the diameter?

9. The square of the time which a body takes to slide down an incline varies as the square of the length and inversely as the height. If the time taken is 1 sec. when the height is 4′ and length 8′, what must be the height of a plane 3′ long so that the body may slide down in $\frac{1}{2}$ sec.?

10. The volume of a cylinder varies jointly as its base and height. Of two cylinders volume of 1st: volume of 2nd = 11: 8, and height of 1st: height of 2nd = 3:4. If the base of the first is 16·5 sq. ft., find the base of the second.

11. The volume of a gas varies inversely as the pressure and directly as the absolute temperature.

What would be the volume at 0°C. and 760mm. pressure of a mass of oxygen whose volume is 60cc. at 40C., under a pressure of 750mm. of mercury?

12. At what temperature must a gas be so that its volume will be 15 litres when the pressure is 800mm., if its volume is 175 litres when its temperature is 100°C., and the pressure 700mm.?

13. The electrical resistance of a wire varies directly as the length and inversely as the square of the diameter of the wire. Its weight varies jointly as the length and the square of the diameter.

If a pound of wire of diameter ·06″ has a resistance of ·25 ohms, what is the resistance of a pound of wire of the same material, the diameter being ·01″?

CHAPTER XXI.

GEOMETRICAL PROGRESSION.

184. **Amount.** If I deposit \$100 in a savings bank which pays interest annually at 4%, I will be entitled to \$4 interest at the end of the first year. If I choose to leave this interest on deposit my bank account would then be \$104. This sum, representing the **principal plus the interest,** is said to be the **amount** of \$100 in one year.

Consider the following examples.

Example 1:

Find the amount of \$100 in 3 years at 6% per annum, compounded yearly.

The interest at the end of the first year $= \frac{6}{100}$ of \$100.

The sum itself $= \frac{100}{100}$ of \$100.

$\therefore$ the amount at the end of the first year $= \frac{106}{100}$ of \$100.

$$= \$100(1 \cdot 06).$$

The interest at the end of the second year $= \frac{6}{100}$ of $\$100(1 \cdot 06)$.

$\therefore$ the amount at the end of the second year

$$= \frac{106}{100} \text{ of } \$100(1 \cdot 06).$$

$$= \$100(1 \cdot 06)^2.$$

The interest at the end of the third year $= \frac{6}{100}$ of $\$100(1 \cdot 06)^2$.

$\therefore$ the amount at the end of the third year $= \frac{106}{100}$ of $\$100(1 \cdot 06)^2$

$$= \$100(1 \cdot 06)^3$$

$$= \$119 \cdot 10.$$

Example 2:

A man saves \$200 a year for 4 years. If each year he invests it at 6% per annum, what are his accumulated savings 4 years from the date of his first investment?

The amount of the first \$200 = \$200(1·06)^4.

The amount of the second \$200 = \$200(1·06)^3.

The amount of the third \$200 = \$200(1·06)^2.

The amount of the fourth \$200 = \$200(1·06).

$$\begin{aligned}\text{Accumulated savings} &= \$200(1\cdot06)^4+\$200(1\cdot06)^3\\ &\quad+\$200(1\cdot06)^2+\$200(1\cdot06),\\ &=\$200(1\cdot06+1\cdot12360+1\cdot19102+1\cdot26248).\\ &=\$200\times4\cdot63710.\\ &=\$927\cdot42.\end{aligned}$$

If in the preceding example the time had been 15 years, instead of 4 years, it is apparent that considerable work would be involved. The following discussion will develop a formula for shortening the work.

Consider the Series:

$$a+ar+ar^2+ar^3\ldots\ldots\ldots\ldots ar^{n-1}.$$

It is apparent that (1) each term is obtained from the preceding by multiplying by r, called the common ratio, (2) in the second term r is raised to the first power, in the third term to the second power, ∴ in the nth term to the $(n-1)^{\text{th}}$ power, (3) there are n terms in the series.

Such a series is called a **Geometrical Series** and the terms in the series are said to be in **Geometrical Progression.**

Let $S=a+ar+ar^2\ldots\ldots\ldots\ldots ar^{n-2}+ar^{n-1}$.

$\therefore\ rS=\quad ar+ar^2\ldots\ldots\ldots\ldots\quad ar^{n-1}+ar^n$.

$\therefore\ rS-S=ar^n-a$.

$\therefore\ S\ (r-1)=a\ (r^n-1)$.

$$\therefore\ S=\frac{a(r^n-1)}{r\ -1}.$$

We here observe that in the above formula a is the first term, r the common ratio, and n the number of terms.

Returning to our previous difficulty in finding the accumulated savings at the end of 15 years, we have as in Example 2.

Accumulated Savings

$$= \$200(1\cdot 06)^{15} + \$200(1\cdot 06)^{14} \ldots . \$200(1\cdot 06)$$

$$= \$200\{1\cdot 06 + (1\cdot 06)^2 \ldots . (1\cdot 06)^{15})\}.$$

The series within the brackets is evidently a geometrical series, the first term being $1\cdot 06$, the common ratio $1\cdot 06$, and the number of terms 15.

$$\therefore \text{Sum} = 1\cdot 06\left\{\frac{(1\cdot 06)^{15}-1}{1\cdot 06-1}\right\} = 1\cdot 06\left\{\frac{2\cdot 39656-1}{1\cdot 06-1}\right\}.$$

$$\therefore \text{Accumulated Savings} = \frac{\$200 \times 1\cdot 06 \times 1\cdot 39656}{\cdot 06}$$

$$= \$4934.88.$$

Note.—The value of $(1\cdot 06)^{15}$ may be obtained from interest tables or by the use of logarithms.

185. **Present Worth.** If a person owes me \$106 a year from now, and money is worth 6%, I might as well accept \$100 now.

The \$100 is here called the **Present Worth** of \$106 and we may, therefore, define **the present worth of a future payment as the sum which will at the given rate, amount to the payment when due.**

Example:

A man wills his son \$1000 a year for 10 years. What is this legacy worth now, if money is worth 5% per annum?

The amount of \$1000 one year hence $= \$1000(1\cdot 05)$.

$\therefore$ \$1000$(1\cdot 05)$ due one year hence has for present worth \$1000.

$\therefore$ \$1000 due one year hence has for present worth $\dfrac{\$1000}{1\cdot 05}$.

The amount of \$1000 two years hence $= \$1000(1\cdot 05)^2$.

$\therefore$ \$1000$(1\cdot05)^2$ due two years hence has for present worth \$1000.

$\therefore$ \$1000 due two years hence has for present worth $\frac{\$1000}{(1\cdot05)^2}$.

Similarly \$1000 due three years hence has for present worth $\frac{\$1000}{(1\cdot05)^3}$.

$\therefore$ Present Worth of all the payments equals

$$\frac{\$1000}{1\cdot05}+\frac{\$1000}{(1\cdot05)^2}+\frac{\$1000}{(1\cdot05)^3}\cdots\cdots+\frac{\$1000}{(1\cdot05)^{10}}.$$

$$=\frac{\$1000}{(1\cdot05)^{10}}\left\{1+(1\cdot05)+(1\cdot05)^2\cdots\cdots+(1\cdot05)^9\right\}$$

$$=\frac{\$1000}{(1\cdot05)^{10}}\left[1\left\{\frac{(1\cdot05)^{10}-1}{1\cdot05-1}\right\}\right]$$

$$=\frac{\$1000}{(1\cdot05)^{10}}\times\frac{1\cdot62889-1}{1\cdot05-1}=\frac{\$1000}{(1\cdot05)^{10}}\times\frac{\cdot62889}{\cdot05}=\$7721\cdot54.$$

Exercises CXIV.

1. Find the amount of \$450 if left on deposit in a bank for 3 years, if interest at 3% per annum compounded half-yearly be allowed?

2. What sum of money loaned at 5% per annum will in 7 years yield \$407·10 interest?

3. What sum will amount to \$1986·86 in $17\frac{1}{2}$ years at 4% per annum?

4. A person deposits \$100 in a savings bank on January 1st, 1911, and the same sum each year until January 1st, 1921. If banks pay 3% per annum, compounded half-yearly, what sum stands to his credit just after making the deposit on January 1st, 1921?

5. A person holds \$6000 in bonds paying 5% per annum. He dies leaving the income for the first 10 years to his son. If money is worth 6% per annum, what is the present worth of the legacy?

6. A mortgage for \$5000 with interest at 6% per annum has 5 years to run. It is necessary to realize on the mortgage. What sum should a person pay for it, if he wishes to make 7% on his money?

Under what conditions would it be worth \$5000?

7. On November 1st, 1920, Brown invests \$6000 in Victory Bonds, due November 1st, 1933, paying $5\frac{1}{2}\%$ per annum payable half-yearly. If money is worth 6% per annum compounded half-yearly, what is the present worth of the bonds? (November 1st, 1921).

8. A man deposits \$200 a year with a loan company which pays 4% per annum compounded quarterly. What sum stands to his credit at the end of 5 years?

9. A man dies leaving an annuity of \$500 to his eldest son for 10 years and then to his second son for the following 10 years. If money is worth 6% per annum, what is the present worth of each legacy?

10. A municipality borrows \$60,000 at 5% per annum. What amount must be collected each year so that the debt may be discharged in 10 equal annual payments, if money is worth 6% per annum?

11. A man invests \$500 in a business which pays 5% per annum. Each year he invests an amount 10% greater than the previous year. What amount stands to his credit at the end of 10 years, if he reinvests his dividends in the business?

12. A man takes a 20-year endowment policy of \$1000 on which the annual premium is \$48·50. If he dies just after the twelfth payment, how much more will his heirs receive than if he had invested the money at 5% per annum? If he had lived, how much less will he receive than if he had invested the money as above?

13. A man takes a straight life policy for \$5000 on which the annual premium is \$136. If he dies just before making the 25th payment, compare the financial returns with having invested the premium each year at 6%.

MISCELLANEOUS EXERCISES.

1. A man ordered $11\frac{1}{2}$ tons of coal. The first load contained 9500 lb., the second 7000 lb. How many tons remain to be delivered?

2. A tank containing 400 gallons has two pipes opening from it. One pipe can empty it in 2 hours, the other in $2\frac{1}{2}$ hours. If both pipes be opened for 15 minutes, how many gallons are left in the tank?

3. The recent summer vacation extended for 72 days. A boy spent $\frac{1}{6}$ of it in the country, $\frac{3}{8}$ of it camping, and the remainder in the city. How many days did he spend in the city?

4. A bricklayer received 90c. an hour for an 8-hour day. In the last year he worked 221 days, what was his total income?

5. A gang of men working on the roadway place $24\frac{1}{8}$ cu. yd. of concrete in 1 hour. How many cu. yd. do they place in 3 days of $8\frac{1}{2}$ hours each?

6. An alloy contains $\frac{19}{28}$ copper, $\frac{1}{7}$ tin, and the balance zinc. How many lb. of each are there in an alloy of 336 lb.?

7. If beef is worth $21\frac{3}{5}$c. per lb., what is the value of beef weighing 532 lb.?

8. One pipe can empty a tank in $3\frac{1}{3}$ hr., and another pipe can empty it in $2\frac{3}{5}$ hr. In what time can they both empty it if running together?

9. A drill with a feed of ·01″ per revolution is making 80 R.P.M. How long will it take to drill 25 holes through a $\frac{1}{2}$″ plate if 15 seconds be lost in setting for each hole?

10. A contractor is to finish a piece of roadway in 20 days. Twelve men work for 8 days and do $\frac{1}{3}$ of the work. How many men must be employed for the balance of the time to finish according to contract?

11. A certain pump delivers 1·43 gallons per stroke. If a gallon of water weighs 10 lb., what weight of water will be delivered in 218 strokes?

12. A drill with a feed of 100 is making 50 R.P.M. If $\frac{1}{3}$ of the time is used in setting, how many holes can be drilled in a $\frac{1}{2}$″ plate in 2 hours?

13. A gallon of water contains 277·274 cu. in. What is the percentage error in taking $6\frac{1}{4}$ gallons as equivalent to 1 cu. ft.?

14. A reamer 9″ long is 1·375″ in diameter at one end and 1·125″ at the other end. What is the taper per foot?

15. How long will it take to excavate to a depth of 4′ for a building having a frontage of 74′ 0″, and a depth of 243′ 0″, using a steam shovel if the bucket holds two-thirds of a cu. yd. and is filled 3 times every 2 minutes?

16. The American gallon contains 231 cu. in. What per cent. is it of the English gallon?

17. In building a roadway containing 1000 cu. yd., the total cost was as follows:—4000 bags of cement at \$1.20; 1000 cu. yd. of stone at \$2.00; 450 cu. yd. of sand at \$1.80; lumber \$225; labor \$3240, tools, etc., \$200. Find the average cost per cu. yd.

18. A pressure of 48·5 lb. per sq. ft. is how many ounces per sq. in.?

19. A man walks $2\frac{1}{4}$ miles. If his average step is 2′ $7\frac{1}{2}$″, how many steps does he take?

20. A room is to be floored with $\frac{7}{8}$″ tongued and grooved flooring taken out of 6″ material, allowing $\frac{1}{2}$″ in width for the dressing and tongue. The room is 12′ 6″×15′ 6″, and has a square bay window 10′ 3″ wide × 3′ 0″ deep. How many square feet are in the room and what number of feet run of flooring would be required?

21. Reduce 13 quarts to the decimal of a bushel.

22. If sound travels 1120 ft. per sec., how long will it take to hear the report of a gun fired at a distance of $8\frac{1}{4}$ miles?

23. An electric-light bill was $16\frac{2}{3}$% higher this month than last. It was \$3.42 last month, what is it this month?

24. In a certain machine $\frac{3}{8}$ of the power supplied is lost in friction, etc. What is the percentage efficiency of the machine?

25. In a room 15′ 3″×22′ 6″ it is required to lay a clear quarter cut white oak floor $\frac{7}{8}$″×$1\frac{1}{2}$″ face measure. Allowing 30% for loss in dressing and working tongue, how much would the flooring cost at \$275 per thousand sq. ft.?

26. In a vernier caliper the reading shows 1″, 2 tenths, 1 small division, while the 8th division of the vernier is in line with a beam division. In reading the instrument I neglect to add the vernier reading. What is the percentage error?

27. A steel hook when immersed in water displaced $1\frac{1}{2}$ quarts. If the specific gravity of steel is 7·8, find the weight of the hook.

28. When casting iron pipe, an allowance of $\frac{1}{8}$″ per foot is allowed for shrinkage. What percent. is this?

29. If the specific gravity of ice is ·921 and of salt water 1·024, what part of an iceberg is below the surface of the water when floating?

30. An alloy contains 22·5% of nickel and 1·2% of carbon. How many pounds of each are there in 500 pounds of the alloy?

31. Find the cost, at \$60 per M, of building a walk 60′ long by 6′ wide; plank to be 2″ thick and laid crosswise on 3 pieces, 4″×4″.

32. A mixture for casting contains 5 parts copper, 4 parts lead, and 3 parts tin. What is the percentage composition?

33. Find the cost of shingling a roof 16′×20′ with shingles laid $4\frac{1}{2}$″ to the weather, if the cost of material and labour is \$12 per square of shingles.

34. A road bed rises 2′ 3″ in 300′. What percent. grade is this?

35. Find the length of the longest straight line that can be drawn in a room 20′ long 16′ wide and 12′ high.

36. A barn is 40′ 0″ wide and 60′ 0″ long, with a roof $\frac{1}{3}$ pitch. The rafter heel and projections at ends are each 2′. Find the cost of covering with 1″ sheeting at \$60 per M, allowing 5% for waste.

37. Find the weight of a hexagonal bar of iron $\frac{5}{8}$″ to a side and 6′ long. (1 cu. in. = ·26 lb.).

38. Find the size of tap drill for a $\frac{9}{16}$″, 12 pitch, sharp "V" thread nut.

39. An equilateral triangle has an area of 32·54 sq. in. What is the length of a side?

40. In a steel plate 4′×2′ 3″ and $\frac{1}{2}$″ thick, 10 round holes are bored each $1\frac{1}{2}$″ in diameter. Find the weight of the plate after boring.

41. The diameter of the safety valve in a boiler is $3\frac{1}{2}$″. If the pressure of the steam is 150 lb. per sq. in., find the total pressure on the valve.

42. A well is to be sunk 4′ in outside diameter and 24′ deep. If the carts used for carting away the earth hold $1\frac{1}{2}$ cu. yd., and excavated earth increases in bulk 15%, how many cart loads will there be? A concrete slab 8″ thick is placed on the bottom, and the sides are bricked, using 4 bricks per superficial foot. How many cu. yd. of concrete would be used and how many bricks?

43. The driving wheels of a locomotive are 4′ in diameter, and the speed of the locomotive is 40 miles per hour. How many revolutions must the drivers make per minute?

44. A tank 7′ in diameter is bound by 4 wrought-iron hoops 2″ wide and $\frac{1}{16}$″ thick. Find their weight.

45. The maximum speed for an emery wheel is a mile a minute. Find the maximum number of revolutions per sec. for an emery wheel 8″ in diameter.

46. A semi-circular platform with a diameter of 18′ has a table 2′ wide around its semi-circumference. Find the area of the table and the available space for seating accommodation.

47. The length of the shadow of a 2′ rod is 1′ 6″ and at the same time the shadow of a tree is found to be 30′. Find the height of the tree.

48. What is the offset of the tail stock for turning a taper 18″ long on a bar 30″ long if the diameters at the ends of the taper are $3\frac{1}{2}$″ and $2\frac{1}{2}$″?

49. Find the size of the largest square timber which can be cut from a log 18″ in diameter.

50. Find the diameter of a tap drill for a Whitworth nut for a screw of outside diameter $1\frac{1}{8}$″, double threaded and 14 pitch.

51. Three circles each of radius 6″ are enclosed in an equilateral triangle; find the side of the triangle.

52. In riding a certain bicycle one revolution of the pedals gives two revolutions of the wheels. If the wheels are 26″ in diameter, how many revolutions of the pedals per min. will give a speed of 12 miles per hour?

53. The diameter of a cylindrical winch barrel for a crane is 10″. If a rope with a diameter of $\frac{7}{8}$″ be used, how many coils of rope and what length of barrel would be necessary to raise a load 30′?

54. If the speed of a point on the circumference of a flywheel must not exceed 5000 ft. per min., find the maximum diameter for the wheel in order to make 120 R.P.M.

55. A steel bar of square cross-section is to be equal in area to a rod 6″ in diameter. Find a side of the square.

56. A triangular steel plate is to have its sides 13″, 14″, and 15″, and to weigh 2·98 lb.; find its thickness.

57. An elliptical steel plate has a major axis of 12″ and a minor axis of 10″. It is $\frac{3}{8}$″ thick; find its weight.

58. A cylinder 14″ in diameter fits in a cubical box. Calculate the percentage void.

59. The commutator of a dynamo is 24″ in diameter and 16″ long. Find the radiating surface in sq. ft. (lateral surface).

60. Find the diameter of a circular plate equal in area to an elliptical plate major and minor axis 18″ and 12″ respectively.

61. The shaft of a square-headed bolt is 1″ in diameter, the head being $\frac{7}{8}$″ thick and $1\frac{1}{2}$″ to the side. Determine the length of the shaft to have twice the weight of the head. (1 cu. in. = ·26 lb.).

62. Find to the nearest sixteenth of an inch the length of a $\frac{7}{8}$″ steel rod that is turned per min., if the cutting speed is 40 ft. per min. and the feed 24.

63. The length of the core of a dynamo is 20″ and it must have a radiating surface of 500 sq. in. Find its minimum diameter.

64. An elliptical funnel has a major axis of 18′ and a minor axis of 12′. Find the discharge of smoke in cu. ft. per min., if at a rate of 12 ft. per sec.

65. The pulley on the armature shaft of a dynamo is $3\frac{1}{2}$″ in diameter. This is belted to a driving shaft which makes 400 R.P.M. The speed of the dynamo must be 1800 R.P.M. What sized pulley must be placed on the driving shaft?

66. Find the weight of a coil of copper wire 400′ long, if the area of the cross-section of the wire is 40,000 circular mils. (1 circular mil = area of a circle one mil, or ·001″, in diameter).

67. Two hundred and forty-five sq. ft. of zinc are required in lining the sides and bottom of a cubical vessel. How many cu. ft. of water will it hold?

68. Four concrete abutments are to be built for a bridge. Each abutment is to be 8′ 0″ high, 3′ 0″ thick, 18′ 0″ long at the bottom and 12′ 0″ at the top. If one cu. yd. of concrete requires 25 cu. ft. of stone, 12 cu. ft. of sand, and 4 bags of cement (1 bag = 1 cu. ft.), find the quantity of each necessary for the job.

69. Find the total cost of building a rubble stone wall for a basement 30′ 0″ by 26′ 0″ by 8′ 0″ high, the wall being 18″ thick, if the stone costs $60 a toise and the labour including sand and lime, is 30c. per cu. ft. (Exact length of wall is taken with no allowance for openings).

70. It is required to build a brick wall on the front and one side of a lot 35′ 0″ in frontage and 128′ 0″ in depth. The wall is to be 7′ 0″ high and single brick 9″ in thickness There are two gates one 12′ 0″×7′ 0″ and the other 3′ 0″×7′ 0″.

Piers for strengthening the wall add 12% to its cubic contents. Allowing 15 bricks per cu. ft., at \$18 per thousand, and \$12 per thousand for laying, find the total cost.

71. A vessel to hold 10 quarts is to have an elliptical cross-section with major and minor axis 12″ and 8″ respectively. Find the height and the number of sq. ft. of tin in the vessel.

72. A garage 10′ 0″ wide and 16′ 0″ long is to have a gable roof, $\frac{1}{3}$ pitch. The following material is supplied:

Rafters 2″×4″, 2′ on centre, 18″ heel—\$65 per M.
Sheeting, 1″ thick, projecting 6″ on ends—\$63 per M.
Shingles laid $4\frac{1}{2}$″ to the weather at \$9·50 per square.

Find the cost of the above material.

73. In drilling in mild steel a $1\frac{1}{2}$″ twist drill makes 40 R.P.M., with a feed of 72. How many cu. in. will be cut away in 5 minutes?

74. A chimney is to be built in a residence with the following dimensions: height from basement floor to first floor level is 9′ 0″; from first to second floor level 10′ 6″; from second to third floor 9′ 6″; and from third floor to top of stack 15′ 0″. The size of the stack from basement to second floor level is 7′ 6″ by 2′ 7″; from second floor to third floor 4′ 10″ by 2′ 7″; from third floor up 4′ 1″ by 2′ 7″. From basement to top of stack a 1′ 1″ by 1′ 1″ furnace flue is run. On first floor a fire-place opening 3′ 0″ by 2′ 6″ by 1′ 10″ deep is to be built. From top of fire-place opening a 9″ by 1′ 1″ flue is carried up to top of stack. On the second floor a fire-place opening 1′ 10″ by 2′ 6″ by 1′ 1″ deep is to be built having a 9″ by 9″ flue carried from top of opening to top of stack. Allowing 15 bricks to the cu. ft., how many bricks would be necessary to build the stack?

75. Find the number of minutes required for turning a shaft 5″ in diameter and 6′ long, the cutting speed being 40 ft. per min. and the feed 100.

76. Two wheels 8″ and 6″ in diameter are running on parallel shafts 4′ apart. Find the length of an open belt connecting the two wheels. (1) Using the formula $3\frac{1}{4}(R+r)+2d$. (2) Using the exact method. Hence find the percentage error in the formula.

77. The circumference of the base of a church spire in the form of a cone is 42′ and the height is 80′. Find the cost of covering with sheet-iron at 30c. a sq. ft.

78. A chimney shaft 70′ 0″ high is to be erected having a flue averaging 3′ 0″ × 3′ 0″ from bottom to top. The shaft is square and for the first 14′ 0″ the walls are 1′ 10″ thick; the next 14′ 0″ is 1′ 6″ thick; the next 20′ 0″ is 1′ 1″ thick and the remaining portion 9″ thick. How many bricks would be required allowing 15 bricks to the cu. ft.?

79. How many sq. ft. of sheet-iron will it take to roof a hemispherical dome 30′ in diameter?

80. A building 24′ 0″ wide and 36′ 0″ long is to have a hip roof, $\frac{1}{4}$ pitch, with an 18″ overhang, measured horizontally (formed by extending the rafters). Find the cost of the following material:

Hip Rafters, 2″ × 6″ – $55 per M.
Rafters, 2″ × 6″ (2′ on centre), – $55 per M.
Square sheeting, $\frac{7}{8}$″ thick (10% added for cutting), – $53 per M sq. ft.
Slate, gauge $7\frac{1}{2}$″, – $30 per square.

81. A hollow copper sphere used as a float weighs 1 lb., and is 6″ in diameter. How heavy a weight will it support in the water?

82. Grain dumped in a pile makes an angle of 30° with the horizontal. How many bushels will there be if the pile forms a regular cone 10′ in diameter?

83. A tank 10′ long and 2′ in diameter is in the form of a cylinder with hemispherical ends. How many gallons will it hold?

84. A steel pin 6″ long and 1″ in diameter at the large end has a B. & S. taper. Find its weight.

85. A chimney shaft 80′ 0″ high is to be erected, having a flue averaging 3′ 0″ in diameter from bottom to top. The shaft is circular and for the first 15′ 0″ the wall is 2′ 0″ thick, the next 15′ 0″ is 1′ 6″ thick, the next 25′ 0″ is 1′ 0″ thick, and the remainder is 9″ thick. How many bricks will be required allowing 15 bricks per cu. ft.?

86. An oil tank in the form of a cylinder 15′ long and 3′ in diameter is lying on its side. It is filled to a depth of 30″. How many gallons of oil does it contain and what is the surface of the tank not in contact with the oil?

87. A ring of outer diameter 16″ is made of round cast-iron $\frac{7}{16}$″ in diameter. Find its total surface and weight.

88. A water pail has a base 12″ in diameter and a top 16″ in diameter. The height of the pail is 18″. Find the capacity in gallons and the sq. ft. of material used in construction.

89. A sphere 8″ in diameter is penetrated axially by a cylindrical hole 4″ in diameter. Find the volume of the remaining solid.

90. A tank is in the form of a cylinder with segments of spheres for ends. The total length is 8′, the cylindrical part 7′, and the diameter 2′. Find the capacity in gallons.

91. A building 24′ 0″ wide and 40′ 0″ long is to have a hip roof, $\frac{1}{3}$ pitch, with a 2′ overhang, measured horizontally (formed by extending the rafters). Find the cost of the following material:

Hip rafters, 2″×6″ – \$60 per M.
Rafters, 2″×6″(2′ on centre), – \$50 per M.
Square sheeting, 1″ thick (8% for cutting), – \$45 per M.
Shingles, $4\frac{1}{4}$″ to the weather, at \$9.50 per square of shingles.

92. In a room 16′ 4″ by 20′ 8″ it is required to lay a quarter cut clear white oak floor 3″×$1\frac{1}{2}$″, face measure. Allowing 30% for loss in dressing and working tongue, find the cost at \$250 per M square feet.

93. A life-buoy elliptical in cross-section has major and minor diameters of 5″ and 3″ respectively. If the mean diameter be 30″, find the volume in cu. in.

94. A conical tent 9′ high is to be of such a size that a man 6′ high can stand erect anywhere within 3′ of the centre pole. How many yd. of canvas 27″ wide does it contain?

95. Find the weight of a sheet of metal weighing 650 oz. per sq. ft., if the equidistant half ordinates at 1·5′ intervals are 0, 1·75, 2·25, 3, 4·25, 6·35, 6 ft.

96. Two wheels 9″ and 8″ in diameter are running on parallel shafts 5′ apart. Find the length of a crossed belt connecting the two wheels. (1) Using the formula $3\frac{3}{8}(R+r)+2d$. (2) Using the exact method. Hence, find the percentage error in the formula.

97. A room 12′ 0″ wide and 17′ 0″ long is to be floored with No. 1 quality birch, $\frac{7}{8}$″×$2\frac{1}{4}$″, to cost 30c. a sq. ft.; $33\frac{1}{3}$% being added for dressing and working the tongue. It is also to be paneled to a height of 4′ 0″ at a cost of 80c. a sq. ft. There are 2 doors each 2′ 8″ wide and 4 windows each 2′ 6″ wide, the sills being 2′ 0″ from the floor. Find the total cost.

98. A lot 50′ 0″ × 160′ 0″ is to be enclosed by a picket fence. The pickets are 4′ 0″ long, 3″ wide and 1″ thick, and are placed 3″ apart. The posts are placed 8′ 0″ apart, scantling 2″ × 4″ being used at top and bottom for railing, and a base-board 10″ wide. If the posts cost 30c each, the lumber \$52 per M, and the pickets \$10 per hundred, find the total cost of material.

99. With a feed of $\frac{1}{8}''$ per revolution, how fast is it necessary to run a bar, to turn 40″ long in 10 minutes?

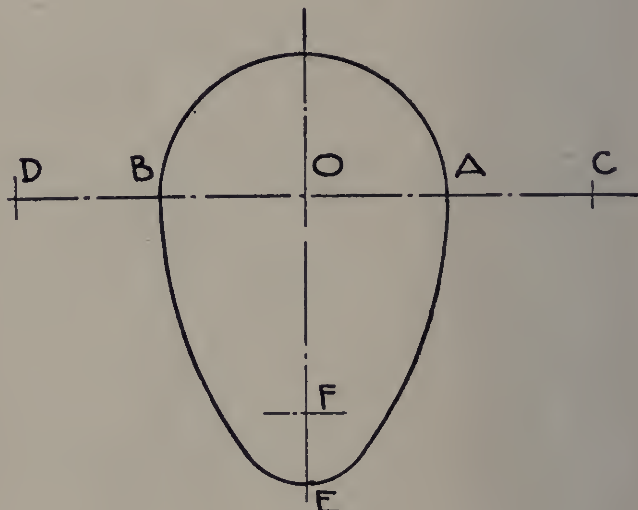


FIG. 131

100. The above figure represents a cross-section of an egg-shaped sewer. OE is the right bisector of AB and equal in length to AB. The semi-circular top has a radius $OA = 12''$. The sides are arcs with radii CB and DA each equal 36″. The small end is an arc with radius $FE = 6''$. Find the area of the cross-section.

MISCELLANEOUS EXERCISES.

1. The area of a circle is πr^2; express the diameter in terms of the area.

2. Express the area of a rectangle in terms of its perimeter when the length is twice the width.

3. Three circles are to touch one another and have their centres 3″, 4″ and 5″ apart. Find the diameters of the circles.

4. The stress in a tie bar is $f=\frac{W}{A}$ where $A=\frac{\pi d^2}{4}$. Find d when $f=7000$ and $W=2100$.

5. The difference of the acute angles of a right-angled triangle is 15°. Find the angles.

6. The weight of a body varies inversely as the square of its distance from the centre of the earth. If a man weighs 190 lb. at the earth's surface, what would he weigh on the top of a mountain $4\frac{1}{2}$ miles high (radius of earth = 4000 miles).

7. The volume of a sphere is $\frac{4}{3}\pi r^3$. Find the radius when the volume is 616 cu. in.

8. The stress in a beam is given by $f=\frac{A}{z}$. Find A when $f=6000$, $z=\frac{cd^2}{6}$, $c=3$, $d=5$.

9. The time of vibration of a pendulum varies as the square root of its length, for a given latitude. A pendulum 39·1″ long vibrates once per second. What is the length of a pendulum that vibrates three times per second?

10. An exterior angle at the base of an isosceles triangle is 108°; find all the angles.

11. The velocity of flow of water under a head h is $V=a\sqrt{2gh}$. Find h when $V=36$, $g=32$, $a=\cdot 5$.

12. In an isosceles right-angled triangle the perp. from the vertex on the hypotenuse is 8″, find all the sides.

13. The twisting moment on a solid circular shaft is given by $T=\frac{\pi f d^3}{16}$. Find d when $T=150000$, $f=6000$.

14. A chord 8″ long is 6″ from the centre of a circle. Find the radius of the circle.

15. The moment of inertia of a body about an axis is given by $I=\frac{wk^2}{g}$. Find k when $I=\cdot 5$, $w=38$, $g=32$.

16. The radius of a circle is 16″. How far from the centre is a chord 8″ in length?

17. The law of a machine is given by $P=x+yw$. Find x when $P=6\cdot48$, $y=\cdot2$, $w=62$.

18. The length and breadth of a rectangular floor differ by 6′; the area is 72 sq. ft., find the perimeter.

19. An equilateral triangle has sides 20″ in length. Find the radius of the inscribed and circumscribed circles.

20. The coefficient of self-induction of a coil of wire is given by $L=\frac{4\pi An^2}{l10^9}$.

Find n, when $A=\pi r^2$, $r=2\cdot5$, $L=\cdot015$, $l=40$.

21. The difference between the lengths of the parallel sides of a trapezium is 4; the area is 100, and the sum of the parallel sides 20. Find the dimensions.

22. The lifting power of an electro-magnet is given by

$$P=\frac{B^2A}{8\pi},$$

P being the pull in dynes. Find P, when $B=14000$, $A=30$.

23. I have to walk a distance of 144 miles, and I find that if I increase my speed by $1\frac{1}{2}$ miles per hour I can walk the distance in 14 hours less than if I walk my usual rate. Find my usual rate.

24. In measuring a rectangle the length is measured $1\frac{1}{2}\%$ too small and the width 2% too large. Find the percentage error in the area.

25. The perpendicular from the vertex of the right angle of a right-angled triangle to the hypotenuse is $2\frac{2}{5}$″. The hypotenuse is 5″. Find the other sides.

26. The length of a cylinder is twice its diameter. Find the diameter so that the cylinder may contain three times as many cu. ft. as a sphere 6″ in diameter.

27. A ton of lead is rolled into a sheet $\frac{1}{8}$″ thick. Find the area of the sheet if a cu. ft. of lead weighs 712 lb.

28. A rectangular piece of tin has an area of 195 sq. in., and its perimeter is 56″. Find its dimensions.

29. Around the outside of a square garden a path 3′ wide is made. If the path contains 516 sq. ft., find a side of the garden.

30. An open box is made from a square piece of tin by cutting out a 2″ square from each corner and turning up the sides. How large is the original square if the box contains 1152 cu. in.?

31. A man whose eye is 5′ 6″ above the ground, sights over the top of a 12′ pole and just sees the top of a tower. If he is 7′ from the pole and 63′ from the tower, find the height of the tower.

32. A derrick for hoisting coal has its arm 24′ long. It swings over an opening 20′ from the base of the arm. How far is the top of the arm above the opening?

33. Show that the area of a triangle is $\sqrt{s(s-a)(s-b)(s-c)}$, where a, b, and c are the sides and s half their sum.

34. An open box is made from a rectangular piece of tin twice as long as it is wide, by cutting out a 2″ square from each corner and turning up the sides. If the total surface is 56 sq. in., find the dimensions of the original piece.

35. At a school entertainment the price of the tickets for the second performance was reduced 20%, which resulted in an increase in receipts of 10%. What was the percentage increase in the number of tickets sold?

36. Two branches of an iron water pipe are respectively $1\frac{1}{2}$″ and $2\frac{1}{2}$″ in diameter. Find the diameter of a pipe that will just carry away the water from both branches.

37. If a man spent $\frac{1}{3}$ of his salary for board, $\frac{1}{3}$ of the remainder for other expenses, and saved annually $400, what was his salary?

38. The resistance offered by the air to the passage of a bullet through it varies jointly as the square of its diameter and the square of its velocity. If the resistance to a bullet whose diameter is ·25″ and whose velocity is 1600′ per second is 48·5 oz., what will be the resistance to a bullet whose diameter is ·4″ and whose velocity is 1550′ per second?

39. The sides of the base of a triangular prism are as 3 : 4 : 5, and its volume is 270 cu. in. If the altitude is 5″, find the sides of the base.

40. The ends of a frustum of a cone are respectively 8″ and 2″ in diameter. If the lateral surface is equal to the area of a circle whose radius is 5″, find the height of the frustum.

41. The safe distributed load in wood beams is given by $S.L. = \frac{c \times b \times d^2}{L}$, where c is a const., b the breadth in in., d the depth in in., L the length in ft.

Solve for c, b, d and L.

If $c = 100$, $b = 10''$, $d = 6''$, $L = 10'$ find $S.L.$

42. The horse-power required to drive air through a pipe is given by $H.P. = \frac{Q^3 L}{41 \cdot 3 d^5}$, where Q is the volume in cu. ft. per sec., L the length in ft., d the diameter in in.

Solve for Q, L and d.

If $Q = 10$ cu. ft., $L = 16'$, $d = 10''$, find $H.P.$

Decimal Equivalents of Parts of an Inch.

$\frac{1}{64}$	·01563	$\frac{21}{64}$	·32813	$\frac{45}{64}$	·70313
$\frac{1}{32}$	·03125	$\frac{11}{32}$	·34375	$\frac{23}{32}$	·71875
$\frac{3}{64}$	·04688	$\frac{23}{64}$	·35938	$\frac{47}{64}$	·73438
$\frac{1}{16}$	·0625	$\frac{3}{8}$	·375	$\frac{3}{4}$	·75
$\frac{5}{64}$	·07813	$\frac{25}{64}$	·39063	$\frac{49}{64}$	·76563
$\frac{3}{32}$	·09375	$\frac{13}{32}$	·40625	$\frac{25}{32}$	·78125
$\frac{7}{64}$	·10938	$\frac{27}{64}$	·42188	$\frac{51}{64}$	·79688
$\frac{1}{8}$	·125	$\frac{7}{16}$	·4375	$\frac{13}{16}$	·8125
$\frac{9}{64}$	·14063	$\frac{29}{64}$	·45313	$\frac{53}{64}$	·82813
$\frac{5}{32}$	·15625	$\frac{15}{32}$	·46875	$\frac{27}{32}$	·84375
$\frac{11}{64}$	·17188	$\frac{31}{64}$	·48438	$\frac{55}{64}$	·85938
$\frac{3}{16}$	·1875	$\frac{1}{2}$	·5	$\frac{7}{8}$	·875
$\frac{13}{64}$	·20313	$\frac{33}{64}$	·51563	$\frac{57}{64}$	·89063
$\frac{7}{32}$	·21875	$\frac{17}{32}$	·53125	$\frac{29}{32}$	·90625
$\frac{15}{64}$	·23438	$\frac{35}{64}$	·54688	$\frac{59}{64}$	·92188
$\frac{1}{4}$	·25	$\frac{9}{16}$	·5625	$\frac{15}{16}$	·9375
$\frac{17}{64}$	·26563	$\frac{37}{64}$	·57813	$\frac{61}{64}$	·95313
$\frac{9}{32}$	·28125	$\frac{19}{32}$	·59375	$\frac{31}{32}$	·96875
$\frac{19}{64}$	·29688	$\frac{39}{64}$	·60938	$\frac{63}{64}$	·98438
$\frac{5}{16}$	·3125	$\frac{5}{8}$	·625	1	1·00000
		$\frac{41}{64}$	·64063		
		$\frac{21}{32}$	·65625		
		$\frac{43}{64}$	·67188		
		$\frac{11}{16}$	·6875		

The following Tables are from Kent's Engineers' Pocket Book:

WEIGHT AND SPECIFIC GRAVITY OF STONE, BRICK, CEMENT, ETC.

	Specific Gravity	Weight in lbs. per cubic foot
Asphaltum	1·39	87
Brick, Soft	1·6	100
" Common	1·79	112
" Hard	2·0	125
" Pressed	2·16	135
" Fire	2·24 to 2·4	140 to 150
" Sand-Lime	2·18	136
Brickwork in Mortar	1·6	100
" in Cement	1·79	112
Portland Cement (loose)	....	92
" " (in barrel)	....	115
Clay	1·92 to 2·4	120 to 150
Concrete	1·92 to 2·48	120 to 155
Earth, loose	1·15 to 1·28	72 to 80
" rammed	1·44 to 1·76	90 to 110
Granite	2·56 to 2·72	160 to 170
Gravel	1·6 to 1·92	100 to 120
Lime, Quick, in bulk	·8 to ·96	50 to 60
Limestone	2·3 to 2·9	140 to 185
Marble	2·56 to 2·88	160 to 180
Masonry, dry rubble	2·24 to 2·56	140 to 160
" dressed	2·24 to 2·68	140 to 180
Mortar	1·44 to 1·6	90 to 100
Pitch	1·15	72
Plaster of Paris	1·50 to 1·81	93 to 113
Quartz	2·64	165
Sand	1·44 to 1·76	90 to 100
Sandstone	2·24 to 2·4	140 to 150
Slate	2·72 to 2·88	170 to 180
Stone, various	2·16 to 3·4	135 to 200
Tile	1·76 to 1·92	110 to 120

WEIGHT AND SPECIFIC GRAVITY OF WOOD.

	Specific Gravity Mean Value	Weight in lbs. per cubic foot
Ash	·72	45
Bamboo	·35	22
Beech	·73	46
Birch	·65	41
Cedar	·62	39
Cherry	·66	41
Chestnut	·56	35
Cypress	·53	33
Ebony	1·23	76
Elm	·61	38
Fir	·59	37
Hemlock	·38	24
Mahogany	·81	51
Maple	·68	42
Oak, White	·77	48
Oak, Red	·74	46
Pine, White	·45	28
Pine, Yellow	·61	38
Poplar	·48	30
Spruce	·45	28
Teak	·82	51
Walnut	·58	36
Willow	·54	34

WEIGHT AND SPECIFIC GRAVITY OF METALS.

	Specific Gravity Mean Value	Weight in lbs. per cubic foot	Weight in lbs. per cubic inch
Aluminum	2·67	166·5	·0963
Brass:—Cu. + Zn.			
80 20	8·6	536·3	·3103
70 30	8·40	523·8	·3031
60 40	8·36	521·3	·3017
50 50	8·2	511·4	·2959
Bronze.................	8·853	552	·3195
Copper	8·853	552	·3195
Iron, Cast.............	7·218	450	·2604
Iron, Wrought	7·7	480	·2779
Lead	11·38	709·7	·4106
Magnesium.............	1·75	109	·0641
Mercury	13·6	848	·4908
Nickel	8·8	548·7	·3175
Platinum..............	21·5	1347	·7758
Silver.................	10·505	655·1	·3791
Steel	7·854	489·6	·2834
Tin	7·35	458·3	·2652
Tungsten	17·3	1078·7	·6243
Zinc	7·00	436·5	·2526

Logarithms.

	0	1	2	3	4	5	6	7	8	9	Mean Differences								
											1	2	3	4	5	6	7	8	9
10	00000	00432	00860	01284	01703						42	85	127	170	212	254	297	339	381
						02119	02531	02938	03342	03743	40	81	121	162	202	242	283	323	364
11	04139	04532	04922	05308	05690						37	77	116	154	193	232	270	309	348
						06070	06446	06819	07188	07555	37	74	111	148	185	222	259	296	333
12	07918	08279	08636	08991	09342						36	71	106	142	177	213	248	284	319
						09691	10037	10380	10721	11059	34	68	102	136	170	204	238	272	307
13	11394	11727	12057	12385	12710						33	66	98	131	164	197	229	262	295
						13033	13354	13672	13988	14301	32	63	95	126	158	190	221	253	284
14	14613	14922	·15229	15534	15836						30	61	91	122	152	183	213	244	274
						16137	16435	16732	17026	17319	29	59	88	118	147	177	206	236	265
15	17609	17898	18184	18469	18752						28	57	85	114	142	171	199	228	256
						19033	19312	19590	19866	20140	28	55	83	110	138	165	193	221	248
16	20412	20683	20951	21219	21484						27	53	80	107	134	160	187	214	240
						21748	22011	22272	22531	22789	26	52	78	104	130	156	182	208	233
17	23045	23300	23553	23805	24055						26	50	76	101	126	151	176	201	227
						24304	24551	24797	25042	25285	25	49	73	98	122	147	171	196	220
18	25527	25768	26007	26245	26482						24	48	71	95	119	143	167	190	214
						26717	26951	27184	27416	27646	23	46	69	93	116	139	162	185	208
19	27875	28103	28330	28556	28780						23	45	68	90	113	135	158	180	203
						29003	29226	29447	29667	29885	22	44	66	88	110	132	154	176	198
20	30103	30320	30535	30750	30963	31175	31387	31597	31806	32015	21	43	64	85	106	127	148	170	190
21	32222	32428	32634	32838	33041	33244	33445	33646	33846	34044	20	40	60	80	100	120	140	160	180
22	34242	34439	34635	34830	35025	35218	35411	35603	35793	35984	20	39	58	77	97	116	135	154	174
23	36173	36361	36549	36736	36922	37107	37291	37475	37658	37840	19	37	56	74	93	111	130	148	167
24	38021	38202	38382	38561	38739	38917	39094	39270	39445	39620	18	35	53	71	89	106	124	142	159
25	39794	39967	40140	40312	40483	40654	40824	40993	41162	41330	17	34	51	68	85	102	119	136	153
26	41497	41664	41830	41996	42160	42325	42488	42651	42813	42975	16	33	49	66	82	98	115	131	148
27	43136	43297	43457	43616	43775	43933	44091	44248	44404	44560	16	32	47	63	79	95	111	126	142
28	44716	44871	45025	45179	45332	45484	45637	45788	45939	46090	15	30	46	61	76	91	107	122	137
29	46240	46389	46538	46687	46835	46982	47129	47276	47422	47567	15	29	44	59	74	88	103	118	132
30	47712	47857	48001	48144	48287	48430	48572	48714	48855	48996	14	29	43	57	72	86	100	114	129
31	49136	49276	49415	49554	49693	49831	49969	50106	50243	50379	14	28	41	55	69	83	97	110	124
32	50515	50650	50786	50920	51054	51188	51322	51455	51587	51720	13	27	40	.54	67	80	94	107	121
33	51851	51983	52114	52244	52375	52504	52634	52763	52892	53020	13	26	39	52	65	78	91	104	117
34	53148	53275	53403	53529	53656	53782	53908	54033	54158	54283	13	25	38	50	63	76	88	101	113
35	54407	54531	54654	54777	54900	55023	55145	55267	55388	55509	12	24	37	49	61	73	85	98	110
36	55630	55751	55871	55991	56110	56229	56348	56467	56585	56703	12	24	36	48	60	71	83	95	107
37	56820	56937	57054	57171	57287	57403	57519	57634	57749	57864	12	23	35	46	58	70	81	93	104
38	57978	58092	58206	58320	58433	58546	58659	58771	58883	58995	11	23	34	45	57	68	79	90	102
39	59106	59218	59329	59439	59550	59660	59770	59879	59988	60097	11	22	33	44	55	66	77	88	99
40	60206	60314	60423	60531	60638	60745	60853	60959	61066	61172	11	22	33	44	55	66	77	88	99
41	61278	61384	61490	61595	61700	61805	61909	62014	62118	62221	10	21	31	42	53	63	74	84	95
42	62325	62428	62531	62634	62737	62839	62941	63043	63144	63246	10	20	31	41	51	61	71	82	92
43	63347	63448	63548	63649	63749	63849	63949	64048	64147	64246	10	20	30	40	50	60	70	80	90
44	64345	64444	64542	64640	64738	64836	64933	65031	65128	65225	10	20	29	39	49	59	68	78	88
45	65321	65418	65514	65610	65706	65801	65896	65992	66087	66181	10	19	29	38	48	57	67	76	86
46	66276	66370	66464	66558	66652	66745	66839	66932	67025	67117	9	19	28	37	47	56	65	74	84
47	67210	67302	67394	67486	67578	67669	67761	67852	67943	68034	9	18	27	36	46	55	64	73	82
48	68124	68215	68305	68395	68485	68574	68664	68753	68842	68931	9	18	27	36	45	53	63	72	81
49	69020	69108	69197	69285	69373	69461	69548	69636	69723	69810	9	18	26	35	44	53	62	70	79

Logarithms.

	0	1	2	3	4	5	6	7	8	9	Mean Differences								
											1	2	3	4	5	6	7	8	9
50	69897	69984	70070	70157	70243	70329	70415	70501	70586	70672	9	17	26	34	43	52	60	69	77
51	70757	70842	70927	71012	71096	71181	71265	71349	71433	71517	8	17	25	34	42	50	59	67	76
52	71600	71684	71767	71850	71933	72016	72099	72181	72263	72346	8	17	25	33	42	50	58	66	75
53	72428	72509	72591	72673	72754	72835	72916	72997	73078	73159	8	16	24	32	41	49	57	65	73
54	73239	73320	73400	73480	73560	73640	73719	73799	73878	73957	8	16	24	32	40	48	56	64	72
55	74036	74115	74194	74273	74351	74429	74507	74586	74663	74741	8	16	23	31	39	47	55	63	70
56	74819	74896	74974	75051	75128	75205	75282	75358	75435	75511	8	15	23	31	39	46	54	62	69
57	75587	75664	75740	75815	75891	75967	76042	76118	76193	76268	8	15	23	30	38	45	53	60	68
58	76343	76418	76492	76567	76641	76716	76790	76864	76938	77012	7	15	22	30	37	44	52	59	67
59	77085	77159	77232	77305	77379	77452	77525	77597	77670	77743	7	15	22	29	37	44	51	58	66
60	77815	77887	77960	78032	78104	78176	78247	78319	78390	78462	7	14	22	29	36	43	50	58	65
61	78533	78604	78675	78746	78817	78888	78958	79029	79099	79169	7	14	21	28	36	43	50	57	64
62	79239	79309	79379	79449	79518	79588	79657	79727	79796	79865	7	14	21	28	35	41	48	55	62
63	79934	80003	80072	80140	80209	80277	80346	80414	80482	80550	7	14	20	27	34	41	48	54	61
64	80618	80686	80754	80821	80889	80956	81023	81090	81158	81224	7	13	20	27	34	40	47	54	60
65	81291	81358	81425	81491	81558	81624	81690	81757	81823	81889	7	13	20	26	33	40	46	53	59
66	81954	82020	82086	82151	82217	82282	82347	82413	82478	82543	7	13	20	26	33	39	46	52	59
67	82607	82672	82737	82802	82866	82930	82995	83059	83123	83187	6	13	19	26	32	38	45	51	58
68	83251	83315	83378	83442	83506	83569	83632	83696	83759	83822	6	13	19	25	32	38	44	50	57
69	83885	83948	84011	84073	84136	84198	84261	84323	84386	84448	6	12	19	25	31	37	43	50	56
70	84510	84572	84634	84696	84757	84819	84880	84942	85003	85065	6	12	19	25	31	37	43	50	56
71	85126	85187	85248	85309	85370	85431	85491	85552	85612	85673	6	12	18	24	31	37	43	49	55
72	85733	85794	85854	85914	85974	86034	86094	86153	86213	86273	6	12	18	24	30	36	42	48	54
73	86332	86392	86451	86510	86570	86629	86688	86747	86806	86864	6	12	18	24	30	35	41	47	53
74	86923	86982	87040	87099	87157	87216	87274	87332	87390	87448	6	12	17	23	29	35	41	46	52
75	87506	87564	87622	87679	87737	87795	87852	87910	87967	88024	6	12	17	23	29	35	41	46	52
76	88081	88138	88195	88252	88309	88366	88423	88480	88536	88593	6	11	17	23	29	34	40	46	51
77	88649	88705	88762	88818	88874	88930	88986	89042	89098	89154	6	11	17	22	28	34	39	45	50
78	89209	89265	89321	89376	89432	89487	89542	89597	89653	89708	6	11	17	22	28	33	39	44	50
79	89763	89818	89873	89927	89982	90037	90091	90146	90200	90255	6	11	17	22	28	33	39	44	50
80	90309	90363	90417	90472	90526	90580	90634	90687	90741	90795	5	11	16	22	27	32	38	43	49
81	90848	90902	90956	91009	91062	91116	91169	91222	91275	91328	5	11	16	21	27	32	37	42	48
82	91381	91434	91487	91540	91593	91645	91698	91751	91803	91855	5	11	16	21	27	32	37	42	48
83	91908	91960	92012	92064	92117	92169	92221	92273	92324	92376	5	10	16	21	26	31	36	42	47
84	92428	92480	92531	92583	92634	92686	92737	92788	92840	92891	5	10	15	20	26	31	36	41	46
85	92942	92993	93044	93095	93146	93197	93247	93298	93349	93399	5	10	15	20	26	31	36	41	46
86	93450	93500	93551	93601	93651	93702	93752	93802	93852	93902	5	10	15	20	25	30	35	40	45
87	93952	94002	94052	94101	94151	94201	94250	94300	94349	94399	5	10	15	20	25	30	35	40	45
88	94448	94498	94547	94596	94645	94694	94743	94792	94841	94890	5	10	15	20	25	29	34	39	44
89	94939	94988	95036	95085	95134	95182	95231	95279	95328	95376	5	10	15	19	24	29	34	39	44
90	95424	95472	95521	95569	95617	95665	95713	95761	95809	95856	5	10	14	19	24	29	34	38	43
91	95904	95952	95999	96047	96095	96142	96190	96237	96284	96332	5	9	14	19	24	28	33	38	42
92	96379	96426	96473	96520	96567	96614	96661	96708	96755	96802	5	9	14	19	24	28	33	38	42
93	96848	96895	96942	96988	97035	97081	97128	97174	97220	97267	5	9	14	18	23	28	32	38	42
94	97313	97359	97405	97451	97497	97543	97589	97635	97681	97727	5	9	14	18	23	28	32	37	42
95	97772	97818	97864	97909	97955	98000	98046	98091	98137	98182	5	9	14	18	23	27	32	36	41
96	98227	98272	98318	98363	98408	98453	98198	98543	98588	98632	5	9	14	18	23	27	32	36	41
97	98677	98722	98767	98811	98856	98900	98945	98989	99034	99078	4	9	13	18	22	27	31	36	40
98	99123	99167	99211	99255	99300	99344	99388	99432	99476	99520	4	9	13	18	22	26	31	35	40
99	99564	99607	99651	99695	99739	99782	99826	99870	99913	99957	4	9	13	17	22	26	31	35	39

Antilogarithms.

	0	1	2	3	4	5	6	7	8	9	Mean Differences 1	2	3	4	5	6	7	8	9
.00	10000	10023	10046	10069	10093	10116	10139	10162	10186	10209	2	5	7	9	12	14	16	19	21
.01	10233	10257	10280	10304	10328	10351	10375	10399	10423	10447	2	5	7	10	12	14	17	19	21
.02	10471	10495	10520	10544	10568	10593	10617	10641	10666	10691	2	5	7	10	12	15	17	20	22
.03	10715	10740	10765	10789	10814	10839	10864	10889	10914	10940	3	5	8	10	13	15	18	20	23
.04	10965	10990	11015	11041	11066	11092	11117	11143	11169	11194	3	5	8	10	13	15	18	20	23
.05	11220	11246	11272	11298	11324	11350	11376	11402	11429	11455	3	5	8	11	13	16	18	21	24
.06	11482	11508	11535	11561	11588	11614	11641	11668	11695	11722	3	5	8	11	13	16	19	21	24
.07	11749	11776	11803	11830	11858	11885	11912	11940	11967	11995	3	5	8	11	14	16	19	22	25
.08	12023	12050	12078	12106	12134	12162	12190	12218	12246	12274	3	6	8	11	14	17	20	22	25
.09	12303	12331	12359	12388	12417	12445	12474	12503	12531	12560	3	6	9	11	14	17	20	23	26
.10	12589	12618	12647	12677	12706	12735	12764	12794	12823	12853	3	6	9	12	15	18	21	24	26
.11	12882	12912	12942	12972	13002	13032	13062	13092	13122	13152	3	6	9	12	15	18	21	24	27
.12	13183	13213	13243	13274	13305	13335	13366	13397	13428	13459	3	6	9	12	15	18	21	25	28
.13	13490	13521	13552	13583	13614	13646	13677	13709	13740	13772	3	6	9	13	16	19	22	25	28
.14	13804	13836	13868	13900	13932	13964	13996	14028	14060	14093	3	6	10	13	16	19	22	26	29
.15	14125	14158	14191	14223	14256	14289	14322	14355	14388	14421	3	7	10	13	16	20	23	26	30
.16	14454	14488	14521	14555	14588	14622	14655	14689	14723	14757	3	7	10	13	17	20	24	27	30
.17	14791	14825	14859	14894	14928	14962	14997	15031	15066	15101	3	7	10	14	17	21	24	28	31
.18	15136	15171	15205	15241	15276	15311	15346	15382	15417	15453	4	7	11	14	18	21	25	28	32
.19	15488	15524	15560	15596	15631	15668	15704	15740	15776	15812	4	7	11	14	18	22	25	29	32
.20	15849	15885	15922	15959	15996	16032	16069	16106	16144	16181	4	7	11	15	18	22	26	30	33
.21	16218	16255	16293	16331	16368	16406	16444	16482	16520	16558	4	8	11	15	19	23	26	30	34
.22	16596	16634	16672	16711	16749	16788	16827	16866	16904	16943	4	8	12	15	19	23	27	31	35
.23	16982	17022	17061	17100	17140	17179	17219	17258	17298	17338	4	8	12	16	20	24	28	32	36
.24	17378	17418	17458	17498	17539	17579	17620	17660	17701	17742	4	8	12	16	20	24	28	32	36
.25	17783	17824	17865	17906	17947	17989	18030	18072	18113	18155	4	8	12	17	21	25	29	33	37
.26	18197	18239	18281	18323	18365	18408	18450	18493	18535	18578	4	8	13	17	21	25	30	34	38
.27	18621	18664	18707	18750	18793	18836	18880	18923	18967	19011	4	9	13	17	22	26	30	35	39
.28	19055	19099	19143	19187	19231	19275	19320	19364	19409	19454	4	9	13	18	22	26	31	35	40
.29	19498	19543	19588	19634	19679	19724	19770	19815	19861	19907	5	9	14	18	23	27	32	36	41
.30	19953	19999	20045	20091	20137	20184	20230	20277	20324	20370	5	9	14	19	23	28	32	37	42
.31	20417	20464	20512	20559	20606	20654	20701	20749	20797	20845	5	10	14	19	24	29	33	38	43
.32	20893	20941	20989	21038	21086	21135	21184	21232	21281	21330	5	10	15	19	24	29	34	39	44
.33	21380	21429	21478	21528	21577	21627	21677	21727	21777	21827	5	10	15	20	25	30	35	40	45
.34	21878	21928	21979	22029	22080	22131	22182	22233	22284	22336	5	10	15	20	25	31	36	41	46
.35	22387	22439	22491	22542	22594	22646	22699	22751	22803	22856	5	10	16	21	26	31	37	42	47
.36	22909	22961	23014	23067	23121	23174	23227	23281	23336	23388	5	11	16	21	27	32	37	43	48
.37	23442	23496	23550	23605	23659	23714	23768	23823	23878	23933	5	11	16	22	27	33	38	44	49
.38	23988	24044	24099	24155	24210	24266	24322	24378	24434	24491	6	11	17	22	28	34	39	45	50
.39	24547	24604	24660	24717	24774	24831	24889	24946	25003	25061	6	11	17	23	29	34	40	46	51
.40	25119	25177	25236	25293	25351	25410	25468	25527	25586	25645	6	12	18	23	29	35	41	47	53
.41	25704	25763	25823	25882	25942	26002	26062	26122	26182	26242	6	12	18	24	30	36	42	48	54
.42	26303	26363	26424	26485	26546	26607	26669	26730	26792	26853	6	12	18	24	31	37	43	49	55
.43	26915	26977	27040	27102	27164	27227	27290	27353	27416	27479	6	13	19	25	31	38	44	50	56
.44	27542	27606	27669	27733	27797	27861	27925	27990	28054	28119	6	13	19	26	32	39	45	51	58
.45	28184	28249	28314	28379	28445	28510	28576	28642	28708	28774	7	13	20	26	33	39	46	52	59
.46	28840	28907	28973	29040	29107	29174	29242	29309	29376	29444	7	13	20	27	34	40	47	54	60
.47	29512	29580	29648	29717	29785	29854	29923	29992	30061	30130	7	14	21	28	34	41	48	55	62
.48	30200	30259	30339	30409	30479	30549	30620	30690	30761	30832	7	14	21	28	35	42	49	56	63
.49	30903	30974	31046	31117	31189	31261	31333	31405	31477	31550	7	14	22	29	36	43	50	58	65

Antilogarithms.

	0	1	2	3	4	5	6	7	8	9	Mean Differences								
											1	2	3	4	5	6	7	8	9
.50	31623	31696	31769	31842	31916	31989	32063	32137	32211	32285	7	15	22	29	37	44	52	59	66
.51	32359	32434	32509	32584	32659	32735	32809	32885	32961	33037	8	15	23	30	38	45	53	60	68
.52	33113	33189	33266	33343	33420	33497	33574	33651	33729	33806	8	15	23	31	39	46	54	62	69
.53	33884	33963	34041	34119	34198	34277	34356	34435	34514	34594	8	16	24	32	40	47	55	63	71
.54	34674	34754	34834	34914	34995	35075	35156	35237	35318	35400	8	16	24	32	40	48	56	65	73
.55	35481	35563	35645	35727	35810	35892	35975	36058	36141	36224	8	16	25	33	41	50	58	66	74
.56	36308	36392	36475	36559	36644	36728	36813	36898	36983	37068	8	17	25	34	42	51	59	68	76
.57	37154	37239	37325	37411	37497	37584	37670	37757	37844	37931	9	17	26	35	43	52	61	69	78
.58	38019	38107	38194	38282	38371	38459	38548	38637	38726	38815	9	18	27	35	44	53	62	71	80
.59	38905	38994	39084	39174	39264	39355	39446	39537	39628	39719	9	18	27	36	45	54	63	72	82
.60	39811	39902	39994	40087	40179	40272	40365	40458	40551	40644	9	19	28	37	46	56	65	74	83
.61	40738	40832	40926	41020	41115	41210	41305	41400	41495	41591	9	19	28	38	47	57	66	76	85
.62	41687	41783	41879	41976	42073	42170	42267	42364	42462	42560	10	19	29	39	49	58	68	78	87
.63	42658	42756	42855	42954	43053	43152	43251	43351	43451	43551	10	20	30	40	50	60	70	80	89
.64	43652	43752	43853	43954	44055	44157	44259	44361	44463	44566	10	20	30	41	51	61	71	81	91
.65	44668	44771	44875	44978	45082	45186	45290	45394	45499	45604	10	21	31	42	52	62	73	83	94
.66	45709	45814	45920	46026	46132	46238	46345	46452	46559	46666	11	21	32	43	53	64	75	85	96
.67	46774	46881	46989	47098	47206	47315	47424	47534	47643	47753	11	22	33	44	54	65	76	87	98
.68	47863	47973	48084	48195	48306	48417	48529	48641	48753	48865	11	22	33	45	56	67	78	89	100
.69	48978	49091	49204	49317	49431	49545	49659	49774	49888	50003	11	23	34	46	57	68	80	91	103
.70	50119	50234	50350	50466	50582	50699	50816	50933	51050	51168	12	23	35	47	58	70	82	93	105
.71	51286	51404	51523	51642	51761	51880	52000	52119	52240	52360	12	24	36	48	60	72	84	96	108
.72	52481	52602	52723	52845	52966	53088	53211	53333	53456	53580	12	24	37	49	61	73	85	98	110
.73	53703	53827	53951	54075	54200	54325	54450	54576	54702	54828	13	25	38	50	63	75	88	100	113
.74	54954	55081	55208	55336	55463	55590	55719	55847	55976	56105	13	26	38	51	64	77	90	102	115
.75	56234	56364	56494	56624	56754	56885	57016	57148	57280	57412	13	26	39	52	66	79	92	105	118
.76	57544	57677	57810	57943	58076	58210	58345	58479	58614	58749	13	27	40	54	67	80	94	107	121
.77	58884	59020	59156	59293	59429	59566	59704	59841	59979	60117	14	27	41	55	69	82	96	110	123
.78	60256	60395	60534	60674	60814	60954	61094	61235	61376	61518	14	28	42	56	70	84	98	112	126
.79	61659	61802	61944	62087	62230	62373	62517	62661	62806	62951	14	29	43	58	72	86	101	115	130
.80	63096	63241	63387	63533	63680	63826	63973	64121	64269	64417	15	29	44	59	74	88	103	118	132
.81	64565	64714	64863	65013	65163	65313	65464	65615	65766	65917	15	30	45	60	75	90	105	120	135
.82	66069	66222	66374	66527	66681	66834	66988	67143	67298	67453	15	31	46	62	77	92	108	123	139
.83	67608	67764	67920	68077	68234	68391	68549	68707	68865	69024	16	32	47	63	79	95	110	126	142
.84	69183	69343	69503	69663	69823	69984	70146	70307	70469	70632	16	32	48	64	81	97	113	129	145
.85	70795	70958	71121	71285	71450	71614	71779	71945	72111	72277	17	33	50	66	83	99	116	132	149
.86	72444	72611	72778	72946	73114	73282	73451	73621	73790	73961	17	34	51	68	85	101	118	135	152
.87	74131	74302	74473	74645	74817	74989	75162	75336	75509	75683	17	35	52	69	87	104	121	138	156
.88	75858	76033	76208	76384	76560	76736	76913	77090	77268	77446	18	35	53	71	89	107	125	142	159
.89	77625	77804	77983	78163	78343	78524	78705	78886	79068	79250	18	36	54	72	91	109	127	145	163
.90	79433	79616	79799	79983	80168	80353	80538	80724	80910	81096	19	37	56	74	93	111	130	148	167
.91	81283	81470	81658	81846	82035	82224	82414	82604	82794	82985	19	38	57	76	95	113	132	151	170
.92	83176	83368	83560	83753	83946	84140	84333	84528	84723	84918	19	39	58	78	97	116	136	155	175
.93	85114	85310	85507	85704	85901	86099	86298	86497	86696	86896	20	40	60	79	99	119	139	158	178
.94	87096	87297	87498	87700	87902	88105	88308	88512	88716	88920	20	41	61	81	102	122	142	162	183
.95	89125	89331	89536	89743	89950	90157	90365	90573	90782	90991	21	42	62	83	104	125	146	166	187
.96	91201	91411	91622	91833	92045	92257	92470	92683	92897	93111	21	42	64	85	106	127	149	170	191
.97	93325	93541	93756	93972	94189	94406	94624	94842	95060	95280	22	43	65	87	109	130	152	174	195
.98	95499	95719	95940	96161	96383	96605	96828	97051	97275	97499	22	44	67	89	111	133	155	178	200
.99	97724	97949	98175	98401	98628	98855	99083	99312	99541	99770	23	46	68	91	114	137	160	182	205

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4.—1. 11413. *2.* 23437. *3.* 16438. *4.* 392·26. *5.* 365·09. *6.* 1907·80. *7.* 79·07. *8.* 1158·94. *9.* 2220·65. *10.* 8380·882. *11.* 35880·920. *12.* 10964·8526. *13.* \$1056·95. *14.* \$2221·39. *15.* \$2253·15.

5.—1. 7535. *2.* 2·36. *3.* 364·947. *4.* 139·20. *5.* 34408·79. *6.* 3·207. *7.* 103·30. *8.* 2·4116. *9.* 11·232. *10.* 217·209.

7.—1. 1557. *2.* 1309. *3.* 6152. *4.* 952. *5.* 55368. *6.* 2960. *7.* 34500. *8.* 764·5. *9.* 7864. *10.* 11·800. *11.* 1788. *12.* 17880. *13.* 7864. *14.* ·0469. *15.* 4·69. *16.* 4290·5. *17.* 18408. *18.* 78. *19.* 76·14. *20.* 1180. *21.* 236896000. *22.* 2538. *23.* 1018400. *24.* 6885.

8.—1. 3276. *2.* 6344. *3.* 4796. *4.* 31433. *5.* 67392. *6.* 3213. *7.* 1910520. *8.* 2809566. *9.* 5049668. *10.* 699678. *11.* 494·34. *12.* 26430·588. *13.* 1299·276. *14.* 69·765. *15.* ·021112. *16.* ·038934. *17.* 28·58392. *18.* ·7916832. *19.* 50092·640436. *20.* 1·3255716. *21.* 11·109280. *22.* 1·47504. *23.* 78·028125. *24.* 2112·280092. *25.* 95·797296. *26.* 3·38928. *27.* 2971·6736. *28.* 19·25625. *29.* ·0000625. *30.* 1·55697696.

10.—1. 32987. *2.* 180912. *3.* 1·764. *4.* $11\frac{2}{14}$. *5.* ·00158. *6.* 1·76. *7.* ·01325. *8.* 1·22. *9.* ·025. *10.* 85·1. *11.* 17·02. *12.* $31196\frac{1}{9}$. *13.* 46·42.

13.—1. 65·02. *2.* 64. *3.* 679·65. *4.* 2119·92. *5.* 134·29. *6.* 278·10. *7.* 3·04. *8.* 7·97. *9.* 14·19. *10.* 4071·17. *11.* 297·99. *12.* 1·033.

14.—1. 18. *2.* 20. *3.* 44. *4.* 70. *5.* 6891. *6.* 585. *7.* 183.

15.—1. 28. *2.* $3\frac{5}{9}$. *3.* 9. *4.* 240000. *5.* $230\frac{4}{55}$. *6.* $\frac{86}{225}$. *7.* $\frac{76}{121}$. *8.* $2\frac{27}{80}$. *9.* 8. *10.* 48. *11.* 76·8. *12.* 115·2. *13.* 2·5. *14.* $22\frac{1}{2}$. *1.* 1133 lb.

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16.—2. 22·9175. *3.* 252 ft. *4.* 3199. *5.* 3773, 3507. *6.* 46548 lb. 26772 lb. *7.* 236 lb. *8.* 144. *9.* 14 rem. 2 ft. *10.* $177\frac{7}{8}$ miles. *11.* 60. *12.* $25\frac{1}{4}$ miles. *13.* \$10287·16. *14.* 2400.

17.—15. 8. *16.* 72.

20.—1. $\frac{6}{14}$. *2.* $\frac{20}{24}$. *3.* $\frac{45}{50}$. *4.* $\frac{21}{75}$. *5.* $\frac{15}{40}$. *6.* $\frac{49}{56}$. *7.* $\frac{102}{108}$. *8.* $\frac{68}{144}$. *9.* $\frac{32}{252}$. *10.* $\frac{72}{189}$.

20.—1. $\frac{3}{4}$. *2.* $\frac{1}{2}$. *3.* $\frac{1}{4}$. *4.* $\frac{2}{3}$. *5.* $\frac{2}{3}$. *6.* $\frac{37}{145}$. *7.* $\frac{242}{311}$. *8.* $\frac{107}{369}$. *9.* $\frac{9}{100}$. *10.* $\frac{5}{16}$.

21.—1. $3\frac{1}{2}$. *2.* 4. *3.* $7\frac{1}{9}$. *4.* 6. *5.* $8\frac{3}{4}$. *6.* $17\frac{6}{7}$. *7.* $29\frac{8}{9}$. *8.* $3\frac{3}{11}$. *9.* $11\frac{2}{11}$. *10.* $27\frac{3}{16}$. *11.* 2500. *12.* $9\frac{3}{13}$. *13.* 53. *14.* 9. *15.* $\frac{10}{3}$. *16.* $\frac{97}{15}$. *17.* $\frac{80}{11}$. *18.* $\frac{3687}{32}$. *19.* $\frac{487}{4}$. *20.* $\frac{275}{3}$. *21.* $\frac{2158}{5}$. *22.* $\frac{160021}{40}$. *23.* $\frac{6506}{9}$. *24.* $\frac{3535}{9}$.

22.—1. $\frac{11}{15}$. *2.* $\frac{26}{35}$. *3.* $\frac{1}{16}$. *4.* $1\frac{23}{48}$. *5.* $\frac{139}{176}$. *6.* $\frac{31}{48}$. *7.* $1\frac{1}{18}$. *8.* $1\frac{53}{64}$. *9.* $\frac{19}{64}$. *10.* $11\frac{7}{8}$. *11.* $3\frac{23}{24}$. *12.* $3\frac{89}{504}$. *13.* $19\frac{23}{30}$. *14.* $8\frac{115}{144}$.

23.—1. $33\frac{3}{4}$ lb. *2.* $\frac{13}{16}$. *3.* $2\frac{3}{16}''$. *4.* $2\frac{13}{64}''$. *5.* $24\frac{5}{16}$. *6.* $19\frac{1}{8}''$.

24.—7. $3\frac{3}{4}''$. *8.* $16\frac{9}{16}''$, $1\frac{1}{4}''$. *9.* $2\frac{5}{12}'$, $2\frac{3}{4}'$, $1\frac{11}{12}'$. *10.* $1\frac{5}{6}'$.

26.—1. $2\frac{2}{9}$. *2.* $2\frac{2}{9}$. *3.* $\frac{16}{27}$. *4.* $\frac{3}{16}$. *5.* $\frac{5}{64}$. *6.* $\frac{3}{16}$. *7.* $\frac{5}{64}$. *8.* $\frac{9}{320}$. *9.* $1\frac{1}{2}$. *10.* $\frac{27}{40}$. *11.* $\frac{135}{32768}$. *12.* $1\frac{3}{65}$. *13.* 500. *14.* $\frac{15}{64}$. *15.* 62. *16.* 25. *17.* 3. *18.* 1. *19.* $5\frac{5}{21}$. *20.* $39\frac{45}{64}$. *21.* 49. *22.* $4\frac{2}{3}$. *23.* 1. *24.* $27\frac{56}{57}$.

27.—1. $\frac{3}{20}$. *2.* $\frac{7}{16}$. *3.* $\frac{3}{256}$. *4.* $\frac{7}{48}$. *5.* $\frac{7}{12}$. *6.* $\frac{15}{23}$. *7.* $3\frac{1}{2}$. *8.* $376\frac{1}{2}$. *9.* 6. *10.* $\frac{7}{10}$. *11.* $7\frac{13}{30}$. *12.* $\frac{11}{50}$. *13.* $12\frac{115}{144}$. *14.* $\frac{13}{112}$. *15.* $1\frac{7}{95}$. *16.* $16\frac{19}{21}$.

27.—1. $\frac{43}{100}$. *2.* $\frac{1}{25}$. *3.* $\frac{94}{125}$. *4.* $\frac{3567}{5000}$. *5.* $\frac{251}{500}$. *6.* $\frac{1}{125}$. *7.* $\frac{141}{200}$. *8.* $\frac{617}{5000}$. *9.* $\frac{2}{125}$. *10.* $\frac{31}{200000}$.

28.—1. ·25. *2.* ·5. *3.* ·375. *4.* ·6875. *5.* ·6. *6.* ·992. *7.* ·96875. *8.* ·9375. *9.* ·96. *10.* ·9921875. *11.* ·74. *12.* ·508.

29.—1. $\cdot\dot{5}$. *2.* $\cdot 08\dot{3}$. *3.* $\cdot\dot{1}4285\dot{7}$. *4.* $\cdot 1\dot{3}$. *5.* $\cdot 0\dot{5}$. *6.* $\cdot\dot{1}\dot{8}$. *7.* $\cdot\dot{3}0769\dot{2}$.

30.—1. $\frac{5}{9}$. *2.* $\frac{4}{11}$. *3.* $\frac{11}{30}$. *4.* $\frac{17}{111}$. *5.* $\frac{61}{165}$. *6.* $\frac{41}{111}$. *7.* $3\frac{19}{75}$. *8.* $\frac{151}{600}$. *9.* $\frac{1}{7}$. *10.* $2\frac{23}{30}$. *11.* $2\frac{589}{1110}$. *12.* $\frac{787}{16650}$. *13.* $\frac{11}{3000}$. *14.* $\frac{71}{1665}$.

31.—2. 4, 2, $22\frac{1}{2}$, 60. *3.* 50%, 25%, $12\frac{1}{2}$%. *4.* 50%, 20%, $33\frac{1}{3}$%. *5.* $416\frac{2}{3}$%, $4\frac{17}{30}$%. *6.* 15, 35, 70, 85. *7.* 80. *8.* 70.

32.—9. \$8·$07\frac{1}{2}$. *10.* 30 gallons.

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33.—*1.* 10, 5, 10. *2.* \$78·30. *3.* $11\frac{1}{21}$ hr. *4.* \$167·47. *5.* first by $4\frac{1}{2}$c. *6.* 21·2 lb. *7.* \$72·25. *8.* $9\frac{28}{33}$. *9.* 4. *10.* \$840·00. *11.* \$1692·90. *12.* \$3125·00. *13.* \$25631·41. *14.* $\frac{4}{7}$, $\frac{2}{7}$, $\frac{1}{7}$.

34.—*15.* $\frac{49}{64}$ in. *16.* $\frac{1}{6}$. *17.* $8\frac{3}{4}$. *18.* 10 lb. *19.* \$27·$03\frac{1}{8}$. *20.* 73. *21.* \$8·44. *22.* 54·94 lb. copper, 27·06 lb. zinc. *23.* $9\frac{87}{107}$ days. *24.* \$360·00. *25.* 63·295. *26.* 309·462. *27.* $232\frac{1}{2}$ oz. *28.* $6\frac{6}{79}$ in.

35.—*29.* 55·97″. *30.* 83·27. *31.* 4·5225″. *32.* 96. *33.* $83\frac{1}{3}$. *34.* 78. *35.* ·25, \$4000. *36.* 1062·5, 187·5. *37.* 14 in. *38.* 822·64 cu. in. *39.* 72. *40.* 8000 lb. *41.* $11\frac{1}{2}$ in.

36.—*42.* $1\frac{3}{8}$ in. *43.* \$5775·00. *44.* \$4100·00, \$6150·00, \$6150·00. *45.* $107\frac{7}{12}$, $35\frac{31}{36}$%. *46.* $65\frac{5}{8}$. *47.* 10383·36.

38.—*1.* 1760. *2.* 5280. *3.* $\cdot 02\dot{7}$. *4.* ·003125. *5.* 681″. *6.* $16339\frac{1}{2}$′. *7.* ·000125. *8.* $\cdot 001\dot{2}\dot{6}$. *9.* 415 li. *10.* $1\frac{77}{396}$ ch. *11.* 2874·96′. *12.* $6\frac{5}{12}$ ch. *13.* 52·37 miles.

39.—*5.* 14′, 4·4406″. *6.* 9525 cm., ·09525 dm. *7.* ·06096 Km.

40.—*8.* 981·456 cm. *9.* 1·34 Km. *10.* 134·112 cm.

41.—*4.* 4840 sq. yd. *5.* 1321 sq. ft. *6.* $\cdot 03\dot{3}$ ac. *7.* 6930 sq. in. *8.* ·51 sq. yd. *9.* $2\frac{1}{2}$ ac. *10.* ·803 sq. ft. *11.* $\frac{49}{64}$ sq. ft. *12.* 198. *13.* 96. *14.* $57\frac{1}{27}$. *15.* 2·16 ac.

42.—*16.* 24012. *17.* \$373·33. *18.* \$6·53. *19.* 72. *20.* $54\frac{23}{45}$.

43.—*3.* 225 cu. ft. *4.* 143432 cu. in. *5.* 6790·363 cc. *6.* $1\frac{27}{28}$. *7.* $3\frac{11}{108}$. ·37. *8.* $21\frac{5}{27}$, 104. *9.* \$10·67. *10.* \$143·44. *11.* 129.

44.—*12.* 442·45. *13.* 19′ 6″. *14.* \$112·12, \$128·34. *15.* $\frac{1}{4}$″. *16.* 36300.

46.—*1.* 58·35 cu. ft. *2.* 1·83.

47.—*3.* 1·03. *4.* 90864·018 tons. *5.* ·79. *1.* 196. *2.* 484 days. *3.* 45563 min. *4.* 1496.

49.—*3.* 343 l, 343000 g. *4.* 3875 cu. in., 1096·625 lb. *5.* 2268 Kgm. *6.* 52310. *7.* 6271·98. *8.* 12145·14. *9.* $3600\frac{5}{6}$ lb., $6\frac{5}{24}$.

51.—*1.* 45. *2.* 199. *3.* 123. *4.* 327. *5.* 37. *6.* 1·732. *7.* 3·4908. *8.* 11·9668. *9.* ·7370. *10.* ·2507. *11.* 220 yd. *12.* 11·67″. *13.* 30·59″. *14.* The square pipe.

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52.—15. 15·87″. *1.* 14·14″. *2.* 48·82′. *3.* 51·38′. *4.* 127·27′. *5.* 469·04′.

53.—6. 121·96′. *7.* 20·85′.

54.—1. 6·39. *2.* 5·93. *3.* (*a*) \$159·05. (*b*) \$146·33.

55.—4. 228·97. 572·42, 1144·48. *5.* $2809\frac{7}{9}$.

56.—2. 21409. *3.* 30491. *4.* \$220·20, \$313·63. *5.* 66248. *6.* \$2647·17.

59.—5. 6, 72. *6.* $13\frac{1}{3}$, 160. *7.* \$143·60. *8.* \$139·78. *9.* \$44·69.

65.—2. \$5·46, \$11·76, \$24·49. *3.* \$8·00, \$14·93, \$34·99.

66.—4. \$25·00, \$33·86, \$52·33. *5.* \$95·63, \$114·55, \$496·40. *6.* \$47·12.

67.—7. \$64·83. *1.* (*a*) \$57·02, (*b*) \$88·59, (*c*) \$197·50. *2.* (*a*) \$91·04, (*b*) \$92·16, (*c*) \$173·40. *3.* \$180·21.

68.—4. \$375·28. *5.* \$800·42.

69.—1. \$6·97, \$10·39, \$8·25.

70.—2. \$4·97, \$2·20, \$10·20. *3.* \$10·11, \$8·99, \$8·50.

89.—1. 70′, 14′. *2.* 320. *3.* 620′. *4.* 160. *5.* 5760 sq. ft.

90.—6. $150\frac{3}{4}$″. *7.* 120′. *8.* 12. *9.* \$6000·00. *10.* 4800 cu. ft., 1200 cu. ft. *11.* 15′. *12.* 18″, 6″. *13.* 48c., 32c., *14.* 65c., 26c. *15.* 2 years. *16.* 3 days. *17.* $9\frac{1}{11}$ min. *18.* $\frac{12}{25}$ day.

91.—19. 25 miles, 30 miles. *20.* $16\frac{2}{3}$. *21.* \$200000. *22.* $68\frac{2}{11}$. *23.* 2100 gals. *24.* \$68750·00. *25.* \$18000·00. *26.* $4\frac{1}{2}$′ from fulcrum. *27.* 10 cm.

93.—11. $19\frac{4}{15}$. *12.* $-\frac{1}{12}$. *13.* $4\frac{29}{50}$. *14.* $1\frac{3}{4}$.

94.—1. $-2a$. *2.* $2a-3b-3c$. *3.* $-8x+23y+3z$. *4.* $7xy-yz+10zx$. *5.* $3x^2+3x-6$. *6.* $7a$. *7.* $6x^3-2x^2-x+2$. *8.* $4y^3+y^2+3$.

102.—1. $x+4$. *2.* $a+1$. *3.* $a-2$. *4.* $x-7$. *5.* $3x-4$. *6.* $3-2a$. *7.* $2+x$. *8.* $x+y$. *9.* $5-3a$. *10.* $2x^2+7$. *11.* $x+b$. *12.* x^2+xy+y^2. *13.* a^2-ab+b^2. *14.* $x^3+3x^2y+3xy^2+y^3$. *15.* $a^2+b^2+c^2-ab-bc-ca$.

104.—2. $\frac{1}{2}$, 88, 37·5. *3.* ·0315, 47·7, 238·1. *4.* 26775, 480, $\frac{1}{2}$. *5.* 15, 3. *6.* $8\frac{8}{11}$, $73\frac{1}{3}$, $2\frac{1}{5}$, 55, 110.

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105.—7. 26·832, 24·4, ·000327. *8.* 1·04, ·024. *9.* 2318·4, 14. *10.* ·448, 254·375, 20·56, 115·5. *11.* 65·625, ·375, 40533⅓, ·75, 78·75, 10·83.

106.—12. 4·42, 22·38, 186·5. *13.* 1584, 34·72, 52·09, *14.* 300, 20, 25. *15.* 4·76, 33·98, 34·375, 4033·3.

107.—16. 34$\frac{2}{7}$, 291⅔, 4½, 171·81.

113.—3. 6, 1·81, 139·4.

114.—5. 5½ ft. *6.* 72. *7.* \$258·00. *8.* 192 sq. in. *9.* 10 lb. *10.* 57·12. *11.* \$157·28. *12.* \$404·60.

115.—13. \$10·80. *14.* \$9·73. *15.* ·7936 acres. *16.* 5$\frac{13}{16}$ sq. ft. *17.* 1·37218, $\sqrt{3}$ half the base. *19.* 23·382 sq. in. *20.* 13·93 sq. in. *21.* 15. *22.* 60. *23.* 22·4. *24.* 11¼″.

116.—25. 10·825 sq. in., 62·352 sq. in., 73·177 sq. in. *26.* 8·75 sq. in. *27.* 41 sq. in. *28.* 77 sq. ch. *29.* 3·75 sq. ft. *30.* 312 acres. *31.* 142·5 sq. ft. *32.* ·6605.

117.—33. 7·2703. *34.* 3·4208. *35.* 1777.

118.—36. 158·1. *37.* 14·9′, 9·4′, 828$\frac{1}{12}$ sq. ft. *38.* 8912 sq. ft.

123.—2. 131·95″. *3.* 4·71′. *4.* 4618, 52·48. *5.* 59·9″. *6.* 240. *7.* 3769·92. *8.* 1884·96. *9.* 15·28″. *10.* 18041·8 miles. *11.* 14″. *12.* 140, 360. *13.* 300 R.P.M. *14.* 13″.

124.—18. 314·16, 64403 lb. *19.* 9·63 sq. in.

125.—20. The 4″ pipe. *21.* 4, 9. *22.* 5·383″. *23.* 12·36″. *24.* 15·6″. *26.* 14·92 sq. in. *27.* 87·965 sq. in. *28.* 6387·27 sq. yd. *29.* 34·56 sq. in. *30.* 7·61 sq. in. *31.* \$9414·91. *32.* 20·11 sq. ft.

126.—33. 18·7″. *34.* 525 sq. in. *35.* 41·33 sq. ft. *36.* 67·2 sq. in.

132.—3. 549·78′, 23562 sq. ft. *4.* 44·11 lb. *5.* ·22″. *6.* 28″. *7.* ·649 sq. in. *8.* 13·25.

133.—9. 123·11 sq. ft. *10.* ·373 lb. *11.* 10·454′. *12.* ·866 sq. ft. *16.* 598·113 sq. in. *17.* 202·1 sq. ft.

137.—1. 4:3. *2.* 5:2. *3.* 3:5.

138.—4. 168, 112. *5.* 335:6. *6.* 20:49. *7.* 196$\frac{4}{11}$. *8.* 175. *9.* \$144. *10.* \$56·25. *11.* 11$\frac{3}{7}$. *12.* \$53·34. *13.* 83·3′. *14.* 51$\frac{3}{7}$′. *15.* 54′. *16.* 124, 93, 31. *17.* 60, 24, 12.

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139.—18. $4\frac{4}{13}'$. *19.* $28\frac{4}{5}''$. *20.* $\frac{3}{100}$. *21.* 15′. *22.* $1\frac{1}{4}$, 80′. *23.* 1056′.

143.—1. 1200, 800. *2.* $\frac{30}{7}$, $-\frac{50}{7}$. *3.* $66\frac{2}{3}$, $133\frac{1}{3}$, *4.* $5\frac{1}{3}$, $10\frac{2}{3}$. *5.* $\frac{1}{10}$, 5, $P=\frac{1}{10}W+5$. *6.* 6000, 20. *7.* ·046, 5. *8.* 21·6, ·0044. *9.* 1957, 1521. *10.* 345, 245.

144.—11. 100, 200. *12.* $4\frac{7}{12}$, $5\frac{1}{4}$. *13.* 106·15, 72·21. *14.* 10·42, ·0088. *15.* $21000000, $1890000. *1.* 1·35, 3·26, 32. *2.* 9·4, 15.

145.—3. 102, 210, −2·13, 14. *4.* 1190·4, 20, 10. *5.* ·548, $106\frac{2}{3}$, 63, 21504, ·658. *6.* 30062·1, ·377, ·65.

146.—7. 13·4, 12·2, 7, 16. *8.* 6283200, ·31416, ·079. *9.* ·58, 1·125, ·7. *10.* $17\cdot\dot{2}$, 161·6, 24·8. *11.* 111·2, 217, ·06, 6·66.

147.—12. 171″, 44·5″. *13.* 245·25″, 31·5″. *14.* 6·66, 1069·2. *15.* 18·6″, 34·9, 4400. *16.* 11″, 58·18, 3300.

148.—17. 42·215, 35·9, 6·9. *18.* 1·0472, 14·3, 86450. *19.* 667·6, 18390, 149·21.

149.—20. 3, 1181·9.

179.—1. 19·64. *2.* 68·75.

180.—3. 63·63. *4.* 392·85. *5.* 22·92. *6.* 9·55″. *7.* 23·86. *8.* 11·45″. *9.* 2357+. *10.* 76·36.

181.—1. 2304. *2.* 9·60 min. *3.* 16 min. *4.* 11·31 min. *5.* $\frac{105}{352}$. *6.* 5 min. *7.* 0028″. *8.* 5 min. *9.* 79·2. *10.* 33·94.

187.—1. $\frac{3}{8}''$. *2.* $\frac{7}{8}''$. *3.* ·8625″. *4.* 12″. *5.* 5″. *6.* $7\frac{1}{2}''$. *7.* No. 0 Morse. *8.* B. & S. *9.* ·604″. *10.* No. 0 Morse. *11.* $\frac{1}{2}''$. *12.* Jarno. *14.* $\frac{15}{64}''$. *15.* $\frac{1}{4}''$. *16.* $\frac{7}{32}''$. *17.* $\frac{5}{12}''$.

188.—18. (*a*) $\frac{15}{64}''$, (*b*) $\frac{3}{16}''$, (*c*) $\frac{1}{4}''$, (*d*) $\frac{3}{8}''$. *19.* $1\frac{1}{2}''$. *20.* ·4″. *21.* $3\frac{5}{12}''$, $\frac{5}{12}''$. *22.* 2·1″. *23.* 1·68″. *24.* 2° 59′, 2° 52′ 24″, 2° 23′, 2° 51′ 50″.

191.—2. 12. *3.* 10. *4.* $\frac{11}{16}$. *5.* $\frac{57}{64}$. *6.* $\frac{27}{64}$. *7.* $\frac{1}{5}$. *8.* $3\frac{1}{2}$. *9.* 1″. *10.* 1·615″.

193.—1. ·1625″. *2.* ·4541″. *3.* $\frac{1}{10}$. *4.* 4″. *5.* $\frac{1}{13}$. *6.* 4. *7.* 3. *8.* $\frac{61}{64}$. *9.* $\frac{1}{2}$. *10.* $\frac{27}{32}$.

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195.—1. 1·187″. *2.* 2·167″. *3.* ½. *4.* 3″. *6.* 1⅓. *7.* ½. *8.* 4. *9.* 6.

197.—1. ·05336″, ·01144″. *2.* $\frac{1}{10}$, ·62193″. *3.* ·08004″, ·83992. *4.* ·0915″, $\frac{1}{7}$, ·0196″. *5.* 4½, 1·7154″, ·02948″. *6.* ·04″, ·295″, ·00858″.

202.—1. 24, 64. *2.* 32, 48. *3.* 45. *4.* 16. *5.* 24, 72. *6.* 36. *7.* $\frac{1}{6}$. *8.* 24. *9.* 72. *10.* 42, 98.

205.—1. 4½. *2.* 24 stud, 92 lead, 36 inside c., 72 outside c.

206.—3. 48 stud, 28 lead, 36 inside c., 72 outside c. *4.* 24 stud, 96 lead, 72 inside c., 36 outside c. *5.* 42 stud, 98 lead. *6.* 24 stud, 96 lead. *7.* 64 stud, 40 lead. *8.* 112. *9.* Equal gears. *10.* 36 inside, 72 outside.

210.—1. 8″. *2.* ·2618″. *3.* ·1122″. *4.* ·1541″. *5.* ·0982″. *6.* ·1348″. *7.* 5·166″. *8.* 5·5″. *9.* 70. *10.* 46. *11.* 10. *12.* 7″.

212.—1. 419. *2.* 40. *3.* 4·09″. *4.* 42. *5.* 30·4. *6.* 5·09″. *7.* ¾″ per min. *8.* 2″ per min.

213.—9. 5. *10.* ¼″.

225.—2. 15″. *4.* 15″. *5.* 21·333″. *6.* 21·5″. *7.* 49° 57′.

226.—8. 22° 48′. *9.* 1″. *10.* 14·945″. *1.* ¼″. *2.* ⅓. *3.* 10. *4.* ·1443″. *5.* 9. *6.* ·3466″, ⅕. *7.* $9\frac{3}{7}$ min. *8.* 71⅓. *9.* ½″, 2° 23′. *10.* ⅜″. *11.* 5·487″. *12.* $\frac{1}{10}$. *13.* 28.

227.—14. 8. *15.* 72. *16.* 36. *17.* 11¼. *18.* 5. *19.* 36 inside, 72 outside. *20.* 6·5″. *21.* 11″. *22.* 3·82″. *23.* 6⅔. *24.* 24 worm, 32 second stud, 64 first stud, 72 screw. *25.* 33° 8′.

239.—1. 3·0099. *2.* ·4420. *3.* ·08836. *4.* ·89827. *5.* 7·5229. *6.* 21·015. *7.* 3·3332. *8.* ·58798. *9.* ·0055873. *10.* ·0007237. *11.* 5800·9. *12.* 4·4419. *13.* 1·2057. *14.* 1·878. *15.* ·2999. *16.* ·25507. *17.* 2·37. *18.* 1·80. *19.* 1·68.

240.—20. 279·04 sq. yd. *21.* 188·86 sq. in. *22.* 43·301 acres. *23.* 13·221. *24.* 475·4′. *25.* 50·77″. *26.* 16495 sq. in. *27.* 30·73. *28.* 13·365. *29.* 40101+. *30.* 592·21.

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242.—2. \$27·20. *3.* 68·35. *4.* ·472. *5.* 10·57 sq. ft., 2·29 cu. ft. *6.* 5·41.

244.—3. 128·18 lb. *4.* 11846+. *5.* \$26·18.

245.—6. 28·49 lb. *2.* 502·65 sq. in.

246.—3. 667·98 lb. *4.* 2127·89 lb. *5.* 522·78 lb. *6.* ·280 lb.

248.—2. 48 sq. in., 11·46 sq. in. *3.* 50·03 lb. *4.* 16·15 lb. *5.* (*a*) \$71·97, (*b*) 22200+.

249.—6. 70·86.

250.—2. 31·196 lb. *3.* 131·4 sq. in., 24·11 lb. *4.* 7856·4, \$49·74. *5.* 3·59 cu. in.

254.—2. 91 sq. in. *3.* 496·61 lb. *4.* 48·01 lb. *5.* 30·73 cu. in. *6.* 2·36 cu. in.

256.—3. 67·02. *4.* 41·34 lb. *5.* 1·55 lb. *6.* 2·29 lb.

257.—1. 907 sq. ft. *2.* 611·3 sq. ft.

258.—3. 74·22 sq. ft., 41·86 cu. ft. *4.* 352·19 sq. in., 490·09 cu. in. *5.* 837·76 cu. in.

259.—1. 30 lb. *2.* 2988 tons. *3.* 3325·64 lb. *4.* 7854 lb. *5.* 165·93. *6.* 10102+ lb. *1.* 1·41″. *2.* 821·6 lb.

260.—3. 5·72″. *4.* ·153. *5.* 16·53′. *6.* 2·02 lb. *7.* 70·68 lb. *8.* 25·01′. *9.* 3291·48 sq. ft. *10.* 4·84. *11.* 22·53. *12.* 51·30 sq. in. *13.* 20 min. *14.* 13675+ lb. *15.* 603·19 sq. in. *16.* \$17·00. *17.* 163·17.

261.—18. 25·79. *19.* 2787 cu. in. *20.* \$255·00. *21.* 26·51 lb. *22.* 8774·30 lb. *23.* ·243. *24.* 2·65″. *25.* 22·15 lb. *26.* 5·01″. *27.* 11581+. *28.* 648000. *29.* 1942·1 sq. ft., 10603+ cu. ft. *30.* \$73·67.

262.—31. 12·57 cu. in. *32.* 3·32″. *33.* 19215+ lb. *34.* 79·79. *35.* 376·99 sq. in. *36.* 4·69 lb. *37.* 14·50″. *38.* 13·76. *39.* 12·84″. *40.* 9·59″.

264.—11. $(a-b)(x-y)$. *12.* $(x+z)(x-y)$. *13.* $(3+a)(x-y)$. *14.* $(x^2-y)(x-2)$. *15.* $(ax-b)(bx-a)$. *16.* $(a^4+1)(a+1)$. *17.* $(a^2-b)(1+c)$. *18.* $(a+3)(2a^2-c)$. *19.* $(x+m^2)(x+m)$. *20.* $(a+b)(x+y-z)$.

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267.—*5.* $(a+b+c)(a-b-c)$. *6.* $(x^2+y^2)(x+y)(x-y)$.
7. $(x^4+y^4)(x^2+y^2)(x+y)(x-y)$. *8.* $(a+b+c)(a-b-c)$.
9. $(x+y+a+b)(x+y-a-b)$. *10.* $(x+y)(x+y)(x-y)(x-y)$.

268.—*11.* $(x+y-z)(x-y+z)$. *12.* $(1-a-b)(1+a+b)$.
13. $(a^8+1)(a^4+1)(a^2+1)(a+1)(a-1)$.
14. $(x-y+a+b)(x-y-a-b)$. *17.* $5(a-b+2c)(a-b-2c)$. *18.* $(4-a+b)(4+a-b)$.
19. $(2x^2+xy+3y^2)(2x^2-xy+3y^2)$.
20. $(x^2-3x+9)(x^2+3x+9)$.
21. $(x^2+2xy+2y^2)(x^2-2xy+2y^2)$.
22. $(x^2-x-1)(x^2+x-1)(x^4+3x^2+1)$.
23. $(2x+y)(2x-y)(x-3y)(x+3y)$.
24. $(x^2+2x+4)(x^2-2x+4)$.
25. $(3x-y)(3x+y)(x+y)(x-y)$.
26. $(2x+3y)(2x-3y)(x+y)(x-y)$.
27. $(x^2-xy+3y^2)(x^2+xy+3y^2)$.
28. $(x^2-3x+5)(x^2+3x+5)$.

269.—*3.* $(x^2+1)(x^4-x^2+1)$. *4.* $(a-b)(a+b)(a^2+ab+b^2)(a^2-ab+b^2)$. *5.* $(x+2)(x-2)(x^2+2x+4)(x^2-2x+4)$.
6. $(a-6)(a^2+6a+36)$. *7.* $3(1-3x)(1+3x+9x^2)$.
8. $x(x-3)(x^2+3x+9)$. *9.* $2(x+5)(x^2-5x+25)$.
10. $(x-y)(x^2+xy+y^2)(x+y)(x^2-xy+y^2)(x^2+y^2)(x^4-x^2y^2+y^4)$. *11.* $(a+b-c)(a^2+2ab+b^2+ac+bc+c^2)$.
12. $2b(3a^2+b^2)$.

272.—*1.* $\frac{1}{x^2}$. *2.* $\frac{1}{p^6}$. *3.* x^3. *4.* 18. *5.* $\frac{3}{2}$. *6.* 1. *7.* 1. *8.* a^2.
9. $\frac{1}{a^3}$. *10.* $\frac{1}{2^9}$. *11.* 1. *12.* a^3.

273.—*1.* $\frac{a^3}{b^3}$. *2.* a. *3.* $\frac{1}{a^3}$. *4.* $\frac{c^4d^5}{a^2b^3}$. *5.* $\frac{y^3}{2x}$. *6.* $\frac{6}{x^{\frac{1}{2}}}$. *7.* $\frac{2}{a^{\frac{1}{x}}}$.
8. $2x^{\frac{3}{4}}$. *9.* $\frac{2}{a^{\frac{1}{2}}}$. *10.* $\frac{1}{x^{\frac{3}{2}}}$. *11.* x. *12.* $2y^{\frac{3}{2}}$. *13.* $\frac{4}{x^{\frac{1}{2}}}$.
14. $\frac{21}{a^{\frac{3}{2}}}$. *15.* $\frac{1}{3a^{\frac{3}{2}}}$ *16.* 8. *17.* $\frac{1}{8}$. *18.* $\frac{1}{8}$. *19.* 64. *20.* 8.
21. $\frac{1}{64}$. *22.* $\frac{1}{2^9}$. *23.* 8. *24.* $\frac{1}{2}$. *25.* 8. *26.* 5. *27.* 2.
28. $\frac{3}{2}$. *29.* $\frac{1}{16^4}$. *30.* $\frac{2187}{128}$. *31.* $\frac{27}{8}$. *32.* $\frac{1}{216}$. *36.* $\sqrt[20]{19863}$.

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273.—*37.* $\sqrt[6]{2187}$. *38.* $\sqrt[5]{9}$. *39.* $x-y$. *40.* $x^{\frac{3}{2}}+xy^{\frac{1}{2}}x^{\frac{1}{2}}y-y^{\frac{3}{2}}$. *41.* 8. *42.* $\frac{1}{16}$. *43.* $\frac{1}{64}$. *44.* 81.

275.—*1.* $\sqrt[12]{3^6}$, $\sqrt[12]{4^4}$, $\sqrt[12]{6^3}$. *2.* $\sqrt[6]{5^3}$, $\sqrt[6]{11^2}$, $\sqrt[6]{13}$. *3.* $\sqrt[6]{2^2}$, $\sqrt[6]{2^9}$, $\sqrt[6]{2^2}$. *4.* $\sqrt[12]{8^3}$, $\sqrt[12]{3^4}$, $\sqrt[12]{6^6}$. *5.* $\sqrt{6}$. *6.* $\sqrt{30}$. *7.* $\sqrt{\frac{16}{5}}$. *8.* $\sqrt[6]{4500}$. *9.* $2\sqrt[6]{9}$.

276.—*1.* $25\sqrt{7}$. *2.* $8\sqrt{11}$. *3.* $14\sqrt{2}$. *4.* $5\sqrt{5}$. *5.* 16·97. *6.* 12·12. *7.* 15·81. *8.* 36·74. *9.* 78·26. *10.* 31·75. *11.* 7. *12.* 2·65. *13.* −25·98. *14.* 7·35. *15.* 16·26. *16.* $4\sqrt[3]{4}$. *17.* $6\sqrt[3]{2}$. *18.* $5\sqrt[4]{5}$. *19.* $-9\sqrt[3]{3}$. *20.* 34·29. *21.* 96. *22.* 61·24. *23.* 41·57. *24.* 118·78. *25.* 51·96. *26.* 332·55.

278.—*1.* 8·66. *2.* 1·155. *3.* 9·794. *4.* 2·683. *5.* ·204. *6.* 19·596. *7.* ·403. *8.* ·045. *9.* ·257. *10.* ·315. *11.* 1·702. *12.* ·822. *13.* 8·989. *14.* ·809. *15.* 1·869. *16.* 2·184. *17.* ·101. *18.* 1·768. *19.* 1·294. *20.* 46·647.

282.—*1.* $\frac{11}{5}$, −5. *2.* $\frac{13}{3}$, $-\frac{11}{3}$. *3.* $\frac{5}{8}$, −3. *4.* $2\frac{1}{2}$, −1. *5.* −3, ·8. *6.* $3\frac{1}{2}$, 1. *7.* 1·72, 1·28. *8.* 6·19, ·807. *9.* 2·38, 4·62. *10.* $\frac{9}{10}$, $-\frac{5}{6}$. *11.* $\frac{8}{3}$, $-\frac{3}{4}$. *12.* 3, −1. *13.* 2, $\frac{1}{3}$. *14.* 13, $\frac{2}{3}$. *15.* 12, −2. *16.* 3, $-\frac{1}{2}$. *17.* 4, $\frac{4}{3}$. *18.* $\frac{3a}{7}$, $\frac{-a}{3}$. *19.* $\frac{-5k}{4}$, $\frac{-2k}{3}$. *20.* $3a$, $\frac{3a}{2}$.

285.—*1.* 5, $\frac{2}{3}$. *2.* $-\frac{1}{2}$, −9. *3.* $\frac{1}{2}$, 2. *4.* 2, −1·72. *5.* $\frac{3}{7}$, −5. *6.* 3, −2. *7.* $\frac{9}{5}$, $-\frac{1}{2}$. *8.* ±3, ±2. *9.* $\frac{9}{10}$, $-\frac{5}{6}$. *10.* ±2, −5, −1. *11.* ±3, ±4. *12.* 3, −2. *13.* $2\frac{1}{2}$, 4. *14.* 6″, 8″. *15.* 5″. *16.* 3·23 rods. *17.* 3·82″, 6·18″. *18.* 4·37 sec. *19.* 7·59 sec.

287.—*1.* $x=17$, 11, $y=11$, 17. *2.* $x=14$, −9, $y=9$, −14. *3.* $x=27$, −19, $y=19$, −27. 4. $x=71$, 13, $y=13$, 71. *5.* $x=\pm 5$, $y=\pm 7$, $x=\pm 7$, $y=\pm 5$. *6.* $x=\pm 8$, ±5, $y=\pm 5$, ±8. *7.* $x=13$, 3, $y=3$, 13. *8.* $x=11$, −8, $y=8$, −11. *9.* $x=1$, $y=1$. *10.* 4″, 3″. *11.* 5″, 7″. *12.* 24″, 7″.

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290.—1. 20 tons. *2.* 181·7 lb. *3.* 480 ft. per sec. *4.* $126\frac{2}{3}$ ft. per min.

291.—5. $\frac{8}{9}''$. *6.* 9·97 lb. *7.* 20·2″. *8.* 208·9 lb. *9.* 23361 + *H.P.* *10.* 10·55″. *11.* 2·406″. *12.* ·285″.

294.—1. 11·91′. *2.* 11·36′. *3.* 25·39 miles per hr. *4.* $9\frac{1}{7}$.

295.—5. $37\frac{1}{3}$. *6.* 30·7 ohms. *7.* $2 \times 3^3 = 1 \times (3{\cdot}78)^3$ approx. *8.* 1·5 lb. *9.* $2\frac{1}{4}'$. *10.* 9 sq. ft. *11.* 51·66 cc.

296.—12. −236·5° C. *13.* 324 ohms.

300.—1. \$492·05. *2.* \$1000. *3.* \$1000. *4.* \$1150·60. *5.* \$2208·05. *6.* \$4794·96, 6% interest.

301.—7. \$5761·04. *8.* \$1128·54. *9.* \$3680·40, \$2055·00. *10.* \$7413·40. *11.* \$10,132·70. *12.* \$228·03, \$683·98. *13.* Amount of premiums invested = \$7325·60.

301.—1. $3\frac{1}{4}$. *2.* 310.

302.—3. 33. *4.* \$1591·20. *5.* $615\frac{3}{16}$. *6.* 228, 40, 60. 7. \$114·91. *8.* $1\frac{41}{89}$ hr. *9.* $21\frac{7}{8}$ min. *10.* 16. *11.* 3117·4 lb. *12.* 80. *13.* ·286. *14.* $\frac{1}{3}''$. *15.* 44·4 hr. *16.* 83·3.

303.—17. $\$11{\cdot}27\frac{1}{2}$. *18.* 5·39. *19.* 4526. *20.* $224\frac{1}{2}$, 492. *21.* ·406. *22.* 38·9 sec. *23.* \$3·99. *24.* $62\frac{1}{2}$. *25.* \$122·67. *26.* ·65%. *27.* 29·25 lb. *28.* 1·04. *29.* ·899. *30.* $112\frac{1}{2}$, 6.

304.—31. \$57·00. *32.* $41\frac{2}{3}$, $33\frac{1}{3}$, 25. *33.* \$31·99. 34. $\frac{3}{4}$. *35.* 28·28′. *36.* \$210·32. *37.* 19 lb. *38.* $\frac{27}{64}$. *39.* 8·67″. *40.* 181·13 lb. *41.* 1443·17 lb. *42.* 8·6, ·31, 312. *43.* 280. *44.* 36·97 lb.

305.—45. 42+. *46.* 50·27 sq. ft., 76·97 sq. ft. *47.* 40′. *48.* ·83″. *49.* 12·73″. *50.* ·942″. *51.* 32·78 sq. in. *52.* 77·6. *53.* 11·46, 10″. *54.* 13·26′. *55.* 5·32″. *56.* ·125″. *57.* 10·16 lb. *58.* 21·5. *59.* 8·38.

306.—60. 14·7″. *61.* 5·01″. *62.* $7\frac{1}{4}''$. *63.* 7·96″. *64.* 122140. *65.* 15·75″. *66.* 48·26 lb. *67.* 343. *68.* $1333\frac{1}{3}$, 640, $213\frac{1}{3}$. *69.* \$852·71. *70.* \$389·63.

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307.—*71.* 9·2″, 2·53″. *72.* \$42·56. *73.* 4·91. *74.* 8183. *75.* 235+. *76.* 118·75″, 118·06″, ·58. *77.* \$507·52.

308.—*78.* 22135. *79.* 1413·72. *80.* \$523·93. *81.* 3·08−. *82.* 58·87. *83.* 182·72. *84.* 1·02 lb. *85.* 19867. *86.* 587·7, 39·7 sq. ft. *87.* 67·259 sq. in., 1·91 lb.

309.—*88.* 10·06, 6·31. *89.* 174·13 cu. in. *90.* 147·66. *91.* \$241·83. *92.* \$109·71. *93.* 1110·3. *94.* 53·31. *95.* 2575·7 lb. *96.* 148·68″, 147·92″, ·52. *97.* \$234·13.

310.—*98.* \$142·72. *99.* 32 R.P.M. *100.* 661·59 sq. in. *1.* $2\sqrt{\frac{A}{\pi}}$. *2.* $\frac{p^2}{18}$.

311.—*3.* 2″, 3″, 1″. *4.* ·615. *5.* 52° 30′, 37° 30′. *6.* 189·55 lb. *7.* 5·28″. *8.* 75000. *9.* 4·35″. *10.* 72°, 72°, 36°. *11.* 81. *12.* 16″, 11·314″. *13.* 5·03. *14.* 6·48″. *15.* ·649. *16.* 15·49″.

312.—*17.* −5·92. *18.* 36′. *19.* 5·77″, 11·54″. *20.* 1559·38. *21.* Sides 8, 12, perp. 10. *22.* 23395×10^4. *23.* 3·25 miles. *24.* ·47% too great. *25.* 3″, 4″. *26.* 6″. *27.* 269·66 sq. ft. *28.* 13″, 15″. *29.* 40′.

313.—*30.* 28″. *31.* 64′. *32.* 13·27′. *34.* 6″×12″. *35.* 37½. *36.* 2·92″. *37.* \$900·00. *38.* 116·52 oz. *39.* 9″, 12″, 15″. *40.* 4″.

314.—*41.* 3600. *42.* ·0039.

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