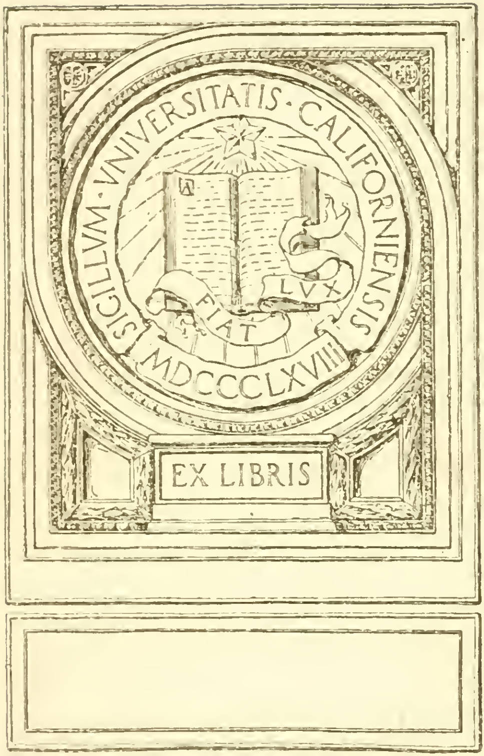
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TRIGONOMETRY FOR BEGINNERS

BY THE

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REVISED AND ENLARGED

FOR THE USE OF AMERICAN SCHOOLS

BY

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PREFACE

The revision of the "Trigonometry for Beginners" differs from that of the original work, chiefly in the following particulars:

The subject matter of Chapter VII. formerly followed that of Chapters VIII. and IX.; the addition formulæ are proved for angles of any magnitude, and for more than two angles; a chapter on Inverse Trigonometric Functions and two chapters on Spherical Trigonometry have been added; logarithmic and trigonometric tables have been inserted. The rearrangement has necessitated minor changes in almost every chapter. Throughout the book, the question of ambiguity of solution has received careful attention. It is believed that the clear, simple presentation which characterized the original work has been retained.

It has been the endeavor to make definitions that need not be unlearned later; to give proofs, rigorous for the general plane angle; to present as much material as the student will master in a first course; and to present such material as will serve him best in his later studies. The proofs of many propositions are left as exercises for the student. These are formulated, and placed in the body of the text.

The lists of examples in the plane trigonometry are, for the most part, those of the original work. Some of the exercises in spherical trigonometry are selected from other texts.

Those desiring a shorter course may omit the chapters and the articles marked with an asterisk.

I acknowledge my indebtedness to Dr. Frank L. Sevenoak, who kindly permitted the use of his tables, and to friends who aided me by suggestions.

JOHN A. MILLER.

JUNE, 1896.

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TRIGONOMETRY FOR BEGINNERS

CHAPTER I

DEFINITIONS

1. The primary object of Trigonometry was, as its name implies, to measure (or solve) triangles; *i.e.* having given the measure of certain parts of a triangle, *e.g.* two sides and its included angle, to compute the remaining parts. In a broader and now universally accepted sense, Trigonometry embraces, in addition to the solution of triangles, all investigations of the relations existing among certain ratios intimately associated with an angle. These ratios are defined in Art. 26.

This branch of the subject is sometimes called **Angular Analysis.**

2. The **figures** with which we shall be concerned in our study of Trigonometry are, with the exception of the *line* and the *angle*, the same as those of Geometry; *i.e.* they are subject to the same limitations and possess the same properties as those of Geometry. For example, the sum of the interior angles of a triangle equals two right angles in Trigonometry as well as in Geometry.

3. The **line** of Trigonometry differs from the line of Geometry, in that, in Trigonometry, it is sometimes of advantage to distinguish between lines drawn in *opposite* directions. [See Art. 47.] For the present, however, we shall not make this distinction.

4. By the **angle** XOP (Fig. 1, Fig. 2, Fig. 3), in Trigonometry, is not meant the *present inclination* of the lines OX and OP as in Geometry, but the amount of turning which OP has done about the point O, in coming from its *initial* position OX, to its *final* position OP.

Illustration. Suppose a race to be run around a circular course. The position of any one of the competitors would be known, if we know that he has described a certain angle about the centre of the course. Thus, if the distance to be run is three times around, the line joining each competitor to the centre would have to describe an angle of 12 right angles.

When we say that a competitor has described an angle of $6\frac{2}{3}$ right angles, we give not only his present position, but the total distance he has gone. He would, in such a case, have gone a little more than one and a half times around the course.

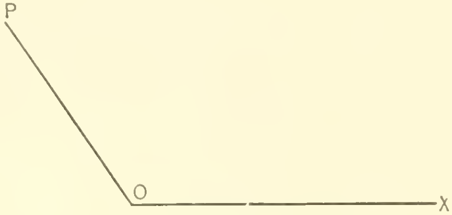


Fig. 1.

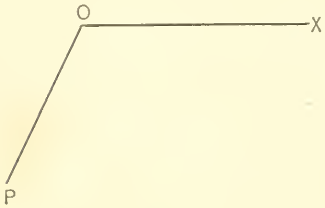


Fig. 2.

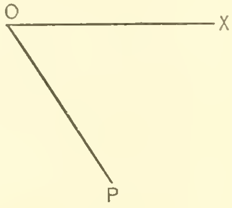


Fig. 3.

It is evident from this definition that a trigonometrical angle may have any magnitude however great. It is well to notice that angle XOP is the amount of turning that *has been done*. In other words, it is the *result* of the turning, not the *process*.

5. The geometrical **representation** of a trigonometric angle depends only on the initial and final positions of the line OP. Hence the figure XOP (Fig. 1, Art. 4) may be the geometrical representative of an unlimited number of trigonometrical angles.

(i.) The angle XOP may represent the angle less than two right angles, as in Geometry.

In this case, OP has turned from the position OX into the position OP by turning about O in the direction *contrary* to that of the hands of a watch.

(ii.) The angle XOP may represent the angle described by OP in turning from the position OX into the position OP in the *same* direction as that of the hands of a watch.

In the first case it is usual to say that the angle XOP is a *positive* angle; in the second case it is a *negative* angle.

(iii.) The angle XOP may be the geometrical representation of any of the trigonometrical angles formed by any number of complete revolutions in the *positive* or in the *negative* direction, added to either of the first two angles mentioned in (i.) and (ii.).

We may express (iii.) thus: XOP is the geometrical representative of $XOP + 4n$ right angles, where n is any integer.

6. DEFINITIONS. O is called the **origin.** The line OX is the **initial line.** The line OP is the **revolving line or radius vector.** When referring to the angle XOP the lines OX and OP are called the **sides** of the angle, and O is called its **vertex.**

7. To add angle $X'OP'$ to angle XOP, both being positive, revolve OP from its final position when it represents angle XOP,

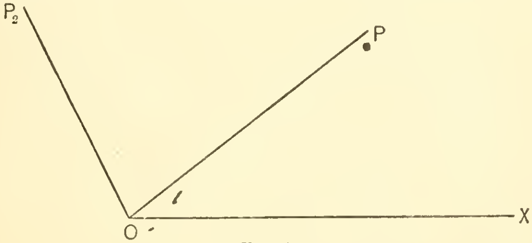


FIG. 4.

through an angle equal to angle $X'OP'$; call this position OP_2. Then,

$$\angle XOP_2 = \angle XOP + \angle X'OP'.$$

EXAMPLES. I.

Give a geometrical representation of each of the following angles, the initial line being drawn in each case from the origin towards the right:

1. $+3$ right angles.
2. $+5$ right angles.
3. $+4\frac{1}{2}$ right angles.
4. $+7\frac{1}{4}$ right angles.
5. -1 right angle.
6. $10\frac{2}{3}$ right angles.
7. $-10\frac{1}{3}$ right angles.
8. $+4$ right angles.
9. -4 right angles.
10. $4n$ right angles.
11. $(4n+2)$ right angles.
12. $-(4n+\frac{1}{2})$ right angles.
13. $1\frac{1}{2}$ right angles $+ 2\frac{1}{2}$ right angles.
14. $3\frac{1}{2}$ right angles $- \frac{1}{2}$ right angle.

8. Certain propositions which the student has proved while studying plane Geometry will be referred to very frequently, and quoted without proof.

The principal ones are:

a. The Pythagorean theorem.

b. Conditions under which two triangles are similar.

c. Homologous sides of similar triangles are proportional, and homologous angles are equal.

d. I. The ratio $\frac{\text{circumference of a circle}}{\text{diameter of a circle}}$ is a certain fixed number.

II. It is an *incommensurable* number.

III. It is 3.14159265+ ⋯.

9. When we say that this number is incommensurable, we mean that its exact value cannot be stated as an *arithmetical* fraction.

It also happens that we have no short *algebraical* expression such as a surd, or combination of surds, which represents it exactly, so that we have no *numerical* expression whatever, arithmetical nor algebraical, to represent *exactly* the ratio of the circumference of a circle to its diameter.

Hence the universal custom has arisen, of denoting its *exact* value by the letter π.

Thus π stands *always* for **the exact value of a certain incommensurable number,** whose approximate value is 3.14159265, and which **is the ratio of the circumference of any circle to its diameter.**

It cannot be too carefully impressed on the student's memory that π stands for this number 3.14159265 ⋯, etc., and for nothing else; just as 180 stands for the number one hundred and eighty, and for nothing else.

We may notice that $\frac{22}{7} = 3.\dot{1}4285\dot{7}$.

So that $\frac{22}{7}$ and π differ by less than a thousandth part of their value.

CHAPTER II

MEASUREMENT OF ANGLES

10. It is usual to say that we have **measured** any concrete quantity, when we have found **how many** times it contains some familiar quantity of the same kind.

We say, for example, that we have measured a line, when we have found *how many* feet it contains. We say that we have measured a field, when we have found out *how many* acres or how many square yards it contains.

To know the measurement of any quantity, then, we must have two things. First, we must have a *unit*, or standard of reference, of the *same kind* as the thing measured. Secondly, we must have the *measure*, or the *number of times* the thing measured contains the unit, or standard quantity.

Hence, the **measure** of a quantity is the **number**, and the **unit** is the **concrete quantity**, by means of which it is measured.

Example 1. *A line contains* 261 feet; that is, 261 times a foot. Here the *measure* or number is 261, and the *unit* a foot.

EXAMPLES. II.

1. What is the measure of 1 mile when a chain of 66 feet is the unit?

2. What is the measure of an acre when a square whose side is 22 yards is the unit?

3. The length of an Atlantic cable is 2300 miles and the length of the cable from England to France is 21 miles. Express the length of the first in terms of the second as a unit.

4. The measure of a certain field is 22 and the unit 1100 square yards: express the area of the field in acres.

5. Find the measure of a miles when b yards is the unit

6. The measure of a certain distance is a when the unit is c feet. Express the distance in yards.

11. Measurement of angles. There are two common methods of measuring angles.

(i.) The rectangular measure.

(ii.) The circular measure.

RECTANGULAR MEASURE

12. Angles are always measured *in practice* with the **right angle** (or part of the right angle) as unit.

The reasons why the right angle is chosen for a unit are:

(i.) All right angles are equal to one another.

(ii.) A right angle is practically easy to draw.

(iii.) It is an angle whose size is very familiar.

13. The right angle is divided into 90 equal parts, each of which is called a **degree**; each degree is subdivided into 60 equal parts, each of which is called a **minute**; and each minute is again subdivided into 60 equal parts, each of which is called a **second**.

Instruments used for measuring angles are subdivided accordingly; and the size of an angle is known when, with such an instrument, it has been observed that the angle contains a certain number of degrees, and a certain number of minutes beyond the number of complete degrees, and a certain number of seconds beyond the number of complete minutes.

Thus an angle might be recorded as containing 79 degrees + 18 minutes + 36.4 seconds.

Degrees, minutes, and seconds are indicated respectively by the symbols °, ′, ″, and the above angle would be written $79° 18' 36.4''$.

14. An angle given in degrees, minutes, and seconds may be expressed as the decimal of a right angle by the usual method.

EXAMPLE. Express $39° 4' 27''$ as the decimal of a right angle.

$$\begin{array}{r|l} 60 & 27 \text{ seconds} \\ \hline 60 & \;4.45 \text{ minutes} \\ \hline 90 & 39.7416666 \text{ etc. degrees} \\ \hline & .4415\dot{7}4\dot{0}74 \text{ etc. right angles} \end{array}$$

$.4415\dot{7}4\dot{0}74$ of a right angle, *Ans.*

NOTE. The French proposed to call the 100th part of a right angle a **grade** (written 3^{g}), the 100th part of a grade a minute (written $3^{\backslash}$), the 100th part of a minute a second (written $3^{\backslash\backslash}$). So that 1.437275 right angles would be read $143^{g}\, 72^{\backslash}\, 75^{\backslash\backslash}$. The decimal method of subdividing the right angles has never been used.

CIRCULAR MEASURE

15. DEFINITION. A **radian** is an angle at the centre of a circle, subtended by an arc equal in length to the radius of the circle. Thus if in the circle *RPS*, whose centre is *O*, arc RP = radius OR, then, angle ROP is a radian.

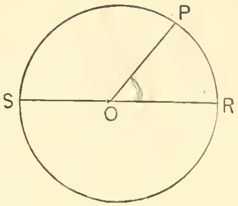


FIG. 5.

16. We shall now prove that the radian is a constant angle; or stating the same thing differently, we are about to prove that if we take any number of different circles, and measure on the circumference of each an arc equal in length to its radius, then the angles at the centres of these circles which stand on these arcs respectively, will be all of the same size.

17. *To prove that all radians are equal to one another.*

Since the radian at the centre of a circle stands on an arc equal in length to the radius,

and an angle of two right angles at the centre of a circle stands on half the circumference,

and since angles at the centre of a circle are to one another as the arcs on which they stand (Geom.),

then,
$$\frac{\text{a radian}}{\text{2 right angles}} = \frac{\text{radius}}{\text{semi-circumference}} = \frac{\text{diameter}}{\text{circumference}} = \frac{1}{\pi}.$$

Therefore a radian $= \frac{1}{\pi}$ of 2 right angles,

= a certain fixed fraction of 180°. (Art. 8.)

Thus the radian possesses the qualification most essential in a unit; viz. it is always the same.

18. The reasons why a radian is used as a unit are:

(i.) All radians are equal to one another.

(ii.) Its use simplifies many formulæ in Theoretical Trigonometry.

19. Since $1 \text{ radian} = \frac{2 \text{ rt.} \angle s}{\pi}$, (Art. 17.)

$\therefore$ 1 radian = 57.2957°.

20. Since $1 \text{ radian} = \frac{2 \text{ rt.} \angle s}{\pi}$,

$$\therefore \pi \text{ radians} = 2 \text{ rt.} \angle s = 180^\circ,$$

$$\frac{\pi}{2} \text{ radians} = 1 \text{ rt.} \angle = 90^\circ,$$

$$\frac{3\pi}{2} \text{ radians} = 3 \text{ rt.} \angle s = 270^\circ,$$

and so on.

21. A "c" is used to indicate radians, just as "°" is used to indicate degrees. Thus θ^c is read "θ radians," just as $A°$ is read "A degrees." When the measure of an angle is expressed as some *multiple* of π, *e.g.* $\frac{\pi}{2}$, *the unit being the radian*, common usage has sanctioned the omission of the "c" after the measure; thus, $\frac{\pi^c}{2}$ is usually written $\frac{\pi}{2}$. This practice sometimes confuses the beginner. Yet, with a little care no confusion need arise. We need only remember that π is a mere number (Art. 8); that in the expression "the angle π" or kindred expressions *some unit must* be implied, and that by common agreement this unit is the radian, so that π is often used for 180°, meaning, then, π radians (Art. 20). This does not exclude the angle $\pi°$; but if a degree is the unit, it is so expressed.

Another agreement sometimes made, is that the Greek letters α, β, etc., are used to express the measure of angles when the radian is the unit of measure; while, the Roman letters A, B, C, etc., are used when the unit of measure is the degree. The distinction, however, is not universally observed.

22. *To find the number of degrees in an angle containing a given number of radians.*

Let D = number of degrees in the angle.
Let α = number of radians in the angle.

$$1^c = \left(\frac{180}{\pi}\right)^\circ. \qquad \text{(Art. 17.)}$$

$$\therefore\ \alpha^c = \left(\frac{\alpha \times 180}{\pi}\right)^\circ.$$

$$\therefore\ \frac{\alpha \times 180}{\pi} = D, \text{ the } \textit{number} \text{ of degrees} \qquad (1)$$

which was to be found.

We may in a similar manner *find the number of radians in an angle containing a given number of degrees.* Or, simply solving (1) for α, we have $\alpha = \frac{\pi D}{180}$, the number of radians in an angle containing D degrees.

The problem may be thus solved: $\frac{D°}{180°} = \frac{\alpha^c}{\pi^c}$, for each fraction is the *ratio* of the same angle to two right angles.

EXAMPLE. Find number of degrees in two radians. We have

$$\frac{D}{180}=\frac{2}{\pi}. \therefore D=\frac{360}{\pi}.$$

23. EXAMPLES. III.

I. Express each of the following angles as the decimal of a right angle:

1. $8° 15' 27''$. **3.** $97° 5' 15''$. **5.** $132° 6'$.

2. $6° 4' 30''$. **4.** $16° 14' 19''$. **6.** $49°$.

Express in degrees, minutes, and seconds:

7. .01375 right angles. **10.** .240025 right angles.

8. .0875 right angles. **11.** .180115 right angles.

9. 1.704535 right angles. **12.** .35 right angles.

II. Express the following angles in rectangular measure:

1. π. **4.** 3^c. **7.** θ.

2. $\frac{3\pi}{4}$. **5.** 3.14159265^c, etc. **8.** $.00314159^c$, etc.

3. 1^c. **6.** $\frac{2^c}{\pi}$. **9.** 10π.

III. Express the following angles in circular measure:

1. $180°$. **4.** $22\frac{1}{2}°$. **7.** $n°$.

2. $360°$. **5.** $1°$. **8.** $\frac{90°}{\pi}$.

3. $60°$. **6.** $57.295°$, etc. **9.** A.

IV. Give a geometrical representation of the following trigonometrical angles. Draw the initial line from O, horizontally, to the right.

1. $\frac{\pi}{2}$. **2.** 8π. **3.** 5π.

4. $(2n+1)\pi$. **5.** $(2n+\frac{1}{2})\pi$.

V. Find the ratio of

1. $45°$ to $\frac{3\pi}{4}$. **2.** $1°$ to 1^c. **3.** 1.75^c to $\frac{100°}{\pi}$.

24. Since angles at the centre of a circle are to one another as the arcs on which they stand (Geom.), therefore (see Fig. 6)

$$\frac{\text{an angle } ROP}{\text{one radian}}=\frac{\text{arc } RP}{\text{arc } RS}=\frac{\text{arc } RP}{\text{the radius}}.$$

Hence the angle $ROP=\frac{\text{arc } RP}{\text{the radius}}$ radians.

So that the **circular measure** of an angle (at the centre of a circle) is the **ratio of its arc to the radius.**

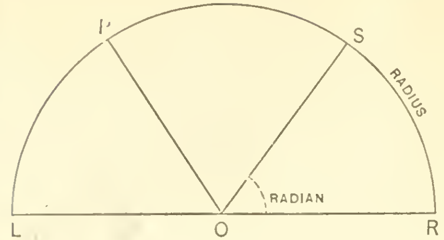


Fig. 6.

Example. Find the number of degrees in the angle subtended by an arc 46 ft. 9 in. long, at the centre of a circle whose radius is 25 feet.

The angle stands on an arc of $46\frac{3}{4}$ feet, and the radian, at the centre of the same circle, stands on an arc of 25 feet.

$$\therefore \text{ the angle} = \frac{46\frac{3}{4}}{25} \text{ radians}, = \frac{187}{100} \times \frac{2 \text{ right angles}}{\pi},$$

$$= \frac{187}{100} \times \frac{180^\circ}{\pi} = 105.8^\circ \text{ nearly.}$$

25. EXAMPLES. IV.

(In the Answers $\frac{22}{7}$ is used for π.)

1. Find the number of radians in an angle at the centre of a circle of radius 25 feet, which stands on an arc of $37\frac{1}{2}$ feet.

2. Find the number of degrees in an angle at the centre of a circle of radius 10 feet, which stands on an arc of 5π feet.

3. Find the number of right angles in the angle at the centre of a circle of radius $3\frac{2}{11}$ inches, which stands on an arc of 2 feet.

4. Find the length of the arc subtending an angle of $4\frac{1}{2}$ radians at the centre of a circle whose radius is 25 feet.

5. Find the length of an arc of 80 degrees on a circle of 4 feet radius.

6. The angle subtended by the diameter of the sun at the eye of an observer is $32'$; find approximately the diameter of the sun if its distance from the observer be 90,000,000 miles.

7. A railway train is travelling on a curve of half a mile radius at the rate of 20 miles an hour; through what angle has it turned in 10 seconds?

8. A railway train is travelling on a curve of two-thirds of a mile radius, at the rate of 60 miles an hour; through what angle has it turned in a quarter of a minute?

9. Find approximately the number of seconds contained in the angle which subtends an arc one mile in length at the centre of a circle whose radius is 4000 miles.

10. If the radius of a circle be 4000 miles, find the length of an arc which subtends an angle of $1''$ at the centre of the circle.

11. If in a circle whose radius is 12 ft. 6 in. an arc whose length is .6545 of a foot subtends an angle of 3 degrees, what is the ratio of the diameter of a circle to its circumference?

12. If an arc 1.309 feet long subtend an angle of $7\frac{1}{2}$ degrees at the centre of a circle whose radius is 10 feet, find the ratio of the circumference of a circle to its diameter.

13. On a circle 80 feet in radius it was found that an angle of $22° 30'$ at the centre was subtended by an arc 31 ft. 5 in. in length; hence calculate to four decimal places the numerical value of the ratio of the circumference of a circle to its diameter.

14. If the diameter of the moon subtend an angle of $30'$, at the eye of an observer, and the diameter of the sun an angle of $32'$, and if the distance of the sun be 375 times the distance of the moon, find the ratio of the diameter of the sun to that of the moon.

15. Find the number of radians (*i.e.* the circular measure of) in $10''$ correct to 3 significant figures. (Use $\frac{355}{113}$ for π.)

16. Find the radius of a globe such that the distance measured upon its surface between two places in the same meridian, whose latitudes differ by $1° 10'$, may be one inch.

17. By construction prove that the unit of circular measure is less than $60°$.

18. On the 31st December the sun subtends an angle of $32' 36''$, and on 1st July an angle of $31' 32''$; find the ratio of the distances of the sun from the observer on those two days.

19. Show that the measure of the angle at the centre of a circle of radius r, which stands on an arc a, is $\frac{k \cdot a}{r}$, where k depends solely on the unit of angle employed.

Find k when the unit is (i.) a radian, (ii.) a degree.

20. The difference of two angles is $\frac{1}{9}\pi$, and their sum $56°$; find them.

21. Find the number of radians in an angle of n'.

22. Express in right angles and in radians the angles

(i.) of a regular hexagon,
(ii.) of a regular octagon,
(iii.) of a regular quindecagon.

23. Taking for unit the angle between the side of a regular quindecagon and the next side produced, find the measures (i.) of a right angle, (ii.) of a radian.

24. Find the unit when the sum of the measures of a degree and of the hundredth part of a right angle is 1.

25. What is the unit when the sum of the measures of a right angles and of b degrees is c?

26. The three angles of a triangle have the same measure when the units are $\frac{1}{90}$ of a right angle, $\frac{1}{100}$ of a right angle, and a radian respectively; find the measure.

27. The interior angles of an irregular polygon are in A.P.; the least angle is $120°$; the common difference is $5°$; find the number of sides.

CHAPTER III

THE TRIGONOMETRICAL RATIOS

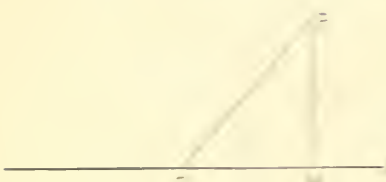


Fig. 5

Fig. 6

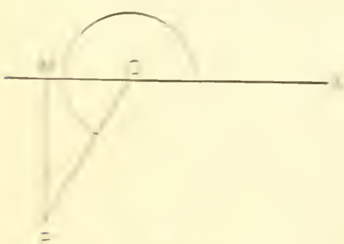


Fig. 7

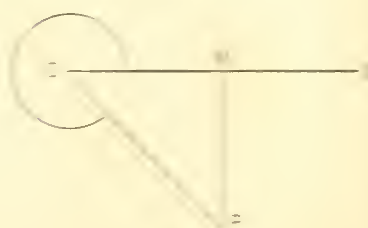


Fig. 8

26. Given any angle XOP as in the figures. From P, any point in OP, the revolving line, a perpendicular PM is let fall on the initial line OX, as in Figs. 5 and 6, or on the initial line produced, as in Figs. 7 and 8. Six ratios may now be formed, viz.,

27. I. $\dfrac{MP}{OP}$, II. $\dfrac{OM}{OP}$, III. $\dfrac{MP}{OM}$, IV. $\dfrac{OP}{MP}$, V. $\dfrac{OP}{OM}$, VI. $\dfrac{OM}{MP}$.

IV., V., VI. are respectively the reciprocals of I., II., and III.

28. These ratios are of much importance in all mathematical investigations. For convenience, they have been given the following names. If XOP be the angle A, then

I., $\frac{MP}{OP}$ is called the **sine of angle *A***, and is read, sine A;

II., $\frac{OM}{OP}$ is called the **cosine of angle *A***, and is read, cosine A;

III., $\frac{MP}{OM}$ is called the **tangent of angle *A***, and is read, tangent A;

IV., $\frac{OP}{MP}$ is called the **cosecant of angle *A***, and is read, cosecant A;

V., $\frac{OP}{OM}$ is called the **secant of angle *A***, and is read, secant A;

VI. $\frac{OM}{MP}$ is called the **cotangent of *A***, and is read, cotangent A.

29. The expressions *sine A*, *cosine A*, *tangent A*, *cosecant A*, *secant A*, *cotangent A* are abbreviated into *sin A*, *cos A*, *tan A*, *csc A* or *cosec A*, *sec A*, *cot A*, respectively.

The powers of the trigonometric ratios are expressed as follows:

$(\sin A)^2 = \sin A \times \sin A$, is written $\sin^2 A$;

$(\cos A)^3 = \cos A \cdot \cos A \cdot \cos A$, is written $\cos^3 A$;

$(\tan A)^n$ is written $\tan^n A$,

and so on.

The student must notice that "$\sin A$" is a *single symbol*. It is the *name* of a *number*, or *fraction*, belonging to the angle A; and if it be at any time convenient, we may denote *sin A* by a *single letter*, such as s or x. Also $\sin^2 A$ is an abbreviation for $(\sin A)^2$, that is, for $(\sin A) \times (\sin A)$. Such abbreviations are used because they are *convenient*.

The student who succeeds in the study of trigonometry must commit the preceding definitions to memory.

CHAPTER IV

THE ACUTE ANGLE

30. Thus far we have placed no limitations on the magnitude of the angle under consideration at any time. In the present chapter we shall confine our attention to angles lying *between* 0° *and* 90°. We shall, in Chapter VII., return to the consideration of the general angle.

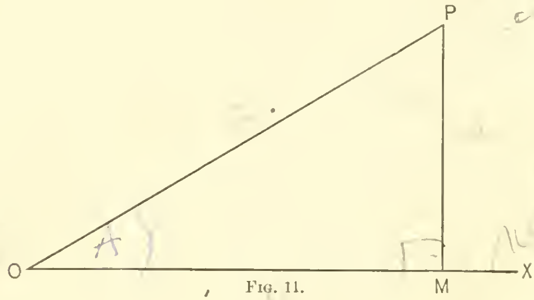


Fig. 11.

31. Given a right triangle OMP with the right angle at M. Then MOP is acute, as is also OPM. Let us consider angle MOP; call $\angle MOP$, A; then,

$$\sin A = \frac{MP}{OP} = \frac{\text{side of triangle opposite angle under consideration}}{\text{hypotenuse}},$$

or, briefly, $$\sin A = \frac{\text{opposite side}}{\text{hypotenuse}};$$

$$\cos A = \frac{OM}{OP}$$

$$= \frac{\text{side of triangle adjacent to the angle under consideration}}{\text{hypotenuse}},$$

or, briefly, $$\cos A = \frac{\text{adjacent side}}{\text{hypotenuse}};$$

$$\tan A = \frac{MP}{OM} = \frac{\text{opposite side}}{\text{adjacent side}};$$

and similar expressions for the other ratios.

Exercise. Write the trigonometrical ratios of angle P.

32. Assuming that the angle XOP is less than 90°, we shall show

I. *That so long as*[1] *the angle remains unchanged, the ratios remain unchanged.*

II. *That a small change in the angle produces a change in each of the ratios.*

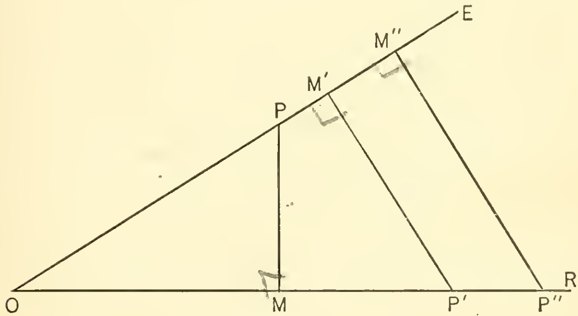


Fig. 12.

I. Take any angle ROE; let P be any point in OE, one of the lines containing the angle, and let P', P'' be any two points in OR, the other line containing the angle. Draw PM perpendicular to OR, and $P'M'$, $P''M''$ perpendiculars to OE.

Then the three triangles OMP, $OM'P'$, $OM''P''$ each contain a right angle, and they have the angle at O common; therefore their third angles must be equal.

Thus the three triangles are equiangular.

Therefore the ratios $\frac{MP}{OP}$, $\frac{M'P'}{OP'}$, $\frac{M''P''}{OP''}$ are all equal. (Geom.)

But each of these ratios is $\frac{\text{opposite side}}{\text{hypotenuse}}$ *with reference to the angle at* O; that is, they are each sin ROE.

Thus, sin ROE is the same *whatever* be the position of the point P on *either* of the lines containing the angle ROE.

Therefore sin ROE is always the same.

II. Let XOP and XOP' be two angles nearly equal. (See Fig. 13).

[1] We shall show (Art. 63) that this change must be, in general, less than 90°. However, our proof is rigorous for the proposition stated.

Let $OP = OP'$. (See I.)

Let $P'Q'$ and PQ be perpendicular to OX.

Since $\angle XOP'$[1] $\neq \angle XOP$; $P'Q' \neq PQ$ (Geometry);

$\therefore M'P' \neq MP$ (Geometry); $\therefore OM' \neq OM$ (Geometry);

$$\sin XOP' = \frac{OP'}{M'P'}, = \frac{M'P'}{OP'} \qquad \sin XOP = \frac{MP}{OP}.$$

But,
$$\frac{M'P'}{OP'} \neq \frac{MP}{OP}.$$

$$\therefore \sin XOP' \neq \sin XOP.$$

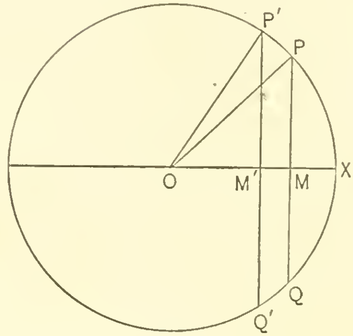


FIG. 13.

EXERCISE I. Prove Proposition I. of this article for the remaining ratios.

EXERCISE II. Prove Proposition II. of this article for the remaining ratios.

33. The student should observe carefully

(i.) that each *ratio*, such as $\frac{\text{opposite side}}{\text{hypotenuse}}$, is a mere *number*;

(ii.) that we have proved, in Art. 32, these ratios remain unchanged as long as the angle remains unchanged;

(iii.) that if the angle be altered ever so slightly, there is a consequent alternation in the value of these ratios;

(iv.) that since these ratios are all numbers, they are therefore algebraic quantities, and hence obey all the laws of ordinary algebra.

(v.) that there is a right angle in the same triangle as the angle referred to.

EXAMPLE. In Fig. 14, in which BDA is a right angle, find $\sin DBA$ and $\cos DBA$.

In this case

(i.) DBA is the *angle*.

(ii.) BDA is a *right angle* in the same triangle as the angle DBA.

(iii.) DA is the *side* opposite DBA and is perpendicular to BD.

(iv.) BA is the *hypotenuse*.

(v.) BD is the *adjacent side*.

[1] We shall use the sign $\neq$, $\not<$, $\not>$, to mean "is different from," "is not less than," "is not greater than," respectively.

Therefore $\sin DBA$, which is $\dfrac{\text{opposite side}}{\text{hypotenuse}}, = \dfrac{DA}{BA}$;

$\cos DBA$, which is $\dfrac{\text{adjacent side}}{\text{hypotenuse}}, = \dfrac{BD}{BA}$.

34. EXAMPLES. V.

1. In the triangle ABC, C being a right angle, $AB = 25$, $CB = 16$; find $\sin A$, $\cos A$.

2. If in the triangle ABC, C being a right angle, $AC = 2$, $BC = 4$, find $\sin B$, $\cos B$, and $\cot B$.

3. Let ACB be any angle, and let ABC and BDC be right angles (see Fig. 14). Write down *two* values for each of the following ratios: (i.) $\sin ACB$, (ii.) $\cos ACB$, (iii.) $\tan ACB$, (iv.) $\sin BAC$, (v.) $\cos BAC$, (vi.) $\tan BAC$.

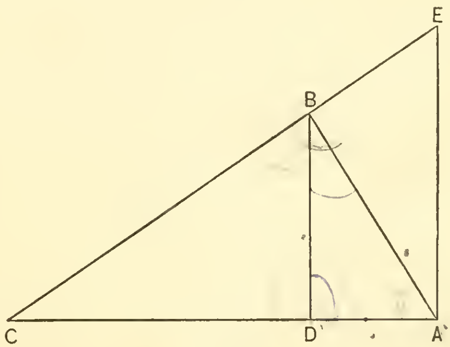


FIG. 14.

4. In Fig. 14 BDC, CBA, and EAC are right angles.
Write down (i.) $\sin DBA$, (ii.) $\sin BEA$, (iii.) $\sin CBD$, (iv.) $\cos BAE$, (v.) $\cos BAD$, (vi.) $\cos CBD$, (vii.) $\tan BCD$, (viii.) $\tan DBA$, (ix.) $\tan BEA$, (x.) $\tan CBD$, (xi.) $\sin DAB$, (xii.) $\sin BAE$.

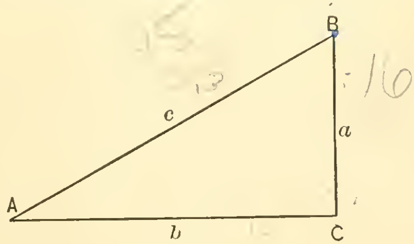


FIG. 15.

5. If ABC be any right-angled triangle with a right angle at C, and we let A, B, and C stand for the angles at A, B, and C respectively, and let a, b, and c be the measures of the sides opposite the angles A, B, and C respectively:

Show that $\sin A = \frac{a}{c}$, $\cos A = \frac{b}{c}$, $\tan A = \frac{a}{b}$.

Show also that $\sin^2 A + \cos^2 A = 1$.

Show also that (i.) $a = c \cdot \sin A$, (ii.) $b = c \cdot \sin B$, (iii.) $a = c \cdot \cos B$, (iv.) $b = c \cdot \cos A$, (v.) $\sin A = \cos B$, (vi.) $\cos A = \sin B$, (vii.) $\tan A = \cot B$.

6. The sides of a right-angled triangle are in the ratio 5 : 12 : 13; find the sine, cosine, and tangent of each acute angle of the triangle.

7. The sides of a right-angled triangle are in the ratio 1 : 2 : $\sqrt{3}$; find the sine, cosine, and tangent of each acute angle of the triangle.

8. Prove that if A be either of the angles of the above two triangles, $\sin^2 A + \cos^2 A = 1$.

9. ABC is a right-angled triangle, C being the right angle. AB is 2 feet and AC is 1 foot; find the length of BC, and thence find the value of $\sin A$, $\cos A$, and $\tan A$.

10. ABC is a right-angled triangle, C being the right angle, $AB = \sqrt{2}$ ft. and $AC = 1$ foot; prove that $\sin A = \cos A = \sin B = \cos B$.

11. ABC is a right-angled triangle, C being the right angle; $BC = 1$ foot, and $AB = \sqrt{3}$ feet; find AC and $\sin A$ and $\sin B$.

CHAPTER V

ON THE TRIGONOMETRICAL RATIOS OF CERTAIN ANGLES

35. The trigonometrical ratios of an angle are *numerical quantities simply*, as their name ratio implies. They are in nearly all cases incommensurable numbers.

Their practical value has been found for all angles between 0° and 90°, which differ by 1′; and a list of these values will be found in any volume of Mathematical Tables.

The general method of finding trigonometrical ratios belongs to a more advanced part of the subject than the present, but there are certain angles whose ratios can be found in a simple manner.

36. *To find the* **sine, cosine,** *and* **tangent** *of an angle of* **45°**.

When one angle of a right-angled triangle is 45°, that is, the half of a right angle, the third angle must also be 45°. Hence 45° is one angle of an *isosceles* right-angled triangle.

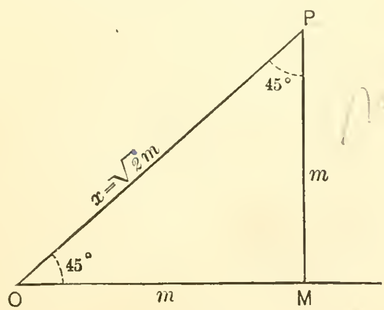


Fig. 16.

Let POM be an isosceles triangle such that PMO is a right angle, and $OM = MP$. Then $POM = OPM = 45°$.

Let the measures of OM and of MP each be m. Let the measure of OP be x.

Then $$x^2 = m^2 + m^2 = 2\,m^2.$$

$$\therefore\ x = \sqrt{2} \cdot m.$$

Hence, $\sin 45° = \sin POM = \frac{MP}{OP} = \frac{m}{\sqrt{2} \cdot m} = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2},$

$$\cos 45° = \cos POM = \frac{OM}{OP} = \frac{m}{\sqrt{2} \cdot m} = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2},$$

$$\tan 45° = \tan POM = \frac{MP}{OM} = \frac{m}{m} = \frac{1}{1} = 1.$$

37. *To find the* **sine, cosine,** *and* **tangent** *of an angle of* **60°.**

Each angle in an *equilateral* triangle is 60°, because they are each one-third of 180°. If we draw a perpendicular from one of the angular points of the triangle to the side opposite, we get a right-angled triangle in which one angle is 60°.

Let OPQ be an equilateral triangle. Draw PM perpendicular to OQ. Then OQ is bisected in M.

Let the measure of OM be m; then that of OQ is $2\,m$, and therefore that of OP is $2\,m$.

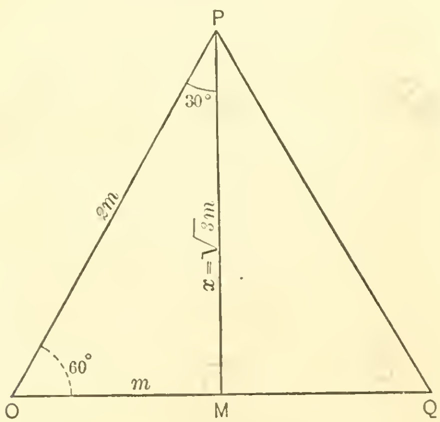


Fig. 17.

Let the measure of MP be x.

Then $$x^2 = (2\,m)^2 - m^2 = 4\,m^2 - m^2 = 3\,m^2.$$

$$\therefore\ x = \sqrt{3} \cdot m.$$

Hence, $\sin 60° = \sin POM = \frac{MP}{OP} = \frac{\sqrt{3} \cdot m}{2m} = \frac{\sqrt{3}}{2}$,

$$\cos 60° = \cos POM = \frac{OM}{OP} = \frac{m}{2m} = \frac{1}{2},$$

$$\tan 60° = \tan POM = \frac{MP}{OM} = \frac{\sqrt{3} \cdot m}{m} = \frac{\sqrt{3}}{1} = \sqrt{3}.$$

38. *To find the* **sine, cosine,** *and* **tangent** *of an angle of* **30°**.

With the same figure and construction as above we have the angle $OPM = 30°$, since it is half of OPQ, *i.e.* of 60°.

Hence, $\sin 30° = \sin OPM = \frac{MO}{PO} = \frac{m}{2m} = \frac{1}{2}$,

$$\cos 30° = \cos OPM = \frac{PM}{PO} = \frac{\sqrt{3} \cdot m}{2m} = \frac{\sqrt{3}}{2},$$

$$\tan 30° = \tan OPM = \frac{MO}{PM} = \frac{m}{\sqrt{3} \cdot m} = \frac{1}{\sqrt{3}} = \frac{1}{3}\sqrt{3}.$$

39. *To find the* **sine, cosine,** *and* **tangent** *of an angle of* **0°**.

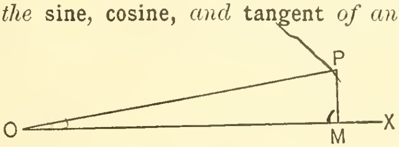


Fig. 18.

Let XOP be a small angle. Draw PM perpendicular to OX, and let OP be always of the same length, so that P lies on the circumference of a circle whose centre is O.

Then if the angle XOP be diminished, we can see that MP is diminished also, and that consequently $\frac{MP}{OP}$, which is $\sin XOP$, is diminished. And by diminishing the angle XOP sufficiently, *we can make MP as small as we please*, and therefore *we can make* sin XOP *smaller than any assignable number however small that number may be.*

This is what is meant when it is said that the value to which sin XOP approaches as the angle is diminished, is 0. This is expressed by saying,

$$\sin 0° = 0. \quad \text{(i.)}$$

Again, as the angle XOP diminishes, OM approaches OP in length; and cos XOP, which is $\frac{OM}{OP}$, approaches in value to $\frac{OP}{OP}$, *i.e.* to 1.

This is expressed by saying,

$$\cos 0^\circ = 1. \qquad \text{(ii.)}$$

Also, tan XOP is $\frac{MP}{OM}$; and we have seen that MP approaches 0, while OM does not; $\therefore$ tan XOP approaches 0.

This is expressed by saying,

$$\tan 0^\circ = 0. \qquad \text{(iii.)}$$

40. *To find the* **sine, cosine,** *and* **tangent** *of* **90°.**

Let XOY be a right angle $= 90°$.

Draw XOP nearly a right angle; draw PM perpendicular to OX, and let OP be always of the same length, so that P lies on the circumference of a circle whose centre is O.

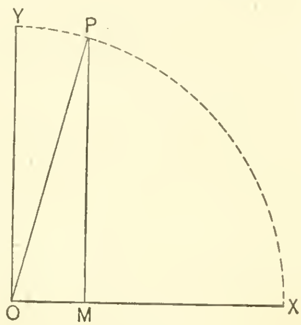


Fig. 19.

Then, as the angle XOP approaches to XOY, we can see that MP approaches OP, while OM continually diminishes.

Hence when XOP approaches 90°, sin XOP, which is $\frac{MP}{OP}$, approaches in value to $\frac{OP}{OP}$, that is to $\frac{1}{1}$, *i.e.* to 1.

Hence we say that $\quad \sin 90^\circ = 1. \qquad$ (i.)

Again, when XOP approaches 90°;

cos XOP, which is $\frac{OM}{OP}$, approaches in value to $\frac{0}{OP}$, that is to 0.

Hence we say that $\quad \cos 90^\circ = 0. \qquad$ (ii.)

Again, when XOP approaches 90°, tan XOP, which is $\frac{MP}{OM}$, approaches in value to $\frac{OP}{\text{a quantity which approaches 0}}$.

But in any fraction whose numerator does not diminish, the smaller the denominator, the greater the value of that fraction; and if the denominator continually diminishes, the value of the fraction continually increases.

Hence, *tan XOP can be made larger than any assigned number by making the angle XOP approach near enough to* 90°.

This is what we mean when we say that

$$\tan 90° \text{ is } \textbf{infinity}, \text{ or, } \tan 90° = \infty. \qquad \text{(iii.)}$$

41. The following table exhibits the above results:

angle	0°	30°	45°	60°	90°
sine	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
cosine	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0
tangent	0	$\frac{1}{3}\sqrt{3}$	1	$\sqrt{3}$	∞

The student may notice that the sine increases with the angle, while the cosine diminishes as the angle increases.

Also that the squares of the sines of 0°, 30°, 45°, 60°, and 90° are respectively 0, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, and $\frac{4}{4}$, and that the squares of the cosines of the same angles are $\frac{4}{4}$, $\frac{3}{4}$, $\frac{2}{4}$, $\frac{1}{4}$, and 0.

EXAMPLES. VI.

If $A = 90°$, $B = 60°$, $C = 30°$, $D = 45°$, prove the following:

1. $\cos^2 B - \sin^2 B = 1 - 2\sin^2 B$.
2. $\sin B \cdot \cos C + \sin C \cdot \cos B = \sin A$.
3. $2(\cos B \cdot \cos D + \sin B \cdot \sin D)^2 = 1 + \cos C$.
4. $2(\sin D \cdot \cos C - \sin C \cdot \cos D)^2 = 1 - \cos C$.
5. $\sin 30° = .5$. 6. $\tan 60° = 1.732 \cdots$. 7. $\sin 45° = .7071 \cdots$.
8. $\sin 60° = .8660 \cdots$. 9. $\tan 30° = .5773 \cdots$.

CHAPTER VI

PRACTICAL APPLICATIONS

42. The actual measurement of the *line* joining two points which are any considerable distance apart, is a very tedious and difficult operation, especially when great accuracy is required; while the accurate measurement of an *angle* can, with proper instruments, be made with comparative ease and quickness.

43. A **Sextant** is an instrument for measuring the angle between the two lines drawn from the observer's eye to each of two distant objects respectively.

A **Theodolite** is an instrument for measuring angles in a horizontal plane; also for measuring "angles of elevation" and "angles of depression."

44. The angle made with the horizontal plane, by the line joining the observer's eye with a distant object, is called

(i.) its **angle of elevation,** when the object is *above* the observer;

(ii.) its **angle of depression,** when the object is *below* the observer.[1]

45. **Trigonometry** enables us, by measuring certain *angles*, to deduce other distances from one known distance, or, by the measurement of a *convenient* line, to deduce by the measurement of *angles* the lengths of lines whose actual measurement is difficult or impossible.

46. For this purpose we require the numerical values of the Trigonometrical Ratios of the angles observed. Accordingly, mathematical tables have been compiled, giving these ratios. These tables constitute a sort of numerical **Dictionary**, in which we can find the

[1] In measuring the angle of *depression* the telescope is turned from a horizontal position *downwards*. See Ex. VII. 3.

numerical values of the trigonometrical ratios of any required angle.

Example 1. At a point 100 feet from the foot of a tower, the angle of elevation of the top of the tower is observed to be 60°. Find the *height* of the top of the tower above the point of observation.

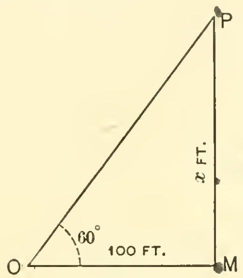


Fig. 20.

Let O be the point of observation; let P be the top of the tower; let a horizontal line through O meet the foot of the tower at the point M. Then $OM = 100$ feet, and the angle $MOP = 60°$. Let MP contain x feet.

Then $$\frac{MP}{OM} = \tan MOP = \tan 60° = \sqrt{3}.$$

$$\therefore \frac{x}{100} = \sqrt{3}.$$

$$\therefore x = 100 . \sqrt{3} = 100 \times 1.7320,$$
$$= 173.2.$$

Therefore the required height is 173.2.

Example 2. At a point 100 yards from the foot of a building, I measure the angle of elevation of the top, and find that it is 23° 15′; what is the height of the building?

As in Example 1, let the height be x yards.

Then $\frac{x}{100} = \tan 23° 15'$.

From the table of tangents we find that $\tan 23° 15' = .4296339$.

Hence $x = 100 \times .4296339 = 42.96339$.

The height of the building = 43 yards, nearly. *Ans.*

Example 3. A flagstaff, 25 feet high, stands on the top of a cliff; from a point on the seashore the angles of elevation of the highest and lowest points of the flagstaff are observed to be 47° 12′ and 45° 13′ respectively; find the height of the cliff.

Let O be the point of observation, PQ the flagstaff.

Let a horizontal line through O meet the vertical line PQ produced in M.

Then $QP = 25$ feet, $MOP = 47° 12'$, $MOQ = 45° 13'$.

Let $MQ = x$ feet; let $OM = y$ feet.

Then $\frac{MP}{OM} = \tan 47° 12'$, $\therefore \frac{x+25}{y} = \tan 47° 12'$,

and $\frac{MQ}{OM} = \tan 45° 13'$, $\therefore \frac{x}{y} = \tan 45° 13'$.

Hence, by division, $\therefore \frac{x+25}{x} = \frac{\tan 47° 12'}{\tan 45° 13'}$.

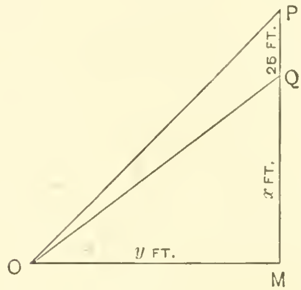


FIG. 21.

In the tables we find that

$$\tan 47° 12' = 1.0799018, \text{ and } \tan 45° 13' = 1.0075918.$$

$$\therefore 1 + \frac{25}{x} = \frac{1.0799018}{1.0075918} = 1 + \frac{.0723100}{1.0075918}.$$

$$\therefore \frac{x}{25} = \frac{1.0075918}{.0723100} = \frac{100759}{7231}.$$

$$\therefore x = \frac{2518975}{7231} = 348, \text{ nearly.}$$

Therefore the cliff is 348 feet high.

EXAMPLES. VII.

NOTE. The answers are given correct to three significant figures.

1. At a point 179 feet in a horizontal line from the foot of a column, the angle of elevation of the top of the column is observed to be 45°; what is the height of the column?

2. At a point 200 feet from, and on a level with the base of a tower, the angle of elevation of the top of the tower is observed to be 60°; what is the height of the tower?

3. From the top of a vertical cliff, the angle of depression of a point on the shore 150 feet from the base of the cliff, is observed to be 30°; find the height of the cliff.

4. From the top of a tower 117 feet high the angle of depression of the top of a house 37 feet high is observed to be 30°; how far is the top of the house from the tower?

5. A man 6 feet high stands at a distance of 4 ft. 9 in. from a lamp-post, and it is observed that his shadow is 19 feet long; find the height of the lamp.

6. The shadow of a tower in the sunlight is observed to be 100 feet long, and at the same time the shadow of a lamp-post 9 feet high is observed to be $3\sqrt{3}$ feet long. Find the angle of elevation of the sun, and the height of the tower.

7. From a point P on the bank of a river, just opposite a post Q on the other bank, a man walks at right angles to PQ to a point R so that PR is 100 yards; he then observes the angle PRQ to be $32° 17'$; find the breadth of the river. ($\tan 32° 17' = .6317667$.)

8. A fine wire 300 feet long is attached to the top of a spire and the inclination of the wire to the horizon when held tight is observed to be $40°$; find the height of the spire. ($\sin 40° = .6428$.)

9. A flagstaff 25 feet high stands on the top of a house; from a point on the plane on which the house stands the angles of elevation of the top and bottom of the flagstaff are observed to be $60°$ and $45°$ respectively; find the height of the house above the point of observation.

10. From the top of a cliff 100 feet high, the angles of depression of two ships at sea are observed to be $45°$ and $30°$ respectively; if the line joining the ships points directly to the foot of the cliff, find the distance between the ships.

11. A tower 100 feet high stands on the top of a cliff; from a point on the sand at the foot of the cliff the angles of elevation of the top and bottom of the tower are observed to be $75°$ and $60°$ respectively; find the height of the cliff. ($\tan 75° = 2 + \sqrt{3}$).

12. A man walking along a straight road observes at one milestone a house in a direction making an angle $30°$ with the road, and that at the next milestone the angle is $60°$; how far is the house from the road?

13. A man stands at a point A on the bank AB of a straight river and observes that the line joining A to a post C on the opposite bank makes with AB an angle of $30°$. He then goes 400 yards along the bank to B and finds that BC makes with BA an angle of $60°$; find the breadth of the river.

14. From the top of a hill the angles of depression of the top and bottom of a flagstaff 25 feet high at the foot of the hill are observed to be $45° 13'$ and $47° 12'$ respectively; find the height of the hill. ($\tan 45° 13' = 1.0075918$. ($\tan 47° 12' = 1.0799018$.)

15. An isosceles triangle of wood is placed on the ground in a vertical position facing the sun. If $2a$ be the base of the triangle, b its height, and $30°$ the altitude of the sun, find the tangent of half the angle at the apex of the shadow.

16. The length of the shadow of a vertical stick is to the length of the stick as $\sqrt{3} : 1$. If the stick be turned about its lower extremity in a vertical plane, so that the shadow is always in the same direction, find what will be the angle of its inclination to the horizon when the length of the shadow is the same as before.

17. What distance in space is travelled in an hour in consequence of the earth's rotation, by a person situated in latitude $60°$? (Earth's radius = 4000 miles.)

CHAPTER VII

USE OF SIGNS + AND −. THE TRIGONOMETRIC RATIOS

47. Lines. The student has learned, in his study of algebra, that every quantitative symbol is affected by one of two signs, + or −; that if b be such a symbol, then $+b$ and $-b$ are of the same absolute magnitude, but that they are *opposite* in *character; i.e. by whatever process* $+b$ *may have been generated,* $-b$ *has been generated by an exactly opposite process.* We express this symbolically thus:

$$a + b - b \doteq a.$$

A line may be regarded as having been generated by the movement of a point. If, then, a line segment generated by the movement of a point in a given direction is represented by $+b$, where b is its measure, then a line segment generated by movement in the *opposite* direction is represented by $-b$, if b be *its* measure.

Thus line segments generated in opposite senses furnish an illustration of this algebraic convention.

48. We add line segments by placing the *beginning* of one segment at the *end* of another, so that all the extremities are collinear; *e.g.* suppose AB, CD, EF are line segments whose measures are a, b, c, respectively.

(1) Suppose AB, CD, EF are all generated in the same sense, then (1) Fig. 22 is the geometrical representation of their sum. The *measure* of their sum is $a + b + c$.

(2) Suppose AB and CD are generated in one sense and EF in the opposite sense, then (2) Fig. 22 is the geometrical representation of their sum. The measure of their sum is $a + b - c$.

(3) Suppose that AB and CD are generated in one sense, and that EF is generated in the opposite sense, and that the measure of EF = the measure of CD; then (3) Fig. 22 is the geometrical rep-

resentation of their sum. The measure of their sum is $a + b - b$. In each case

$$AB + CD + EF = AF.$$

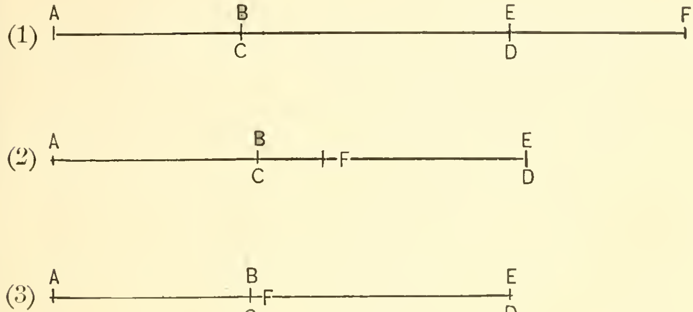


FIG. 22.

Any straight line, OX being given, then it follows that

Lines drawn parallel to OX in **one sense** may be assumed **positive**; that is, are *represented algebraically* by their *measures with the sign + before them;* then

Lines drawn parallel to OX in the **opposite sense** are **negative**; that is, are *represented algebraically* by their *measures with the sign − before them.*

49. In naming a line by the letters at its extremities, we can indicate by the **order of the letters** the direction in which the line is supposed to be drawn.

Hence in using the two letters at its extremities to represent a line, the student will find it advantageous always to pay careful attention to the **order of the letters.**

50. We should notice

(1) That it is immaterial which sense we *choose* as *positive.* But that, when once chosen, the *negative* sense is determined.

(2) That the line is the *result* of the movement, not the process.

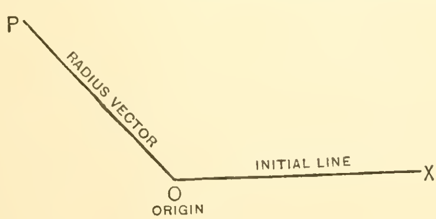


FIG. 23.

51. We have already said in Art. 5

(i.) that, when an angle XOP is described by OP turning about O in the direction *contrary to that of the hands of a watch*, the angle XOP is said to be **positive**; that is, *is represented algebraically* by its *measure with the sign + before it.*

(ii.) that, when an angle XOP is described by OP turning about O in the *same* direction as the hands of a watch, the angle is said to be **negative**; that is, *is represented algebraically* by its *measure with the sign − before it.*

52. Definitions. Quadrants.

Draw through any point O two lines OX, OY perpendicular to each other.

(OX is drawn usually horizontally.)

The plane is divided into four parts called **quadrants.**

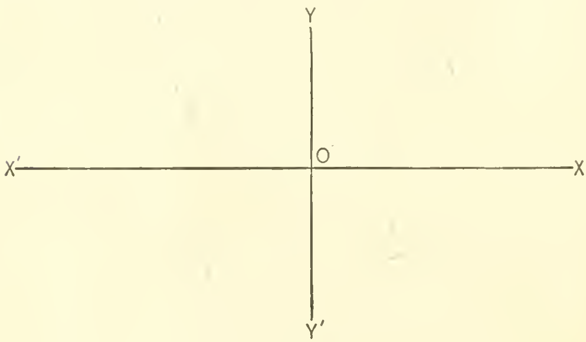


Fig. 24.

The part of the plane lying between

(i.) OX and OY constitutes the *first* **quadrant.**

(ii.) OY and OX' constitutes the *second* **quadrant.**

(iii.) OX' and OY' constitutes the *third* **quadrant.**

(iv.) OY' and OX constitutes the *fourth* **quadrant.**

An **angle** is said to be of the **first quadrant** if the radius vector OP is in the first quadrant; of the second, third, or fourth quadrant according as it is in the second, third, or fourth quadrant.

53. Let A be an angle between 0° and 90°, and let n be any *whole* number, positive, negative, or zero. Then (i.) $2n \times 180° + A$ represents algebraically an angle of the *first* quadrant.

(ii.) $2n \times 180° - A$ represents algebraically an angle of the *fourth* quadrant.

[For $2n \times 180°$ represents some number n of *complete* revolutions of OP; so that after describing $n \times 360°$, OP is again in the position OX.]

(iii.) $(2n+1) \times 180° - A$ represents algebraically an angle of the *second* quadrant.

(iv.) $(2n+1) \times 180° + A$ represents algebraically an angle of the *third* quadrant.

[For after describing $(2n+1) \times 180°$, OP is in the position OX'.]

The corresponding expressions in circular measure are

(i.) $2n\pi + \theta$; (ii.) $2n\pi - \theta$; (iii.) $(2n+1)\pi - \theta$; (iv.) $(2n+1)\pi + \theta$.

EXAMPLES. VIII.

Draw a figure giving the position of the radius vector after it has turned through each of the following angles, and state to what quadrant each angle belongs.

1. $270°$. **3.** $425°$. **5.** $-30°$. **7.** $-480°$.

2. $370°$. **4.** $590°$. **6.** $-330°$. **8.** $-750°$.

9. $\frac{27\pi}{4}$. **11.** $(2n+1)\pi + \frac{\pi}{3}$. **13.** $2n\pi - \frac{\pi}{2}$.

10. $2n\pi + \frac{\pi}{6}$. **12.** $(2n+1)\pi - \frac{\pi}{4}$. **14.** $(2n+1)\pi - \frac{\pi}{2}$.

NOTE. $n\pi$ always stands for a whole number of two right angles.

54. For the remainder of our discussion, we shall make the following agreements:

Choose O as origin; choose OX, drawn horizontally, as initial line.

I. For lines parallel to the initial line OX, the positive direction is from O to X.

It is convenient, when considering one angle only, so to arrange the figure that from O to X is from left to right.

We next *choose* the positive direction of revolution about O for the radius vector.

II. For lines perpendicular to OX, the positive direction is from O to Y, where XOY is a *positive right angle.*

When from O to X is from left to right, and the positive direction of revolution about O is contrary to that of the hands of a watch with its face upwards (counter-clockwise), then from O to Y is in the direction from the bottom of the page to the top.

III. For lines parallel to OP, the positive direction is from O to P.

The radius vector carries its positive direction around with it; hence OP is *always positive*. The direction from O to P, as OP revolves, nowhere undergoes a sudden reversal of direction such as is indicated by a change of sign.

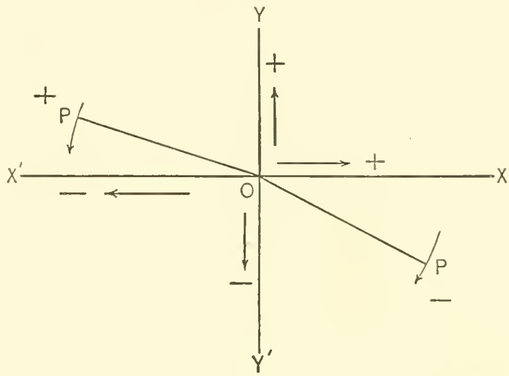


Fig. 25.

As already explained, the *negative* direction is in all cases exactly the reverse of the positive direction.

We have said that the definitions of the Trigonometrical Ratios (Art. 26) apply to angles of any magnitude. We may say also that when the angle exceeds a right angle, a line on which the point P is taken must be considered the radius vector; and that **the order of the letters in** MP, OM, OP **give the directions** of the lines, and therefore the *signs* of their measures.

55. We will now show that the trigonometrical ratios of an angle vary in **Sign** according to the **Quadrant** in which the radius vector of the angle happens to be.

From the definition we have, with the usual letters,

$$\sin XOP = \frac{MP}{OP}, \quad \cos XOP = \frac{OM}{OP}, \quad \tan XOP = \frac{MP}{OM}.$$

I. When OP is in the **first** quadrant (Fig. 26),

MP is *positive* because from M to P is *upwards* (Art. 54, II.);

OM is *positive* because from O to M is *towards the right* (Art. 54, I.);

OP is *positive* (Art. 54, III.).

Hence, if A be any angle of the *first* quadrant,

$$\sin A, \text{ which is } \frac{MP}{OP}, \text{ is } \textit{positive};$$

$$\cos A, \text{ which is } \frac{OM}{OP}, \text{ is } \textit{positive};$$

$$\tan A, \text{ which is } \frac{MP}{OM}, \text{ is } \textit{positive}.$$

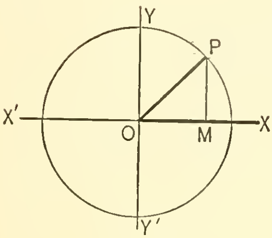


Fig. 26.

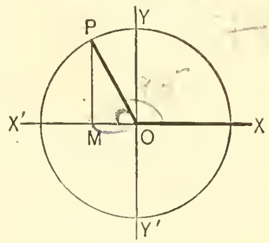


Fig. 27.

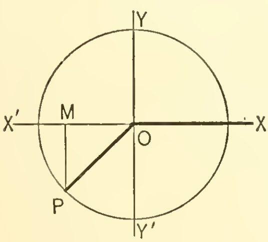


Fig. 28.

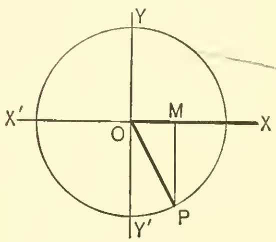


Fig. 29.

II. When OP is in the **second** quadrant (Fig. 27),
MP is *positive*, because from M to P is *upwards*.
OM is *negative*, because from O to M is *towards the left*.
OP is *positive*.
Hence, if A be any angle of the *second* quadrant,

$$\sin A, \text{ which is } \frac{MP}{OP}, \text{ is } \textit{positive};$$

$$\cos A, \text{ which is } \frac{OM}{OP}, \text{ is } \textit{negative};$$

$$\tan A, \text{ which is } \frac{MP}{OM}, \text{ is } \textit{negative}.$$

III. When OP is in the **third** quadrant (Fig. 28), MP is *negative*, OM is *negative*, OP is *positive*.

So that, if A be any angle of the *third* quadrant,

$\sin A$ is *negative*, $\cos A$ is *negative*, $\tan A$ is *positive*.

IV. When OP lies in the **fourth** quadrant (Fig. 29), MP is *negative*, OM is *positive*, OP is *positive*.

So that, if A be any angle of the *fourth* quadrant,

$\sin A$ is *negative*, $\cos A$ is *positive*, $\tan A$ is *negative*.

56. The table given below exhibits the results of the last article:

Quadrant . . .	I.	II.	III.	IV.
Sine	+	+	–	–
Cosine	+	–	–	+
Tangent . . .	+	–	+	–

The student should notice that for any particular quadrant the three *signs* of sine, cosine, and tangent are unlike their signs for any other quadrant.

57. The cosecant, secant, and cotangent of an angle A have the same *sign* as the sine, cosine, and tangent of A respectively.

EXAMPLES. IX.

State the *sign* of the sine, cosine, and tangent of each of the following angles:

1. 60°.	**2.** 135°.	**3.** 265°.
4. 275°.	**5.** – 10°.	**6.** – 91°.
7. – 193°.	**8.** – 350°.	**9.** – 1000°.
10. $2n\pi + \frac{1}{4}\pi$.	**11.** $2n\pi + \frac{3}{4}\pi$.	**12.** $2n\pi - \frac{1}{6}\pi$.

58. *The ratios of an angle of any quadrant may be expressed in terms of the ratios of a positive angle between 0° and 90°.*

The algebraic values of the ratios depend only on the position of the radius vector.

I. Hence the proposition is evident for angles of the first quadrant.

II. Consider an angle of the second quadrant.

Let XOP_2 be an angle of the second quadrant. With OP_2 as adius describe a circle about O as centre. Construct $\angle XOP_1$, P_1 eing on the circumference of the circle [see I., Art. 32], so that

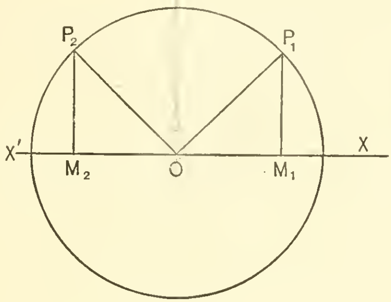


Fig. 30.

he measure of $\angle XOP_1 =$ measure of $\angle X'OP_2$. From P_1 and P_2 et perpendiculars fall on XX', cutting it in M_1 and M_2 respectively.

hen $$\triangle M_1OP_1 = \triangle M_2OP_2;$$

hence $$M_2P_2 = M_1P_1,$$

nd $$OM_2 = -\ OM_1 \text{ (Art. 47).}$$

$$\sin XOP_2 = \frac{M_2P_2}{OP_2};\ \sin XOP_1 = \frac{M_1P_1}{OP_1}.$$

ut $$\frac{M_2P_2}{OP_2} = \frac{M_1P_1}{OP_1};$$

$$\therefore\ \sin XOP_2 = \sin XOP_1;$$

$$\cos XOP_2 = \frac{OM_2}{OP_2} = -\frac{OM_1}{OP_1} = -\cos XOP_1;$$

$$\tan XOP_2 = \frac{M_2P_2}{OM_2} = -\frac{M_1P_1}{OM_1} = -\tan XOP_1.$$

III. Angles of third quadrant.

Let OXP_3 be an angle of the third quadrant (Fig. 31).

By reasoning similar to that in II.,

$$M_3P_3 = -\ M_1P_1;\ OM_3 = -\ OM_1 \text{ (Art. 47).}$$

Hence
$$\sin XOP_3 = \frac{M_3P_3}{OP_3} = -\frac{M_1P_1}{OP_1} = -\sin XOP_1;$$

$$\cos XOP_3 = \frac{OM_3}{OP_3} = -\frac{OM_1}{OP_1} = -\cos XOP_1;$$

$$\tan XOP_3 = -\frac{M_3P_3}{OM_3} = \frac{M_1P_1}{OM_1} = \tan XOP_1.$$

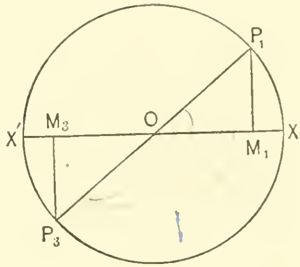


FIG. 31.

IV. We leave as an exercise for the student to prove: If XOP_4 is an angle of the fourth quadrant,

$$\sin XOP_4 = -\sin XOP_1;$$

$$\cos XOP_4 = \cos XOP_1;$$

$$\tan XOP_4 = -\tan XOP_1.$$

We have now proved our theorem.

COROLLARY I. The results of the preceding article may be formulated as follows (see Art. 53):

$$\left.\begin{aligned} \sin(2n\,180^\circ \pm A) &= \pm \sin A; \\ \cos(2n\,180^\circ \pm A) &= \cos A; \\ \tan(2n\,180^\circ \pm A) &= \pm \tan A. \end{aligned}\right\} \text{I.}$$

$$\left.\begin{aligned} \sin[(2n+1)\,180^\circ \pm A] &= \mp \sin A; \\ \cos[(2n+1)\,180^\circ \pm A] &= -\cos A; \\ \tan[(2n+1)\,180^\circ \pm A] &= \pm \tan A. \end{aligned}\right\} \text{II.}$$

where A is an angle between 0° and 90°, and n is any integer positive, or negative, or zero, and either the *upper* sign or the *lower* sign is read on both sides of the equations.

Corollary II. Making $n = 1$ in each of the equations I., we obtain the very important formulæ:

$$\sin(360° \pm A) = \pm \sin A;$$
$$\cos(360° \pm A) = \cos A;$$
$$\tan(360° \pm A) = \pm \tan A.$$

Making $n = 0$ in II.,

$$\sin(180° \pm A) = \mp \sin A;$$
$$\cos(180° \pm A) = -\cos A;$$
$$\tan(180° \pm A) = \pm \tan A.$$

Making $n = 0$ in I., and reading the lower sign, we obtain:

$$\sin(-A) = -\sin A;$$
$$\cos(-A) = \cos A;$$
$$\tan(-A) = -\tan A.$$

59. *If A be any angle,*

$$\sin(180° - A) = \sin A;$$
$$\cos(180° - A) = -\cos A;$$
$$\tan(180° - A) = -\tan A.$$

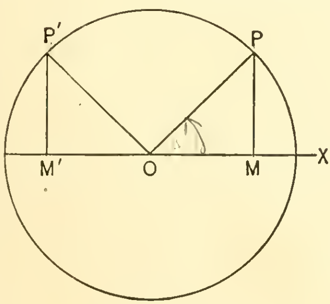


Fig. 32.

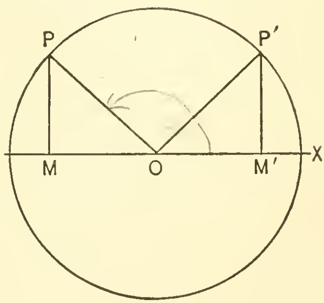


Fig. 33.

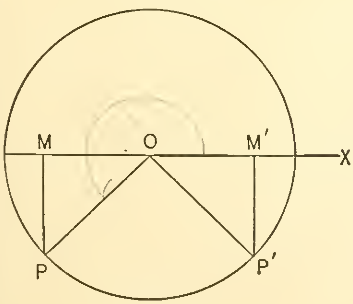


Fig. 34.

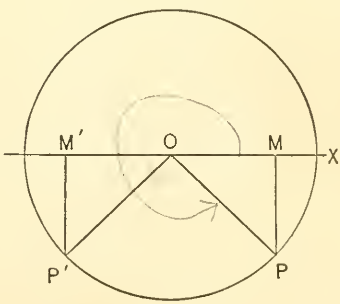


Fig. 35.

Let XOP be any angle, A; see Figs. 32, 33, 34, 35.
Let XOP' be $180° - A$.

It is easily proved that

if A is an angle of 1st quadrant, $180° - A$ is an angle of 2d quadrant;
if A is an angle of 2d quadrant, $180° - A$ is an angle of 1st quadrant;
if A is an angle of 3d quadrant, $180° - A$ is an angle of 4th quadrant;
if A is an angle of 4th quadrant, $180° - A$ is an angle of 3d quadrant;

and as in Art. 58, $P'M' = PM$;

$$OM' = - OM.$$

In each case

$$\sin (180° - A) = \frac{M'P'}{OP'} = \frac{MP}{OP} = \sin A;$$

$$\cos (180° - A) = \frac{OM'}{OP'} = \frac{-OM}{OP} = - \cos A;$$

$$\tan (180° - A) = \frac{M'P'}{OM'} = \frac{MP}{-OM} = - \tan A.$$ Q.E.D.

60. Definition. One angle is said to be the **supplement** of another when their sum is two right angles. The results of Art. 58 may now be stated in words:

The *sine* of the *supplement* of an *angle* = the *sine* of the *angle*.

The *cosine* of the *supplement* of an *angle* = the *negative cosine* of the *angle*.

The *tangent* of the *supplement* of an *angle* = the *negative tangent* of the *angle*.

EXAMPLES. X.

Prove, drawing a separate figure in each case, that

1. $\sin 60° = \sin 120°$.

2. $\sin 340° = \sin (-160°)$.

3. $\sin (-40°) = \sin 220°$.

4. $\cos 320° = - \cos (-140°)$.

5. $\cos (-380°) = - \cos 560°$.

6. $\cos 195° = - \cos (-15°)$.

If A, B, C be the angles of a triangle, prove

7. $\sin A = \sin (B + C)$.

8. $\sin C = \sin (A + B)$.

9. $\cos B = - \cos (A + C)$.

10. $\cos A = - \cos (C + B)$.

Prove by means of a figure that

11. $\sin (-A) = - \sin A$.

12. $\cos (-A) = \cos A$.

DEFINITION. One angle is said to be the **complement** of another if their **sum** = a **right angle.**

EXAMPLE. The complement of 190° is − 100°.

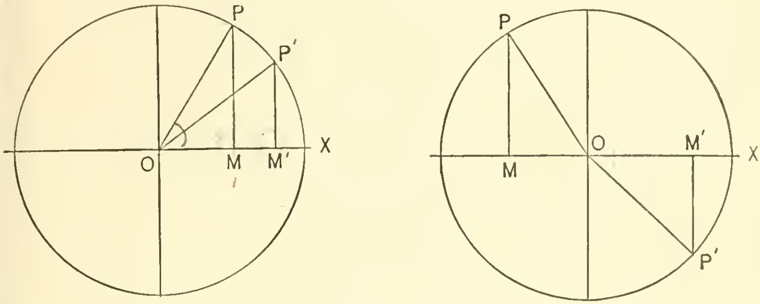


FIG. 36.

61. *To prove if A is an angle, that*

$$\sin(90° - A) = \cos A;$$
$$\cos(90° - A) = \sin A;$$
$$\tan(90° - A) = \cot A.$$

We shall prove the theorem in case A is an angle of the first or of the second quadrant. The student should prove it for the remaining cases.

If A is an angle of the 1st quadrant, $90° - A$ is an angle of the 1st quadrant.

If A is an angle of the 2d quadrant, $90° - A$ is an angle of the 4th quadrant.

Let $XOP = A$, and $XOP' = 90° - A$.

Then, triangles MPO and $M'OP'$ are similar (Fig. 36).

$$\frac{M'P'}{OP'} = \frac{OM}{OP} \text{ (i.)}, \quad \text{and} \quad \frac{OM'}{OP'} = \frac{MP}{OP} \text{ (ii.)}, \quad \text{and} \quad \frac{M'P'}{OM'} = \frac{OM}{MP} \text{ (iii.)};$$

or

$$\sin(90° - A) = \cos A;$$
$$\cos(90° - A) = \sin A;$$
$$\tan(90° - A) = \cot A.$$

EXAMPLES. XI.

Find the complements of

1. 30°. **3.** 90°. **5.** − 25°. **7.** $\frac{3}{4}\pi$.

2. 190°. **4.** 350°. **6.** − 320°. **8.** $-\frac{1}{6}\pi$.

Prove by drawing a figure in each case:

9. sin 70° = cos 20°. **11.** tan 79° = cot 11°.

10. cos 47° 16′ = sin 42° 44′. **12.** sec 36° = cosec 54°.

13. $\cos 105° = \sin - 15°$.

14. $\tan 135° = \cot - 45°$.

15. $\sec A = \operatorname{cosec} (90° - A)$.

16. $\cot A = \tan (90° - A)$.

If A, B, C be the angles of a triangle, so that $A + B + C = 180°$, prove

17 $\cos \frac{1}{2} A = \sin \frac{1}{2} (B + C)$.

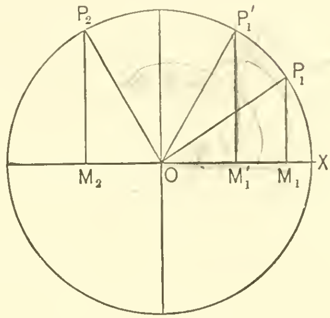
18. $\cos \frac{1}{2} B = \sin \frac{1}{2} (A + C)$.

19. $\sin \frac{1}{2} C = \cos \frac{1}{2} (A + B)$.

20. $\sin \frac{1}{2} A = \cos \frac{1}{2} (B + C)$.

62. *To find the ratios of* $90° + A$.

We shall consider only the case when A is between 0° and 90°.



Let $XOP_1 = A$, and $XOP_2 = 90° + A$.

Construct $XOP'_1 = 90° - A$.

Then the measure of $XOP_2 =$ measure of $180° - XOP'_1$.

$$\therefore \sin XOP_2 = \sin XOP'_1 \text{ (Art. 57).}$$

But $$\sin XOP'_1 = \cos XOP_1 \text{ (Art. 60);}$$

$$\textit{i.e.} \sin (90° + A) = \cos A, \text{ and similarly}$$

$$\cos (90° + A) = - \sin A,$$

$$\tan (90° + A) = - \cot A.$$

The proof for the remaining cases offers little difficulty.

EXAMPLES. XII.

Find the algebraical value of the sine, cosine, and tangent of the following angles:

1. 150°. **2.** 135°. **3.** − 240°. **4.** 330°. **5.** − 45°. **6.** − 300°. **7.** 225°. **8.** − 135°. **9.** 390°. **10.** 750°. **11.** − 840°. **12.** 1020°.

13. $2n\pi + \frac{\pi}{4}$. **14.** $(2n + 1)\pi - \frac{\pi}{3}$. **15.** $(2n - 1)\pi + \frac{\pi}{6}$.

16. $\sin A = \frac{1}{2}$. **17.** $\sin A = \frac{1}{\sqrt{2}}$. **18.** $\sin A = \frac{\sqrt{3}}{2}$. **19.** $\sin A = -\frac{1}{2}$.

Find four angles between zero and + 8 right angles which satisfy the equations:

20. $\sin A = \sin 20°$ **21.** $\sin\theta = -\frac{1}{\sqrt{2}}.$ **22.** $\sin\theta = -\sin\frac{\pi}{7}.$

23. Prove that 30°, 150°, − 330°, 390°, − 210° have the same sine.

24. Show that each of the following angles has the same cosine:

$$-120°,\ 240°,\ 480°,\ -480°.$$

25. The angles 60° and − 120° have one of the Trigonometrical Ratios the same for both; which of the ratios is it?

26. Can the following angles have any one of their Trigonometrical Ratios the same for all? − 23°, 157°, and − 157°.

63. *The ratios of the angles*

$$2n\pi,\ 2n\pi+\frac{\pi}{2},\ (2n+1)\pi,\ (2n+1)\pi+\frac{\pi}{2}.$$

The reasoning by which we obtained the ratios of 0° and 90° applies with equal force for $2n\pi$ and $2n\pi+\frac{\pi}{2}$ respectively, whence we obtain,

$$\sin 2n\pi = 0,\quad \cos 2n\pi = 1,\quad \tan 2n\pi = 0; \qquad \text{(i.)}$$

$$\sin\left(2n\pi+\frac{\pi}{2}\right)=1,\ \cos\left(2n\pi+\frac{\pi}{2}\right)=0,\ \tan\left(2n\pi+\frac{\pi}{2}\right)=\infty. \qquad \text{(ii.)}$$

We will now obtain the ratios of $(2n+1)\pi$.

We have shown that

$$\sin XOP_2 = \sin XOP_1; \qquad \text{(Fig. 30, Art. 58.)}$$
$$\cos XOP_2 = -\cos XOP_1;$$
$$\tan XOP_2 = -\tan XOP_1,$$

however little the difference between 180° and XOP_2 may be. If this difference be very small, then XOP_1 differs from 0° by a very small quantity. Hence

$$\sin XOP_1 = 0,\quad \cos XOP_1 = 1,\quad \tan XOP_1 = 0;$$

whence $\sin XOP_2 = 0,\quad \cos XOP_2 = -1,\quad \tan XOP_2 = 0;$

or $\sin(2n+1)\pi = 0,\ \cos(2n+1)\pi = -1,\ \tan(2n+1)\pi = 0.$ (iii.)

We leave as an exercise to prove that

$$\left.\begin{aligned}\sin\left[(2n+1)\pi+\frac{\pi}{2}\right]=-1,\quad \cos\left[(2n+1)\pi+\frac{\pi}{2}\right]=0,\\ \tan\left[(2n+1)\pi+\frac{\pi}{2}\right]=\infty.^*\end{aligned}\right\} \qquad \text{(iv.)}$$

* See remark under corollary, next page.

COROLLARY. Making (1) $n = 0$ in each of the preceding formulæ, and (2) $n = 1$ in I., we obtain:

Angle	0°	90°	180°	270°	360°
Sine	0	1	0	-1	0
Cosine	1	0	-1	0	1
Tangent . . .	0	∞	0	∞	0

The sign has been omitted from ∞, since all that we know is, when the angle is a little less than 90° or 270°, or a little greater than 90° or 270°, that the tangent is very large *numerically;* that in the first case, viz. the angle is a little less than either 90° or 270°, the tangent is positive; and in the second case, negative.

64. By Art. 32, if A be an angle of the first quadrant,

(1) A small change in A will produce a change in the values of its ratios.

(2) So long as the angle A is unchanged, its ratios are unchanged.

(3) To each value of A there is a definite value for each of its ratios. By Art. 58 the ratios of an angle of *any* quadrant may be expressed in terms of the ratios of an angle of the first quadrant. Hence the theorems of Art. 32 are true for any angle; or, more briefly, *the ratios of an angle are unique.*

DEFINITION. When two quantities are so related that a change in the value of one of them produces a corresponding change in the value of the other, the second is said to be a function of the first.

It appears from this article that the angle and its ratios are so related. For this reason the trigonometrical ratios are generally called trigonometric functions or circular functions.

65. To trace the changes in the magnitude and sign of $\sin A$ as A increases from 0° to 360°.

Construct circles with OP as radius. Let PM be perpendicular to OX. Let $A = XOP$.

Then $$\sin A = \frac{MP}{OP}.$$

When the angle A is 0°, MP is zero, and when A is 90°, MP is equal to OP; and as A continuously increases from 0° to 90°, MP

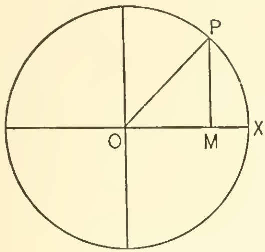


FIG. 37.

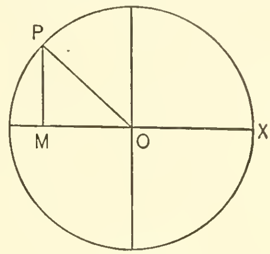


FIG. 38.

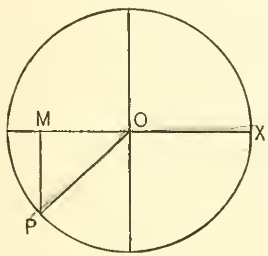


FIG. 39.

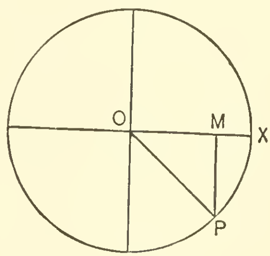


FIG. 40.

increases continuously from zero to OP; also OP is always equal to OX.

Therefore, when $A = 0°$, the fraction $\frac{MP}{OP}$ is equal to $\frac{0}{OP}$, that is 0; when $A = 90°$, the fraction $\frac{MP}{OP}$ is equal to $\frac{OP}{OP}$, that is 1; and as A continuously increases from 0° to 90°, the *numerator* of the fraction $\frac{MP}{OP}$ *continuously increases* from zero to OP, while the denominator is unchanged, and therefore the *fraction* $\frac{MP}{OP}$, which is $\sin A$, *increases continuously* from 0 to 1, and is positive.

As A increases from 0° to 90°, MP increases from zero to OP, and is positive.

Therefore $\sin A$ increases from 0 to 1, and is positive.

As A increases from 90° to 180°, MP decreases from OP to zero, and is positive.

Therefore $\sin A$ decreases from 1 to 0, and is positive.

As A increases from 180° to 270°, MP increases numerically from 0 to OP, and is negative; hence $\sin A$ increases numerically from 0 to 1, and is negative.

As A increases from 270° to 360°, MP decreases from OP to zero, and is negative.

Therefore $\sin A$ decreases numerically from 1 to 0, and is negative.

COROLLARY. Therefore we may conclude that **sin A is never greater than 1**; that **cos A is never greater than 1**. That the *numerical* value of $\sec A$ or of $\operatorname{cosec} A$ is never less than 1. For

$$\sec A = \frac{1}{\cos A}, \quad \operatorname{cosec} A = \frac{1}{\sin A}. \qquad \text{(Def.)}$$

EXAMPLES. XIII.

Trace the changes in sign and magnitude as A increases from 0° to 360° of

1. $\cos A$.	**3.** $\cot A$.	**5.** $\operatorname{cosec} A$.	**7.** $\sin^2 A$.
2. $\tan A$.	**4.** $\sec A$.	**6.** $1 - \sin A$.	**8.** $\sin A \cdot \cos A$.

CHAPTER VIII

ON THE RELATIONS BETWEEN THE TRIGONOMETRIC RATIOS

66.

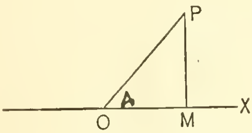


Fig. 41.

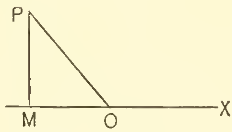


Fig. 42.

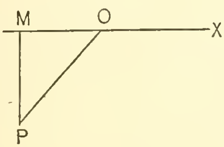


Fig. 43.

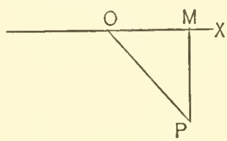


Fig. 44.

Let $XOP = A$, any angle. (The figures are those of Art. 25.)
The following relations are evident from the definitions:

$$\operatorname{cosec} A = \frac{1}{\sin A}; \tag{1}$$

$$\sec A = \frac{1}{\cos A}; \tag{2}$$

$$\cot A = \frac{1}{\tan A}. \tag{3}$$

To prove $$\tan A = \frac{\sin A}{\cos A}.$$

$$\sin A = \frac{MP}{OP}, \quad \cos A = \frac{OM}{OP}.$$

$$\therefore \quad \frac{\sin A}{\cos A} = \frac{\dfrac{MP}{OP}}{\dfrac{OM}{OP}} = \frac{MP}{OM} = \tan A. \tag{4}$$ Q.E.D.

Similarly $$\frac{\cos A}{\sin A} = \cot A. \tag{5}$$

67. To prove $\sin^2 A + \cos^2 A = 1$.

$$\sin^2 A = \frac{MP^2}{OP^2}, \quad \cos^2 A = \frac{OM^2}{OP^2}.$$

Now $\sin^2 A + \cos^2 A = \dfrac{MP^2}{OP^2} + \dfrac{OM^2}{OP^2} = \dfrac{MP^2 + OM^2}{OP^2} = \dfrac{OP^2}{OP^2} = 1,$

since $\qquad MP^2 + OM^2 = OP^2. \qquad$ (Pythagorean Prop.)

Similarly we may prove that

$$1 + \tan^2 A = \sec^2 A,$$

and that $\qquad 1 + \cot^2 A = \operatorname{cosec}^2 A.$

68. The following is a **List of Formulæ** with which the student must make himself familiar:

$$\operatorname{cosec} A = \frac{1}{\sin A}, \quad \sec A = \frac{1}{\cos A},$$

$$\cot A = \frac{1}{\tan A}, \quad \tan A = \frac{\sin A}{\cos A}, \quad \cot A = \frac{\cos A}{\sin A},$$

$$\sin^2 A + \cos^2 A = 1,$$

$$\tan^2 A + 1 = \sec^2 A,$$

$$\cot^2 A + 1 = \operatorname{cosec}^2 A.$$

69. By means of these formulæ we are able to transform a given trigonometrical expression into a great variety of equivalent expressions.

Example. Prove that $\tan A + \cot A = \sec A \cdot \operatorname{cosec} A$.

Since $\quad \tan A = \dfrac{\sin A}{\cos A}, \quad \cot A = \dfrac{\cos A}{\sin A}, \quad \sec A = \dfrac{1}{\cos A}, \quad \operatorname{cosec} A = \dfrac{1}{\sin A},$

we have $\qquad \tan A + \cot A = \dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}$

$$= \frac{\sin^2 A + \cos^2 A}{\cos A \cdot \sin A} = \frac{1}{\cos A \cdot \sin A} = \sec A \cdot \operatorname{cosec} A. \qquad \text{[Art. 67.]}$$

70. It is sometimes convenient to write a given expression in terms of the *sine* only, or in terms of the *cosine* only.

Example I. Prove that $\sin^4\theta + 2\sin^2\theta\cos^2\theta = 1 - \cos^4\theta$.

By Art. 67, we have $\qquad \sin^2\theta = 1 - \cos^2\theta,$

hence
$$\begin{aligned}\sin^4\theta + 2\sin^2\theta\cos^2\theta &= (1 - \cos^2\theta)^2 + 2(1 - \cos^2\theta) \times \cos^2\theta \\ &= (1 - 2\cos^2\theta + \cos^4\theta) + (2\cos^2\theta - 2\cos^4\theta) \\ &= 1 - \cos^4\theta.\end{aligned}$$

EXAMPLE II. Express $\sin^4\theta + \cos^4\theta$ in terms of $\cos\theta$.

$$\begin{aligned}\sin^4\theta + \cos^4\theta &= (1-\cos^2\theta)^2 + \cos^4\theta \\ &= (1 - 2\cos^2\theta + \cos^4\theta) + \cos^4\theta \\ &= 1 - 2\cos^2\theta + 2\cos^4\theta.\end{aligned}$$

NOTE. $(1-\cos\theta)$ is called the **versed sine** of θ, and is written $\text{versin}\,\theta$.

EXAMPLES. XIV.

Prove the following statements :

1. $\cos A \cdot \tan A = \sin A$.
2. $\cot A \cdot \tan A = 1$.
3. $\cos A = \sin A \cdot \cot A$.
4. $\sec A \cdot \cot A = \text{cosec}\, A$.
5. $\text{cosec}\, A \cdot \tan A = \sec A$.
6. $(\tan A + \cot A)\sin A \cdot \cos A = 1$.
7. $(\tan A - \cot A)\sin A \cdot \cos A = \sin^2 A - \cos^2 A$.
8. $\cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$.
9. $(\sin A + \cos A)^2 = 1 + 2\sin A \cdot \cos A$.
10. $(\sin A - \cos A)^2 = 1 - 2\sin A \cdot \cos A$.
11. $\cos^4 B - \sin^4 B = 2\cos^2 B - 1$.
12. $(\sin^2 B + \cos^2 B)^2 = 1$.
13. $(\sin^2 B - \cos^2 B)^2 = 1 - 4\cos^2 B + 4\cos^4 B$.
14. $1 - \tan^4 B = 2\sec^2 B - \sec^4 B$.
15. $(\sec B - \tan B)(\sec B + \tan B) = 1$.
16. $(\text{cosec}\,\theta - \cot\theta)(\text{cosec}\,\theta + \cot\theta) = 1$.
17. $\sin^3\theta + \cos^3\theta = (\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta)$.
18. $\cos^3\theta - \sin^3\theta = (\cos\theta - \sin\theta)(1 + \sin\theta\cos\theta)$.
19. $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta \cdot \cos^2\theta$.
20. $(\sin^6\theta - \cos^6\theta) = (2\sin^2\theta - 1)(1 - \sin^2\theta + \sin^4\theta)$.

1. $\dfrac{\tan A + \tan B}{\cot A + \cot B} = \tan A \cdot \tan B$.

2. $\dfrac{\cot\alpha + \tan\beta}{\tan\alpha + \cot\beta} = \cot\alpha \cdot \tan\beta$.

3. $\dfrac{1-\sin A}{1+\sin A} = (\sec A - \tan A)^2$.

24. $\dfrac{1+\cos A}{1-\cos A} = (\text{cosec}\, A + \cot A)^2$.

25. $2\,\text{versin}\,\theta - \text{versin}^2\theta = \sin^2\theta$.

26. $\text{versin}\,\theta\,(1+\cos\theta) = \sin^2\theta$.

Express in terms of (i.) $\cos\theta$, (ii.) of $\sin\theta$,

27. $\cos^4\theta - \sin^4\theta$.
28. $(\sin^2\theta - \cos^2\theta)^2$.
29. $1 - \tan^4\theta$.
30. $\sin^6\theta + \cos^6\theta$.
31. $\tan^2\theta + \cot^2\theta$.
32. $1 + \cot^4\theta$.
33. $1 + \cot^2\theta - \text{cosec}^2\theta$.
34. $2\tan^4\theta - 4\sin^2\theta$.
35. Show $\cos(90^\circ - \alpha)\sin(180^\circ - \alpha) - \sin(90^\circ - \alpha)\cos(180^\circ - \alpha) = 1$.
36. $\tan(180^\circ + 45^\circ)\tan 45^\circ + 1 = \sec 245^\circ$.
37. $\cot 120^\circ \cdot \cot(-60^\circ) + 1 = \dfrac{1}{\sin^2 60^\circ} = \dfrac{1}{\cos^2 30^\circ}$.
38. $\cos^2[(2n+1)\pi + A] = \dfrac{\cos^2 A + \sin^2(1800 - A)}{\sec^2 A}$.

By the aid of the formulæ of Art. 68 we may express the ratios of any angle in terms of any *one* of its ratios, *e.g.*

To express the other trigonometric ratios of θ in terms of its tangent.

$$\cot\theta = \frac{1}{\tan\theta}.$$

$$\sec\theta = \pm\sqrt{1+\tan^2\theta}.$$

$$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\pm\sqrt{1+\tan^2\theta}}.$$

$$\sin\theta = \tan\theta\ \cos\theta = \frac{\tan\theta}{\pm\sqrt{\tan^2\theta+1}}.$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{\pm\sqrt{1+\tan^2\theta}}{\tan\theta}.$$

71. If θ be less than 90°, the following method can be employed with advantage:

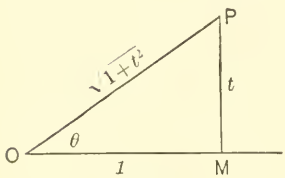


FIG. 45.

Let MOP be a right triangle; Fig. 45. Let $MP = t$, $OM = 1$, $\therefore\ OP = \sqrt{1+t^2}$.

$$\tan\theta = \frac{t}{1} = t.\ \sin\theta = \frac{t}{\sqrt{1+t^2}}.$$

$$\cos\theta = \frac{1}{\sqrt{1+t^2}},$$ and so on. With suitable modifications we may employ this method whatever be the magnitude of θ.

Hence if we are given $\tan\theta$ we can calculate the value of the remaining ratios by either of the above methods.

E.g. suppose $\tan\theta = \sqrt{3}$, then

$$\sin\theta = \frac{\sqrt{3}}{\sqrt{1+3}} = \frac{\sqrt{3}}{2},\ \text{and so on.}$$

72. The expressions obtained for $\sin\theta$, $\cos\theta$, $\sec\theta$, and $\csc\theta$, in Art. 70, are affected with a double sign. Hence these ratios are not *uniquely* determined. If, however, we *know* $\tan\theta$ and in addition to *what quadrant* θ *belongs*, we can determine the respective signs of the ratios named.

E.g. if $\tan\theta = \sqrt{3}$ and we know that θ is of the third quadrant, then $\sin\theta = -\frac{\sqrt{3}}{2}$.

EXAMPLES. XV. (*a*).

1. Express all the other ratios of A in terms of $\cos A$.

2. Express all the ratios of $90° + A$ in terms of $\cot A$; $A < 90°$.

3. Express all the ratios of $180° - A$ in terms of $\sec A$.

4. Express all the other ratios of A in terms of $\csc A$.

5. Use formulæ of Art. 67 to express all the other ratios of A in terms of sin A.

6. Use the method of Art. 71 to express all the ratios of A in terms of cosine A; $A < 90°$.

EXAMPLES. XV. (*b*).

Suppose no angle in the following list is greater than $+ 180°$.

1. If $\sin A = \frac{3}{5}$, find $\tan A$ and $\operatorname{cosec} A$.

2. If $\cos B = \frac{1}{3}$, find $\sin B$ and $\cot B$.

3. If $\tan A = \frac{4}{3}$, find $\sin A$ and $\sec A$.

4. If $\sec \theta = 4$, find $\cot \theta$ and $\sin \theta$.

5. If $\tan \theta = \sqrt{3}$, find $\sin \theta$ and $\cos \theta$.

6. If $\cot \theta = \dfrac{2}{\sqrt{5}}$, find $\sin \theta$ and $\sec \theta$.

7. If $\sin \theta = \dfrac{b}{c}$, find $\tan \theta$.

8. If $\tan \theta = a$, find $\sin \theta$ and $\cos \theta$.

9. If $\sec \theta = a$, find $\sin \theta$ and $\cot \theta$.

10. If $\sin \theta = a$, and $\tan \theta = b$, prove that $(1 - a^2)(1 + b^2) = 1$.

11. If $\cos \theta = h$, and $\tan \theta = k$, find the equation connecting h and k.

12. If $A < 90°$, and $\tan A + \sec A = 2$, prove that $\sin A = \frac{3}{5}$. Find the remaining ratios of A.

13. Show, using the formulæ Art. 67, that the numerical value of

$\sin A \not> 1$. $\quad \cos A \not> 1$. $\quad \sec A \not< 1$. $\quad \operatorname{cosec} A \not< 1$.

CHAPTER IX

THE SOLUTION OF TRIGONOMETRICAL EQUATIONS

73. The formulæ developed in Chapter VII. and the expressions to be proved in Examples XIV. are true for *any* angle.

E.g. $\sin^2 A + \cos^2 A = 1$, whatever be the value of A. $\frac{\sin\theta}{\cos\theta} = \tan\theta$ is true whatever value θ may have.

We shall now consider expressions which are true only for certain values of the angle.

E.g. $\sin\theta = \frac{\sqrt{2}}{2}$ is an equation satisfied by 45° and 135°, and by no other positive angles less than 360°.

The former expressions are called **Trigonometric Identities.**

The latter expressions are called **Trigonometric Equations.**

The **solution** of a trigonometric equation is the process of finding an angle which, if substituted in the equation, satisfies it.

74. Example 1. Solve the equation

$$\sin\theta - \csc\theta + \tfrac{3}{2} = 0.$$

Substitute in the equation $\csc\theta = \frac{1}{\sin\theta}$.

Then $$2\sin^2\theta + 3\sin\theta - 2 = 0,$$

or $$(\sin\theta + 2)(2\sin\theta - 1) = 0.$$

$$\therefore \sin\theta = -2, \quad (1)$$

$$\sin\theta = \tfrac{1}{2}. \quad (2)$$

No angle satisfies (1). [Example 13, XV., (*b*).]

I. Let us suppose that $\theta < 180°$.

Then from (2) $\theta = 30°$, (Art. 41.)

or, $\theta = 150°$. (Art. 58, Cor.)

II. Suppose θ is unrestricted in magnitude.

Then $\theta = 30°,\ 2\pi + 30°,\ 390°, \cdots 2n\pi + 30°$; (Art. 58, Cor.)

or, $\theta = 150°,\ 2\pi + 150° \cdots 2n\pi + 150°$,

or, in a single statement,

$$\theta = 2n\pi + 30°, \text{ or } (2n+1)\pi - 30°.$$

Another series of angles will satisfy equation (2),

viz. $-330°$, *i.e.* $-2\pi + 30°$, and in general $-2n\pi + 30°$, (Art. 58.)

and $-210°$, *i.e.* $-\pi - 30°$, and in general $-(2n+1)\pi - 30°$, (Art. 58.)

n being a *positive* integer.

Another statement combines the results of II. thus,

$$\theta = 2n\pi + 30°,$$

or

$$\theta = (2n+1)\pi - 30°,$$

n being any integer, *positive* or *negative*, or zero.

EXAMPLE 2. What positive angles satisfy the equation

$$\sqrt{2}\cos\theta = \cot\theta?$$

Substitute $$\cot\theta = \frac{\cos\theta}{\sin\theta}.$$

Then $$\sqrt{2}\cos\theta - \frac{\cos\theta}{\sin\theta} = 0,$$

$$\cos\theta\left(\sqrt{2} - \frac{1}{\sin\theta}\right) = 0.$$

$$\therefore \cos\theta = 0, \qquad (1)$$

$$\sin\theta = \frac{1}{\sqrt{2}}. \qquad (2)$$

From (1) $\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$, $2\pi + \frac{\pi}{2}$, $3\pi + \frac{\pi}{2}$, and in general $n\pi + \frac{\pi}{2}$.

From (2) $\theta = 2n\pi + \frac{\pi}{4}$ or $(2n+1)\pi - \frac{\pi}{4}$.

n being a *positive* integer or zero.

EXAMPLES. XVI.

Find all positive angles not greater than 360° satisfying the equations:

1. $\sin\theta = \frac{1}{\sqrt{2}}$.
2. $4\sin\theta = \csc\theta$.
3. $2\cos\theta = \sec\theta$.
4. $4\sin\theta - 3\csc\theta = 0$.
5. $4\cos\theta - 3\sec\theta = 0$.
6. $3\tan\theta = \cot\theta$.
7. $3\sin\theta - 2\cos^2\theta = 0$.
8. $\sqrt{2}\sin\theta = \tan\theta$.
9. $2\cos\theta = \sqrt{3}\cot\theta$.
10. $\tan\theta = 3\cot\theta$.
11. $\tan\theta + \cot\theta = 2$.
12. $2\sin^2\theta + \sqrt{2}\cos\theta = 2$.
13. $2\cos^2\theta + \sqrt{2}\sin\theta = 2$.
14. $3\tan^2\theta - 4\sin^2\theta = 1$.
15. $2\sin^2\theta + \sqrt{2}\sin\theta = 2$.
16. $\cos^2\theta - \sqrt{3}\cos\theta + \frac{3}{4} = 0$.
17. $\cos^2\theta + 2\sin^2\theta - \frac{5}{2}\sin\theta = 0$.

Find all angles, either positive or negative, whose magnitude is not greater than 360°, which satisfy the following equations:

18. $5\tan^2\theta - \sec^2\theta = 11$.
19. $\frac{\sin(180° - \theta)}{2} + \tan\theta = 0$.

Find all angles that satisfy the following equations:

20. $\tan^2\theta = 1$.
21. $\sqrt{3}\sin\theta + 2\cos^2\theta = 2$.
22. $2\sin\theta\cos\theta = 1$.

CHAPTER X

ON THE TRIGONOMETRICAL RATIOS OF TWO OR MORE ANGLES

75. We will now establish the following fundamental formulæ:

$$\left.\begin{aligned}\sin(A+B)&=\sin A\cdot\cos B+\cos A\cdot\sin B\\ \cos(A+B)&=\cos A\cdot\cos B-\sin A\cdot\sin B\\ \sin(A-B)&=\sin A\cdot\cos B-\cos A\cdot\sin B\\ \cos(A-B)&=\cos A\cdot\cos B+\sin A\cdot\sin B\end{aligned}\right\}\quad\text{(i.)}.$$

Here A and B are angles; so that $(A+B)$ and $(A-B)$ are also angles.

Hence, $\sin(A+B)$ is the sine of an angle, and must not be confounded with $\sin A+\sin B$.

$\text{Sin}(A+B)$ is a single fraction.

$\text{Sin}\ A+\sin B$ is the sum of two fractions.

The student should notice that the words of the two proofs of Arts. 76, 77 are very nearly the same.

76. *To prove that*

$$\sin(A+B)=\sin A\cdot\cos B+\cos A\cdot\sin B,$$

and that

$$\cos(A+B)=\cos A\cdot\cos B-\sin A\cdot\sin B.$$

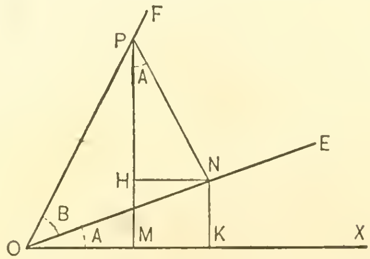


Fig. 46.

Let XOE be the angle A, and EOF the angle B, then XOF is the angle $(A+B)$.

In OF, *the line which is one of the sides of the angle* $(A+B)$, take any point P, and from P draw PM and PN at right angles to OX and OE respectively. Draw NH and NK at right angles to MP and OX respectively. Then the angle

$$HPN = 90^\circ - PNH = HNO = XOE = A.^1$$

Now $$\sin(A+B) = \sin XOF = \frac{MP}{OP} = \frac{MH+HP}{OP} = \frac{KN}{OP} + \frac{HP}{OP}$$

$$= \frac{KN \cdot ON}{ON \cdot OP} + \frac{HP \cdot NP}{NP \cdot OP} = \frac{KN}{ON} \cdot \frac{ON}{OP} + \frac{HP}{NP} \cdot \frac{NP}{OP}$$

$$= \sin XOE \cdot \cos EOF + \cos HPN \cdot \sin EOF$$

$$= \sin A \cdot \cos B + \cos A \cdot \sin B.$$

Also, $$\cos(A+B) = \cos XOF = \frac{OM}{OP} = \frac{OK-MK}{OP} = \frac{OK}{OP} - \frac{HN}{OP}$$

$$= \frac{OK \cdot ON}{ON \cdot OP} - \frac{HN \cdot NP}{NP \cdot OP} = \frac{OK}{ON} \cdot \frac{ON}{OP} - \frac{HN}{NP} \cdot \frac{NP}{OP}$$

$$= \cos XOE \cdot \cos EOF - \sin HPN \cdot \sin EOF$$

$$= \cos A \cdot \cos B - \sin A \cdot \sin B.$$

77. *To prove that*

$$\sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B,$$

and that $$\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B.$$

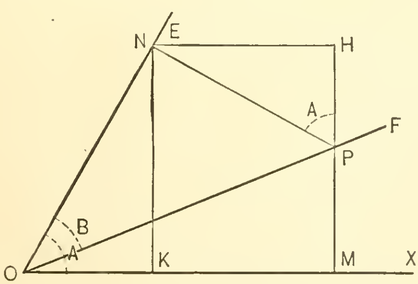


Fig. 47.

[1] Or thus. On OP as diameter, describe a circle. This will pass through M and N, because the angles OMP and ONP are right angles; therefore MPN and MON are angles in the same segment; so that the angle $MPN = MON = A$.

Let XOE be the angle A, and FOE the angle B. Then in the figure, XOF is the angle $(A-B)$.

In OF, *the line which bounds the compound angle* $(A-B)$, take any point P, and from P draw PM, PN at right angles to OX and OE respectively. Draw NH, NK at right angles to MP and OX respectively. Then the angle

$$NPH = 90° - HNP = HNE = XOE = A.[1]$$

Now $\sin(A-B) = \sin XOF = \dfrac{MP}{OP} = \dfrac{MH-PH}{OP} = \dfrac{KN}{OP} - \dfrac{PH}{OP}$

$$= \frac{KN \cdot ON}{ON \cdot OP} - \frac{PH \cdot NP}{NP \cdot OP} = \frac{KN}{ON} \cdot \frac{ON}{OP} - \frac{PH}{NP} \cdot \frac{NP}{OP}$$

$$= \sin XOE \cdot \cos FOE - \cos HPN \cdot \sin FOE$$

$$= \sin A \cdot \cos B - \cos A \cdot \sin B.$$

Also, $\cos(A-B) = \cos XOF = \dfrac{OM}{OP} = \dfrac{OK+KM}{OP} = \dfrac{OK}{OP} + \dfrac{NH}{OP}$

$$= \frac{OK \cdot ON}{ON \cdot OP} + \frac{NH \cdot NP}{NP \cdot OP} = \frac{OK}{ON} \cdot \frac{ON}{OP} + \frac{NH}{NP} \cdot \frac{NP}{OP}$$

$$= \cos XOE \cdot \cos FOE + \sin HPN \cdot \sin FOE$$

$$= \cos A \cdot \cos B + \sin A \cdot \sin B.$$

Example. Find the value of $\sin 75°$.

$$\sin 75° = \sin(45° + 30°)$$
$$= \sin 45° \cdot \cos 30° + \cos 45° \cdot \sin 30°$$
$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$
$$= \frac{\sqrt{3}+1}{2\sqrt{2}} = \frac{\sqrt{2}(\sqrt{3}+1)}{4}.$$

EXAMPLES. XVII.

1. Show that $\cos 75° = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$.
2. Show that $\sin 15° = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$.
3. Show that $\cos 15° = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$.
4. Show that $\tan 75° = 2 + \sqrt{3}$.

[1] Or thus. On OP as diameter, describe a circle. This will pass through M and N, because the angles OMP and ONP are right angles; therefore the angles MPN and MON together make up two right angles, so that the angle $HPN = MON = A$.

5. If $\sin A = \frac{4}{5}$ and $\sin B = \frac{3}{5}$, find a value for $\sin(A + B)$ and for $\cos(A - B)$.

6. If $\sin A = .6$ and $\sin B = \frac{5}{13}$, find a value for $\sin(A + B)$ and for $\cos(A + B)$.

7. When $\sin A = \dfrac{1}{\sqrt{5}}$ and $\sin B = \dfrac{1}{\sqrt{10}}$, then one value of $(A + B)$ is 45°.

8. Prove that $\sin 75° = .9659 \cdots$.

9. Prove that $\sin 15° = .2588 \cdots$.

10. Prove that $\tan 15° = .2679 \cdots$.

11. Calculate $\sin 90°$ and $\cos 90°$, using the ratios of 45°.

* **78.** The proofs given in Arts. 76 and 77 are really, at least so far as the figures are concerned, rigorous only in case $(A + B) < 90°$, and hence A and B are each less than 90°. By a careful regard as to sign of angles and lines, however, the wording will hold for angles of any magnitude. The student may satisfy himself that this is true by constructing suitable figures. The accompanying figure will serve in case $A + B < 180°$ and $A < 90°$, $B > 90°$.

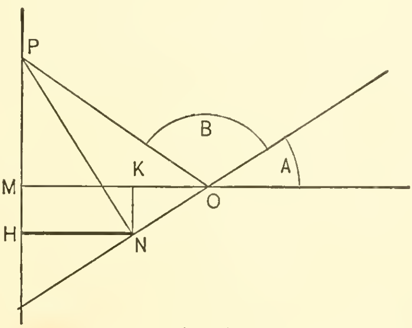


Fig. 48.

* **79.** In order, however, to remove the restrictions as to magnitude, placed on A and B, we shall pursue the following course:

Suppose $\quad 180° > A > 90°$ and $B < 90°$.

Put $\quad A = 90° + A'$; then $A' < 90°$.

$$\sin(A + B) = \sin(90° + A' + B) = \sin[90° + (A' + B)], \quad \text{(Art. 64.)}$$

$$\sin[90° + (A' + B')] = \cos(A' + B). \quad \text{(Art. 62.)}$$

A' and B are each less than 90°. We may therefore write,

$$\cos(A' + B) = \cos A' \cos B - \sin A' \sin B; \quad \text{(Art. 76.)}$$

or

$$\cos(A - 90° + B)$$
$$= \cos(A - 90°)\cos B - \sin(A - 90°)\sin B;$$

since $\cos(A - 90° + B) = \cos[90° - (A + B)]$ (Art. 58 Cor.)

$= \sin(A + B)$; (Art. 61.)

and since $\sin(A - 90°) = -\sin(90° - A)$ (Art. 58.)

$= -\cos A$, (Art. 61.)

we may write,

$$\sin(A + B) = \sin A \cos B + \cos A \sin B.$$

By a repetition of this process A may become an angle of any quadrant, and it is evident the reasoning and wording remain just as above; so that the *magnitude of A is unrestricted.*

$$\therefore \sin(A + B) = \sin A \cos B + \cos A \sin B. \quad \text{(i.)}$$

whatever be the *magnitude of A, provided $B < 90°$.*

There remains yet to inquire what happens if A and B are both greater than $90°$.

Suppose $180° > B > 90°$.

Put $B = 90° + B'$; then $B' < 90°$;

$$\sin(A + B) = \sin(90° + A + B') = \cos(A + B').$$

We can now apply (i.), whence

$$\sin(A + B) = \cos A \cos B' - \sin A \sin B'$$

$$= \cos A \sin B + \sin A \cos B.$$

Equation (1) is therefore true if $B > 90°$ but $< 180°$. As before we need only repeat the process in order to remove all restrictions as to the magnitude of B. Hence we conclude that whatever the values of A and B,

$$\sin(A + B) = \sin A \cos B + \sin B \cos A.$$

By a similar process, the three kindred formulæ may be shown to be true for all angles.

NOTE. See Nixon's *Elementary Trigonometry*, Sec. 18.

For an elegant proof of this theorem by the method of projections, see Hobson's *Plane Trigonometry*, Sec. 40.

80. It is important that the student should become thoroughly familiar with the formulæ of Art. 75, and that he should be able to work examples involving their use.

EXAMPLES. XVIII.

Prove the following statements:

1. $\sin(A+B)+\sin(A-B)=2\sin A\cdot\cos B.$
2. $\sin(A+B)-\sin(A-B)=2\cos A\cdot\sin B.$
3. $\cos(A+B)+\cos(A-B)=2\cos A\cdot\cos B.$
4. $\cos(A-B)-\cos(A+B)=2\sin A\cdot\sin B.$
5. $\dfrac{\sin(A+B)+\sin(A-B)}{\cos(A+B)+\cos(A-B)}=\tan A.$
6. $\tan\alpha+\tan\beta=\dfrac{\sin(\alpha+\beta)}{\cos\alpha\cdot\cos\beta}.$
7. $\tan\alpha-\tan\beta=\dfrac{\sin(\alpha-\beta)}{\cos\alpha\cdot\cos\beta}.$
8. $\cot\alpha+\tan\beta=\dfrac{\cos(\alpha-\beta)}{\sin\alpha\cdot\cos\beta}.$
9. $\cot\alpha-\tan\beta=\dfrac{\cos(\alpha+\beta)}{\sin\alpha\cdot\cos\beta}.$
10. $\tan\alpha+\cot\beta=\dfrac{\cos(\alpha-\beta)}{\cos\alpha\cdot\sin\beta}.$
11. $\dfrac{\tan\theta+\tan\phi}{\tan\theta-\tan\phi}=\dfrac{\sin(\theta+\phi)}{\sin(\theta-\phi)}.$
12. $\dfrac{\tan\theta\cdot\tan\phi+1}{1-\tan\theta\cdot\tan\phi}=\dfrac{\cos(\theta-\phi)}{\cos(\theta+\phi)}.$
13. $\dfrac{\tan\theta+\cot\phi}{\cot\phi-\tan\theta}=\cos(\theta-\phi)\cdot\sec(\theta+\phi).$
14. $\dfrac{\cot\theta+\cot\phi}{\cot\theta-\cot\phi}=-\dfrac{\sin(\theta+\phi)}{\sin(\theta-\phi)}.$
15. $\dfrac{\tan\theta\cdot\cot\phi+1}{\tan\theta\cdot\cot\phi-1}=\dfrac{\sin(\theta+\phi)}{\sin(\theta-\phi)}.$
16. $\dfrac{1+\cot\gamma\cdot\tan\delta}{\cot\gamma-\tan\delta}=\tan(\gamma+\delta).$
17. $\dfrac{1-\cot\gamma\cdot\tan\delta}{\cot\gamma+\tan\delta}=\tan(\gamma-\delta).$
18. $\dfrac{\tan\gamma\cdot\cot\delta-1}{\tan\gamma+\cot\delta}=\tan(\gamma-\delta).$
19. $\dfrac{\tan\gamma\cdot\cot\delta+1}{\cot\delta-\tan\gamma}=\tan(\gamma+\delta).$
20. $\dfrac{\cot\delta-\cot\gamma}{\cot\gamma\cdot\cot\delta+1}=\tan(\gamma-\delta).$
21. $\tan^2\alpha-\tan^2\beta=\dfrac{\sin(\alpha+\beta)\cdot\sin(\alpha-\beta)}{\cos^2\alpha\cdot\cos^2\beta}.$
22. $\cot^2\alpha-\tan^2\beta=\dfrac{\cos(\alpha+\beta)\cdot\cos(\alpha-\beta)}{\sin^2\alpha\cdot\cos^2\beta}.$
23. $\dfrac{\tan^2\alpha-\tan^2\beta}{1-\tan^2\alpha\cdot\tan^2\beta}=\tan(\alpha+\beta)\cdot\tan(\alpha-\beta).$
24. $\sin(\alpha+\beta)\cdot\sin(\alpha-\beta)=\sin^2\alpha-\sin^2\beta=\cos^2\beta-\cos^2\alpha.$
25. $\cos(\alpha+\beta)\cdot\cos(\alpha-\beta)=\cos^2\alpha-\sin^2\beta=\cos^2\beta-\sin^2\alpha.$
26. $\sin(A-45^\circ)=\dfrac{\sin A-\cos A}{\sqrt{2}}.$
27. $\sqrt{2}\cdot\sin(A+45^\circ)=\sin A+\cos A.$
28. $\cos A-\sin A=\sqrt{2}\cdot\cos(A+45^\circ).$
29. $\cos(A+45^\circ)+\sin(A-45^\circ)=0.$
30. $\cos(A-45^\circ)=\sin(A+45^\circ).$
31. $\sin(\theta+\phi)\cdot\cos\theta-\cos(\theta+\phi)\cdot\sin\theta=\sin\phi.$
32. $\sin(\theta-\phi)\cdot\cos\phi+\cos(\theta-\phi)\cdot\sin\phi=\sin\theta.$
33. $\cos(\theta+\phi)\cdot\cos\theta+\sin(\theta+\phi)\cdot\sin\theta=\cos\phi.$
34. $\dfrac{\tan(\theta-\phi)+\tan\phi}{1-\tan(\theta-\phi)\cdot\tan\phi}=\tan\theta.$

35. $\dfrac{\tan(\theta+\phi)-\tan\theta}{1+\tan(\theta+\phi)\cdot\tan\theta}=\tan\phi.$

36. $2\sin\left(\alpha+\dfrac{\pi}{4}\right)\cdot\cos\left(\beta-\dfrac{\pi}{4}\right)=\cos(\alpha-\beta)+\sin(\alpha+\beta).$

37. $2\sin\left(\dfrac{\pi}{4}-\alpha\right)\cdot\cos\left(\dfrac{\pi}{4}+\beta\right)=\cos(\alpha-\beta)-\sin(\alpha+\beta).$

38. $\cos(\alpha+\beta)+\sin(\alpha-\beta)=2\sin\left(\dfrac{\pi}{4}+\alpha\right)\cdot\cos\left(\dfrac{\pi}{4}+\beta\right).$

39. $\cos(\alpha+\beta)-\sin(\alpha-\beta)=2\sin\left(\dfrac{\pi}{4}-\alpha\right)\cdot\cos\left(\dfrac{\pi}{4}-\beta\right).$

40. $\sin nA\cdot\cos A+\cos nA\cdot\sin A=\sin(n+1)A.$

41. $\cos(n-1)A\cdot\cos A-\sin(n-1)A\cdot\sin A=\cos nA.$

42. $\sin nA\cdot\cos(n-1)A-\cos nA\cdot\sin(n-1)A=\sin A.$

43. $\cos(n-1)A\cdot\cos(n+1)A-\sin(n-1)A\cdot\sin(n+1)A=\cos 2nA.$

44. $(\cos A+\sin A)(\cos B+\sin B)=\cos(A-B)+\sin(A+B).$

81. The following formulæ are important:

$$\left.\begin{aligned}\tan(A+B)&=\frac{\tan A+\tan B}{1-\tan A\cdot\tan B};\\ \tan(A-B)&=\frac{\tan A-\tan B}{1+\tan A\cdot\tan B}.\end{aligned}\right\}\qquad\text{(ii.)}$$

The proof of the first is given below. The student may prove the second in a similar manner. Or, since these formulæ are true for all values of A and B, we may substitute $-B$, for B; the formula now becomes $\tan(A-B)=\dfrac{\tan A+\tan(-B)}{1-\tan A\tan(-B)}=\dfrac{\tan A-\tan B}{1+\tan A\tan B}.$

Example. To prove $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\cdot\tan B}.$

(i.) By using the results of Arts. 76, 77, we have

$$\tan(A+B)=\frac{\sin(A+B)}{\cos(A+B)}=\frac{\sin A\cdot\cos B+\cos A\cdot\sin B}{\cos A\cdot\cos B-\sin A\cdot\sin B}.$$

Divide the numerator and the denominator of this fraction each by $\cos A\cdot\cos B$, and we get

$$\tan(A+B)=\frac{\dfrac{\sin A\cdot\cos B}{\cos A\cdot\cos B}+\dfrac{\cos A\cdot\sin B}{\cos A\cdot\cos B}}{\dfrac{\cos A\cdot\cos B}{\cos A\cdot\cos B}-\dfrac{\sin A\cdot\sin B}{\cos A\cdot\cos B}}=\frac{\tan A+\tan B}{1-\tan A\cdot\tan B}.\qquad\text{Q.E.D.}$$

EXAMPLES. XIX.

1. If $\tan A=\frac{1}{2}$ and $\tan B=\frac{1}{4}$, prove that $\tan(A+B)=\frac{6}{7}$, and $\tan(A-B)=\frac{2}{9}$.

2. If $\tan A=1$ and $\tan B=\dfrac{1}{\sqrt{3}}$, prove that $\tan(A+B)=2+\sqrt{3}$.

3. Prove that $\tan 15^\circ=2-\sqrt{3}$.

4. If $\tan A = \frac{5}{6}$ and $\tan B = \frac{1}{11}$, prove that $\tan(A + B) = 1$. What is $(A + B)$ in this case?

5. If $\tan A = m$ and $\tan B = \frac{1}{m}$, prove that $\tan(A + B) = \infty$. What is $(A + B)$ in this case?

Prove the following statements:

6. $\cot(A + B) = \dfrac{\cot A \cdot \cot B - 1}{\cot A + \cot B}$.

7. $\cot(A - B) = \dfrac{\cot A \cdot \cot B + 1}{\cot B - \cot A}$.

8. $\cot\left(\theta - \dfrac{\pi}{4}\right) = \dfrac{\cot\theta + 1}{1 - \cot\theta}$.

9. $\dfrac{\cot\theta - 1}{\cot\theta + 1} = \cot\left(\theta + \dfrac{\pi}{4}\right)$.

10. $\tan\left(\theta - \dfrac{\pi}{4}\right) + \cot\left(\theta + \dfrac{\pi}{4}\right) = 0$.

11. $\cot\left(\theta - \dfrac{\pi}{4}\right) + \tan\left(\theta + \dfrac{\pi}{4}\right) = 0$.

12. If $\tan\alpha = \dfrac{m}{m+1}$ and $\tan\beta = \dfrac{1}{2m+1}$, prove that $\tan(\alpha + \beta) = 1$.

13. $\dfrac{\tan(n+1)\phi - \tan n\phi}{1 + \tan(n+1)\phi \cdot \tan n\phi} = \tan\phi$.

14. $\dfrac{\tan(n+1)\phi + \tan(1-n)\phi}{1 - \tan(n+1)\phi \cdot \tan(1-n)\phi} = \tan 2\phi$.

15. If $\tan\alpha = m$ and $\tan\beta = n$, prove that

$$\cos(\alpha + \beta) = \frac{1 - mn}{\sqrt{(1 + m^2)(1 + n^2)}}.$$

16. If $\tan\alpha = (a + 1)$ and $\tan\beta = (a - 1)$, then $2\cot(\alpha - \beta) = a^2$.

17. If $\alpha + \beta + \gamma = 90°$, then $\tan\gamma = \dfrac{1 - \tan\alpha\tan\beta}{\tan\alpha + \tan\beta}$.

82. From Art. 75 we have

$$\left.\begin{aligned} \sin(A + B) &= \sin A \cdot \cos B + \cos A \cdot \sin B; \\ \sin(A - B) &= \sin A \cdot \cos B - \cos A \cdot \sin B; \\ \cos(A + B) &= \cos A \cdot \cos B - \sin A \cdot \sin B; \\ \cos(A - B) &= \cos A \cdot \cos B + \sin A \cdot \sin B. \end{aligned}\right\} \quad \text{(i.)}$$

From these by addition and subtraction we get

$$\left.\begin{aligned} \sin(A + B) + \sin(A - B) &= 2\sin A \cdot \cos B; \\ \sin(A + B) - \sin(A - B) &= 2\cos A \cdot \sin B; \\ \cos(A + B) + \cos(A - B) &= 2\cos A \cdot \cos B; \\ \cos(A - B) - \cos(A + B) &= 2\sin A \cdot \sin B. \end{aligned}\right\} \quad \text{(ii.)}$$

Now put S for $(A + B)$, and put T for $(A - B)$:

Then $$S + T = 2A, \text{ and } S - T = 2B,$$

so that $$A = \frac{S + T}{2}, \text{ and } B = \frac{S - T}{2}.$$

Hence the above results may be written

$$\left.\begin{aligned}
\sin S + \sin T &= 2\sin\frac{S+T}{2}\cdot\cos\frac{S-T}{2};\\
\sin S - \sin T &= 2\cos\frac{S+T}{2}\cdot\sin\frac{S-T}{2};\\
\cos S + \cos T &= 2\cos\frac{S+T}{2}\cdot\cos\frac{S-T}{2};\\
{}^{1}\cos T - \cos S &= 2\sin\frac{S+T}{2}\cdot\sin\frac{S-T}{2}.
\end{aligned}\right\}\qquad\text{(iii.)}$$

83. The formulæ (iii.) are most important, and the student is recommended to get thoroughly familiar with them in *words*, as follows:

(1) *The sum of the sines of two angles equals twice the sine of half their sum multiplied by the cosine of half their difference.*

(2) *The difference of the sines of two angles equals twice the cosine of half their sum multiplied by the sine of half their difference.*

(3) *The sum of the cosines of two angles equals twice the cosine of half their sum multiplied by the cosine of half their difference.*

(4) *The difference of the cosines*[2] *of two angles equals twice the sine of half their sum multiplied by the sine of half their difference.*

84. It will be convenient to refer to the formulæ (i.) as the 'A, B' formulæ, and to the formulæ (iii.) as the 'S, T' formulæ.

EXAMPLES. XX.

Prove the following statements:

1. $\sin 60^\circ + \sin 30^\circ = 2\sin 45^\circ\cdot\cos 15^\circ$.

2. $\sin 60^\circ + \sin 20^\circ = 2\sin 40^\circ\cdot\cos 20^\circ$.

3. $\sin 40^\circ - \sin 10^\circ = 2\cos 25^\circ\cdot\sin 15^\circ$.

4. $\cos\frac{\pi}{3} + \cos\frac{\pi}{2} = 2\cos\frac{5\pi}{12}\cdot\cos\frac{\pi}{12}$.

5. $\cos\frac{\pi}{3} - \cos\frac{\pi}{2} = 2\sin\frac{5\pi}{12}\cdot\sin\frac{\pi}{12}$.

6. $\sin 3A + \sin 5A = 2\sin 4A\cdot\cos A$.

7. $\sin 7A - \sin 5A = 2\cos 6A\cdot\sin A$.

8. $\cos 5A + \cos 9A = 2\cos 7A\cdot\cos 2A$.

[1] If A and B are each less than 90°, then S, which is their *sum*, is greater than T, their *difference*. Therefore if S be less than 90°, $\cos S$ is less than $\cos T$; so that $\cos T - \cos S$ is *positive*.

[2] The difference of the cosines of two angles is the cosine of the smaller angle − the cosine of the greater angle.

9. $\cos 5A - \cos 4A = -2 \sin\frac{9A}{2} \cdot \sin\frac{A}{2}$.

10. $\cos A - \cos 2A = 2 \sin\frac{3A}{2} \cdot \sin\frac{A}{2}$.

11. $\frac{\sin 2\theta + \sin\theta}{\cos\theta + \cos 2\theta} = \tan\frac{3\theta}{2}$.

12. $\frac{\sin 2\theta - \sin\theta}{\cos\theta - \cos 2\theta} = \cot\frac{3\theta}{2}$.

13. $\frac{\sin 3\theta + \sin 2\theta}{\cos 2\theta - \cos 3\theta} = \cot\frac{\theta}{2}$.

14. $\frac{\sin\theta + \sin\phi}{\cos\theta - \cos\phi} = \frac{\cos\theta + \cos\phi}{\sin\phi - \sin\theta}$.

15. $\cos(60^\circ + A) + \cos(60^\circ - A) = \cos A$.

16. $\cos(45^\circ + A) + \cos(45^\circ - A) = \sqrt{2} \cdot \cos A$.

17. $\sin(45^\circ + A) - \sin(45^\circ - A) = \sqrt{2} \cdot \sin A$.

18. $\cos(30^\circ - A) - \cos(30^\circ + A) = \sin A$.

19. $\frac{\sin\theta - \sin\phi}{\cos\phi - \cos\theta} = \cot\frac{\theta + \phi}{2}$.

20. $\frac{\sin\theta - \sin\phi}{\sin\theta + \sin\phi} = \cot\left(\frac{\theta + \phi}{2}\right) \cdot \tan\left(\frac{\theta - \phi}{2}\right)$.

85. It is important that the student should be thoroughly familiar with the second set of formulæ on Art. 82.

Written as follows, they may be regarded as the inverse of the 'S, T' formulæ.

$$\left.\begin{aligned} 2\sin A \cdot \cos B &= \sin(A + B) + \sin(A - B); \\ 2\cos A \cdot \sin B &= \sin(A + B) - \sin(A - B); \\ 2\cos A \cdot \cos B &= \cos(A + B) + \cos(A - B); \\ 2\sin A \cdot \sin B &= \cos(A - B) - \cos(A + B). \end{aligned}\right\} \quad \text{(iv.)}$$

EXAMPLES. XXI.

Express as the sum or as the difference of two trigonometrical ratios the ten following expressions:

1. $2 \sin\theta \cdot \cos\phi$.

2. $2 \cos\alpha \cdot \cos\beta$.

3. $2 \sin 2\alpha \cdot \cos 3\beta$.

4. $2 \cos(\alpha + \beta) \cdot \cos(\alpha - \beta)$.

5. $2 \sin 3\theta \cdot \cos 5\theta$.

6. $2 \cos\frac{3\theta}{2} \cdot \cos\frac{\theta}{2}$.

7. $\sin 4\theta \cdot \sin\theta$.

8. $\cos\frac{5\theta}{2} \cdot \sin\frac{3\theta}{2}$.

9. $2 \cos 10^\circ \cdot \sin 50^\circ$.

10. $\cos 45^\circ \cdot \sin 15^\circ$.

11. Simplify $2\cos 2\theta \cdot \cos\theta - 2\sin 4\theta \cdot \sin\theta$.

12. Simplify $\sin\frac{5\theta}{2} \cdot \cos\frac{\theta}{2} - \sin\frac{9\theta}{2} \cdot \cos\frac{3\theta}{2}$.

13. Simplify $\sin 3\theta + \sin 2\theta + 2\sin\frac{3\theta}{2} \cdot \cos\frac{\theta}{2}$.

14. Prove that $\sin\frac{11\theta}{4} \cdot \sin\frac{\theta}{4} + \sin\frac{7\theta}{4} \cdot \sin\frac{3\theta}{4} = \sin 2\theta \cdot \sin\theta$.

***86.** Since A and B are any angles, we may substitute for A, $\alpha+\beta$, and for B, γ; then

$$\begin{aligned}\sin(A+B) &= \sin(\alpha+\beta+\gamma)\\ &= \sin(\alpha+\beta)\cos\gamma+\cos(\alpha+\beta)\sin\gamma\\ &= (\sin\alpha\cos\beta+\cos\alpha\sin\beta)\cos\gamma\\ &\quad+(\cos\alpha\cos\beta-\sin\alpha\sin\beta)\sin\gamma.\end{aligned}$$

$$\begin{aligned}\therefore\ \sin(\alpha+\beta+\gamma) &= \sin\alpha\cos\beta\cos\gamma+\sin\beta\cos\alpha\cos\gamma\\ &\quad+\cos\alpha\cos\beta\sin\gamma-\sin\alpha\sin\beta\sin\gamma. \qquad (1)\end{aligned}$$

It is evident that these formulæ may be extended in this way to include any number of angles that we choose.

EXAMPLES. XXII.

Find the expressions similar to (1) for

1. $\cos(\alpha+\beta+\gamma)$.

2. $\sin(\alpha+\beta-\gamma)$.

3. $\cos(\alpha+\beta-\gamma)$.

4. $\sin(\alpha\pm\beta\pm\gamma)$.

***87.** The formulæ of Arts. 86 and 75 are called the addition formulæ of the Trigonometric functions.

CHAPTER XI

ON THE TRIGONOMETRICAL RATIOS OF MULTIPLE ANGLES AND SUBMULTIPLE ANGLES

88. To express the **trigonometrical ratios** of the **angle $2A$** in terms of those of the angle A.

Since $\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$;

$\therefore \sin(A+A) = \sin A \cdot \cos A + \cos A \cdot \sin A$;

$$\therefore \sin 2A = 2 \sin A \cdot \cos A. \qquad (1)$$

Also, since $\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$;

$\therefore \cos(A+A) = \cos A \cdot \cos A - \sin A \cdot \sin A$;

$$\therefore \cos 2A = \cos^2 A - \sin^2 A. \qquad (2)$$

But $1 = \cos^2 A + \sin^2 A$;

$\therefore 1 + \cos 2A = 2\cos^2 A$,

and $1 - \cos 2A = 2\sin^2 A$.

The last two results are usually written

$$\cos 2A = 2\cos^2 A - 1, \qquad (3)$$

$$\text{and } \cos 2A = 1 - 2\sin^2 A. \qquad (4)$$

Again, $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$;

$\therefore \tan(A+A) = \dfrac{\tan A + \tan A}{1 - \tan A \cdot \tan A}$;

$$\therefore \tan 2A = \frac{2\tan A}{1 - \tan^2 A}. \qquad (5)$$

89. These five formulæ are very important,

$$\left.\begin{aligned}\sin 2A &= 2\sin A\cdot\cos A; &&(1)\\ \cos 2A &= \cos^2 A - \sin^2 A; &&(2)\\ \cos 2A &= 2\cos^2 A - 1; &&(3)\\ \cos 2A &= 1 - 2\sin^2 A; &&(4)\\ \tan 2A &= \frac{2\tan A}{1-\tan^2 A}. &&(5)\end{aligned}\right\}\text{(v.)}$$

90. The following result is important,

$$\frac{\sin 2A}{1+\cos 2A} = \frac{2\sin A\cdot\cos A}{2\cos^2 A} = \tan A.$$

91. Submultiple Angles. In formulæ (v.) the angle A is *any* angle. Hence we may write $A = \frac{\alpha}{2}$.

The formulæ (v.) now become

$$\sin\alpha = 2\sin\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2};$$

$$\cos\alpha = \cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2};$$

$$\cos\alpha = 2\cos^2\frac{\alpha}{2} - 1;$$

$$\cos\alpha = 1 - 2\sin^2\frac{\alpha}{2};$$

$$\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}}.$$

***92.** An examination of formulæ (v.) shows that if $\sin A$ or $\cos A$ be given, $\cos 2A$ is uniquely determined. The converse is not true; *i.e.* if $\cos 2A$ is given, $\sin A$ and $\cos A$ have a sign ambiguity.

We have $$\cos^2 A = \frac{\cos 2A + 1}{2}.$$

$$\therefore\ \cos A = \pm\sqrt{\frac{\cos 2A + 1}{2}}.$$

Similarly, $$\sin A = \pm\sqrt{\frac{1-\cos 2A}{2}};$$

$$\tan A = \pm\sqrt{\frac{1-\cos 2A}{1+\cos 2A}}.$$

If $\sin 2A$ *be given*, we have

$$\sin 2A + 1 = 2\sin A\cos A + \cos^2 A + \sin^2 A.$$

Since $\sin^2 A + \cos^2 A = 1,$

$$\therefore \pm\sqrt{\sin 2A + 1} = \cos A + \sin A.$$

Similarly, $\pm\sqrt{1 - \sin 2A} = \sin A - \cos A,$

whence $$2\sin A = \pm\sqrt{\sin 2A + 1} \pm \sqrt{1 - \sin 2A}.$$

This presents a fourfold sign ambiguity.

Similar remarks apply with equal force to the submultiple angles.

EXAMPLES. XXIII.

Prove the following identities:

1. $2\csc 2A = \sec A \cdot \csc A.$
2. $\dfrac{\operatorname{cosec}^2 A}{\operatorname{cosec}^2 A - 2} = \sec 2A.$
3. $\dfrac{2 - \sec^2 A}{\sec^2 A} = \cos 2A.$
4. $\cos^2 A(1 - \tan^2 A) = \cos 2A.$
5. $\cot 2A = \dfrac{\cot^2 A - 1}{2\cot A}.$
6. $\dfrac{2\tan B}{1 + \tan^2 B} = \sin 2B.$
7. $\tan B + \cot B = 2\operatorname{cosec} 2B.$
8. $\dfrac{1 - \tan^2 B}{1 + \tan^2 B} = \cos 2B.$
9. $\cot B - \tan B = 2\cot 2B.$
10. $\dfrac{\cot^2 B + 1}{\cot^2 B - 1} = \sec 2B.$
11. $\left(\sin\dfrac{\theta}{2} + \cos\dfrac{\theta}{2}\right)^2 = 1 + \sin\theta.$
12. $\left(\sin\dfrac{\theta}{2} - \cos\dfrac{\theta}{2}\right)^2 = 1 - \sin\theta.$
13. $\cos^2\dfrac{\theta}{2}\left(1 + \tan\dfrac{\theta}{2}\right)^2 = 1 + \sin\theta.$
14. $\sin^2\dfrac{\theta}{2}\left(\cot\dfrac{\theta}{2} - 1\right)^2 = 1 - \sin\theta.$
15. $\left(\dfrac{\tan\frac{\theta}{2} + 1}{\tan\frac{\theta}{2} - 1}\right)^2 = \dfrac{1 + \sin\theta}{1 - \sin\theta}.$
16. $\dfrac{\sin\beta}{1 + \cos\beta} = \tan\dfrac{\beta}{2}.$
17. $\dfrac{\sin\beta}{1 - \cos\beta} = \cot\dfrac{\beta}{2}.$
18. $\dfrac{1 - \cos\beta}{1 + \cos\beta} = \tan^2\dfrac{\beta}{2}.$
19. $\dfrac{1 + \sec\beta}{\sec\beta} = 2\cos^2\dfrac{\beta}{2}.$
20. $\operatorname{cosec}\beta - \cot\beta = \tan\dfrac{\beta}{2}.$
21. $\dfrac{\cos 2x}{1 + \sin 2x} = \dfrac{1 - \tan x}{1 + \tan x}.$
22. $\dfrac{\cos x}{1 - \sin x} = \dfrac{1 + \tan\frac{x}{2}}{1 - \tan\frac{x}{2}}.$
23. $\dfrac{\cos x}{1 + \sin x} = \dfrac{\cot\frac{x}{2} - 1}{\cot\frac{x}{2} + 1}.$
24. $\dfrac{\cos x}{1 - \sin x} = \dfrac{\cot\frac{x}{2} + 1}{\cot\frac{x}{2} - 1}.$
25. $\dfrac{1 + \sin x + \cos x}{1 + \sin x - \cos x} = \cot\dfrac{x}{2}.$
26. $\dfrac{\cos^3\alpha + \sin^3\alpha}{\cos\alpha + \sin\alpha} = \dfrac{2 - \sin 2\alpha}{2}.$
27. $\dfrac{\cos^3\alpha - \sin^3\alpha}{\cos\alpha - \sin\alpha} = \dfrac{2 + \sin 2\alpha}{2}.$
28. $\cos^4\alpha - \sin^4\alpha = \cos 2\alpha.$
29. $\cos^6\alpha + \sin^6\alpha = \dfrac{1 + 3\cos^2 2\alpha}{4}.$

30. $\cos^6 \alpha - \sin^6 \alpha = \dfrac{(3 + \cos^2 2\alpha)\cos 2\alpha}{4}$.

31. $\dfrac{\sin 3\beta}{\sin \beta} - \dfrac{\cos 3\beta}{\cos \beta} = 2$.

32. $\dfrac{\cos 3\beta}{\sin \beta} + \dfrac{\sin 3\beta}{\cos \beta} = 2 \cot 2\beta$.

33. $\dfrac{\sin 4\beta}{\sin 2\beta} = 2 \cos 2\beta$.

34. $\dfrac{\sin 5\beta}{\sin \beta} - \dfrac{\cos 5\beta}{\cos \beta} = 4 \cos 2\beta$.

35. $\dfrac{\sin \frac{5\pi}{12}}{\sin \frac{\pi}{12}} - \dfrac{\cos \frac{5\pi}{12}}{\cos \frac{\pi}{12}} = 2\sqrt{3}$.

36. $\tan (45° + A) - \tan (45° - A) = 2 \tan 2A$.

37. $\tan (45° - A) + \cot (45° - A) = 2 \sec 2A$.

38. $\dfrac{\tan^2 (45° + A) - 1}{\tan^2 (45° + A) + 1} = \sin 2A$.

39. $\dfrac{\sec A + \tan A}{\sec A - \tan A} = \tan\left(45° + \dfrac{A}{2}\right) \cdot \cot\left(45° - \dfrac{A}{2}\right)$.

40. $\dfrac{\cos (A + 45°)}{\cos (A - 45°)} = \sec 2A - \tan 2A$.

41. $\tan B = \dfrac{\sin B + \sin 2B}{1 + \cos B + \cos 2B}$.

42. $\tan B = \dfrac{\sin 2B - \sin B}{1 - \cos B + \cos 2B}$.

93. The following two formulæ should be remembered:

$$\left.\begin{aligned} \sin 3A &= 3 \sin A - 4 \sin^3 A; \\ \cos 3A &= 4 \cos^3 A - 3 \cos A. \end{aligned}\right\} \qquad \text{(vi.)}$$

Note. The similarity of these two results is likely to cause confusion. This may be avoided by observing that the second formula must be true when $A = 0°$; and then $\cos 3A = \cos 0° = 1$. In which case the formula gives $\cos 0° = 4 \cos 0° - 3 \cos 0°$, or $1 = 4 - 3$, which is true.

The first formula may be proved thus:

$$\begin{aligned} \sin 3A = \sin (2A + A) &= \sin 2A \cdot \cos A + \cos 2A \cdot \sin A \\ &= (2 \sin A \cdot \cos A) \cos A + (1 - 2 \sin^2 A) \sin A \\ &= 2 \sin A \cdot \cos^2 A + \sin A - 2 \sin^3 A \\ &= 2 \sin A (1 - \sin^2 A) + \sin A - 2 \sin^3 A \\ &= 2 \sin A - 2 \sin^3 A + \sin A - 2 \sin^3 A \\ &= 3 \sin A - 4 \sin^3 A. \end{aligned}$$

The second formula may be proved in a similar manner.

Example. Prove that

$$\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}.$$

$$\tan 3A = \tan (2A + A) = \frac{\tan 2A + \tan A}{1 - \tan 2A \cdot \tan A}$$

$$= \frac{\dfrac{2 \tan A}{1 - \tan^2 A} + \tan A}{1 - \dfrac{2 \tan A}{1 - \tan^2 A} \cdot \tan A} = \frac{2 \tan A + \tan A - \tan^3 A}{1 - \tan^2 A - 2 \tan^2 A}$$

$$= \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}.$$

EXAMPLES. XXIV.

Prove the following statements:

1. $\dfrac{\sin 3A}{\sin A} = 2\cos 2A + 1.$

2. $\dfrac{\cos 3A}{\cos A} = 2\cos 2A - 1.$

3. $\dfrac{3\sin A - \sin 3A}{\cos 3A + 3\cos A} = \tan^3 A.$

4. $\cot 3A = \dfrac{\cot^3 A - 3\cot A}{3\cot^2 A - 1}.$

5. $\dfrac{\sin 3A - \sin A}{\cos 3A + \cos A} = \tan A.$

6. $\dfrac{\sin 3A - \cos 3A}{\sin A + \cos A} = 2\sin 2A - 1.$

7. $\dfrac{\sin 3A + \cos 3A}{\cos A - \sin A} = 2\sin 2A + 1.$

8. $\dfrac{1}{\tan 3A - \tan A} + \dfrac{1}{\cot A - \cot 3A} = \cot 2A.$

9. $\left(\dfrac{3\sin A - \sin 3A}{3\cos A + \cos 3A}\right)^2 = \left(\dfrac{\sec 2A - 1}{\sec 2A + 1}\right)^3.$

10. $\dfrac{1 - \cos 3A}{1 - \cos A} = (1 + 2\cos A)^2.$

CHAPTER XII

INVERSE TRIGONOMETRIC FUNCTIONS

94. From the equation $y = \sin\phi$, we know that ϕ is an angle whose sine is y. The last statement is expressed by the notation $\phi = \sin^{-1}y$.

Hence $\sin^{-1}y$ is an angle.

$\sin^{-1}y$ is sometimes read, "an angle whose sine is y," sometimes "arc-sine y," but more frequently "anti-sine y." But it must be remembered that it means "an angle whose sine is y."

$\cos^{-1}y$ means "an angle whose **cosine** is y."

$\tan^{-1}y$ means "an angle whose **tangent** is y."

$\csc^{-1}y$ means "an angle whose **cosecant** is y."

$\sec^{-1}y$ means "an angle whose **secant** is y."

$\cot^{-1}y$ means "an angle whose **cotangent** is y."

These are read "anti-cosine y," "anti-tangent y," "anti-cosecant y," "anti-secant y," "anti-cotangent y," respectively.

EXAMPLE. $30^\circ = \sin^{-1}\frac{1}{2}$; $45^\circ = \sin^{-1}\frac{1}{\sqrt{2}}$.

95. The expressions $\sin^{-1}y$, $\cos^{-1}y$, etc., are called the *Inverse Trigonometric Functions* or *Inverse Circular Functions.*

96. In Art. 58 we showed that an infinite number of angles, differing by 2π, have the same ratios. Accordingly an infinite number of angles will satisfy an equation of the form $\phi = \sin^{-1}y$, $\theta = \cos^{-1}x$, etc. Accordingly for the sake of definiteness we shall (unless otherwise stated) make the following conventions:

(1) When we are given either of the equations $\phi = \sin^{-1}y$, $\phi = \tan^{-1}y$, $\phi = \csc^{-1}y$, $\phi = \cot^{-1}y$, we shall understand ϕ to be an angle, either *positive* or *negative*, whose magnitude is not greater than 90°.

(2) When we are given the equations $\phi = \cos^{-1}y$, $\phi = \sec^{-1}y$, we shall limit ϕ to a *positive* angle whose magnitude is not greater than 180°.

With these agreements one value and but one will satisfy any of these equations.

If, however, ϕ *is given,* we can always write *a definite equation.*

For example, $225° = \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$. But if we had given us

$$\phi = \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right),$$

and had known nothing else whatever of ϕ, by our agreements we would have concluded $\phi = -45°$.

Example 2. Given $\phi = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.

By our agreements we know that $\phi = 135°$. We can now write

$$135° = \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right).$$

If, on the other hand, we are given

$$\phi = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right), \text{ we conclude } \phi = 45°.$$

97. *To express the inverse ratios in terms of a given one.*

e.g. if $\theta = \tan^{-1}x$, to express the inverse trigonometric functions in terms of x.

$$x = \tan\theta; \qquad \therefore\ \theta = \tan^{-1}x.$$

$$\frac{1}{x} = \cot\theta; \qquad \therefore\ \theta = \cot^{-1}\frac{1}{x}.$$

$$\sqrt{1+x^2} = \sec\theta; \qquad \therefore\ \theta = \sec^{-1}\sqrt{1+x^2}.$$

$$\frac{1}{\sqrt{1+x^2}} = \cos\theta; \qquad \therefore\ \theta = \cos^{-1}\frac{1}{\sqrt{1+x^2}}.$$

$$\frac{x}{\sqrt{1+x^2}} = \sin\theta; \qquad \therefore\ \theta = \sin^{-1}\frac{x}{\sqrt{1+x^2}}.$$

$$\frac{\sqrt{1+x^2}}{x} = \csc\theta; \qquad \therefore\ \theta = \csc^{-1}\frac{\sqrt{1+x^2}}{x}.$$

No ambiguity of sign exists.

For if x is positive, $\sin\theta$ is positive; therefore $\sqrt{1+x^2}$ is positive. If x is negative, $\sin\theta$ is negative; hence $\sqrt{1+x^2}$ is positive. Then by convention $\cos\theta$ is positive; therefore $\sqrt{1+x^2}$ is positive.

Exercise. If $x = \cos\theta$, express the inverse trigonometric functions in terms of x.

98. Let $x = \sin\theta$.

Then
$$x = \cos\left(\frac{\pi}{2} - \theta\right);$$
$$\therefore\ \theta = \sin^{-1} x.$$
$$\frac{\pi}{2} - \theta = \cos^{-1} x;$$
$$\therefore\ \theta = \frac{\pi}{2} - \cos^{-1} x;$$
$$\therefore\ \boldsymbol{\sin^{-1} x = \frac{\pi}{2} - \cos^{-1} x.}$$

Exercise. Show
$$\tan^{-1} x = \frac{\pi}{2} - \cot^{-1} x;$$
$$\sec^{-1} x = \frac{\pi}{2} - \csc^{-1} x.$$

99. *To express two inverse trigonometric functions as a single inverse function;*

e.g. consider $\sin^{-1} x + \sin^{-1} y$.

Put
$$x = \sin A;\ \therefore\ A = \sin^{-1} x.$$
$$y = \sin B;\ \therefore\ B = \sin^{-1} y;$$
$$\therefore\ \sin(A + B) = x\sqrt{1 - y^2} + y\sqrt{1 - x^2}.$$
$$A + B = \sin^{-1} x + \sin^{-1} y = \sin^{-1}\left(x\sqrt{1 - y^2} + y\sqrt{1 - x^2}\right).$$

Exercise. Express $\tan^{-1} x + \tan^{-1} y$ as a single inverse trigonometric function.

EXAMPLES. XXV.

1. $\phi = \tan^{-1}\left(\frac{1}{3}\right)$; find $\cos\phi$.

2. $\phi = \cos^{-1}\left(-\frac{1}{2}\right)$ find $\csc\phi$.

3. $\phi = \sin^{-1}\left(-\frac{1}{2}\right)$ find $\sec\phi$; find $\tan\phi$.

4. $\phi = \tan^{-1}\frac{m}{m+1}$, $\theta = \tan^{-1}\frac{1}{2m+1}$; show that $(\theta + \phi) = \frac{\pi}{4}$.

5. $\sin^{-1}\frac{3}{5} + \cos^{-1}\frac{3}{5} = \frac{\pi}{2}$.

6. $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$.

7. $2\tan^{-1} x = \tan^{-1}\frac{2x}{1 - x^2}$.

8. $\sin(2\sin^{-1} x) = 2x\sqrt{1 - x^2}$.

9. $\sin^{-1}\sqrt{\frac{x - a + b}{2b}} = \cos^{-1}\sqrt{\frac{b + a - x}{2b}}$.

10. $\cos^{-1} x \pm \cos^{-1} y = \cos^{-1}\left(xy \mp \sqrt{(1 - x^2)(1 - y^2)}\right)$.

11. $\tan^{-1}\frac{1}{2} = \tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9}$.

12. $\tan^{-1} 2 + \tan^{-1}\frac{1}{2} = \frac{\pi}{2}$.

13. Find value of $\sin\left(\sin^{-1}\dfrac{\sqrt{3}}{2} + \cos^{-1}\dfrac{\sqrt{3}}{2}\right)$.

14. $\sin^{-1}\frac{3}{5} - \sin^{-1}\frac{5}{13} = \sin^{-1}\frac{16}{65}$.

15. If $\tan^{-1} ky = \tan^{-1} kz - gkx$, then $y = \dfrac{1}{k} \cdot \dfrac{2k - \tan gkx}{1 + 2k \tan kgx}$.

16. $\sin(\sin^{-1} x + \sin^{-1}\sqrt{1 - x^2}) = 1$.

17. $\sin^{-1}\left(\tan\dfrac{\pi}{4}\right) = 2\cos^{-1}\left(\dfrac{1}{\sqrt{2}}\right)$.

CHAPTER XIII

ON THE RELATIONS BETWEEN THE SIDES AND ANGLES OF A TRIANGLE

100. The **three sides** and the **three angles** of any **triangle** are called its six **parts**.

By the letters A, B, C we shall indicate

geometrically, the three angular points of the triangle ABC;

algebraically, the three angles at those angular points respectively.

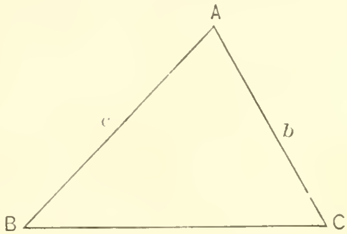


Fig. 49.

By the letters a, b, c, we shall indicate the measures of the sides BC, CA, AB, opposite the angles A, B, C, respectively.

101. I. We know that $A + B + C = 180°$. (Geom.)

102. Also if A be an angle of a triangle, then A may have any value between $0°$ and $180°$. Hence,

(i.) **sin *A*** must be positive (and less than 1);

(ii.) **cos *A*** may be positive or negative (but must be numerically less than 1);

(iii.) **tan *A*** may have any value whatever, positive or negative.

103. Also, if we are given the value of

(i.) **sin *A***, there are **two** angles, each less than 180°, which have the given positive value for their sine.

(ii.) $\cos A$, or (iii.) $\tan A$, then there is only one value of A, which value can be found from the Tables.

104. $\frac{A}{2}+\frac{B}{2}+\frac{C}{2}=90°$. Therefore $\frac{A}{2}$ is less than $90°$, and its Trigonometrical Ratios are all positive. Also, $\frac{A}{2}$ is known, when the value of any one of its ratios is given. Similar remarks of course apply to the angles B and C.

Example 1. *To prove* $\sin(A+B)=\sin C$.

$$A+B+C=180°;\ \therefore A+B=180°-C,$$

and $$\therefore \sin(A+B)=\sin(180°-C)=\sin C. \qquad \text{(Art. 59.)}$$

Example 2. *To prove* $\sin\frac{A+B}{2}=\cos\frac{C}{2}$.

Now $$\frac{A+B+C}{2}=90°;\ \therefore \frac{A+B}{2}=90°-\frac{C}{2},$$

and $$\therefore \sin\frac{A+B}{2}=\sin\left(90°-\frac{C}{2}\right)=\cos\frac{C}{2}. \qquad \text{(Art. 61.)}$$

EXAMPLES. XXVI.

Find A from each of the six following equations, A being an angle of a triangle:

1. $\cos A=\frac{1}{2}$.
2. $\cos A=-\frac{1}{2}$.
3. $\sin A=\frac{1}{2}$.
4. $\tan A=-1$.
5. $\sqrt{2}\sin A=1$.
6. $\tan A=-\sqrt{3}$.

Prove the following statements, A, B, C being the angles of a triangle:

7. $\sin(A+B+C)=0$.
8. $\cos(A+B+C)=-1$.
9. $\sin\frac{1}{2}(A+B+C)=1$.
10. $\cos\frac{1}{2}(A+B+C)=0$.
11. $\tan(A+B)=-\tan C$.
12. $\cot\frac{1}{2}(B+C)=\tan\frac{1}{2}A$.
13. $\cos(A+B)=-\cos C$.
14. $\cos(A+B-C)=-\cos 2C$.
15. $\tan A-\cot B=\cos C\cdot\sec A\cdot\csc B$.
16. $\frac{\sin A-\sin B}{\sin A+\sin B}=\tan\frac{C}{2}\cdot\tan\frac{A-B}{2}$.
17. $\frac{\sin 3B-\sin 3C}{\cos 3C-\cos 3B}=\tan\frac{3A}{2}$.

105. II. *To prove* $a=b\cos C+c\cos B$.

From A, any one of the angular points, draw AD perpendicular to BC, or to BC produced if necessary.

There will be three cases. Fig. i. when both B and C are acute angles; Fig. ii. when one of them (B) is obtuse; Fig. iii. when one of them (B) is a right angle. Then,

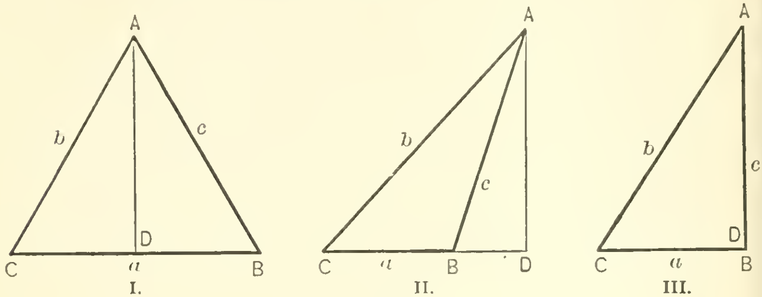


Fig. 50.

Fig. i. $\dfrac{CD}{CA} = \cos ACD$; or, $CD = b \cos C$,

and $\dfrac{DB}{AB} = \cos ABD$; or, $DB = c \cos B$.

$\therefore\ a = CD + DB = b \cos C + c \cos B.$

Fig. ii. $\dfrac{CD}{CA} = \cos ACD$; or, $CD = b \cos C$,

$\dfrac{BD}{AB} = \cos ABD$; or, $BD = c \cos (180^\circ - B)$,

$\therefore\ a = CD - BD = b \cos C - c \cos (180^\circ - B)$

$= b \cos C + c \cos B.$

Fig. iii. $a = CB = b \cos C$

$= b \cos C + c \cos B.$ [For, $\cos B = \cos 90^\circ = 0$.]

Similarly it may be proved that,

$$b = c \cos A + a \cos C;\ c = a \cos B + b \cos A.$$

106. III. *To prove that, in any triangle, the sides are proportional to the sines of the angles opposite; or, To prove that*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

From A, any one of the angular points, draw AD perpendicular to BC, or to BC produced if necessary. Then,

I. Fig. 50. $AD = b \sin C$; for, $\dfrac{AD}{AC} = \sin C$ [Def.];

also $AD = c \sin B$; for, $\frac{DA}{AB} = \sin B$.

$$\therefore\ b \sin C = c \sin B;$$

or,
$$\frac{b}{\sin B} = \frac{c}{\sin C}.$$

II. Fig. 50. $$AD = b \sin C,$$

and
$$AD = c \sin ABD = c \sin (180° - B).$$

$$\therefore\ AD = c \sin B;$$

$$\therefore\ b \sin C = c \sin B;$$

or,
$$\frac{b}{\sin B} = \frac{c}{\sin C}.$$

III. Fig. 50. $AB = AC \cdot \sin C$; or, $c = b \sin C$;

$$\therefore\ \frac{c}{\sin C} = \frac{b}{\sin B}. \quad [\text{For } \sin B = \sin 90° = 1.]$$

Similarly it may be proved that

$$\frac{a}{\sin A} = \frac{b}{\sin B};$$

$$\therefore\ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$ Q.E.D.

107. IV. To prove that $a^2 = b^2 + c^2 - 2\,bc \cos A$.

Take one of the angles A. Then of the other two, one must be acute. Let B be an acute angle. From C draw CF perpendicular to BA, or to BA produced if necessary.

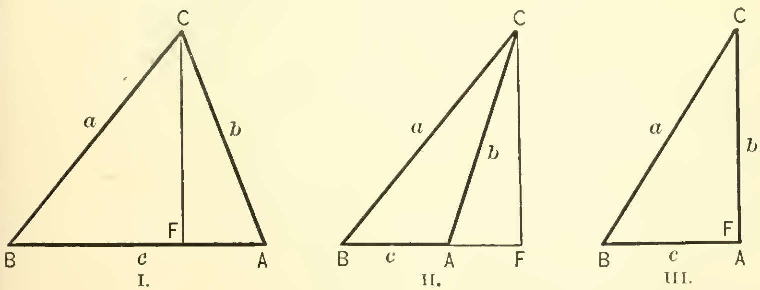


FIG. 51.

There will be three figures according as A is less, greater than, or equal to a right angle. Then,

I. Fig. 51. $BC^2 = CA^2 + AB^2 - 2 \cdot BA \cdot FA$; (Geom.)

or, $a^2 = b^2 + c^2 - 2\,c \cdot FA$

$= b^2 + c^2 - 2\,cb \cos A$. (For $FA = b \cdot \cos A$.)

II. Fig. 51. $BC^2 = CA^2 + AB^2 + 2 \cdot BA \cdot AF$; (Geom.)

or, $a^2 = b^2 + c^2 + 2\,cb \cos FAC$

$= b^2 + c^2 - 2\,bc \cos A$. (For $FAC = 180° - A$.)

III. Fig. 51. $BC^2 = CA^2 + AB^2$; (Geom.)

or, $a^2 = b^2 + c^2 - 2\,bc \cos A$. (For $\cos A = \cos 90° = 0$.)

Similarly it may be proved that

$$b^2 = c^2 + a^2 - 2\,ca \cos B,$$

and that
$$c^2 = a^2 + b^2 - 2\,ab \cos C.$$

108. V. Hence,

$$\cos A = \frac{b^2 + c^2 - a^2}{2\,bc}, \quad \cos B = \frac{c^2 + a^2 - b^2}{2\,ca}, \quad \cos C = \frac{a^2 + b^2 - c^2}{2\,ab}.$$

***109.** The formulæ of Art. 108 may be obtained directly from those of Art. 105.

$$a = b \cos C + c \cos B. \quad (1)$$

$$b = c \cos A + a \cos C. \quad (2)$$

$$c = a \cos B + b \cos A. \quad (3)$$

Multiplying (1), (2), (3) by a, b, c respectively and adding, we obtain

$$a^2 + b^2 + c^2 = 2\,a\,(b \cos C + c \cos B) + 2\,bc \cos A = 2\,a^2 + 2\,bc \cos A.$$

$$\therefore \cos A = \frac{b^2 + c^2 - a^2}{2\,bc}.$$

Exercise I. Find the two corresponding expressions, viz., for $\cos B$ and $\cos C$.

Exercise II. If $a = 5$, $b = 6$, $c = 7$, find $\cos A$.

110. VI. Let s stand for half the sum of a, b, c; so that

$(a + b + c) = 2\,s$.

Then, $(b + c - a) = (b + c + a - 2\,a) = (2\,s - 2\,a) = 2\,(s - a)$,

and $(c + a - b) = (c + a + b - 2\,b) = (2\,s - 2\,b) = 2\,(s - b)$,

and $(a + b - c) = (a + b + c - 2\,c) = (2\,s - 2\,c) = 2\,(s - c)$.

111. VII. To prove that

$$\sin\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{bc}} \text{ and that } \cos\frac{A}{2}=\sqrt{\frac{s(s-a)}{bc}},$$

where s stands for half the sum of the sides a, b, c.

Now, since

$$\cos A=\frac{b^2+c^2-a^2}{2\,bc} \text{ and } 1-\cos A=2\sin^2\frac{A}{2}, \quad \text{(Art. 88.)}$$

$$\therefore\ 2\sin^2\frac{A}{2}=1-\cos A=1-\frac{b^2+c^2-a^2}{2\,bc}$$

$$=\frac{2\,bc-(b^2+c^2-a^2)}{2\,bc}=\frac{a^2-(b^2-2\,bc+c^2)}{2\,bc}$$

$$=\frac{a^2-(b-c)^2}{2\,bc}=\frac{\{a-(b-c)\}\{a+(b-c)\}}{2\,bc};$$

$$\therefore\ \sin^2\frac{A}{2}=\frac{(a+c-b)(a+b-c)}{4\,bc};$$

$$\therefore\ \sin\frac{A}{2}=\sqrt{\frac{(2\,s-2\,b)(2\,s-2\,c)}{4\,bc}}=\sqrt{\frac{(s-b)(s-c)}{bc}}.$$

Again, since $2\cos^2\frac{A}{2}=1+\cos A$, (Art. 88.)

$$\therefore\ 2\cos^2\frac{A}{2}=1+\cos A=1+\frac{b^2+c^2-a^2}{2\,bc}$$

$$=\frac{(b+c)^2-a^2}{2\,bc}=\frac{(b+c+a)(b+c-a)}{2\,bc};$$

$$\therefore\ \cos\frac{A}{2}=\sqrt{\frac{2\,s\cdot(2\,s-2\,a)}{4\,bc}}=\sqrt{\frac{s(s-a)}{bc}}.$$

112. $$\tan\frac{A}{2}=\frac{\sin\frac{A}{2}}{\cos\frac{A}{2}}=\frac{\sqrt{\frac{(s-b)(s-c)}{bc}}}{\sqrt{\frac{s(s-a)}{bc}}}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}.$$

Example. Write down the corresponding formulæ for

$$\sin\frac{B}{2}, \text{ for } \cos\frac{B}{2}, \text{ and for } \tan\frac{B}{2}.$$

113. VIII. Again,

$$\sin A=2\sin\frac{A}{2}\cdot\cos\frac{A}{2}; \quad \text{(Art. 91.)}$$

$$\therefore\ \sin A=2\sqrt{\frac{(s-b)(s-c)}{bc}}\cdot\sqrt{\frac{s(s-a)}{bc}}$$

$$\therefore\ \sin A=\frac{2}{bc}\sqrt{s(s-a)(s-b)(s-c)}.$$

The letter S usually stands for $\sqrt{s(s-a)(s-b)(s-c)}$, so that the above may be written $\frac{\sin A}{a} = \frac{2S}{abc}$.

Similarly, $$\frac{\sin B}{b} = \frac{2S}{abc} = \frac{\sin C}{c}.$$

114. IX. To prove that

$$\frac{b-c}{b+c} \cdot \cot \frac{A}{2} = \tan \frac{B-C}{2}.$$

Since $\frac{b}{\sin B} = \frac{c}{\sin C}$ (Art. 106), let each of these fractions $= d$.

Then $\qquad b = d \sin B$, and $c = d \sin C$.

$$\therefore \frac{b-c}{b+c} = \frac{d \sin B - d \sin C}{d \sin B + d \sin C} = \frac{\sin B - \sin C}{\sin B + \sin C}$$

$$= \frac{2 \sin \frac{B-C}{2} \cdot \cos \frac{B+C}{2}}{2 \sin \frac{B+C}{2} \cdot \cos \frac{B-C}{2}}$$

$$= \frac{\tan \frac{B-C}{2}}{\tan \frac{B+C}{2}} \qquad \text{(Art. 82.)}$$

$$\therefore \frac{b-c}{b+c} = \frac{\tan \frac{B-C}{2}}{\cot \frac{A}{2}}. \quad \left[\text{Since } \tan \frac{B+C}{2} = \tan\left(90^\circ - \frac{A}{2}\right).\right]$$

$$\therefore \boldsymbol{\frac{b-c}{b+c} \cdot \cot \frac{A}{2} = \tan \frac{B-C}{2}}. \qquad \text{Q.E.D.}$$

Similarly, $$\frac{c-a}{c+a} \cot \frac{B}{2} = \tan \frac{C-A}{2};$$

$$\frac{a-b}{a+b} \cot \frac{C}{2} = \tan \frac{A-B}{2}.$$

115. The student is advised to make himself thoroughly familiar with the following formulæ:

$$\boldsymbol{\sin A = \frac{2}{bc} \sqrt{s(s-a)(s-b)(s-c)} = \frac{2S}{bc}}; \qquad \text{(i.) (Art. 113.)}$$

$$\boldsymbol{a = b \cos C + c \cos B}; \qquad \text{(ii.) (Art. 105.)}$$

$$\boldsymbol{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = \frac{abc}{2S}}; \qquad \text{(iii.) (Art. 106.)}$$

$$\boldsymbol{\cos A = \frac{b^2 + c^2 - a^2}{2bc}}; \qquad \text{(iv.) (Art. 108.)}$$

$$\sin\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{bc}};\qquad \text{(v.) (Art. 111.)}$$

$$\cos\frac{A}{2}=\sqrt{\frac{s(s-a)}{bc}};\qquad \text{(vi.) (Art. 111.)}$$

$$\tan\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}};\qquad \text{(vii.) (Art. 112.)}$$

$$\tan\frac{B-C}{2}=\frac{b-c}{b+c}\cot\frac{A}{2}.\qquad \text{(viii.) (Art. 114.)}$$

116. The sign of the radicals in v., vi., and vii. of Art. 115 is positive, because $\frac{A}{2}<90°$.

$\text{Sin}\frac{A}{2}$, $\cos\frac{A}{2}$, $\tan\frac{A}{2}$ cannot be imaginary, since, then, either $s-a$, $s-b$, $s-c$, is negative; which is impossible.

EXAMPLES. XXVII.

In any triangle ABC prove the following statements :

1. $\frac{\sin A+2\sin B}{a+2b}=\frac{\sin C}{c}$.

2. $\frac{\sin^2 A-m\cdot\sin^2 B}{a^2-m\cdot b^2}=\frac{\sin^2 C}{c^2}$.

3. $a\cos A+b\cos B-c\cos C=2c\cos A\cdot\cos B$.

4. $(a+b)\sin\frac{C}{2}=c\cos\frac{A-B}{2}$.

5. $(b-c)\cos\frac{A}{2}=a\sin\frac{B-C}{2}$.

6. $a\sin(B-C)+b\sin(C-A)+c\sin(A-B)=0$.

7. $\frac{a-b}{c}=\frac{\cos B-\cos A}{1+\cos C}$.

8. $\frac{b+c}{a}=\frac{\cos B+\cos C}{1-\cos A}$.

9. $\sqrt{bc\sin B\cdot\sin C}=\frac{b^2\sin C+c^2\sin B}{b+c}$.

10. $a+b+c=(b+c)\cos A+(c+a)\cos B+(a+b)\cos C$.

11. $b+c-a=(b+c)\cos A-(c-a)\cos B+(a-b)\cos C$.

12. $\tan A=\frac{a\sin C}{b-a\cos C}$.

13. $\frac{\tan B}{\tan C}=\frac{a^2+b^2-c^2}{a^2-b^2+c^2}$.

In solving the following list of examples, the student will select from the formulæ of Art. 115 those best suited to his purpose.

Example. Given $a=2$, $b=\sqrt{6}$, $c=1+\sqrt{3}$. To find the angles of the triangle.

It is evident that we may apply either iv., v., vi., or vii. ; but that ii., iii., and viii. contain *two* angles, and hence cannot be used. We shall employ iv. We find

$$\cos A=\frac{1}{\sqrt{2}},\quad\therefore\ A=45°;$$

$$\cos B=\tfrac{1}{2},\quad\therefore\ B=60°;$$

$$C=180°-(A+B)=75°.$$

EXAMPLES. XXVIII.

1. Simplify the formulæ

$$\cos A = \frac{b^2 + c^2 - a^2}{2\,bc}, \quad \cos \tfrac{1}{2} A = \sqrt{\left\{\frac{s(s-a)}{bc}\right\}}$$

in the case of an equilateral triangle.

2. The sides of a triangle are as $2 : \sqrt{6} : 1 + \sqrt{3}$; find the angles.

3. The sides of a triangle are as $4,\ 2\sqrt{2},\ 2(\sqrt{3} - 1)$; find the angles.

4. Given $C = 120°$, $c = \sqrt{19}$, $a = 2$; find b.

5. Given $A = 60°$, $b = 4\sqrt{7}$, $c = 6\sqrt{7}$; find a.

6. Given $A = 45°$, $B = 60°$, and $a = 2$; find c.

7. The sides of a triangle are as $7 : 8 : 13$; find the greatest angle.

8. The sides of a triangle are 1, 2, $\sqrt{7}$; find the greatest angle.

9. The sides of a triangle are as $a : b : \sqrt{(a^2 + ab + b^2)}$; find the greatest angle.

10. When $a : b : c$ as $3 : 4 : 5$, find the greatest and least angles; given $\cos 36° 52' = .8$.

11. If $a = 5$ miles, $b = 6$ miles, $c = 10$ miles, find the greatest angle. [$\cos 49° 33' = .65$.]

12. If $a = 4$, $b = 5$, $c = 8$, find C. Given that $\cos 54° 54' = .575$.

13. $a : b = \sqrt{3} : 1$, and $C = 30°$; find the other angles.

14. If $b = 3$, $C = 120°$, $c = \sqrt{13}$, find a and the sines of the other angles.

15. Given $A = 105°$, $B = 45°$, $c = \sqrt{2}$; solve the triangle.

16. Given $B = 75°$, $C = 30°$, $c = \sqrt{8}$; solve the triangle.

17. Given $B = 45°$, $c = \sqrt{75}$, $b = \sqrt{50}$; solve the triangle.

18. Two sides of a triangle are $3\sqrt{6}$ yards and $3\sqrt{3} + 1$ yards, and the included angle is $45°$; solve the triangle.

19. If the angles adjacent to the base of a triangle are $22\frac{1}{2}°$ and $112\frac{1}{2}°$, show that the perpendicular altitude will equal half the base.

20. If $A = 45°$ and $B = 60°$, show that $2\,c = a(1 + \sqrt{3})$.

21. The cosines of two of the angles of a triangle are $\frac{1}{2}$ and $\frac{2}{3}$; find the ratio of the sides.

CHAPTER XIV

LOGARITHMS

117. Before proceeding to the problem known as the solution of triangles, we shall discuss *very briefly* the use of common logarithms and certain mathematical tables. These tables may be found on the last sixty-five pages of this book.

NOTE. — The discussion of logarithms belongs properly to Algebra, to which the student is referred for a more general treatment.

The student will observe that computation by means of logarithms is a mere combination of exponents.

118. In Algebra it is explained that when different powers or roots of the same number are concerned,

(i.) *multiplication* is effected by *adding* the *indices;*

(ii.) *division* is effected by *subtracting* the *indices;*

(iii.) *involution* and *evolution* are respectively effected by the *multiplication* and *division* of the *indices.*

EXAMPLE **1.** Let $m = a^h$, $n = a^k$;

then
$$m \times n = a^h \times a^k = a^{h+k}, \quad \text{(i.)}$$
$$m \div n = a^h \div a^k = a^{h-k}, \quad \text{(ii.)}$$
$$\left.\begin{aligned} m^3 &= (a^h)^3 = a^{3h}, \\ \sqrt[4]{m} &= m^{\frac{1}{4}} = (a^h)^{\frac{1}{4}} = a^{\frac{h}{4}}. \end{aligned}\right\} \quad \text{(iii.)}$$

119. DEFINITION. The **logarithm of a number,** n, to a certain **base,** b, is the index with which it is necessary to affect b to produce n.

E.g. suppose $b^l = n$; then logarithm n to base b is l for b, raised to lth power, produces n. This is expressed in the following **notation:**

$$\log_b n = l.$$

and is read "logarithm of n to the base b equals l." Or, if no ambiguity arises, simply "$\log n = l$."

EXAMPLES. $\log_2 8 = 3$; for $2^3 = 8$, or **2** must be raised to **power 3** to produce **8.**

$$\log_{10} 100 = 2.$$

120. The use of logarithms is based upon the following propositions:

I. *The logarithm of the* **product** *of two numbers is equal to the logarithm of one of the numbers plus the logarithm of the other.*

For, let $\log_b m = x$; then $m = b^x$. (Def.)

and let $\log_b n = y$; then $n = b^y$. (Def.)

$$(mn) = (b^x b^y) = (b^{x+y}).$$

$$\therefore \log_b(mn) = \log_b(b^{x+y}) = x + y.$$

But $x + y = \log_b m + \log_b n$;

or, $\log_b(mn) = x + y = \log_b m + \log_b n$. Q.E.D.

II. *The logarithm of the* **quotient** *of two numbers is the logarithm of the* **dividend** *minus the logarithm of the* **divisor.**

For $\left(\frac{m}{n}\right) = \left(\frac{b^x}{b^y}\right) = b^{(x-y)}$.

But $x - y = \log_b m - \log_b n$.

$$\therefore \log_b\left(\frac{m}{n}\right) = x - y = \log_b m - \log_b n.$$

III. The logarithm of a number raised to a power k is κ times the logarithm of the number.

For, $(m^k) = (b^x)^k$.

Now, $\log_b b^{kx} = kx = k \log_b m$.

Examples. Given $\log_{10} 2 = .30103$, $\log_{10} 3 = .47712$, $\log_{10} 7 = .84509$; find the values of the following:

(i.) $\log_{10} 6 = \log_{10}(2 \times 3) = \log_{10} 2 + \log_{10} 3$

$= .30103 + .47712 = .77815.$ [by I.]

(ii.) $\log_{10} \frac{7}{3} = \log_{10} 7 - \log_{10} 3 = .84509 - .47712 = .36797.$ [by II.]

(iii.) $\log_{10} 3^5 = 5$ times $\log_{10} 3 = 5 \times .30103 = 1.50515.$ [by III.]

(iv.) $\log_{10} \sqrt[3]{\frac{3 \times 4}{7}} = \log_{10}\left(\frac{3 \times 4}{7}\right)^{\frac{1}{3}} = \frac{1}{3}$ of $\log_{10} \frac{3 \times 4}{7}$ [by III.]

$= \frac{1}{3}$ of $(\log 3 + \log 4 - \log 7) = \frac{1}{3}$ of $\{.47712 +$ twice $.30103 - .84509\}$

$= \frac{1}{3}$ of $.23408 = .07802.$ [by I. and II.]

(v.) $\log_{10} 5 = \log_{10} \frac{10}{2} = \log_{10} 10 - \log_{10} 2 = 1 - .30103 = .69897.$

EXAMPLES. XXIX.

1. Find the logarithms to the base a of a^3, $a^{\frac{10}{3}}$, $\sqrt[4]{a}$, $\sqrt[3]{a^2}$, $\frac{1}{a^{\frac{5}{2}}}$.
2. Find the logarithms to the base 2 of 8, 64, $\frac{1}{2}$, .125, .015625, $\sqrt[3]{64}$.
3. Find the logarithms to the base 3 of 9, 81, $\frac{1}{3}$, $\frac{1}{27}$, $\frac{1}{81}$.

4. Find the value of $\log_2 8$, $\log_2 .5$, $\log_3 243$, $\log_5 .04$, $\log_{10} 1000$, $\log_{10} .001$.

5. Find the value of $\log_a a^{\frac{4}{3}}$ $\log_b \sqrt[3]{b^2}$ $\log_8 2$, $\log_{27} 3$, $\log_{100} 10$.

6. Prove that $\log(\sqrt[3]{2} \times \sqrt[4]{7} \div \sqrt[5]{9}) = \frac{1}{3}\log 2 + \frac{1}{4}\log 7 - \frac{2}{5}\log 3$.

121. That system of logarithms whose base is 10 is called the **common** system of logarithms.

In speaking of logarithms hereafter, *common* logarithms are referred to unless the contrary is expressly stated.

We shall assume that an index of 10 can be found such that 10 affected with this index is practically equivalent to any number.

The indices of these powers of 10, *i.e.* the common logarithms, are in general *incommensurable* numbers.

Now, the greater the index with which 10 is affected, the greater will be the value of the equivalent expression; and the less the index, the less will be the numerical value of the expression.

Hence, if one number be less than another, the logarithm of the first will be less than the logarithm of the second.

But the student should notice that logarithms (or indices) are *not proportional* to the corresponding numbers.

EXAMPLE. 1000 is less than 10000; and the logarithm to base 10 of the first is 3 and of the second is 4.

But 1000, 10000, 3, 4 are not in proportion.

122. PROPOSITION. *If two numbers expressed in the decimal notation have the same digits arranged in the same order (so that they differ only in the position of the decimal point), their logarithms to the base* 10 *differ only by an integer.*

The decimal point in a number is moved by multiplying or dividing the number by some power of 10.

Let the numbers be m and n; then $m = n \times 10^k$ when k is a whole number (positive or negative); then

$$\log m = \log(n \times 10^k) = \log n + \log 10^k = \log n + k. \quad \text{(Art. 120.)}$$

That is, $\log m$ and $\log n$ differ by an integer. Q.E.D.

EXAMPLE 1. $\log 1679.2 = \log\{(1.6792) \times 10^3\} = \log 1.6792 + \log 10^3$
$= \log 1.6792 + 3.$

EXAMPLE 2. Given that $\log 1.7692 = .247776$;

find (i.) $\log 17692$, (ii.) $\log .0017692$, (iii.) $\log 176.92$.

Here $\log 17692 = \log(1.7692 \times 10^4) = 4.247776$,

$\log .0017692 = \log(1.7692 \times 10^{-3}) = -3 + .247776$,

$\log 176.92 = \log(1.7692 \times 10^2) = 2.247776$.

123. We know from Algebra that $1 = 10^0$,

$$10 = 10^1 \text{ and that } .1 = \tfrac{1}{10} = 10^{-1}$$
$$100 = 10^2 \text{ and that } .01 = \tfrac{1}{100} = 10^{-2}$$
$$1000 = 10^3 \text{ and that } .001 = \tfrac{1}{1000} = 10^{-3}$$
$$10000 = 10^4 \text{ and that } .0001 = \tfrac{1}{10000} = 10^{-4}$$

and so on.

Hence, the logarithm of 1 is 0.

The logarithm of any number greater than 1 is *positive*.

The logarithm of any **positive** number less than 1 is *negative*.

124. Observe also

that the logarithm of any number between 1 and 10 is a positive decimal fraction;

that the logarithm of any number between 10 and 100, *i.e.* between 10^1 and 10^2, is 1 + a decimal fraction;

that the logarithm of any number between 1000 and 10000, *i.e.* between 10^3 and 10^4, is 3 + a decimal fraction; and so on.

125. Observe also

that the logarithm of any number between 1 and .1, *i.e.* between 10^0 and 10^{-1}, can be written in the form $-1 +$ **a decimal fraction**;

that the logarithm of any number between .1 and .01, *i.e.* between 10^{-1} and 10^{-2}, can be written in the form $-2 +$ **a decimal fraction**; and so on.

Example 1. How many digits are contained in the integral part of the number whose logarithm is 3.67192?

The number is $10^{3.67192}$ and this is greater than 10^3, *i.e.* greater than 1000, and it is less than 10^4, *i.e.* less than 10000. Therefore the number lies between 1000 and 10000, and therefore the integral part of it contains four figures.

Example 2. Given that $3 = 10^{.4771213}$, find the number of the digits in the integral part of 3^{20}.

We have $$3 = 10^{.4771213},$$

$$\therefore\ 3^{20} = (10^{.47712})^{20} = 10^{9.54242}.$$

Therefore there are 10 digits in the integral part of 3^{20}; for it is greater than 10^9 and less than 10^{10}.

Example 3. Suppose that the decimal part of the logarithm is to be kept positive, find the integral part of the logarithm of .0001234.

This number is greater than .0001, *i.e.* than 10^{-4} and less than .001, *i.e.* than 10^{-3}.

Therefore its logarithm lies between -3 and -4, and therefore it is $-4 +$ a fraction; the integral part is therefore -4.

126. From Art. 120–125 it is evident that the logarithm of any positive number may be written as an **integer** + a decimal fraction.

The *integral* part of the logarithm is called the **characteristic.** The *decimal* part of the logarithm is called the **mantissa.** For *convenience*, the mantissæ of common logarithms are always kept **positive.** In this way the mantissæ of the logarithms of numbers consisting of the same digits, arranged in the same order, are *always* the *same* (Art. 120); because removing the decimal place to the right or to the left is equivalent to multiplying the number by 10^k, where k is a positive or negative *integer*, as the case may be.

Example. The mantissa of $\log 3.456$ = mantissa of $\log (345.6)$.

The student cannot observe too carefully that the mantissa is always positive.

The mantissæ have been calculated and arranged in convenient tables. See table I.

127. It is evident from Arts. 120–125, that the characteristic of a logarithm can be obtained by the following rule:

Rule. The characteristic of the logarithm of a number greater than unity is **one less** than the number of figures in the integral part of the number.

The characteristic of a number less than unity is negative, and (when the number is expressed as a decimal) is **one more** than the number of ciphers between the decimal point and the first significant figure to the right of the decimal point.

When the characteristic is negative, as for example in the logarithm $-3 + .17609$, the logarithm is abbreviated thus, $\bar{3}.17609$.

Example **1.** The characteristics of 36741, 36.741, .0036741, 3.6741, and .36741 are respectively 4, 1, -3, 0, and -1.

Example **2.** Given that the mantissa of the logarithm of 36741 is 56515, we can at once write down the logarithm of any number whose digits are 36741.

Thus

$$\log 3674100 = 6.56515,$$
$$\log 36741 = 4.56515,$$
$$\log 367.41 = 2.56515,$$
$$\log .36741 = \bar{1}.56515,$$
$$\log .00036741 = \bar{4}.56515,$$

and so on.

128. We have said that logarithms are in general incommensurable numbers. Their values can, therefore, only be given approximately.

If the value of any number is given to seven significant figures, then the **error** (*i.e.* the difference between the *given* value and the *exact* value of the number) is less than a millionth part of the number.

EXAMPLE. 3.141592 is the value of π correct to seven significant figures. The *error* is less than .000001; for π is less than 3.141593, and greater than 3.141592.

The ratio of .000001 to 3.141592 is equal to 1 : 3141592. The ratio of .000001 to π is less than this; *i.e.* much less than the ratio of one to one million.

129. An actual measurement of any kind must be made with the greatest care, with the most accurate instruments, by the most skilful observers, if it is to attain to anything like the accuracy represented by 'seven significant figures'; and, indeed, the value of any quantity given correct to 'four significant figures' is exact for most practical purposes.

130. A five-place table of logarithms is placed at the end of the book. (See table I.) Page 1 of this table contains the logarithms, to five places of decimals, of all numbers from 1 to 100. Pages 2–16 contain the mantissæ, to five decimal places, of the logarithms all numbers from 100 to 10000. But all numbers from 0 to ∞ is one of these numbers multiplied by ten affected with either a positive or negative index,

$$e.g.\ 4628326 = 4.628326 \times 10^6;\ \ .03986 = 3.986 \times 10^{-2}.$$

Hence by prefixing to the mantissæ the proper characteristic (see Art. 126) we obtain the logarithm of any number, of not more than four significant figures, from 0 to ∞.

131. *To find the logarithm of a given number.* (*a*) If the number contains not more than four significant figures. Find the mantissæ from the table corresponding to these four significant figures and prefix the proper characteristic. The result is the logarithm required.

EXAMPLE 1. To find the logarithm of 4064.

$$4064 = 4.064 \cdot 10^3.$$

Referring to the table I., page 8, we find, at the intersection of the row headed 406 and the column headed 4, the number .60895.

$\therefore$ log 4064. = 3.60895.

EXAMPLE 2. To find log .04064. This logarithm differs from the former as to the characteristic, which is -2.

$\therefore$ log .04064 = $\bar{2}$.60895.

(*b*) To find the logarithm of a number of more than four digits.

We shall **assume** that *if the difference is small the difference between numbers is proportional to the difference of their logarithms.* This proposition is proved in works on Algebra. If there are more than four digits in the number, we cannot obtain its logarithm directly from the table, but must *interpolate.* This is illustrated by the following example. To find the logarithm of 3456.4. This number lies between 3457 and 3456. Its logarithm therefore lies between log 3457 and log 3456.

$$\log 3457 - \log 3456 = 3.53870 - 353857 = .00013.$$

If $\log 3456.4 = l$,

$$\log 3456.4 - \log 3456 = l - 3.53857.$$

By the theorem stated above,

$$1 : .00013 = .4 : l - 3.53857;$$
$$\therefore\ l = .4 \times .00013 + 3.53857;$$
$$\therefore\ \log 3456.4 = 3.53862.$$

Or we may reason thus:

Since increasing 3456 by 1 increases its logarithm by .00013 increasing 3456 by .4 of 1 increases its logarithm by .4 of .00013 or by .000052.

In forming such products as $.4 \times .00013$, we retain only five decimal places. We increase the number occupying the fifth place by unity if the succeeding number is equal to or is greater than 5. We neglect the number occupying the sixth place if it is less than 5.

Example. To find the logarithm of 56.452.

$$\log 56.46 = 1.75174$$
$$\log 56.45 = 1.75166$$
$$8$$

$$.2 \times 8 = 1.6. \quad \therefore \log 56.452 = 1.75168.$$

EXAMPLES. XXX.

Find logarithms of the following numbers:

1. 3562; 7.456; .00432.
2. 86421; 96.204; .00352.
3. .045624; .035421; .0072345.
4. 6789000; 32,456,000.

132. *To find the number whose logarithm is given.* The method of procedure is just the reverse of that of Art. 131, and will be illustrated by the following examples:

antilogarithms

Example 1. To find the number whose logarithm is 3.41447.

Looking in the body of the table of logarithms of numbers, we find, at the intersection of the line headed 259 and the column headed 7, the given mantissa 41447.

$\therefore \log 2.597 = .41447$. The characteristic 3 shows there are four figures to the left of the decimal point. Hence the number required is 2597.

Example 2. The number whose logarithm is 5.41447 is 259700.

The number whose logarithm is $\bar{2}.41447$ is .02597.

It may happen that the mantissa of the given logarithm does not occur in the table. We then proceed as follows:

To find the number whose logarithm is .43563.

The mantissæ next lower and next higher than the given mantissa are .43553 and .43569 respectively.

$$\log 2.727 = .43569.$$
$$\log 2.726 = .43553.$$
$$.43569 - .43553 = .00016.$$
$$.43562 - .43553 = .00009.$$

Using the proportion of Art. 132,

$$.001 : .00016 = n - 2.726 : .00009,$$

where n is the desired number:

$$\therefore n = 2.72656.$$

Or thus:

Since the mantissa of the number is .00009 greater than the mantissa of log 2.726, and the mantissa of log 2.727 is .00016 greater, the number must be $\frac{9}{16}$ of .001 greater than 2.726.

The number obtained is not accurate to more than five significant figures. If the characteristic indicates that more than five figures are to the left of the decimal point, the remaining places are filled with ciphers. Thus in the problem solved we can rely only on 27265. The 6 is questionable.

EXAMPLES. XXXI.

1. Find the number whose logarithm is .56867.
2. Find the number whose logarithm is 4.66029.
3. Find the number whose logarithm is 6.39669.
4. Find the number whose logarithm is $\bar{4}.64311$.
5. Find the number whose logarithm is .75504.

EXAMPLES. XXXII.

Find the values of the following correct to four significant figures:

1. $\sqrt[3]{451}$.
2. $\sqrt[5]{802}$.
3. $(273)^{\frac{4}{9}} \times (234)^{\frac{1}{4}}$.
4. $(451)^{\frac{3}{5}} \times (231)^{\frac{4}{3}}$.
5. $\left(\frac{192.5}{84}\right)^3$.
6. $\frac{(34.79)^{\frac{2}{3}}}{(41.25)^{\frac{3}{2}}}$.
7. $\frac{(24.76)^{\frac{2}{7}}}{(.0045)^{\frac{3}{2}}}$.
8. $\frac{7.89}{.0345} \times (89130)^{\frac{1}{7}}$.
9. $\frac{\frac{3}{2}\sqrt{(5.2)}}{5\sqrt{(11.31)}} \times \left(\frac{3}{7}\right)^{-\frac{1}{2}}$.
10. $\sqrt[5]{\left\{\frac{2\sqrt{(34)}}{3\sqrt{(791)}}\right\}}$.
11. $\frac{\sqrt[4]{3}}{\sqrt[6]{3}}$.
12. $\left(\frac{21^3 \times 45^5}{2^7 \times 3^9}\right)^{\frac{1}{2}}$.

Solve the equations correct to four figures:

13. $10^x = 421$.
14. $\left(\frac{21}{20}\right)^x = 3$.
15. $\left(\frac{203}{200}\right)^{2x} = 2$.

133. Since the trigonometric ratios are numbers we can find their logarithms. Since the sine of an angle is never greater than 1, the characteristic of its logarithm is negative (except for $\sin\theta = 1$).

To avoid the use of negative characteristics it is usual to add ten to the actual logarithm of $\sin\theta$ and call the result $\log\sin\theta$, *e.g.* the actual logarithm of $\sin 7°$ is $\bar{1}.08589$, but as explained above it is written $\log\sin 7° = 9.08589$. Similar remarks apply to the logarithm of the cosine of an angle, to the logarithm of tangents of angles from 0° to 45°, and to the logarithm of cotangents of angles from 45° to 90°.

Table II. contains the logarithmic sine, tangent, cotangent, and cosine for every ten seconds from 0° to 2°, and for every minute from 1° to 89°. From these we can find the logarithms of the trigonometric ratios of any angle, because (Art. 58) the ratios of any angle can be expressed in terms of the ratios of angles of the first quadrant.

134. *To find $\log\sin\theta$, having given θ.* We shall illustrate the method of procedure by some examples.

EXAMPLE 1. To find $\log\sin 15° 25'$, see tables, page 41, at the intersection of the line headed 25, in left margin, and the column headed $\log\sin\theta$ under 15°, we find 9.42416, *i.e.* $\log\sin 15° 25' = 9.42461$.

EXAMPLE 2. To find $\log\sin 74° 20' 40''$.

$\log\sin 74° 20' 40''$ cannot be found directly in the tables. Hence we must *interpolate*.

We assume the theorem, *A very small change in an angle is proportional to the corresponding change in its sine.*[1]

[1] We have added a proof of this theorem in Arts. 137–139.

From the tables, at the intersection of the line headed 20 in the *right* margin and column headed log sin θ *above* 74°, we find 9.98356, *i.e.*

$$\log \sin 74° 20' = 9.98356$$
$$\text{Similarly } \log \sin 74° 21' = 9.98359$$
$$3$$

Hence the difference of 1′ (*i.e.* 60″) in the angle corresponds to a difference .00003 in the log sin of its angle. Hence by theorem quoted above, to a difference of 40″ in the angle corresponds a difference of $\frac{40}{60}$ of .00003 = .00002.

$$\therefore \log \sin 74° 20' 40'' = 9.98356 + .00002$$
$$= 9.98358.$$

135. To find θ, having given $\log \sin \theta$. This is the converse of Art. 134. The reasoning is not essentially different from that of Art. 131.

136. The sine, cosine, etc., are sometimes called the natural sine, natural cosine, etc. Table III., *a, b, c, d*, contains the natural trigonometric functions from 0° to 90° at intervals of 6′.

* **137.** To prove $\sin \theta < \theta \tan \theta$. Let $P'XP$ be a circle with centre O, and radius OP. Draw the tangents PI and $P'I$. Connect PP'.

Let θ be the *circular* measure of XOP. Then 2θ is the circular measure of $P'OP$. Now

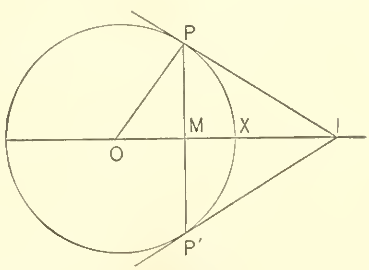


FIG. 52.

$$PP' < \text{arc } PP' < (PI + IP'). \qquad \text{(Geom.)}$$

$$\therefore PM < \text{arc } XP < PI.$$

$$\frac{PM}{OP} < \frac{\text{arc } XP}{OP} < \frac{PI}{OP}. \qquad (1)$$

But $\qquad \text{arc } XP = OP \cdot \theta.$

Substituting in (1), $\quad \sin \theta < \theta < \tan \theta. \qquad (2)$ Q.E.D.

*138. *To prove when θ is very small.*

$$\frac{\sin\theta}{\theta} = 1, \text{ approximately;}$$

$$\frac{\tan\theta}{\theta} = 1, \text{ approximately.}$$

Divide (2) of the last article by $\sin\theta$.

$$1 < \frac{\theta}{\sin\theta} < \cos\theta.$$

By Art. 39, when θ is very small, $\cos\theta$ approaches 1.

But $\frac{\theta}{\sin\theta}$ lies between 1 and $\cos\theta$.

Hence, approximately, $\frac{\theta}{\sin\theta} = 1$ when θ is *very small.*

$$\frac{\theta}{\tan\theta} = \frac{\theta}{\sin\theta}\cos\theta. \quad \text{Hence, as before,}$$

$$\frac{\theta}{\tan\theta} = 1 \text{ when } \theta \text{ is } \textit{very small.}$$

It must not be forgotten that θ is given in circular measure.

*139. $$\frac{\sin(\theta + x) - \sin\theta}{\sin(\theta + y) - \sin\theta} = \frac{\sin\theta\cos x + \cos\theta\sin x - \sin\theta}{\sin\theta\cos y + \cos\theta\sin y - \sin\theta}.$$

If now x is very small, and y is very small, we may write $\cos x = 1$, $\sin x = x$; and if y is very small, $\cos y = 1$, $\sin y = y$.

$\therefore$ when x and y are very small, $\frac{\sin(\theta + x) - \sin\theta}{\sin(\theta + y) - \sin\theta} = \frac{x}{y}$. (1)

Or, a very small change in an angle is proportional to the corresponding change in the sine of that angle.

In a similar way this law of proportional change may be established for each of the natural functions.

CHAPTER XV

ON THE SOLUTION OF TRIANGLES

140. The problem known as the **solution of triangles** may be stated thus: *When a sufficient number of the parts of a triangle are given, to find the magnitude of each of the other parts.*

141. Solution of the right triangle (see Ch. VI.).

Let ACB be a right-angled triangle with the right angle at C.

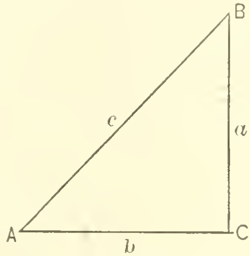


FIG. 53.

We have

$$a = c \sin A = c \cos B;$$

$$b = c \cos A = c \sin B;$$

$$a = b \tan A = b \cot B.$$

The signs of the ratios are all positive.

$$\therefore \log a = \log c + \log \sin A - 10; \qquad \text{(Art. 132.)}$$

$$\log b = \log c + \log \cos A - 10; \qquad \text{(Art. 132.)}$$

$$\log a = \log b + \log \tan A.[1] \qquad \text{(Art. 132.)}$$

EXAMPLES. XXXIII.

1. Given $a = 12562$, $A = 12°$. Find B, b, and c.
2. Given $c = 35$, $B = 37° 10' 5''$. Solve the triangle.
3. Given $b = 100$, $B = 42°$. Find c and a.

[1] See footnote, page 94.

142. The student has proved, while studying Geometry, that a triangle is uniquely determined when there are given:

I. Three sides.

II. One side and two angles.

III. Two sides and the included angle. And

IV. That either one, two, or no triangles are determined when two sides and the angle opposite one of them is given.

When, therefore, three parts (one of which is a side) are given, the other parts can be calculated. There are four cases.

Case I.

143. Given **three sides,** a, b, c. (Art. 141, I.)

We find two of the angles from the formulæ

$$\tan\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}};$$

$$\tan\frac{B}{2}=\sqrt{\frac{(s-c)(s-a)}{s(s-b)}}.$$

The third angle $C=180^\circ-A-B$.

144. In practical work we proceed as follows:

$$\log\tan\frac{A}{2}=\log\sqrt{\frac{(s-b)(s-c)}{s(s-a)}};$$

or,

$$\log\tan\frac{A}{2}=\tfrac{1}{2}\{\log(s-b)+\log(s-c)-\log s-\log(s-a)\}.$$

Similarly,

$$\log\tan\frac{B}{2}=\tfrac{1}{2}\{\log(s-c)+\log(s-a)-\log s-\log(s-b)\}.$$

145. Either of the formulæ

$$\sin\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{bc}},\ \cos\frac{A}{2}=\sqrt{\frac{s(s-a)}{bc}}$$

may also be used as above.

The $\sin\frac{A}{2}$ and the $\cos\frac{A}{2}$ formulæ are either of them as convenient as the $\tan\frac{A}{2}$ formula, when *one* of the angles only is to be found. If *all* the angles are to be found, the tangent formulæ are convenient, because we can find the log tangents of two half angles from the same *four* logs, viz. $\log s$, $\log(s-a)$, $\log(s-b)$, $\log(s-c)$. To find the log sines of *two* half angles we require the *six* logarithms, viz. $\log(s-a)$, $\log(s-b)$, $\log(s-c)$, $\log a$, $\log b$, $\log c$.

EXAMPLE. Given $a = 275.35$, $b = 189.28$, $c = 301.47$ chains; find A and B.

Here, $s = 383.05$, $s - a = 107.70$, $s - b = 193.77$, $s - c = 81.58$.

Then $\log \tan \frac{A}{2} = \frac{1}{2}\{\log 193.77 + \log 81.58 - \log 383.05 - \log 107.70\}$

$= \frac{1}{2}\{2.28728 + 1.91158 - 2.58325 - 2.03221\}$

$= 9.79169$[1] (Art. 132), [From the tables.]

whence $\frac{A}{2} = 31° 45' 28.5''$; $\therefore A = 63° 30' 57''$. Again,

$\log \tan \frac{B}{2} = \frac{1}{2}\{\log 81.58 + \log 107.70 - \log 383.05 - \log 193.77\}$

$= 9.53663 = \log \tan 18° 59' 9.8''$;

$\therefore B = 37° 58' 20''$; $C = 180° - A - B = 78° 30' 43''$.

146. This case may also be solved by the formula

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}.$$

But this formula is not adapted for logarithmic calculation, and therefore is seldom used in practice.

It may sometimes be used with advantage, when the given lengths of a, b, c are small.

EXAMPLE. Find the greatest angle of the triangle whose sides are 13, 14, 15.

Let $a = 15$, $b = 14$, $c = 13$. Then the greatest angle is A.

Now, $\cos A = \frac{14^2 + 13^2 - 15^2}{2 \times 14 \times 13} = \frac{140}{2 \times 14 \times 13} = \frac{5}{13} = .384615$

$= \cos 67° 23'$, nearly. [By the table of natural cosines.]

$\therefore$ the greatest angle $= 67° 23'$.

EXAMPLES. XXXIV.

1. If $a = 352.25$, $b = 513.27$, $c = 482.68$ yards, find the angles A and B.

2. Find the two largest angles of the triangle whose sides are 484, 376, 522 feet.

3. Given $a = .641$, $b = .529$, $c = .702$; find two largest angles of the triangle.

4. Given $a = 2$, $b = 1.64$, $c = 2.075$; find B.

5. If $a = 3811$, $b = 3850$, $c = 4090$ yards, find C.

6. The sides are 2, $\sqrt{2}$, and $\sqrt{3} - 1$; find the angles.

[1] The result here is really 1.79169, but we have added ten to it in order to make this number agree with the one given in the table (see Art. 132). In the example, Art. 148, each of the values of $\log \sin A$ and $\log \sin B$ are ten too large; since one is negative and the other positive the result is not affected. Little difficulty will be experienced, if careful attention is given to Art. 132.

Case II.

147. Given **one side** and **two angles**, as a, B, C.

First, $A = 180° - B - C$; which determines A.

Next, $$\frac{b}{\sin B} = \frac{a}{\sin A}, \quad \text{or, } b = \frac{a \cdot \sin B}{\sin A};$$

and, $$\frac{c}{\sin C} = \frac{a}{\sin A}, \quad \text{or, } c = \frac{a \cdot \sin C}{\sin A}.$$

These determine b and c.

148. In practical work we proceed as follows:

Since $$b = \frac{a \cdot \sin B}{\sin A},$$

$$\therefore \log b = \log \frac{a \cdot \sin B}{\sin A},$$

$$\therefore \log b = \log a + \log \sin B - \log \sin A.$$

Similarly, $\log c = \log a + \log \sin C - \log \sin A.$

EXAMPLE. Given that $c = 1764.3$ feet, $C = 18° 27'$, and $B = 66° 39'$; find b.

From the tables we find $\log 1764.3 = 3.24657$.

$$\log \sin 18° 27' = 9.50034, \ \log \sin 66° 39' = 9.96289;$$

$$\therefore \log b = 3.24657 + 9.96289 - 9.50034$$

$$= 3.70912 = \log 5118.2;$$

$$\therefore b = 5118.2 \text{ feet.}$$

EXAMPLES. XXXV.

1. $A = 53° 24'$, $B = 66° 27'$, $c = 338.65$ yards. Find a and C.
2. Find c, having given $a = 1000$, $A = 50°$, $C = 66°$.
3. Find b, having given $B = 32° 15'$, $C = 21° 47' 20''$, $a = 34$ feet.
4. Given $c = .0161$, $A = 35° 15'$, $C = 123° 39'$; solve the triangle.

Case III.

149. Given **two sides** and the **included angle**, as b, c, A.

First, $B + C = 180° - A$. Thus $(B + C)$ is determined.

Next, $$\tan \frac{B - C}{2} = \frac{b - c}{b + c} \cot \frac{A}{2}.$$

Thus $(B - C)$ is determined.

And B and C can be found when the values of $(B+C)$ and $(B-C)$ are known.

Lastly, $$\frac{a}{\sin A}=\frac{b}{\sin B}, \text{ or } a=\frac{b\cdot\sin A}{\sin B}.$$

Whence a is determined.

150. In practical work we proceed as follows:

Since $$\tan\frac{B-C}{2}=\frac{b-c}{b+c}\cot\frac{A}{2},$$

$$\therefore\ \log\left(\tan\frac{B-C}{2}\right)=\log(b-c)-\log(b+c)+\log\left(\cot\frac{A}{2}\right).$$

Also, since $$a=\frac{b\cdot\sin A}{\sin B},$$

$$\therefore\ \log a=\log b+\log\sin A-\log\sin B, \text{ as in Case II.}$$

EXAMPLE. Given $b=456.12$ chains, $c=296.86$ chains, and $A=74°\,20'$; find the other angles.

Here, $$b-c=159.26,\ b+c=752.98.$$

From the table we find

$$\log 159.26=2.2021, \text{ and } \log 752.98=2.87678,$$
$$\log\cot 37°\,10'=.1202\,;$$ [1]

$$\therefore\ \log\tan\frac{B-C}{2}=2.20210-2.87678+.12026=9.44558 = \log\tan 15°\,35'\,18'';$$ [1]

$$\therefore\ B-C=31°\,10'\,36'', \text{ and } B+C=180°-74°\,20'.$$

Thus $$B+C=105°\,40'\,;$$

$$\therefore\ 2B=136°\,50'\,36''\,;\ 2C=74°\,29'\,24'',$$

or $$B=68°\,25'\,18''\,;\ \text{or, } C=37°\,14'\,42''.$$

151. The formula $a^2=b^2+c^2-2\,bc\cos A$ may be used in simple cases.

EXAMPLE. If $b=35$ feet, $c=21$ feet, and $A=50°$, find a, given that $\cos 50°=.643$.

Here $$a^2=35^2+21^2-2\times 35\times 21\times\cos 50°\,;$$

$$\therefore\ \frac{a^2}{7^2}=5^2+3^2-2\times 5\times 3\times\cos 50°,\ =25+9-30\times .643,\ =14.71.$$

$$\frac{a}{7}=3.82 \text{ nearly}\,;\ \text{or, } a=26.74=\text{about } 26\tfrac{3}{4} \text{ feet.}$$

[1] See footnote, page 94.

EXAMPLES. XXXVI.

1. $b = 19$ feet, $c = 20$ feet, $A = 60°$; find B and C.

2. $a = 376.375$ feet, $b = 251.765$ feet, $C = 78° 26'$; find B and A.

3. $a = .3$, $b = .363$, $C = 124° 56'$; solve the triangle.

4. $a = 135$, $b = 105$, $C = 60°$; find A.

5. $b = 8$, $c = 11$, $A = 93° 35'$; find a.

6. In a certain triangle two of the sides are 12 and 16 respectively; the angle included by them is 160.5°; find the other angles.

Case IV.

152. Given **two sides** and the **angle opposite** one of them, as b, c, B.

First, since $\frac{c}{\sin C} = \frac{b}{\sin B}$; $\therefore \sin C = \frac{c \sin B}{b}$.

C must be found from this equation.

When C is known, $A = 180° - B - C$,

and $$a = \frac{b \sin A}{\sin B}.$$

Which solves the triangle.

153. It should be noticed, however, that the angle C, found from the **trigonometrical** equation *sin* $C =$ *a given quantity*, where C is an angle of a triangle, has **two** values, one less than 90°, and one greater than 90° (Art. 103).

The question arises, Are both these values admissible?

This may be decided as follows:

If B is not less than 90°, C must be less than 90°; and the smaller value for C **only** is admissible.

If B is less than 90° we proceed thus:

1. If b is less than $c \sin B$, then $\sin C$, which $= \frac{c \sin B}{b}$, is greater than 1. This is impossible. Therefore if b is less than $c \sin B$, there is **no solution** whatever.

2. If b is equal to $c \sin B$, then $\sin C = 1$, and therefore $C = 90°$; and there is only **one value** of C, viz. 90°.

3. If b is greater than $c \sin B$, and less than c, then B is less than C, and C may be obtuse or acute. In this case C may have either of the values found from the equation $\sin C = \frac{c \sin B}{b}$. Hence there are **two solutions**, and the triangle is said to be **ambiguous**.

4. If b is equal to or greater than c, then B is equal to or greater than C, so that C must be an acute angle; and the smaller value for C only is admissible.

154. The same results may be obtained **geometrically**.

Construction. Draw $AB = c$; make the angle $ABD =$ the given angle B; with centre A and radius $= b$ describe a circle; draw AD perpendicular to BD.

Then $AD = c \sin B$.

1. If b is less than $c \sin B$, *i.e.* less than AD, the circle will not cut BD at all, and the construction **fails**. (Fig. I.)

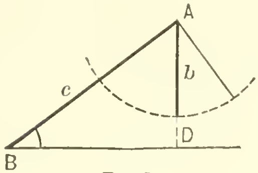


FIG. I.

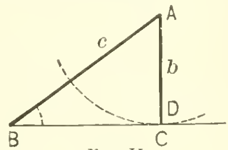


FIG. II.

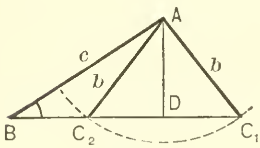


FIG. III.

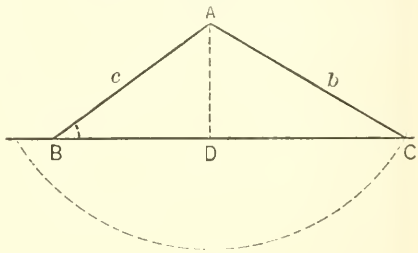


FIG. IV.

2. If b is equal to AD, the circle will *touch* the line BD in the point D, and the required triangle is the **right-angled** triangle ABD. (Fig. II.)

3. If b is greater than AD and less than AB, *i.e.* than c, the circle will cut the line BD in *two* points C_1, C_2, each on the same side of B. And we get **two triangles** ABC_1, ABC_2, each satisfying the given condition. (Fig. III.)

4. If b is equal to c, the circle cuts BD in B and in one other point C; if b is greater than c, the circle cuts BD in *two* points, but on *opposite* sides of B. In either case there is only **one** triangle satisfying the given condition. (Fig. IV.)

155. We may also obtain the same results **algebraically** from the formula $b^2 = c^2 + a^2 - 2\,ca \cos B$.

In this b, c, B are given, a is unknown. Write x for a, and we get the **quadratic equation**,

$$x^2 - 2\,c \cos B \cdot x = b^2 - c^2.$$

Whence, $x^2 - 2c\cos B \cdot x + c^2\cos^2 B = b^2 - c^2 + c^2\cos^2 B$

$$= b^2 - c^2\sin^2 B;$$

$$\therefore\ x = c\cos B \pm \sqrt{b^2 - c^2\sin^2 B}.$$

Let a_1, a_2 be the two values of x thus obtained; then

$$\left.\begin{aligned} a_1 &= c\cos B + \sqrt{b^2 - c^2\sin^2 B}; \\ a_2 &= c\cos B - \sqrt{b^2 - c^2\sin^2 B}. \end{aligned}\right\}$$

Which of these two solutions is admissible may be decided as follows:

1. When b is less than $c\sin B$, then $(b^2 - c^2\sin^2 B)$ is negative, so that a_1, a_2 are **impossible** quantities.

2. When b is equal to $c\sin B$, then $(b^2 - c^2\sin^2 B) = 0$, and $a_1 = a_2$; thus the *two* solutions become **one**.

3. When b is greater than $c\sin B$, then the **two** values a_1, a_2 are different and positive unless

$$\sqrt{b^2 - c^2\sin^2 B} \text{ is } > c\cos B;$$

i.e. unless $\quad b^2 - c^2\sin^2 B > c^2\cos^2 B,$

i.e. unless $\quad b^2 > c^2.$

4. When b is equal to c, then $a_2 = 0$; if b is greater than c, then a_2 is negative, and is therefore inadmissible. In either of these cases a_1 is the **only** available solution.

156. We give two examples. In the first there are two solutions; in the second there is only one.

Example 1. Find A and C, having given that $b = 379.41$ chains, $c = 483.74$ chains, and $B = 34° 11'$.

$$\begin{aligned} \log\sin C &= \log c + \log\sin B - \log b \\ &= 2.68461 + 9.74961 - 2.57910 \\ &= 9.85511 = L\sin 45° 45'; \end{aligned}$$

$$\therefore\ C = 45° 45', \text{ or } 180° - 45° 45' = 134° 15'.$$

Since b is less than c, each of these values is admissible.

When $C = 45° 45'$, then $A = 100° 4'$.

When $C = 134° 15'$, then $A = 11° 34'$.

Example 2. Find A and C, when $b = 483.74$ chains, $c = 379.14$ chains, and $B = 34° 11'$.

$$\begin{aligned} \log\sin C &= \log c + \log\sin B - \log b \\ &= 2.5791 + 9.7496148 - 2.68461 \\ &= 9.64411 = \log\sin 26° 9'; \end{aligned}$$

$$\therefore\ C = 26° 9', \text{ or } 180° - 26° 9' = 153° 51'.$$

Since b is greater than c, C must be less than 90°, and the larger value for C is inadmissible.

[It is also clear that $(153° 51' + 34° 11')$ is $> 180°$.]

$$\therefore\ C = 26° 9',\ A = 119° 40'.$$

EXAMPLES. XXXVII.

1. Discuss the following problems, using the method of Art. 153.

(a) $B = 45°$; $c = 12$; $b = \sqrt{50}$. (b) $B = 45°$; $c = 10$; $b = \sqrt{50}$.

(c) $B = 45°$; $c = \sqrt{40}$; $b = \sqrt{50}$. (d) $B = 45°$; $c = \sqrt{75}$; $b = \sqrt{50}$.

2. If $B = 40°$, $b = 140.5$ ft., $a = 170.6$ ft. Find A and C.

3. Find B, C, and c, having given that $A = 50°$, $b = 97$, $a = 119$. (See Ex. 2.)

4. If $C = 30°$, $b = 100$, $c = 45$, is the triangle ambiguous?

157. It is sometimes easier to use natural sines, cosines, and tangents (Art. 136), than to use processes of computation involving logarithms. The theory of the solution of triangles is the same in either case, and it is only a question as to which process involves the greater amount of labor, to perform the indicated arithmetical processes, or to use logarithms.

Example. $a = 50$, $A = 78°$, $B = 27°$; find b.

We have
$$b = a\frac{\sin B}{\sin A} = 50 \times \frac{\sin 27°}{\sin 78°} = \frac{50 \times 4540}{9781}.$$

$$\therefore\ b = 23.1.$$

MISCELLANEOUS EXAMPLES. XXXVIII.

1. Find A when $a = 374.5$, $b = 576.2$, $c = 759.3$ feet.
2. Find B when $a = 4001$, $b = 9760$, $c = 7942$ yards.
3. Find C when $a = 8761.2$, $b = 7643$, $c = 4693.8$ chains.
4. Find B when $A = 86° 19'$, $b = 4930$, $c = 5471$ chains.
5. Find C when $B = 32° 58'$, $c = 1873.5$, $a = 764.2$ chains.
6. Find c when $C = 108° 27'$, $a = 36541$, $b = 89170$ feet.
7. Find c when $B = 74° 10'$, $C = 62° 45'$, $b = 3720$ yards.
8. Find b when $B = 100° 19'$, $C = 41° 59'$, $a = 1000$ chains.
9. Find a when $B = 123° 7' 20''$, $C = 15° 9'$, $c = 9964$ yards.

Find the other two angles in the six following triangles:

10. $C = 100° 37'$, $b = 1450$, $c = 6374$ chains.

11. $C = 52° 10'$, $b = 643$, $c = 872$ chains.

12. $A = 76° 2' 30''$, $b = 1000$, $a = 2000$ chains.

13. $C = 54° 23'$, $b = 873.4$, $c = 752.8$ feet.

14. $C = 18° 21'$, $b = 674.5$, $c = 269.7$ chains.

15. $A = 29° 11' 43''$, $b = 7934$, $a = 4379$ feet.

16. The difference between the angles at the base of a triangle is $17° 48'$, and the sides subtending those angles are 105.25 feet and 76.75 feet; find the third angle.

17. If $b : c = 4 : 5$, $a = 1000$ yards and $A = 37° 19'$; find b.

The student will find some Examples of Solution of Triangles without the aid of logarithms, in Examples XLII.

158. *To find the* Area *of a Triangle.*

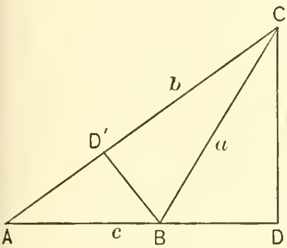


Fig. 54.

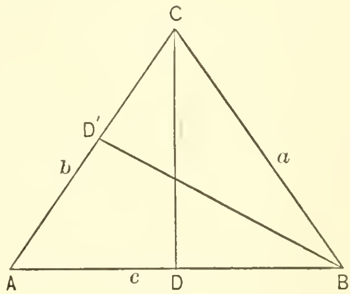


Fig. 55.

In either figure

Let Δ = area of triangle ABC.

Let CD be perpendicular to AB.

Let length of $CD = p$.

$$\Delta = \tfrac{1}{2}cp; \qquad \text{(Geom.)}$$

$$\therefore \ \Delta = \tfrac{1}{2}a \sin B \cdot c = \tfrac{1}{2}ac \sin B;$$

$$\text{or} \ \ \Delta = \tfrac{1}{2}b \sin A \cdot c = \tfrac{1}{2}bc \sin A.$$

Similarly, if BD' is perpendicular to AC,

$$\Delta = \tfrac{1}{2}ab \sin C.$$

In words:

The area of a triangle equals the continued product of any two sides and the sine of the angle included by them.

159. By i., Art. 115, $\sin A = \dfrac{2}{bc} \times \sqrt{s(s-a)(s-b)(s-c)}$;

$$\therefore \ \Delta = \sqrt{s(s-a)(s-b)(s-c)} = S. \qquad \text{(Art. 113.)}$$

160. Data, sufficient for the solution of a triangle, may be given in other terms than that of sides and angles, as in the four cases considered.

e.g. The triangle is determined when there are given area and two angles.

For, suppose in triangle ABC, we are given Δ, A, B.

$$ac \sin B = 2\,\Delta. \quad \therefore\ ac = \frac{2\,\Delta}{\sin B}.$$

$$a = \frac{c \sin A}{\sin C}. \quad \therefore\ c^2 = \frac{2\,\Delta \sin C}{\sin A \sin B}. \qquad \text{Q.E.D.}$$

See the following list of exercises for similar examples:

EXAMPLES. XXXIX.

1. Show that the triangle is determined when there are given: area, one angle, and side opposite the angle.

2. Find the area of the triangle ABC when

(i.) $a = 4$ feet, $b = 10$ feet, $C = 30°$.

(ii.) $b = 5$ inches, $c = 20$ inches, $A = 60°$.

(iii.) $c = 66\frac{2}{3}$ yards, $a = 15$ yards, $B = 17° 14'$ [$\sin 17° 14' = .29626$].

(iv.) $a = 13$, $b = 14$, $c = 15$.

(v.) $a = 10$ feet, the perpendicular from A on $BC = 20$ feet.

(vi.) $a = 625$, $b = 505$, $c = 904$.

CHAPTER XVI

ON THE MEASUREMENT OF HEIGHTS AND DISTANCES

161. By the aid of the solution of triangles

we can find the distance between points which are inaccessible;

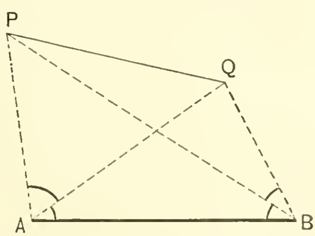
we can calculate the magnitude of angles which cannot be practically observed;

we can find the relative heights of distant and inaccessible points.

The method on which the trigonometrical survey of a country is conducted affords the following illustration:

162. *To find the distance between two distant objects.*

Two convenient positions A and B, on a level plain as far apart as possible, having been selected, the distance between A and B is



measured with the greatest possible care. This line AB is called the base line.

Next, the two distant objects, P and Q (church spires, for instance), visible from A and B, are chosen.

The angles PAB, PBA are observed. Then by Case II. Ch. XV., the lengths of the lines PA, PB are calculated.

Again, the angles QAB, QBA are observed; and by Case II. the lengths of QA and QB are calculated.

Thus the lengths of PA and QA are found.

The angle PAQ is observed; and then by Case III. the length of PQ is calculated.

Thus we are able to find not only the length PQ, but the angle which PQ makes with any line in the figure. The points P and Q are not necessarily accessible, the only condition being that P and Q must be *visible* from both A and B.

163. In practice, the points P and Q will generally be accessible, and then the line PQ, whose length has been calculated, may be used as a new base to find other distances.

164. *To find the height of a distant object above the point of observation.*

Let B be the point of observation; P the distant object. From B measure a base line BA of any convenient length, in any convenient direction; observe the angles PAB, PBA, and by Case II. Ch. XV.

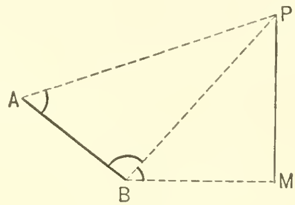


Fig. 56.

calculate the length of BP. Next observe at B the 'angle of elevation' of P; that is, the angle which the line BP makes with the horizontal line BM, M being the point in which the vertical line through P cuts the horizontal plane through B.

Then PM, which is the vertical height of P above B, can be calculated, for $PM = BP \cdot \sin MBP$.

Example 1. *The distance between a church spire A and a milestone B is known to be 1764.3 feet; C is a distant spire. The angle CAB is $94° 54'$, and the angle CBA is $66° 39'$; find the distance of C from A.*

ABC is a triangle, and we know one side, c, and two angles (A and B), and therefore it can be solved by Case II. Ch. XV.

The angle $\quad ACB = 180° - 94° 54' - 66° 39' = 18° 27'$.

Therefore the triangle is the same as that solved in Art. 148. Therefore $AC = 5118.2$ feet.

Example 2. *If the spire C, in the last example, stands on a hill, and the angle of elevation of its highest point is observed at A to be $4° 19'$; find how much higher C is than A.*

The required height $x = AC \cdot \sin 4° 19'$ and AC is 5118.2 feet;

$$\therefore \log x = \log (AC \cdot \sin 4° 19')$$
$$= \log 5118.2 + \log \sin 4° 19' - 10$$
$$= 3.70911 + 8.87661$$
$$= 2.58573 = \log 385.24.$$
$$\therefore x = 385 \text{ ft. } 3 \text{ in. nearly.}$$

EXAMPLES. XL.

(Examples VII. consist of Easy Examples on this Subject.)

1. Two straight roads, inclined to one another at an angle of 60°, lead from a town A to two villages B and C; B on one road distant 30 miles from A, and C on the other road distant 15 miles from A. Find the distance from B to C.

Ans. 25.98 m.

2. Two ships leave harbor together, one sailing N.E. at the rate of $7\frac{1}{2}$ miles an hour, and the other sailing north at the rate of 10 miles an hour. Prove that the distance between the ships after an hour and a half is 10.6 miles.

3. A and B are two consecutive milestones on a straight road, and C is a distant spire. The angles ABC and BAC are observed to be 120° and 45° respectively. Show that the distance of the spire from A is 3.346 miles.

4. If the spire C in the last question stands on a hill, and its angle of elevation at A is 15°, show that it is .896 of a mile higher than A.

5. If in Question (3) there is another spire D such that the angles DBA and DAB are 45° and 90° respectively and the angle DAC is 45°, prove that the distance from C to D is $2\frac{3}{4}$ miles very nearly.

6. A and B are two consecutive milestones on a straight road, and C is the chimney of a house visible from both A and B. The angles CAB and CBA are observed to be 36° 18′ and 120° 27′, respectively. Show that C is 2639.5 yards from B.

7. A and B are two points on opposite sides of a mountain, and C is a place visible from both A and B. It is ascertained that C is distant 1794 feet and 3140 feet from A and B, respectively, and the angle ACB is 58° 17′. Show that the angle which the line pointing from A to B makes with AC is 86° 55′ 49″.

8. A and B are two hill-tops 34920 feet apart, and C is the top of a distant hill. The angles CAB and CBA are observed to be 61° 53′ and 76° 49′, respectively. Prove that the distance from A to C is 51515 feet.

$\log 34920 = 4.54307$; $\log \sin 76° 49' = 9.98840$;
$\log 51515 = 4.71193$; $\log \operatorname{cosec} 41° 18' = 10.18045$.

9. From two stations A and B on shore, 3742 yards apart, a ship C is observed at sea. The angles BAC, ABC are simultaneously observed to be 72° 34′ and 81° 41′, respectively. Prove that the distance from A to the ship is 8522.7 yards.

$\log 3742 = 3.57310$; $\log \sin 81° 41' = 9.99540$;
$\log 8522.7 = 3.90057$; $\log \operatorname{cosec} 25° 45' = 10.36206$.

10. The distance between two mountain peaks is known to be 4970 yards, and the angle of elevation of one of them when seen from the other is $9° 14'$. How much higher is the first than the second? $\sin 9° 14' = .16045$.

Ans. 797.5 yards.

11. Two straight railways intersect at an angle of $60°$. From their point of intersection two trains start, one on each line, one at the rate of 40 miles an hour. Find the rate of the second train that at the end of an hour they may be 35 miles apart. *Ans.* Either 25 or 15 miles an hour. (Art. 153.)

12. A and B are two positions on opposite sides of a mountain; C is a point visible from A and B; AC and BC are 10 miles and 8 miles, respectively, and the angle BCA is $60°$. Prove that the distance between A and B is 9.165 miles.

13. A and B are consecutive milestones on a straight road; C is the top of a distant mountain. At A the angle CAB is observed to be $38° 19'$; at B the angle CBA is observed to be $132° 42'$, and the angle of elevation of C at B is $10° 15'$; show that the top of the mountain is 1243.5 yards higher than B.

$$\log \sin 38° 19' = 9.79239; \qquad \log 1760 = 3.24551;$$
$$\log \operatorname{cosec} 8° 59' = 10.80646; \qquad \log 1243.5 = 3.09465;$$
$$\log \sin 10° 15' = 9.25028.$$

14. A base line AB, 1000 feet long, is measured along the straight bank of a river; C is an object on the opposite bank; the angles BAC and CBA are observed to be $65° 37'$ and $53° 4'$ respectively; prove that the perpendicular breadth of the river at C is 829.87 feet.

15. A is the foot of a vertical pole, B and C are due east of A, and D is due south of C. The elevation of the pole at B is double that at C, and the angle subtended by AB at D is $\tan^{-1}\frac{1}{3}$. Also $BC = 20$ feet, $CD = 30$; find the height of the pole. — *Hobson's Trig.*

16. Two towers, one 200 feet high, the other 150 feet high, standing on a horizontal plane, subtend, at a point in the plane, angles of $30°$ and $60°$ respectively. The horizontal angle that their bases subtend at the same point is $120°$; how far are the two towers apart?

17. The diagonals of a parallelogram are in length d_1 and d_2, the angle between them is ϕ; show that the area of a parallelogram is $\frac{1}{2} d_1 d_2 \sin \phi$.

18. A man walking along a straight road at the rate of three miles an hour sees in front of him at an elevation of $60°$ a balloon which is travelling horizontally in the same direction at the rate of six miles an hour; ten minutes after he observes that the elevation is $30°$; prove that the height of the balloon above the road is $440\sqrt{3}$ yards.

19. A person standing at a point A, due south of a tower built on a horizontal plain, observes the altitude of the tower to be $60°$. He then walks to a point B due west from A and observes the altitude to be $45°$, and then at the point C in AB produced he observes the altitude to be $30°$; prove that $AB = BC$.

20. The angle of elevation of a balloon, which is ascending uniformly and vertically, when it is one mile high is observed to be $35° 20'$; 20 minutes later the elevation is observed to be $55° 40'$. How fast is the balloon moving?

Ans. $3(\sin 20° 20')(\sec 55° 40')(\operatorname{cosec} 35° 20')$ miles per hour.

21. A tower stands at the foot of an inclined plane whose inclination to the horizon is 9° ; a line is measured up the incline from the foot of the tower of 100 feet in length. At the upper extremity of this line the tower subtends an angle of 54° ; find the height of the tower. *Ans.* 114.4 feet.

22. The altitude of a certain rock is observed to be 47°, and after walking 1000 feet towards the rock, up a slope inclined at an angle of 32° to the horizon the observer finds that the altitude is 77° ; prove that the vertical height of the rock above the first point of observation is 1034 feet. Sin 47° = .73135.

23. At the top of a chimney 150 feet high standing at one corner of a triangular yard, the angle subtended by the adjacent sides of the yard are 30° and 45° respectively ; while that subtended by the opposite side is 30° ; show that the lengths of the sides are 150 feet, 86.6 feet, and 106 feet respectively.

24. A flagstaff h feet stands on the top of a tower. From a point in the plane on which the tower stands, the angles of elevation of the top and bottom of the flagstaff are observed to be α and β respectively ; prove that the height of the tower is $\frac{h \sin \beta \cos \alpha}{\sin (\alpha - \beta)}$ feet.

25. The angular elevation of the top of a steeple at a place due south of it is 45°, and at another place due west of the former station and distant a feet from it the elevation is 15° ; show that the height of the steeple is $\frac{a}{2}(3^{\frac{1}{4}} - 3^{-\frac{1}{4}})$ feet.

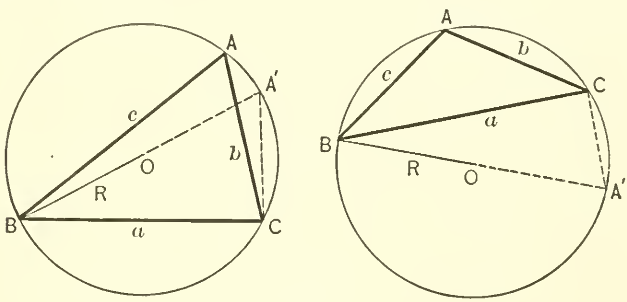
*CHAPTER XVII

MISCELLANEOUS THEOREMS

165. To find the radius of the **Circumscribing Circle.**

Let a circle $AA'CB$ be described about the triangle ABC. Let R stand for its radius. Let O be its centre. Join BO, and produce it to cut the circumference in A'. Join $A'C$.

Then, Fig. I., the angles BAC, $BA'C$ in the same segment are equal; Fig. II., the angles BAC, $BA'C$ are supplementary; also the angle $BC'A$ in a semicircle is a right angle.



I. Fig. 57. II.

Therefore, $$\frac{CB}{A'B} = \sin CA'B = \sin CAB = \sin A,$$

or, $$\frac{a}{2\,R} = \sin A; \quad \therefore\ 2\,R = \frac{a}{\sin A}.$$

166. Similarly, it may be proved that

$$2\,R = \frac{b}{\sin B}; \text{ and that } 2\,R = \frac{c}{\sin C}.$$

Hence, $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2\,R.$$

Thus d, the value of each of these fractions, is the diameter of the circumscribing circle, which is another proof of the "law of sines," viz.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

167. To find the radius of the **Inscribed Circle.**

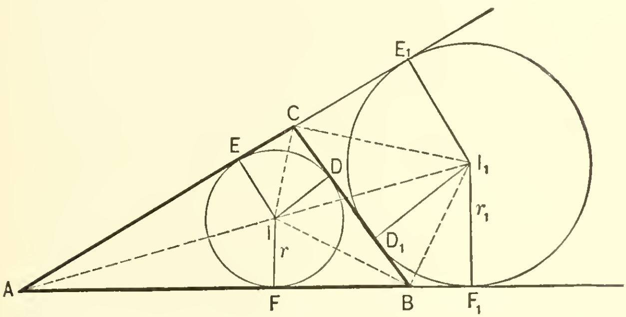


FIG. 58.

Let D, E, F be the points in which the circle inscribed in the triangle ABC touches the sides. Let I be the centre of the circle; let r be its radius. Then $ID = IE = IF = r$.

The area of the triangle ABC

$$= \text{area of } IBC + \text{area of } ICA + \text{area of } IAB.$$

And the area of the triangle $IBC = \frac{1}{2} ID \cdot BC = \frac{1}{2} r \cdot a$;

$$\therefore \text{ area of } ABC = \tfrac{1}{2} ID \cdot BC + \tfrac{1}{2} IE \cdot CA + \tfrac{1}{2} IF \cdot AB$$

$$= \tfrac{1}{2} ra + \tfrac{1}{2} rb + \tfrac{1}{2} rc;$$

or,

$$\Delta = \tfrac{1}{2} r (a + b + c) = \tfrac{1}{2} r \cdot 2s = rs.$$

$$\therefore r = \frac{\Delta}{s} = \frac{S}{s}.$$

168. A circle which touches one of the sides of a triangle and the other two sides produced is called an **Escribed Circle** of the triangle.

169. To find the radius of an **Escribed Circle.**

Let an escribed circle touch the side BC and the sides AC, AB produced in the points D_1, E_1, F_1 respectively. Let I_1 be its centre, r_1 its radius. Then

$$I_1D_1 = I_1E_1 = I_1F_1 = r_1.$$

The area of the triangle ABC

$$= \text{area of } ABI_1C - \text{area of } I_1BC,$$

$$= \text{area of } I_1CA + \text{area of } I_1AB - \text{area of } I_1BC;$$

or, $$\Delta = \tfrac{1}{2} I_1E_1 \cdot CA + \tfrac{1}{2} I_1F_1 \cdot AB - \tfrac{1}{2} I_1D_1 \cdot BC,$$

$$= \tfrac{1}{2} r_1 b + \tfrac{1}{2} r_1 c - \tfrac{1}{2} r_1 a$$

$$= \tfrac{1}{2} r_1(b + c - a) = \tfrac{1}{2} r_1(2s - 2a) = r_1(s - a).$$

$$\therefore\ r_1 = \frac{\Delta}{s-a} = \frac{S}{s-a}.$$

Similarly if r_2 and r_3 be the radii of the other two escribed circles of the triangle ABC, then

$$r_2 = \frac{S}{s-b};\quad r_3 = \frac{S}{s-c}.$$

170. *To calculate the lengths AE, AE_1, AF_1.*

$$AE + EC + CD + DB + BF + FA = 2s. \quad \text{(Fig. 58, Art. 167.)}$$

But $AE = AF$,

$CD = EC$,

$DB = BF$ (tangents to the same circle from a given point).

$$\therefore\ AE + CD + BD = s, \text{ or } AE + a = s.$$

$$\therefore\ AE = s - a.$$

From similar triangles,

$$\frac{AE_1}{AE} = \frac{r_1}{r}.\quad \therefore\ AE_1 = \frac{AE \cdot r_1}{r} = \frac{(s-a)\Delta}{\frac{\Delta}{s}(s-a)} = s.$$

$$\therefore\ AE_1 = s = AF_1.$$

EXAMPLES. XLI.

1. Show (i.) $CD = EC = (s - c)$; (ii.) $BF = BD = s - b$; (iii.) $DD_1 =$ numerical difference of b and c.

2. Find the radii of the inscribed and each of the escribed circles of the triangle ABC when $a = 13$, $b = 14$, $c = 15$ feet.

3. Show that the triangles in which (i.) $a = 2$, $A = 60°$; (ii.) $b = \frac{2}{3} \cdot \sqrt{3}$, $B = 30°$ can be inscribed in the same circle.

4. Prove that $R = \dfrac{abc}{4S}$; find R in the triangle of (**2**).

5. Prove that if a series of triangles of equal perimeter are described about the same circle, they are equal in area.

6. If $A = 60°$, $a = \sqrt{3}$, $b = \sqrt{2}$, prove that the area $= \frac{1}{4}(3 + \sqrt{3})$.

7. Prove that each of the following expressions represents the area of the triangle ABC:

(i.) $\dfrac{abc}{4R}$. (v.) $\frac{1}{2}a^2 \sin B \cdot \sin C \cdot \operatorname{cosec} A$.

(ii.) $2R^2 \sin A \cdot \sin B \cdot \sin C$. (vi.) $ra \operatorname{cosec}\frac{1}{2}A \cos\frac{1}{2}B \cos\frac{1}{2}C$.

(iii.) rs. (vii.) $(rr_1r_2r_3)^{\frac{1}{2}}$.

(iv.) $Rr(\sin A + \sin B + \sin C)$. (viii.) $\frac{1}{2}(a^2 - b^2)\sin A \cdot \sin B \cdot \operatorname{cosec}(A - B)$.

Prove the following statements:

8. If a, b, c are in A.P., then $ac = 6rR$.

9. The area of the greatest triangle, two of whose sides are 50 and 60 feet, is 1500 sq. feet.

10. If the altitude of an isosceles triangle is equal to the base, R is five-eighths of the base.

171. We give here a geometrical proof of the following propositions.

Prop. I. *To prove that*

$$\cos A = 2\cos^2\tfrac{1}{2}A - 1 = 1 - 2\sin^2\tfrac{1}{2}A. \quad \text{(See Art. 91.)}$$

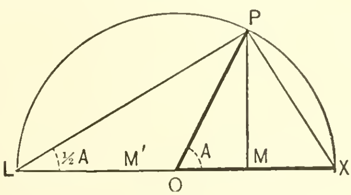


Fig. 59.

Let XOP be the angle A; with O as centre and any radius OX describe the semicircle XPL; join PL, PX, and draw PM perpendicular to LOX.

Then $$POM = OLP + OPL = 2\,OL$$

$$\therefore\ OLP = \tfrac{1}{2}POM = \tfrac{1}{2}A.$$

Now, $$\cos A = \frac{OM}{OP} = \frac{LM - LO}{OP} = \frac{2\,LM}{2\,OP} - \frac{OP}{OP}$$

$$= 2\cdot\frac{LM}{LP}\cdot\frac{LP}{LX} - 1 = 2\cos OLP \cdot \cos OLP - 1$$

$$= 2\cos^2\tfrac{1}{2}A - 1 \qquad \text{(i.)}$$

$$= 2(1 - \sin^2\tfrac{1}{2}A) - 1$$

$$= 1 - 2\sin^2\tfrac{1}{2}A. \qquad \text{(ii.)}$$

PROP. II. *To prove* $\sin A = 2 \sin \frac{1}{2} A \cdot \cos \frac{1}{2} A$,

$$\sin A = \frac{MP}{OP} = 2 \cdot \frac{MP}{LP} \cdot \frac{LP}{2\,OP} = 2 \cdot \frac{MP}{LP} \cdot \frac{LP}{LX}$$

$$= 2 \sin OLP \cdot \cos OLP = 2 \sin \tfrac{1}{2} A \cdot \cos \tfrac{1}{2} A.$$

(See Art. 91.)

PROP. III. *To prove that in any triangle*

$$\tan \frac{B - C}{2} = \frac{b - c}{b + c} \cot \frac{A}{2}.$$

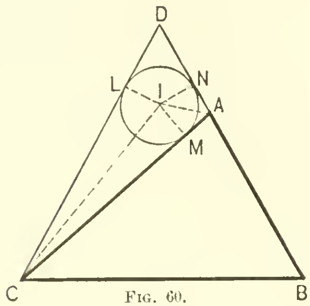


FIG. 60.

Let ABC be a triangle of which the angle B is greater than C.

Make the angle $BCD = B$ and produce BA to D.

In the triangle ACD inscribe the circle LMN, centre I, touching the sides in L, M, N; join IL, IM, IN, IA, IC.

Then $ICM = \frac{1}{2} LCM = \frac{1}{2}(DCB - ACB) = \frac{1}{2}(B - C)$,

$$IAM = \tfrac{1}{2} DAC = \tfrac{1}{2}(180° - CAB) = (90° - \tfrac{1}{2} A),$$

$$CM = CL = CD - LD = BD - ND = BN = BA + AM;$$

$$\therefore\ CM = \tfrac{1}{2}(CM + BA + AM) = \tfrac{1}{2}(AC + AB) = \tfrac{1}{2}(b + c),$$

and $AM = AC - CM = b - \frac{1}{2}(b + c) = \frac{1}{2}(b - c)$.

Hence
$$\frac{\tan \frac{B - C}{2}}{\cot \frac{A}{2}} = \frac{\tan ICM}{\tan (90° - \frac{1}{2} A)} = \frac{\tan ICM}{\tan IAM}$$

$$= \frac{\frac{IM}{CM}}{\frac{IM}{AM}} = \frac{AM}{CM} = \frac{\frac{1}{2}(b - c)}{\frac{1}{2}(b + c)} = \frac{b - c}{b + c}. \qquad \text{Q.E.D.}$$

172. If a line CF, drawn from the vertex C, of a triangle ABC, divides the vertical angle into two angles γ and γ', and the base into two corresponding segments, m, m', and if CF makes the angle θ with the base, then $(m+m')\cos\theta = m\cot\gamma - m'\cot\gamma'$.

(Minchin's Statics, § 36.)

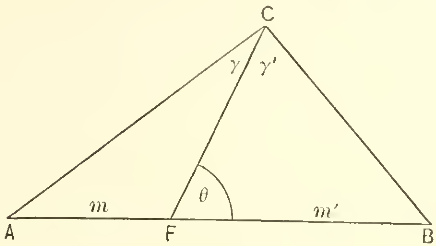


FIG. 61.

$$\frac{CF}{\sin A} = \frac{m}{\sin\gamma}; \therefore\ CF = \frac{m\sin A}{\sin\gamma};$$
$$= \frac{m(\sin\theta\cos\gamma - \sin\gamma\cos\theta)}{\sin\gamma},;$$

also,
$$CF = \frac{m'\sin B}{\sin\gamma'} = \frac{m'(\sin\theta\cos\gamma' + \sin\gamma'\cos\theta)}{\sin\gamma'}.$$

Equate these values of CF.

$$m(\sin\theta\cot\gamma - \cos\theta) = m'(\sin\theta\cot\gamma' + \cos\theta);$$
$$\therefore\ (m+m')\cot\theta = m\cot\gamma - m'\cot\gamma'. \qquad (1)$$ Q.E.D.

COROLLARY. I. Suppose CF is a median; then

$$m = m',\ 2\cot\theta = \cot\gamma - \cot\gamma'. \qquad (2)$$

II. Suppose CF bisects angle C;

$$\frac{m+m'}{m-m'} = \frac{c}{m-m'} = \frac{\cot\frac{C}{2}}{\cot\theta}. \qquad (3)$$

III. Suppose CF is perpendicular to the base; then

$$\cot\theta = 0,\ m'\cot\gamma' = m\cot\gamma. \qquad (4)$$

This is evident geometrically.

EXERCISE. Show that

$$(m+m')\cot\theta = m'\cot A - m\cot B. \qquad (5)$$

173. Let ABC be any triangle, and AD, BE, CF the medians drawn from A, B, C respectively. Let AD divide the angles A into two angles α, α'; BE divide angle B into β, β', and CF divide angle C into γ, γ'.

From (2), $\quad 2\cot\theta = \cot\gamma - \cot\gamma'$.

From (5), $\quad 2\cot\theta = \cot A - \cot B$;

$$\therefore\ \cot A - \cot B = \cot\gamma - \cot\gamma'.$$

Similarly, $$\cot B - \cot C = \cot \alpha - \cot \alpha';$$

$$\cot C - \cot A = \cot \beta - \cot \beta'.$$

Adding these equations, we obtain

$$\cot \alpha + \cot \beta + \cot \gamma = \cot \alpha' + \cot \beta' + \cot \gamma'.$$

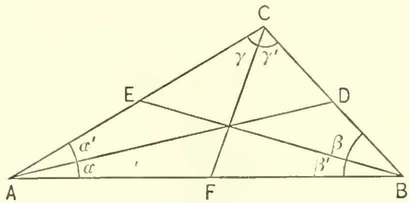


Fig. 62.

EXAMPLES. XLII.

Miscellaneous Exercises.

1. Define the terms sine, cotangent; and prove that if A be *any* angle,

$$\sin^2 A + \cos^2 A = 1.$$

If $\tan A = \frac{3}{4}$, find $\sin A$ and $\cos A$.

2. Find the sine, cosine, and tangent of 30°.

In a triangle ABC, the angle C is a right angle, the angle A is 60°, and the length of the perpendicular let fall from C on AB is 20 feet; find the length of AB.

3. Prove geometrically that $\cos (180° - A) = -\cos A$.

4. Prove (1) $\sin (A + B) \cdot \sin (A - B) = \sin^2 A - \sin^2 B$;

$$(2)\quad \frac{\sin A + \sin B}{\sin A - \sin B} = \frac{\tan \frac{1}{2}(A + B)}{\tan \frac{1}{2}(A - B)}.$$

5. Prove that

$$\cos^2 A - \cos A \cos (60° + A) + \sin^2 (30° - A) = \tfrac{3}{4}.$$

6. Find the greatest side of the triangle of which one side is 2183 feet and the adjacent angles are 78° 14′ and 71° 24′.

---

7. Express the other trigonometrical ratios in terms of the cosine.

8. Prove $$\sin (180° + A) = -\sin A;$$

$$\tan (90° + A) = -\cot A.$$

9. Write down the sines of all the angles which are multiples of 30° and less than 360°.

10. Prove $$\tan^2 A = \frac{1 - \cos 2A}{1 + \cos 2A}.$$

11. If $\tan A + \sec A = 2$, prove that $\sin A = \frac{3}{5}$, when A is less than 90°.

If $\sin A = \frac{4}{5}$, prove that $\tan A + \sec A = 3$, when A is less than 90°.

12. The length of the greatest side of a triangle is 1035.43 feet, and the three angles are 44°, 66°, and 70°. Solve the triangle.

13. Express the other trigonometrical ratios in terms of the cotangent.

14. Prove that $\operatorname{cosec}(180° + A) = -\operatorname{cosec} A$.

15. Write down the tangents of all the angles which are multiples of 30° and less than 360°.

16. If $\tan A + \sec A = 3$, prove that $\sin A = \frac{4}{5}$, when A is less than 90°.

If $\sin A = \frac{3}{5}$, prove that $\tan A + \sec A = 2$, when A is less than 90°.

17. Find the sines of the three angles of the triangle whose sides are 193, 194, and 195 feet.

18. Investigate the following formulæ:

(1) $\cos \frac{3A}{2} = (2 \cos A - 1) \cos \frac{1}{2} A$;

(2) $\cos\theta - \cos(\theta + \delta) = \sin\theta \sin\delta\,(1 + \cot\theta \tan \frac{1}{2}\delta)$.

---

19. Define the secant of an angle.

Prove the formula $\frac{1}{\sec^2 A} + \frac{1}{\operatorname{cosec}^2 A} = 1$.

If $\sin A = \frac{1}{3}$, find $\sec A$.

20. Find the logarithms of $\sqrt{(32)}$; and of .03125 to the base $\sqrt[3]{2}$.

21. Express the sine, cosine, and tangent of each of the angles 1962°, 2376°, 2844°, in terms of the trigonometrical functions of angles lying between 0 and 45°.

22. Prove the formula to express the cosine of the sum of two angles in terms of the sines and cosines of those angles.

Express $\cos 5\alpha$ in terms of $\cos\alpha$.

23. Find solutions of the equations

(i.) $\sec\theta \operatorname{cosec}\theta - \cot\theta = \sqrt{3}$;

(ii.) $\sin 2\theta - \sin\theta = \cos 2\theta + \cos\theta$.

24. A ring 10 inches in diameter is suspended from a point 1 foot above its centre by six equal strings attached to its circumference at equal intervals; find the cosine of the angle between two consecutive strings.

---

25. Define 1°. Assuming that $\frac{22}{7}$ is the circular measure of two right angles, express the angle $A°$ in circular measure.

Find the number of degrees in the angle whose circular measure is .1.

26. Find the trigonometrical ratios of the angle whose cosine is $\frac{3}{5}$.

27. Prove that (1) $\cos(180° + A) = \cos(180° - A)$;

(2) $\tan(90° + A) = \cot(180° - A)$.

28. Prove $$\sin x(2\cos x - 1) = 2\sin\frac{x}{2}\cos\frac{3x}{2}.$$

29. Express $\log_{10}5.832$, $\log_{10}\sqrt[3]{(35)}$, and $\log_{10}.304\dot{8}$ in terms of $\log_{10}2$, $\log_{10}3$, $\log_{10}7$.

30. If the angle opposite the side a be $60°$, and if b, c be the remaining sides of the triangle, prove that

$$(a+b+c)(b+c-a) = 3bc.$$

---

31. Assuming $\frac{22}{7}$ to be the circular measure of two right angles, express in degrees the angle whose circular measure is θ. Find the number of degrees in an angle whose circular measure is $\frac{1}{3}$.

32. Show from the definitions of the trigonometrical functions that

$$\sin^2 A + \cot^2 A + \cos^2 A = \operatorname{cosec}^2 A.$$

Prove that $$\frac{\tan A + \sec A + 1}{\tan A + \sec A - 1} = \frac{\sec A + 1}{\tan A}.$$

33. Prove $$\sin x(2\cos x + 1) = 2\cos\frac{x}{2}\sin\frac{3x}{2}.$$

34. Find the logarithms of $\sqrt{(27)}$ and $.\dot{0}3\dot{7}$ to the base $\sqrt[3]{3}$.

35. If $(\sin A + \sin B + \sin C)(\sin A + \sin B - \sin C) = 3\sin A\sin B$, and $A + B + C = 180°$, prove that $C = 60°$.

36. Given $A = 18°$, $B = 144°$, and $b = 1$; solve the triangle.

---

37. Give the trigonometrical definition of an angle.

What angle does the minute-hand of a clock describe between twelve o'clock and 20 minutes to four?

38. Express the cosine and the tangent of an angle in terms of the sine.

The angle A is greater than $90°$, but less than $180°$, and $\sin A = \frac{1}{3}$; find $\cos A$.

39. Find all the values of θ between 0 and 2π for which $\cos\theta + \cos 2\theta = 0$.

40. If in a triangle $a\cos A = b\cos B$, the triangle will be either isosceles or right-angled.

41. The sides are 1 foot and $\sqrt{3}$ feet respectively, and the angle opposite to the shorter side is $30°$; solve the triangle.

42. The sides of a triangle are 2, 3, 4; find the greatest angle, having given

$$\log 2 = .30103,$$
$$\log 3 = .47712,$$
$$\log\tan 52°\,15' = .11110,$$
$$\log\tan 52°\,14' = .11083.$$

---

43. Distinguish between Euclid's definition of an angle and the trigonometrical definition.

What angle does the minute-hand of a clock describe between half-past four and a quarter-past six?

44. Express the sine and the cosine of an angle in terms of the tangent.

The angle A is greater than $180°$, but less than $270°$, and $\tan A = \frac{1}{2}$. Find $\sin A$.

45. Prove (i.)
$$\sin 2A = \frac{2\cot A}{1+\cot^2 A}.$$

(ii.) Show that if $A + B + C = 90°$,
$$\sin 2A + \sin 2B + \sin 2C = 4\cos A\cos B\cos C.$$

46. Find all the values of θ between 0 and 2π, for which
$$\sin\theta + \sin 2\theta = 0.$$

47. If in a triangle $b\cos A = a\cos B$, show that the triangle is isosceles.

48. The sides are 1 foot and $\sqrt{2}$ feet respectively, and the angle opposite to the shorter side is $30°$. Solve the triangle.

---

49. Express in degrees, minutes, and seconds (1) the angle whose circular measure is $\frac{1}{20}\pi$; (2) the angle whose circular measure is 5.

If the angle subtended at the centre of a circle by the side of a regular heptagon be the unit of angular measurement, by what number is an angle of $45°$ represented?

50. Prove that
$$(\sin 30° + \cos 30°)(\sin 120° + \cos 120°) = \sin 30°.$$

51. Prove the formulæ:

(1) $$\cos^2(\alpha+\beta) - \sin^2\alpha = \cos\beta\cos(2\alpha+\beta);$$

(2) $$1 + \cot\alpha\cot\tfrac{1}{2}\alpha = \operatorname{cosec}\alpha\cot\tfrac{1}{2}\alpha.$$

52. Find solutions of the equations:

(1) $5\tan^2 x - \sec^2 x = 11$; (2) $\sin 5\theta - \sin 3\theta = \sqrt{2}\cdot\cos 4\theta$.

53. Two sides of a triangle are 10 feet and 15 feet in length, and the angle between them is $30°$. What is its area?

54. Given that
$$\sin 40°\,29' = 0.64922,\quad \sin 40°\,30' = 0.64944;$$
find the angle whose sine is 0.64930.

---

55. Express in circular measure (1) $10'$; (2) $\frac{1}{5}$ of a right angle.

If the angle subtended at the centre of a circle by the side of a regular pentagon be the unit of angular measurement, by what number is a right angle represented?

56. If $\sec\alpha = 7$, find $\tan\alpha$ and $\operatorname{cosec}\alpha$.

57. Prove the formulæ:

(1) $$\cos^2(\alpha-\beta) - \sin^2(\alpha+\beta) = \cos 2\alpha\cos 2\beta;$$

(2) $$1 + \tan\alpha\tan\tfrac{1}{2}\alpha = \sec\alpha.$$

58. Find solutions of the equations:

(1) $5\tan^2 x + \sec^2 x = 7$; (2) $\cos 5\theta + \cos 3\theta = \sqrt{2}\cdot\cos 4\theta$.

59. The lengths of the sides of a triangle are 3 feet, 5 feet, and 6 feet. What is its area?

60. Given that

$$\sin 38^\circ 25' = 0.62137\,;\ \ \sin 38^\circ 26' = 0.62160\,;$$

find the angle whose sine is (0.62150).

---

61. Which is greater, 76° or 1.2^c?

62. Determine geometrically $\cos 30^\circ$ and $\cos 45^\circ$.

If $\sin A$ be the arithmetic mean between $\sin B$ and $\cos B$, then

$$\cos 2A = \cos^2(B + 45^\circ).$$

63. Establish the following relations:

(1) $$\tan^2 A - \sin^2 A = \tan^2 A \sin^2 A\,;$$

(2) $$\cot A - \cot 2A = \operatorname{cosec} 2A\,;$$

(3) $$\frac{\sin(x + 3y) + \sin(3x + y)}{\sin 2x + \sin 2y} = 2\cos(x + y).$$

64. Express $\log_{10}\sqrt{(28)}$, $\log_{10} 3.888$, $\log_{10} .174\dot{2}$ in terms of $\log_{10} 3$, $\log_{10} 5$, $\log_{10} 7$.

65. Prove that $\sin(A + B) = \sin A \cos B + \cos A \sin B$, and deduce the expression for $\cos(A + B)$.

Show that

$$\sin A \cos(B + C) - \sin B \cos(A + C) = \sin(A - B)\cos C.$$

66. One side of a triangular lawn is 102 feet long, its inclinations to the other sides being $70^\circ 30'$, $78^\circ 10'$, respectively. Determine the other sides and the area. $\log \sin 70^\circ 30' = 9.974$, $\log 102 = 2.009$, $\log \sin 78^\circ 10' = 9.990$, $\log 185 = 2.267$, $\log \sin 31^\circ 20' = 9.716$, $\log 192 = 2.283$, $\log 2 = .301$, $\log 9234 = 3.965$.

---

67. Which is greater, 126° or the angle whose circular measure is 2.3?

68. Establish the following relations:

(1) $$\cot^2 A - \cos^2 A = \cot^2 A \cos^2 A\,;$$

(2) $$\tan A + \cot 2A = \operatorname{cosec} 2A\,;$$

(3) $$\frac{\cos(x - 3y) - \cos(3x - y)}{\sin 2x + \sin 2y} = 2\sin(x - y).$$

69. Given $\log_{10} 2 = .3010300$, $\log_{10} 9 = .9542425$; find without using tables, $\log_{10} 5$, $\log_{10} 6$, $\log_{10} .0216$, and $\log_{10}\sqrt[5]{(.375)}$.

70. Prove that $\sin 30^\circ + \sin 120^\circ = \sqrt{2}\cos 15^\circ$.

71. Establish the identities:

(1) $1 + \cos A + \sin A = \sqrt{2(1 + \cos A)(1 + \sin A)}\,;$

(2) $\operatorname{cosec} 2A = \dfrac{\operatorname{cosec}^2 A}{2\sqrt{\operatorname{cosec}^2 A - 1}}\,;$

(3) $\sin\dfrac{2\pi}{7} + \sin\dfrac{4\pi}{7} - \sin\dfrac{6\pi}{7} = 4\sin\dfrac{\pi}{7}\sin\dfrac{3\pi}{7}\sin\dfrac{5\pi}{7}.$

72. The sides of a triangular lawn are 102, 185, and 192 feet in length, the smallest angle being approximately 31° 20′. Find its other angles and its area.

$$\log 102 = 2.009, \qquad \log\sin 31^\circ 20' = 9.716,$$
$$\log 185 = 2.267, \qquad \log\sin 70^\circ 30' = 9.974,$$
$$\log 192 = 2.283, \qquad \log\sin 78^\circ 10' = 9.990,$$
$$\log 2 = .301, \ \log 9234 = 3.965.$$

---

73. If the circumference of a circle be divided into five parts in arithmetical progression, the greatest part being six times the least, express in radians the angle each subtends at the centre.

74. Define the sine of an angle, wording your definition so as to include angles of any magnitude.

Prove that $\sin(90^\circ + A) = \cos A,$

and $\cos(90^\circ + A) = -\sin A,$

and by means of these deduce the formulæ

$$\sin(180^\circ + A) = -\sin A, \ \cos(180^\circ + A) = -\cos A.$$

75. Prove the formulæ:

(1) $\cot^2 A = \operatorname{cosec}^2 A - 1$;

(2) $\cot^4 A + \cot^2 A = \operatorname{cosec}^4 A - \operatorname{cosec}^2 A$.

Verify (2) when $A = 30^\circ$.

76. Evaluate to 4 significant figures by the aid of the table of logarithms

$$\frac{7.891}{.0345} \times \sqrt[7]{.(008931)}.$$

77. If $\sin B$ be the geometric mean between $\sin A$ and $\cos A$, then $\cos 2B = 2\cos^2(A + 45^\circ)$.

78. The lengths of two of the sides of a triangle are 1 foot and $\sqrt{2}$ feet respectively, the angle opposite the shorter side is 30°. Prove that there are two triangles which satisfy these conditions; find their angles, and show that their areas are in the ratio $\sqrt{3} + 1 : \sqrt{3} - 1$.

---

79. If the circumference of a circle be divided into six parts in arithmetical progression, the greatest being six times the least, express in radians the angle each subtends at the centre.

80. Define the tangent of an angle, wording your definition so as to include angles of any magnitude.

Prove that $\tan(90^\circ + A) = -\cot A$, and by means of this formula deduce the formula $\tan(180^\circ + A) = \tan A$.

81. Compute by means of tables the value of

$$\frac{6.12}{.4131} \times \sqrt[5]{54.17}.$$

82. Prove that $\cos(A + B) = \cos A\cos B - \sin A\sin B$, and deduce the expression for $\sin(A + B)$.

Show that $\cos A\cos(B + C) - \cos B\cos(A + C) = \sin(A - B)\sin C$.

83. Establish the identities:

(1) $1 + \cos A - \sin A = \sqrt{2(1 + \cos A)(1 - \sin A)}$;

(2) $\sec 2A = \dfrac{\sec^2 A}{2 - \sec^2 A}$;

(3) $\cos\dfrac{2\pi}{7} + \cos\dfrac{4\pi}{7} + \cos\dfrac{6\pi}{7} + 4\cos\dfrac{\pi}{7}\cos\dfrac{3\pi}{7}\cos\dfrac{5\pi}{7} + 1 = 0.$

84. Two adjacent sides of a parallelogram 5 in. and 3 in. long respectively, include an angle of 60°. Find the lengths of the two diagonals and the area of the figure.

---

85. Investigate the following formulæ:

(1) $\sin\dfrac{3A}{2} = (1 + 2\cos A)\sin\frac{1}{2}A$;

(2) $\sin(\theta + \delta) - \sin\theta = \cos\theta\sin\delta\,(1 - \tan\theta\tan\frac{1}{2}\delta)$.

86. Prove that

(1) $\sin 10^\circ + \sin 50^\circ = \sin 70^\circ$;

(2) $\sqrt{3} + \tan 40^\circ + \tan 80^\circ = \sqrt{3}\tan 40^\circ \tan 80^\circ$;

(3) if $A + B + C = 180^\circ$,

$$\frac{\sin A - \sin B\cos C}{\cos B} = \frac{\sin B - \sin A\cos C}{\cos A}.$$

87. Prove by means of the logarithmic table that

$$\frac{1}{73^{-\frac{1}{7}}} = 1.846 \text{ nearly.}$$

88. The length of one side of a triangle is 1006.62 feet and the adjacent angles are 44° and 70°. Solve the triangle, having given

$$L\sin 44^\circ = 9.8417713, \qquad L\sin 70^\circ = 9.9729858,$$
$$L\sin 66^\circ = 9.9607302, \qquad \log 1006.62 = 3.0028656,$$
$$\log 7654321 = 6.8839067, \qquad \log 103543 = 5.0151212.$$

---

89. Find the length of the arc of a circle whose radius is 8 feet which subtends at the centre an angle of 50°, having given

$$\pi = 3.1416.$$

90. Prove that $\sin A = -\sin(A - 180^\circ)$.

Find the sines of 30° and 2010°.

91. Given that the integral part of $(3.1622)^{100000}$ contains fifty thousand digits, find $\log_{10} 31622$ to five places of decimals.

92. Prove that

(1) $\cos^2 A + \cos^2 B - 2\cos A\cos B\cos(A + B) = \sin^2(A + B)$;

(2) $\cos^2 A + \sin^2 A\cos 2B = \cos^2 B + \sin^2 B\cos 2A$.

93. Prove that in any triangle

$$a^2\cos 2B + b^2\cos 2A = a^2 + b^2 - 4ab\sin A\sin B.$$

94. If $a = 123$, $B + 29^\circ\,17'$, $C = 135^\circ$, find c, having given

$$\log 123 = 2.0899051, \qquad \log 2 = .3010300,$$
$$\log 3211 = 4.5066403, \qquad \text{diff. for } 1 = 1352,$$
$$\log\sin 15^\circ\,43' = 9.4327777.$$

CHAPTER XVIII

RELATIONS AMONG THE SIDES AND THE ANGLES OF A SPHERICAL TRIANGLE

174. The succeeding pages contain a brief discussion of some of the properties of spherical triangles.

For the sake of ready reference, we shall enumerate, without proof, some properties of solid figures. The statements contained in sections 175–177 are proved in works on solid geometry, to which the student is referred.

175. Definitions and Theorems. The curve of intersection of a plane and a sphere is a circle.

When the plane of the circle passes through the centre of the sphere, their curve of intersection is called a **great circle.**

One great circle can be passed through any two points on the surface of the sphere, and only one if these points are not extremities of a diameter of the sphere.

A spherical figure is any part of the surface of the sphere bounded by arcs of great circles.

A **spherical polygon** is a spherical figure bounded by more than two arcs. The arcs are called the **sides** of the polygon. The intersections of the arcs are called the **vertices** of the polygon.

176. The **angle** between two great circles is measured by the angle between the tangents drawn to the circles at their point of intersection. This is called a **spherical angle.**

The angle between two great circles equals the angle between their planes.

177. Spherical Triangles. A **spherical triangle** is a spherical polygon of three sides.

Let ABC be a spherical triangle.

Let O be the centre of the sphere.

By the letters A, B, C we shall indicate *geometrically* the three angular points of the triangle ABC; *algebraically*, the three angles at those points respectively. By the letters a, b, c we shall indicate the measures of the sides opposite A, B, C, respectively.

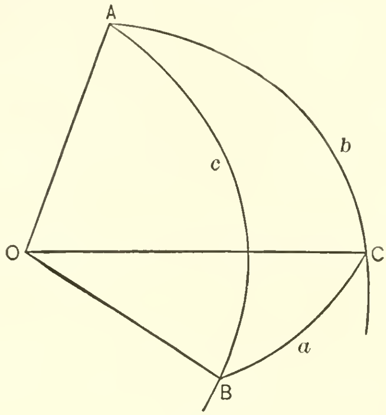


Fig. 63.

a, b, c are measured by the angles at the centre of the sphere, and hence they are measured in angular units; *e.g.* c is measured by angle AOB.

We know, then, the following properties:

1. * The sum of two sides of a triangle is greater than the third.
2. The greatest side is opposite the greatest angle, and conversely.
3. Any angle $A < 180°$.
4. $(A + B + C) < 540°$ and $> 180°$.
5. Any side $a < 180°$.
6. $(a + b + c) < 360°$.
7. $A - B$ and $a - b$ are of the same sign.
8. A side which differs from $90°$ more than another side is in the same quadrant as the angle opposite it.
9. If $A'B'C'$ is the polar triangle of ABC, and if A', B', C' are its angles, and a', b', c' the corresponding sides, then

$$A = 180° - a'; \quad A' = 180° - a;$$
$$B = 180° - b'; \quad B' = 180° - b;$$
$$C = 180° - c'; \quad C' = 180° - c.$$

* Note. We know that in general three great circles intersect in such a way as to form eight spherical triangles; and that at least one of these triangles satisfies the conditions of Art. 177. We shall consider only such triangles.

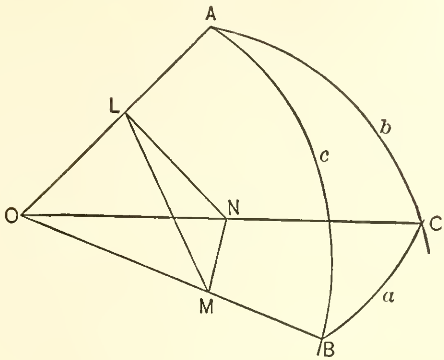


FIG. 64.

178. *To prove* $\cos a = \cos c \cos b + \sin b \sin c \cos A$.

Let ABC be a spherical triangle.

Let O be the centre of the sphere, and $OA = OB = OC =$ radius of the sphere.

In the plane AOB, draw LM perpendicular to OA.

In the plane AOC, draw LN perpendicular to OA.

Then the plane angle $MLN = A$ (Art. 176).

In the plane triangles LMN and MON,

$$\overline{MN}^2 = \overline{LM}^2 + \overline{LN}^2 - 2\,\overline{LM}\ \overline{LN}\cos A. \qquad \text{(Art. 107.)}$$

$$\overline{MN}^2 = \overline{OM}^2 + \overline{ON}^2 - 2\,\overline{OM}\cdot\overline{ON}\cos a.$$

Equate these values of $\overline{MN}^2$,

$$\overline{OM}^2 - \overline{LM}^2 + \overline{ON}^2 - \overline{LN}^2 - 2\,\overline{OM}\cdot\overline{ON}\cos a + 2\,LM\cdot LN\cos A = 0.$$

But $\qquad \overline{OM}^2 - \overline{LM}^2 = \overline{OL}^2$, and $\overline{ON}^2 - \overline{LN}^2 = \overline{OL}^2$;

$$\therefore\ 2\,\overline{OL}^2 - 2\,\overline{OM}\cdot\overline{ON}\cos a + 2\,LM\cdot LN\cos A = 0;$$

or
$$\frac{OL}{OM}\cdot\frac{OL}{ON} + \frac{LM}{OM}\cdot\frac{LN}{ON}\cos A = \cos a;$$

i.e.
$$\cos c\cos b + \sin b\sin c\cos A = \cos a. \qquad (a)$$

179. By reference to Fig. 63 it is evident that the demonstration of the preceding article requires that c and b are each less than 90°, while a is unrestricted.

Suppose, now, that one of the sides, b, say, is greater than 90°. Produce the great circles AC and BC. They intersect again in C', Fig. 65.

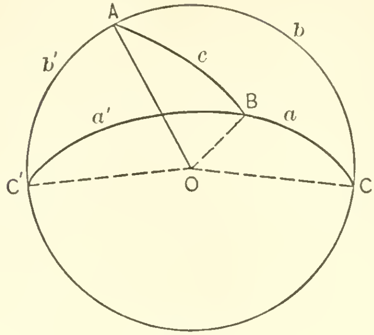


Fig. 65.

Consider now the triangle ABC.

$$AB < 90^\circ \text{ (hypothesis).}$$

$$AC' = (180^\circ - AC) < 90^\circ.$$

We can now apply the formula (a), of the preceding article, to ABC'. Hence

$$\cos a' = \cos b' \cos c + \sin b' \sin c \cos (180^\circ - A). \qquad (b)$$

But $\cos a' = -\cos a$; (Art. 59.)

and $\cos b' = -\cos b$; (Art. 59.)

and $\cos (180^\circ - A) = -\cos A$.

Substituting these values in (b) we obtain

$$\cos a = \cos b \cos c + \sin b \sin c \cos A.$$

Similarly, if both b and c are greater than 90°, it may be shown that $\cos a = \cos b \cos c + \sin b \sin c \cos A.$

So that (a) is true for all spherical triangles which we are considering. Similarly we can express the other angles in terms of the sides. We therefore have this relation involving the sides and one angle.

$$\left.\begin{aligned} \cos a &= \cos b \cos c + \sin b \sin c \cos A, \\ \cos b &= \cos c \cos a + \sin c \sin a \cos B, \\ \cos c &= \cos a \cos b + \sin a \sin b \cos C. \end{aligned}\right\} \qquad (1)$$

Whence

$$\left.\begin{aligned} \cos A &= \frac{\cos a - \cos b \cos c}{\sin b \sin c}, \\ \cos B &= \frac{\cos b - \cos c \cos a}{\sin c \sin a}, \\ \cos C &= \frac{\cos c - \cos a \cos b}{\sin a \sin b}. \end{aligned}\right\} \qquad (2)$$

The last two formulæ of either set can be derived from the first one of that set by making a cyclical interchange of a into b, b into c, c into a, and at the same time changing A into B, B into C, C into A.

180. *To show that*

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}. \qquad (3)$$

This is a relation involving the **sides** and the **opposite angles.**

PROOF. $\text{Sin}^2 A = 1 - \cos^2 A$

$$= \frac{\sin^2 b \sin^2 c - (\cos a - \cos b \cos c)^2}{\sin^2 b \sin^2 c}. \qquad \text{(Art. 179.)}$$

Substitute for $\sin^2 b$, $1 - \cos^2 b$, and $\sin^2 c = 1 - \cos^2 c$, and reducing we obtain

$$\sin^2 A = \frac{1 - \cos^2 b - \cos^2 c - \cos^2 a + 2 \cos a \cos b \cos c}{\sin^2 b \sin^2 c}.$$

Multiplying both sides of the equation by $\frac{1}{\sin^2 a}$, and extracting the square root of each side, we obtain

$$\frac{\sin A}{\sin a} = \frac{\sqrt{1 - \cos^2 b - \cos^2 c - \cos^2 a + 2 \cos a \cos b \cos c}}{\sin a \sin b \sin c}.$$

In a similar way we might solve for $\frac{\sin B}{\sin b}$. But this is *equivalent* to a *cyclical interchange* of the letters as described in Art. 179. But an interchange of the letters changes the left side of the equation into $\frac{\sin B}{\sin b}$, and leaves the right side unchanged. Hence

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

$$= \frac{\sqrt{1 - \cos^2 b - \cos^2 c - \cos^2 a + 2 \cos a \cos b \cos c}}{\sin a \sin b \sin c}, \qquad \text{Q.E.D.}$$

since $A < 180°$, and since $a < 180°$ (Art. 177).

Therefore $\sin A$ and $\sin a$ are positive. Therefore there is no *ambiguity* of sign.

181. We shall add a geometrical proof of the theorem.

Take L any point in OA. From L let a perpendicular fall on plane COB, piercing it in G; from G draw GM and GN perpendicular to OB and OC respectively. Join L and M; join L and N. Then $LM \perp OB$, and $LN \perp OC$. (Geom.)

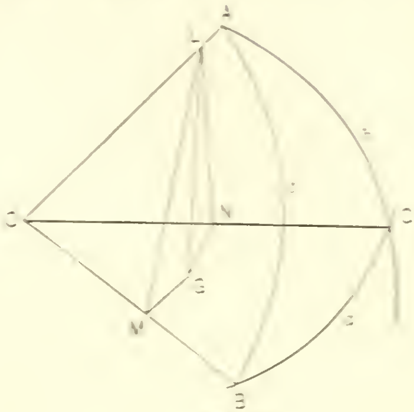


Fig. 66.

Angle B is measured by angle LMG.

Angle C is measured by angle LNG.

$$LM \cdot \sin B = LG = LN \sin C;$$

$$LM = OL \sin AOB = OL \sin c;$$

$$LN = OL \sin AOC = OL \sin b;$$

$$\therefore\ OL \sin c \sin B = OL \sin b \sin C.$$

$$\therefore\ \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}.$$

182. If $A'B'C'$ be the polar triangle of ABC, then (Fig. 67)

$$\left.\begin{aligned} a' &= 180^\circ - A; \\ b' &= 180^\circ - B; \\ c' &= 180^\circ - C. \end{aligned}\right\} \qquad \text{(Art. 177.)}$$

We may now apply formula (1) to $A'B'C'$.

$$\cos a' = \cos b' \cos c' + \sin b' \sin c' \cos A'. \qquad (a)$$

Now, $\cos a' = -\cos A, \quad \sin b' = \sin B;$

$\cos b' = -\cos B, \quad \sin c' = \sin C;$

$\cos c' = -\cos C, \quad \cos A' = -\cos a.$

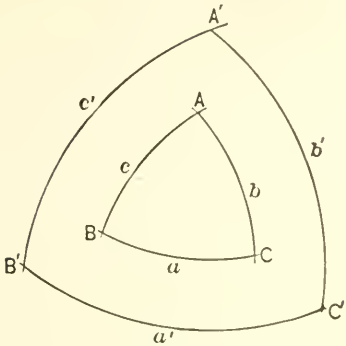


Fig. 67.

Substituting these values in (a), we obtain

$$-\cos A = \cos B \cos C - \sin B \sin C \cos a.$$

Treating the other two formulæ similarly, we obtain

$$\left.\begin{aligned}\cos A &= -\cos B \cos C + \sin B \sin C \cos a;\\ \cos B &= -\cos C \cos A + \sin C \sin A \cos b;\\ \cos C &= -\cos A \cos B + \sin A \sin B \cos c.\end{aligned}\right\} \quad (4)$$

Relation involving the angles and one side.

Solving these equations for $\cos a$, $\cos b$, $\cos c$, we have

$$\left.\begin{aligned}\cos a &= \frac{\cos A + \cos B \cos C}{\sin B \sin C};\\ \cos b &= \frac{\cos B + \cos C \cos A}{\sin C \sin A};\\ \cos c &= \frac{\cos C + \cos A \cos B}{\sin A \sin B}.\end{aligned}\right\} \quad (5)$$

183. From (1),

$$\cos a = \cos b \cos c + \sin b \sin c \cos A.$$

Substituting in this formula the value of $\cos c$ obtained from (1), we obtain

$$\cos a\,(1 - \cos^2 b) = \sin a \sin b \cos b \cos C + \sin b \sin c \cos A;$$

or $\cos a \sin b = \sin a \cos b \cos C + \sin c \cos A$;

and similarly,

$$\left.\begin{aligned}\cos a \sin b &= \sin a \cos b \cos C + \sin c \cos A;\\ \cos b \sin a &= \sin b \cos a \cos C + \sin c \cos B;\\ \cos b \sin c &= \sin b \cos c \cos A + \sin a \cos B;\\ \cos c \sin b &= \sin c \cos b \cos A + \sin a \cos C;\\ \cos c \sin a &= \sin c \cos a \cos B + \sin b \cos C;\\ \cos a \sin c &= \sin a \cos c \cos B + \sin b \cos A.\end{aligned}\right\} \quad (6)$$

(6) is **a relation involving two angles and the sides.**

By treating (3) similarly,

$$\left.\begin{aligned}
\cos A \sin B &= \cos a \sin C - \cos c \cos B \sin A;\\
\cos C \sin B &= \cos c \sin A - \cos a \cos B \sin C;\\
\cos C \sin A &= \cos c \sin B - \cos b \cos A \sin C;\\
\cos B \sin A &= \cos b \sin C - \cos c \cos A \sin B;\\
\cos B \sin C &= \cos b \sin A - \cos a \cos C \sin B;\\
\cos A \sin C &= \cos a \sin B - \cos b \cos C \sin A.
\end{aligned}\right\} \quad (7)$$

(7) is **a relation between the angles and two sides.**

From (6),

$$\cos a \sin b = \sin a \cos b \cos C + \sin c \cos A;$$

$$\therefore \frac{\cos a}{\sin a} \sin b = \cos b \cos C + \frac{\sin c}{\sin a} \cos A;$$

$$\therefore \cot a \sin b = \cos b \cos C + \sin C \cdot \frac{\cos A}{\sin A}; \qquad \text{(Art. 180.)}$$

$$\cot a \sin b = \cos b \cos C + \sin C \cot A. \qquad (8)$$

The student may derive the five corresponding formulæ:

184. *To express* $\cos \frac{1}{2} A$, $\sin \frac{1}{2} A$, $\tan \frac{1}{2} A$ *in terms of the sides.*

$$\cos \tfrac{1}{2} A = \sqrt{\frac{\cos A + 1}{2}}. \qquad \text{(Art. 92.)}$$

From (2), $$\sqrt{\frac{\cos A + 1}{2}} = \sqrt{\frac{\sin b \sin c + \cos a - \cos b \cos c}{2 \sin b \sin c}}$$

$$= \sqrt{\frac{\cos a - \cos(b + c)}{2 \sin b \sin c}} \qquad \text{(Art. 75.)}$$

$$= \sqrt{\frac{\sin\left(\frac{a+b+c}{2}\right)\sin\left(\frac{b+c-a}{2}\right)}{\sin b \sin c}}. \quad (a) \text{(Art. 82.)}$$

If we put $a + b + c = 2s$, (a) may be written:

$$\left.\begin{aligned}
\cos \tfrac{1}{2} A &= \sqrt{\frac{\sin s \cdot \sin(s-a)}{\sin b \cdot \sin c}};\\
\text{similarly,} \quad \cos \tfrac{1}{2} B &= \sqrt{\frac{\sin s \cdot \sin(s-b)}{\sin a \sin c}},\\
\cos \tfrac{1}{2} C &= \sqrt{\frac{\sin s \cdot \sin(s-c)}{\sin a \sin b}},
\end{aligned}\right\} \quad (9)$$

EXERCISE. The student may show that

$$\sin\tfrac{1}{2}A=\sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b\sin c}}, \tag{10}$$

$$\tan\tfrac{1}{2}A=\sqrt{\frac{\sin(s-b)(s-c)}{\sin s\cdot\sin(s-a)}}, \tag{11}$$

with the corresponding formulæ for B and C.

185. *To find* $\cos\frac{1}{2}a$, $\sin\frac{1}{2}a$, $\tan\frac{1}{2}a$. Using the results of (5), the student may show that, if $S=\frac{1}{2}(A+B+C)$,

$$\cos\tfrac{1}{2}a=\sqrt{\frac{\cos(S-B)\cos(S-C)}{\sin B\cdot\sin C}}; \tag{12}$$

$$\sin\tfrac{1}{2}a=\sqrt{\frac{1-\cos a}{2}},$$

$$=\sqrt{\frac{\sin B\sin C-\cos A-\cos B\cos C}{2\sin B\sin C}},$$

$$=\sqrt{\frac{-(\cos(B+C)+\cos A)}{2\sin B\sin C}}.$$

$$\therefore\ \sin\tfrac{1}{2}a=\sqrt{\frac{-\cos S\cos(S-A)}{\sin B\sin C}}, \tag{13}$$

$$\tan\tfrac{1}{2}a=\sqrt{\frac{-\cos S\cos(S-A)}{\cos(S-B)\cos(S-C)}}. \tag{14}$$

$$\sin a=2\sin\tfrac{1}{2}a\cos\tfrac{1}{2}a \qquad \text{(Art. 91.)}$$

$$=\frac{2}{\sin B\sin C}\sqrt{-\cos S\cos(S-A)\cos(S-B)\cos(S-C)}. \tag{15}$$

186. I. Since any side $a<180°$, and any angle $A<180°$,

$$\cos\tfrac{1}{2}a,\ \sin\tfrac{1}{2}a,\ \tan\tfrac{1}{2}a,\ \sin\tfrac{1}{2}A,\ \cos\tfrac{1}{2}A,\ \tan\tfrac{1}{2}A$$

are all positive; hence the sign of the radical in expressions (9)–(15) is positive.

II. Since $s<180°$, $a<180°$, $b<180°$, $c<180°$, and since the differences $s-a$, $s-b$, $s-c$ are less than 180° and positive, the expressions (9)–(11) are real.

III. If a', b', c' are the sides of the polar triangle of A, B, C, then $a'<(b'+c')$;

i.e. $$180°-A<(180°-B+180°-C);$$

i.e. $$B+C-A<180°;$$

i.e. $$S-A<90°;$$

$$\therefore\ \cos(S-A)\text{ is positive.}$$

Since $270° > S° > 180°$, $\cos S$ is negative; *i.e.* $-\cos S$ is positive. Therefore the expressions (12) – (15) are real.

187. $\cos\frac{1}{2}(A+B) = \cos\frac{1}{2}A\cos\frac{1}{2}B - \sin\frac{1}{2}A\sin\frac{1}{2}B$. (Art. 75.)

Substitute in this equation the values of $\cos\frac{1}{2}A$, etc., from (9) and (10).

$$\cos\tfrac{1}{2}(A+B) = \frac{\sin s}{\sin c}\sqrt{\frac{\sin(s-a)\sin(s-b)}{\sin b\sin a}} - \frac{\sin(s-c)}{\sin c}\sqrt{\frac{\sin(s-a)\sin(s-c)}{\sin b\sin a}}.$$

$$\therefore\ \cos\tfrac{1}{2}(A+B) = \frac{\sin s - \sin(s-c)}{\sin c}\sqrt{\frac{\sin(s-a)(s-c)}{\sin b\sin a}}.$$

Now
$$\frac{\sin s - \sin(s-c)}{\sin c} = \frac{2\cos\dfrac{2s-c}{2}\cdot\sin\dfrac{c}{2}}{2\sin\frac{1}{2}c\cos\frac{1}{2}c} \quad \text{(Art. 83.)}$$

$$= \frac{\cos\dfrac{a+b}{2}}{\cos\frac{1}{2}c}. \quad \text{(Art. 91.)}$$

And
$$\sin\frac{C}{2} = \sqrt{\frac{\sin(s-a)\sin(s-b)}{\sin a\sin b}}. \quad \text{(from (10))}$$

$$\therefore\ \cos\frac{A+B}{2} = \frac{\cos\dfrac{a+b}{2}}{\cos\dfrac{c}{2}}\sin\frac{C}{2}. \quad (16)$$

Exercise. The student may prove:

$$\cos\frac{A-B}{2} = \frac{\sin\dfrac{a+b}{2}}{\sin\dfrac{c}{2}}\sin\frac{C}{2}; \quad (17)$$

$$\sin\frac{A+B}{2} = \frac{\cos\dfrac{a-b}{2}}{\cos\dfrac{c}{2}}\cos\frac{C}{2}; \quad (18)$$

$$\sin\frac{A-B}{2} = \frac{\sin\dfrac{(a-b)}{2}}{\sin\dfrac{C}{2}}\cos\frac{C}{2}. \quad (19)$$

Formulæ (16), (17), (18), (19) are known as **Gauss' Formulæ.**

188. The student will prove without difficulty:

$$\tan\frac{A+B}{2}=\frac{\cos\frac{a-b}{2}}{\cos\frac{a+b}{2}}\cot\frac{C}{2};\tag{20}$$

$$\tan\frac{A-B}{2}=\frac{\sin\frac{a-b}{2}}{\sin\frac{a+b}{2}}\cot\frac{C}{2};\tag{21}$$

$$\tan\frac{a+b}{2}=\frac{\cos\frac{A-B}{2}}{\cos\frac{A+B}{2}}\tan\frac{c}{2};\tag{22}$$

$$\tan\frac{a-b}{2}=\frac{\sin\frac{A-B}{2}}{\sin\frac{A+B}{2}}\tan\frac{c}{2}.\tag{23}$$

Formulæ (20), (21), (22), (23) are known as **Napier's Analogies.**

189. In the special case that one of the angles, C, is equal $90°$, we derive from the foregoing formulæ the following:

From (1),
$$\cos c=\cos a\cos b.\tag{24}$$

From (3),
$$\left.\begin{aligned}\sin A&=\frac{\sin a}{\sin c};\\ \sin B&=\frac{\sin b}{\sin c}.\end{aligned}\right\}\tag{25}$$

From (7),
$$\left.\begin{aligned}\cos A&=\cos a\sin B;\\ \cos B&=\cos b\sin A.\end{aligned}\right\}\tag{26}$$

$$\left.\begin{aligned}\therefore\ \cos A&=\frac{\tan b}{\tan c};\\ \cos B&=\frac{\tan a}{\tan c}.\end{aligned}\right\}\tag{27}$$

From (8),
$$\left.\begin{aligned}\tan A&=\frac{\tan a}{\sin b};\\ \tan B&=\frac{\tan b}{\sin a}.\end{aligned}\right\}\tag{28}$$

From (5),

$$\left.\begin{aligned}\cos a &= \frac{\cos A}{\sin B};\\ \cos b &= \frac{\cos B}{\sin A};\\ \therefore\ \cos c &= \frac{\cos A}{\sin A}\cdot\frac{\cos B}{\sin B} = \cot A\cdot\cot B.\end{aligned}\right\}\quad(29)$$

190. Definition. Two angles are said to be of the "same affection," if both are greater or both are less than 90°. They are said to be of **opposite affection** if one is greater and the other less than 90°.

Formula (24) shows that if a and b are of the *same affection*, $c < 90°$; and if *a and b are of opposite affection*, $c > 90°$, C being a right angle. For, if a and b are both of the same affection, $\cos a \cdot \cos b$ is positive, $\therefore$ $\cos c$ is positive, $\therefore$ $c < 90°$. But if they are of opposite affection, $\cos a \cdot \cos b$ is negative, $\therefore$ $\cos c$ is negative; $\therefore$ $c > 90°$.

Exercise 1. Show that in a right-angled triangle an angle and the side opposite are of the same affection.

Exercise 2. Show that a side and the hypotenuse of a right-angled triangle are of the same or opposite affection, according as the included angle is less than or greater than 90°.

Exercise 3. In *any* triangle, $\frac{a+b}{2}$ and $\frac{A+B}{2}$ are of the same affection.

Exercise 4. If C is a right angle, show that

$$2\cos c = \cos(a+b) + \cos(a-b).$$

Exercise 5. If C is a right angle, show that

$$\tan\tfrac{1}{2}(c+a)\tan(c-a) = \tan^2\tfrac{1}{2}b.$$

CHAPTER XIX

SOLUTION OF SPHERICAL TRIANGLES

191. According to propositions proved in Geometry we are able to construct a spherical triangle when *any three* of its parts are given. Accordingly, we propose the problem, to show that if **given any three parts of a spherical triangle, we are able to calculate** the remaining parts.

This problem is known as the solution of spherical triangles.

There are six cases; viz. when there are given:

I. Three sides.
II. Two sides and included angle.
III. Two sides and the angle opposite one of them.
IV. Two angles and a side opposite one of them.
V. One side and two angles adjacent to it.
VI. Three angles.

192. The Right Triangle.

Before considering the solution of the general triangle, we shall discuss the solution of the right triangle. Let C be a right angle.

We have the following cases:

	Given:	To calculate:
I.	a, b.	c, A, B.
II.	a, c.	b, A, B.
	c, b.	a, A, B.
III.	a, A.	b, c, B.
	b, B.	a, c, A.
IV.	a, B.	b, c, A.
	b, A.	a, c, B.
V.	c, A.	a, b, B.
	c, B.	a, b, A.
VI.	A, B.	a, b, c.

193. If any of the parts, when calculated, is found in terms of its sine, then the solution in general is ambiguous; since there are *two* angles less than 180° which have the same sine. See Case III., below.

If, however, all the parts are found in terms of other ratios than their sines, or cosecants, the solution is unique.

194. Discussion of the Different Cases. By referring to Art. 189, the student will verify the following equations. In many instances he will find other sets of equations which will solve the problems.

Case I.

Given a, b; required c, A, B.

$$\cos c = \cos a \cos b;$$

$$\tan A = \frac{\tan a}{\sin b};$$

$$\tan B = \frac{\tan b}{\sin a}.$$

The solution is unique.

Case II.

Given a, c; required b, A, B.

$$\cos b = \frac{\cos c}{\cos a};$$

$$\sin A = \frac{\sin a}{\sin c};$$

$$\cos B = \frac{\tan a}{\tan c}.$$

This case presents an apparent ambiguity. But by Art. 190, a and A are of the same affection. ∴ the solution is unique.

Case III.

Given a, A; required c, b, B.

$$\sin c = \frac{\sin a}{\sin A};$$

$$\sin b = \frac{\tan a}{\tan A};$$

$$\sin B = \frac{\cos A}{\cos a}.$$

B, b, and c are given by their sines; hence there is, apparently at least, a series of ambiguities.

(*a*) If $\sin a = \sin A$, then $a = A$. (Art. 190, Exercise 1.)

$$\therefore \sin c = 1, \sin b = 1, \sin B = 1.$$

Hence in this case the solution is unique.

(*b*) There remain yet two possibilities for c; viz.:

I. c and a of like affection; then $b < 90°$, and $B < 90°$.
(Art. 190, Exercises 1 and 2.)

II. c and a of opposite affection; then $b > 90°$ and $B > 90°$.

These are the only alternatives.

Case IV.

Given a, B; required c, b, A.

$$\tan c = \frac{\tan a}{\cos B};$$

$$\cos A = \cos a \sin B;$$

$$\tan b = \sin a \tan B.$$

The solution is unique. Why?

Case V.

Given c, A; required a, b, B.

$$\sin a = \sin c \sin A;$$

$$\tan b = \tan c \cos A;$$

$$\cot B = \cos c \tan A.$$

The solution is unique. Why?

Case VI.

Given A, B; required a, b, c.

$$\cos a = \frac{\cos A}{\sin B};$$

$$\cos b = \frac{\cos B}{\sin A};$$

$$\cos c = \cot A \cot B.$$

195. Solution of the general triangle.

Case I. (Art. 194.)

Given a, b, c; required A, B, C.

This may be solved by (2),

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}.$$

This formula, however, is not adapted to logarithmic computation. When logarithms are employed, either (9), (10), or (11) may be used. If, however, *all* the angles are desired, (11) serves us best.

The solution is unique. Why?

EXAMPLE. Given

$$a = 93° 45',$$
$$b = 27° 16',$$
$$c = 88° 12'.$$

$$\tan \tfrac{1}{2} A = \sqrt{\frac{\sin (s-b) \sin (s-c)}{\sin s \cdot \sin (s-a)}}.$$

$$\therefore \log \tan \tfrac{1}{2} A = \tfrac{1}{2} [\log \sin (s-b) + \log \sin (s-c) - \log s - \log (s-a)].$$

$$s = 104° 36' 30'',$$
$$s - a = 10° 51' 30'',$$
$$s - b = 77° 20' 30'',$$
$$s - c = 16° 24' 30''.$$

$$\log \sin (s-b) + \log \sin (s-c) = 9.98936 + 9.45099 = 9.44035;$$
$$\log \sin s + \log \sin (s-a) = 9.98572 + 9.27504 = 9.26016;$$
$$\therefore \log \tan \tfrac{1}{2} A = .09009;$$
$$\therefore A = 101° 48' 4''.$$

Case II.

Given a, b, C; required A, B, c.

Formulæ (20) and (21) give

$$\tan \frac{(A+B)}{2} = \frac{\cos \frac{a-b}{2}}{\cos \frac{a+b}{2}} \cot \frac{C}{2};$$

$$\tan \frac{(A-B)}{2} = \frac{\sin \frac{(a-b)}{2}}{\sin \frac{(a+b)}{2}} \cot \frac{C}{2}.$$

Whence $A + B$ and $(A - B)$ can be found, and therefore A and B.

From (16), $$\cos \frac{c}{2} = \frac{\cos \frac{a+b}{2}}{\cos \frac{A+B}{2}} \sin \frac{C}{2},$$

which determines c.

If we wish to determine c independent of A and B, proceed as follows:

$$\cos c = \cos a \cos b + \sin a \sin b \cos C = \cos a (\cos b + \tan a \sin b \cos C).$$

Put $\tan a \cos C = \tan \phi$, which determines ϕ.

$$\therefore \cos c = \cos a \frac{(\cos b \cos \phi + \sin b \sin \phi)}{\cos \phi};$$

$$\therefore \cos c = \frac{\cos a \cos (b - \phi)}{\cos \phi}.$$

This formula is adapted to logarithmic computation.

Case III.

Given a, b, A; required B, c, C.

$$\sin B = \frac{\sin A}{\sin a} \sin b \ (\alpha); \qquad \text{(from (3))}$$

$$\tan \frac{c}{2} = \frac{\cos \frac{A+B}{2}}{\cos \frac{A-B}{2}} \tan \frac{a+b}{2}; \qquad \text{(from (22))}$$

$$\cot \frac{C}{2} = \frac{\cos \frac{a+b}{2}}{\cos \frac{(a-b)}{2}} \tan \frac{(A+B)}{2}. \qquad \text{(from (20))}$$

Discussion of the solution.

I. If $\sin A \sin b > \sin a$, no solution is possible; for $\sin B > 1$.

II. If $\sin a > \sin A \sin b$, equation (α) is satisfied by two supplementary values of B. Now $\frac{A+B}{2}$ and $\frac{a+b}{2}$ are of like affection, hence, if both values of B satisfy this condition, there are two solutions; otherwise, but one. This test requires that we first solve for B before we determine whether there are one or two solutions. There are several methods of determining this by an inspection of given data. The following is one, quoted from Wheeler's Trigonometry, where the reader is referred to Chauvenet's Trigonometry, p. 197:

(1) When b differs more from 90° than a, B must be in the same quadrant as b (Art. 177), and there can be but one solution. It remains to show that (2) when a differs more from 90° than b, there will be necessarily two solutions. We have

$$\sin c = \frac{\cos a - \cos b \cos c}{\sin b \cos A}. \qquad \text{(from (1))}$$

Two solutions exist so long as $\sin c$ is positive. Now, when a differs more from 90° than b, we have (neglecting the signs for a moment),

$$\cos a > \cos b > \cos b \cos c;$$

therefore, the numerator of the above value of $\sin c$ has the sign of $\cos a$. But by Art. 177, a and A are in the same quadrant, and therefore $\cos a$ and $\cos A$ have the same sign; therefore, the numerator and denominator have the same sign, and the value of $\sin c$ is positive, as was to be proved.

Hence, there is but one solution when the side opposite the given angle differs less from 90° than the other given side, and two solutions when the side opposite the given angle differs more from 90° than the other given side.

Case IV.

Given a, A, B; required C, b, c.

$$\sin b = \frac{\sin B}{\sin A} \cdot \sin a;$$

$$\tan\frac{c}{2} = \frac{\cos\frac{A+B}{2}}{\cos\frac{A-B}{2}}\tan\frac{a+b}{2};$$

$$\tan\frac{C}{2} = \frac{\sin\frac{(a-b)}{2}}{\cos\frac{a+b}{2}}\cot\frac{A+B}{2}.$$

This solution is also ambiguous.

If $\sin a \sin B > \sin A$, no solution is possible.

Now it is evident that the triangle ABC, and its polar triangle $A'B'C'$, have the same number of solutions.

The data given in this case determine *two sides* of $A'B'C'$ and *one angle* of $A'B'C'$ opposite one of these sides. Hence, by Case III., we can determine whether there are one or two solutions of $A'B'C'$; and, hence, whether there are one or two triangles of ABC satisfying the given data.

Case V.

Given c, A, B; required C, a, b.

$$\tan\frac{a+b}{2} = \frac{\cos\frac{A-B}{2}}{\cos\frac{A+B}{2}}\tan\frac{c}{2}; \qquad \text{(from (22))}$$

$$\tan\frac{a-b}{2} = \frac{\sin\frac{A-B}{2}}{\sin\frac{A+B}{2}}\tan\frac{c}{2}.$$

This determines $\frac{a+b}{2}$ and $\frac{a-b}{2}$, and hence a and b.

$$\tan\frac{C}{2}=\frac{\cos\frac{a-b}{2}}{\cos\frac{a+b}{2}}\cot\frac{A+B}{2}. \qquad \text{(from (20))}$$

The solution is unique.

Case VI.

Given A, B, C; required a, b, c.

$$\cos\tfrac{1}{2}a=\sqrt{\frac{\cos(S-B)\cos(S-C)}{\sin B\sin C}}; \qquad \text{(from (12))}$$

or, if all the sides are desired,

$$\tan\tfrac{1}{2}a=\sqrt{\frac{-\cos S\cos(S-A)}{\cos(S-B)\cos(S-C)}}. \qquad \text{(from (14))}$$

Formula (3) may be used as a check formula in each of the cases.

EXAMPLES. XLIII.

1. $a=84°\ 30'$; $b=46°\ 10'$; $C=75°\ 46'$. Find A, B, c.

2. In a right-angled triangle calculate A, B, c, having given

$a=86°\ 45'$; $b=108°\ 20'$.

3. In a right-angled triangle,

$a=63°\ 12'$; $B=75°\ 9'$. Find A, B, c.

4. A spherical triangle has,

$a=124°\ 12'\ 30''$; $b=54°\ 18'$; $c=97°\ 12'\ 30''$. Solve the triangle.

5. In a right triangle,

$A=125°$; $B=43°\ 18'$. Solve the triangle.

6. A spherical triangle has,

$a=125°\ 12'$; $b=107°\ 10'$; $c=45°$. Solve the triangle.

7. Having given,

$b=108°\ 30'$; $c=40°\ 50'$; $C=39°\ 50'$. Solve the triangle.

8. A spherical triangle has,

$a=110°\ 30'$; $c=16°\ 12'$; $C=84°\ 2'$. Solve the triangle.

9. Having given,

$A=66°\ 30'$; $B=75°\ 20'$; $C=110°\ 10'$. Solve the triangle.

10. In a spherical triangle,

$A=107°\ 36'\ 9''$; $B=36°\ 17'\ 20''$; $c=56°\ 21'\ 30''$. Solve the triangle.

11. In a right triangle,

$a=75°\ 24'$; $c=69°\ 20'$. Solve the triangle.

12. In a spherical triangle,

$a = 147° 49'$; $A = 137° 36'$; $C = 90°$. Find b, B, c.

13. In a right triangle,

$A = 73° 7'$; $c = 114° 32'$; $C = 90°$. Find a, B, b.

14. A spherical triangle,

$C = 146° 37' 54''$; $B = 55° 36''$; $b = 96° 34' 24''$. Solve the triangle.

15. In a right triangle,

$A = 85° 25'$; $B = 105° 12'$; $C = 90°$. Solve the triangle.

16. In a spherical triangle,

$A = 39° 45'$; $B = 26° 59'$; $c = 154° 47'$. Solve the triangle.

17. A spherical triangle has,

$a = 150° 57'$; $b = 134° 16'$; $A = 144° 23'$. Solve the triangle.

18. In a spherical triangle,

$a = 35° 37'$; $b = 59° 12'$; $C = 124° 18'$. Solve the triangle.

19. In a spherical triangle,

$A = 58° 16'$; $B = 41° 28'$; $c = 75° 54'$. Solve the triangle.

20. In a spherical triangle,

$a = 68° 16'$; $b = 53° 24'$; $C = 86° 40'$. Solve the triangle.

21. In a right triangle,

$A = 29° 33'$; $B = 87° 21'$; $C = 90°$. Solve the triangle.

ANSWERS TO EXAMPLES

II.

1. 80. **2.** 10. **3.** $109\frac{11}{21}$. **4.** 5 acres. **5.** $\frac{1760a}{b}$. **6.** $\frac{a \cdot c}{3}$ yds.

III.

(i.) **1.** .09175 of a right angle.
2. .0675 of a right angle.
3. 1.07875 of a right angle.
4. .180429 of a right angle.
5. $1.46\dot{7}$ of a right angle.
6. .54 of a right angle.

7. $1° 14' 15''$.
8. $7° 52' 30''$.
9. $153° 24' 29.34''$.
10. $21° 36' 8.1''$.
11. $16° 12' 37.26''$.
12. $31° 30'$.

(ii.) **1.** 2 right angles, or 180°.
2. $\frac{3}{2}$ of a right angle.
3. $\frac{2}{\pi}$ right angles.
4. $\frac{6}{\pi}$ right angles.
5. 2 right angles.
6. $\frac{4}{\pi^2}$ right angles.
7. $\frac{2\theta}{\pi}$ right angles.
8. .002 of a right angle.
9. 20 right angles.

(iii.) **1.** π. **2.** 2π. **3.** $\frac{\pi}{3}$. **4.** $\frac{\pi}{8}$. **5.** $\frac{\pi}{180}$. **6.** 1^c. **7.** $\frac{n}{180}\pi$. **8.** $\frac{1}{2}^c$. **9.** $\frac{A\pi}{180}$.

(v.) **1.** $\frac{1}{3}$. **2.** $\frac{\pi}{180}$. **3.** $\frac{63}{20}$.

IV.

1. $\frac{3}{2}$. **2.** 90. **3.** $4\frac{4}{5}$. **4.** $112\frac{1}{2}$ ft.
5. $5\frac{37}{63}$ ft. **6.** 838000 miles.
7. $\frac{1}{9}$ radian $= 6\frac{4}{11}$ degrees.
8. $21\frac{21}{44}$ degrees. **9.** $51\frac{6}{11}''$.
10. about 34 yds. **11.** 1 : 3.1416.
12. 3.1416. **13.** 3.1416.
14. 400 : 1. **15.** .0000484 ⋯.
16. $49\frac{1}{11}$ in. **18.** 473 : 489.
19. (i.) $k = 1$. (ii.) $k = \frac{180}{\pi}$.
20. 38°, 18°. **21.** $\frac{n\pi}{10800}$.
22. (i.) 120°, $\frac{2\pi}{3}$; (ii.) 135°, $\frac{3\pi}{4}$.
(iii.) 156°, $\frac{13}{15}\pi$.
23. (i.) $3\frac{3}{4}$; (ii.) $\frac{15}{2\pi}$. **24.** $\frac{19°}{10}$.
25. $\frac{9a + 10b}{10c}$. **26.** $\frac{1800\pi}{19\pi + 1800}$.
27. 9 or 16.

V.

1. $\sin A = \frac{16}{25}$; $\cos A = \frac{3\sqrt{41}}{25}$.

2. $\frac{1}{\sqrt{5}}$; $\frac{2}{\sqrt{5}}$; 2.

3. (i.) $\frac{DB}{CB}, \frac{BA}{CA}$; (ii.) $\frac{CD}{CB}, \frac{CB}{CA}$; (iii.) $\frac{DB}{CD}, \frac{BA}{CB}$; (iv.) $\frac{DB}{AB}, \frac{BC}{AC}$; (v.) $\frac{AD}{AB}, \frac{AB}{AC}$; (vi.) $\frac{DB}{AD}, \frac{BC}{AB}$.

4. (i.) $\frac{DA}{BA}$; (ii.) $\frac{BA}{EA}$ or $\frac{AC}{EC}$; (iii.) $\frac{DC}{BC}$; (iv.) $\frac{AB}{AE}$; (v.) $\frac{AD}{AB}$ or $\frac{AB}{AC}$; (vi.) $\frac{BD}{BC}$; (vii.) $\frac{DB}{CD}$ or $\frac{BA}{CB}$ or $\frac{AE}{CA}$; (viii.) $\frac{DA}{BD}$; (ix.) $\frac{BA}{EB}$ or $\frac{AC}{EA}$; (x.) $\frac{DC}{BD}$; (xi.) $\frac{DB}{AB}$ or $\frac{BC}{AC}$; (xii.) $\frac{BE}{AE}$.

6. Of the smaller angle, the sine $= \frac{5}{13}$, cosine $= \frac{12}{13}$, tangent $= \frac{5}{12}$.
Of the larger angle, the sine $= \frac{12}{13}$, cosine $= \frac{5}{13}$, tangent $= \frac{12}{5}$.

7. Of the smaller angle, the sine $= \frac{1}{2}$, cosine $= \frac{\sqrt{3}}{2}$, tangent $= \frac{1}{\sqrt{3}}$.
Of the larger angle, the sine $= \frac{1}{2}\sqrt{3}$, cosine $= \frac{1}{2}$, tangent $= \sqrt{3}$.

9. $BC = \sqrt{3}$; $\sin A = \frac{1}{2}\sqrt{3}$; $\cos A = \frac{1}{2}$; $\tan A = \sqrt{3}$.

11. $AC = \sqrt{2}$; $\sin B = \sqrt{\frac{2}{3}}$; $\sin A = \frac{1}{\sqrt{3}}$.

VII.

1. 179 ft.
2. 346 ft.
3. 86.6 ft.
4. 138.5 ft.
5. $7\frac{1}{2}$ ft.
6. 60°; 173 ft.
7. 63.17 yds.
8. 192.8 ft.
9. 34.15 ft.
10. 73.2 ft.
11. 86.6 ft.
12. 1524 yds.
13. 173.2 yds.
14. 373 ft.
15. $\frac{\sqrt{3}\,a}{3\,b}$.
16. 30°.
17. About 523.6 miles.

XI.

1. 60°.
2. − 100°.
3. 0°.
4. − 260°.
5. 115°.
6. 410°.
7. $-\frac{1}{4}\pi$.
8. $\frac{2}{3}\pi$.

XII.

1. $+\frac{1}{2}$; $-\frac{\sqrt{3}}{2}$; $-\frac{1}{\sqrt{3}}$.
2. $\frac{1}{\sqrt{2}}$; $-\frac{1}{\sqrt{2}}$; -1.
3. $+\frac{\sqrt{3}}{2}$; $-\frac{1}{2}$; $-\sqrt{3}$.
4. $-\frac{1}{2}$; $+\frac{\sqrt{3}}{2}$; $-\frac{1}{\sqrt{3}}$.
5. $-\frac{1}{\sqrt{2}}$; $\frac{1}{\sqrt{2}}$; -1.
6. $\frac{\sqrt{3}}{2}$; $+\frac{1}{2}$; $+\sqrt{3}$.
7. $-\frac{1}{\sqrt{2}}$; $-\frac{1}{\sqrt{2}}$; $+1$.
8. $-\frac{1}{\sqrt{2}}$; $-\frac{1}{\sqrt{2}}$; $+1$.
9. $+\frac{1}{2}$; $+\frac{\sqrt{3}}{2}$; $+\frac{1}{\sqrt{3}}$.
10. $+\frac{1}{2}$; $+\frac{\sqrt{3}}{2}$; $+\frac{1}{\sqrt{3}}$.
11. $-\frac{\sqrt{3}}{2}$; $-\frac{1}{2}$; $+\sqrt{3}$.
12. $-\frac{\sqrt{3}}{2}$; $+\frac{1}{2}$; $-\sqrt{3}$.
13. $+\frac{1}{\sqrt{2}}$; $+\frac{1}{\sqrt{2}}$; $+1$.
14. $\frac{\sqrt{3}}{2}$; $-\frac{1}{2}$; $-\sqrt{3}$.
15. $-\frac{1}{2}$; $-\frac{\sqrt{3}}{2}$; $+\frac{1}{\sqrt{3}}$.

16. $36°$; $150°$; $-210°$; $-330°$.
17. $45°$; $135°$; $-225°$; $-315°$.
18. $60°$; $120°$; $-240°$; $-300°$.
19. $-30°$; $-150°$; $210°$; $330°$.
20. $20°$; $160°$; $380°$; $520°$.
21. $\frac{5}{4}\pi$; $\frac{7}{4}\pi$; $\frac{13}{4}\pi$; $\frac{15}{4}\pi$.
22. $\frac{8}{7}\pi$; $\frac{13}{7}\pi$; $\frac{22}{7}\pi$; $\frac{27}{7}\pi$.
25. The tangent.
26. No.

XV. (*a*).

1. $\sin A = \pm\sqrt{1-\cos^2 A}$; $\tan A = \pm\dfrac{\sqrt{1-\cos^2 A}}{\cos A}$;

$\cot A = \pm\dfrac{\cos A}{\sqrt{1-\cos^2 A}}$; $\sec A = \dfrac{1}{\cos A}$; $\csc A = \pm\dfrac{1}{\sqrt{1-\cos^2 A}}$.

2. $\sin(90° + A) = \dfrac{\cot A}{\sqrt{\cot^2 A + 1}}$; $\cos(90° + A) - \dfrac{1}{\sqrt{\cot^2 A + 1}}$;

$\tan(90° + A) = -\cot A$; $\sec(90° + A) = -\sqrt{\cot A + 1}$;

$\csc(90° + A) = \dfrac{\sqrt{\cot^2 A + 1}}{\cot A}$.

3. $\sin(180° - A) = \pm\dfrac{\sqrt{\sec^2 A - 1}}{\sec A}$; $\cos(180° - A) = -\dfrac{1}{\sec A}$;

$\tan(180° - A) = \pm\sqrt{\sec A - 1}$, etc.

4. $\sin A = \dfrac{1}{\csc A}$; $\cos A = \dfrac{\pm\sqrt{\csc^2 A - 1}}{\csc A}$; $\tan A = \dfrac{1}{\pm\sqrt{\csc^2 A - 1}}$, etc.

5. $\sin A = \sin A$; $\cos A = \pm\sqrt{1-\sin^2 A}$; $\tan A = \pm\dfrac{\sin A}{\sqrt{1-\sin^2 A}}$, etc.

6. See answers to Example 1.

(*b*) 1. $\tan A = \pm\frac{3}{4}$; $\operatorname{cosec} A = \frac{5}{3}$.

2. $\sin B = \dfrac{2\sqrt{2}}{3}$; $\cot B = \dfrac{1}{2\sqrt{2}}$.

3. $\sin A = \frac{4}{5}$; $\sec A = \frac{5}{3}$.

4. $\cot\theta = \dfrac{1}{\sqrt{15}}$; $\sin\theta = \dfrac{\sqrt{15}}{4}$.

5. $\sin\theta = \dfrac{\sqrt{3}}{2}$; $\cos\theta = \frac{1}{2}$.

6. $\sin\theta = \dfrac{\sqrt{5}}{3}$; $\sin\theta = \frac{3}{2}$.

7. $\dfrac{b}{\pm\sqrt{c^2 - b^2}}$.

8. $\dfrac{a}{\pm\sqrt{a^2+1}}$; $\dfrac{1}{\sqrt{a^2+1}}$.

9. $\sin\theta = \dfrac{\sqrt{a^2-1}}{a}$, $\cot\theta = \dfrac{1}{\sqrt{a^2-1}}$.

11. $h^2(1 + k^2) = 1$.

12. $\cos\theta = \frac{4}{5}$; $\tan\theta = \frac{3}{4}$; $\cot\theta = \frac{4}{3}$; $\sec\theta = \frac{5}{4}$; $\csc\theta = \frac{5}{3}$.

XVI.

1. $45°$; $135°$.
2. $30°$; $150°$; $210°$; $330°$.
3. $45°$; $135°$; $225°$; $315°$.
4. $60°$; $120°$; $240°$; $300°$.
5. $30°$; $150°$; $210°$; $330°$.
6. $30°$; $150°$; $210°$; $330°$.
7. $30°$; $150°$.
8. $45°$; $315°$; $0°$; $180°$.
9. $90°$; $270°$; $60°$; $120°$.
10. $60°$; $120°$; $240°$; $300°$.
11. $45°$; $225°$.
12. $90°$; $70°$; $45°$; $315°$.
13. $0°$; $180°$; $45°$; $135°$.

14. 45°; 135°; 225°; 315°. **15.** 45°; 135°. **16.** 30°; 330°. **17.** 30°; 150°.

18. ± 60°; ± 120°; ± 240°; ± 300°.

19. 0°; 180°; − 180°; − 360°.

20. $2n\pi \pm \frac{\pi}{4}$; (n any integer).

21. $n\pi$; $2n\pi + \frac{\pi}{3}$; $(2n+1)\pi - \frac{\pi}{3}$.

22. $n\pi + \frac{\pi}{4}$.

XXI.

1. $\sin(\theta + \phi) + \sin(\theta - \phi)$.

2. $\cos(\alpha - \beta) + \cos(\alpha + \beta)$.

3. $\sin(2\alpha + 3\beta) + \sin(2\alpha - 3\beta)$.

4. $\cos 2\alpha + \cos 2\beta$.

5. $\sin 8\theta - \sin 2\theta$. **6.** $\cos\theta + \cos 2\theta$. **7.** $\frac{1}{2}(\cos 3\theta - \cos 5\theta)$.

8. $\frac{1}{2}(\sin 4\theta - \sin\theta)$.

9. $\sin 60° + \sin 40°$.

10. $\frac{1}{2}(\sin 60° - \sin 30°)$.

11. $2\cos 3\theta \cos 2\theta$.

12. $-\cos 4\theta \sin 2\theta$.

13. $4\cos^2\frac{\theta}{2}\sin 2\theta$.

XXV.

1. $\frac{3}{\sqrt{10}}$. **2.** $\frac{2}{\sqrt{3}}$. **3.** $\frac{2}{\sqrt{3}}$. $-\sqrt{\frac{7}{3}}$. **13.** 1.

XXVI.

1. 60°. **2.** 120°. **3.** 30°; 150°. **4.** 135°. **5.** 45°; 135°. **6.** 120°.

XXVIII.

1. $\cos A = \frac{1}{2}$; $\cos\frac{1}{2}A = \frac{1}{2}\sqrt{3}$.

2. 45°; 60°; 75°.

3. 135°; 30°; 15°. **4.** 3. **5.** 14.

6. $1 + \sqrt{3}$. **7.** 120°. **8.** 120°.

9. 120°. **10.** 90°; 36° 52′.

11. 130° 27′. **12.** 125° 6′.

13. 120°. **14.** $a = 1$.

15. $C = 30°$; $a = \sqrt{3} + 1$; $b = 2$.

16. $A = 75°$; $a = b = \sqrt{3} + 1$.

17. $C = 60°$ or 120°.

18. 60°; 75°; 6 yds.

21. $15 : 8\sqrt{3} : 4\sqrt{5} + 6$.

XXIX.

1. 3; $\frac{10}{3}$; $\frac{1}{4}$; $\frac{2}{3}$; $-\frac{5}{2}$.

2. 3; 6; −1; −3; −6, 2.

3. 2; 4; −1; −3; −4.

4. 3; −1; 5; −2; 3; −3.

5. $\frac{4}{3}$; $\frac{2}{3}$; $\frac{1}{3}$; $\frac{4}{3}$; $\frac{1}{2}$.

XXX.

1. 3.55169; .87251; $\bar{3}$.63548.

2. 4.93662; 1.98320; $\bar{3}$.54654.

3. $\bar{2}$.65920; $\bar{2}$.54926; $\bar{3}$.85941.

4. 6.83181; 7.51129.

XXXI.

1. 3.7040. **2.** 45740.2. **3.** 2492830. **4.** .00043965. **5.** 5.68915.

XXXII.

1. 7.669. **2.** 3.809. **3.** 47.32. **4.** 55460. **5.** 12.03. **6.** .04023. **7.** 8287. **8.** 1165. **9.** .3107. **10.** .6731. **11.** 1.096. **12.** 823.6. **13.** 2.624. **14.** 22.51. **15.** 23.28.

XXXIII.

1. $C = 60.42$; $b = 59.1$; $B = 78°$. 2. $b = 21.14$; $A = 52° 49' 55''$; $a = 27.89$.

XXXIV.

1. $A = 41° 16' 52''$.
2. $73° 32' 12''$; $62° 46' 18''$.
3. $60° 50' 51''$; $73° 1' 50''$.
4. $47° 25' 40''$.
5. $64° 31' 58''$.
6. $135°$; $30°$; $15°$.

XXXV.

1. $a = 313.46$ yds. 2. 1192.55 yds. 3. 22.415 ft. 4. $a = .01116$; $b = .006962$.

XXXVI.

1. $57° 27' 25''$; $62° 32' 35''$.
2. $64° 26' 47''$; $37° 7' 13'$.
3. $A = 24° 41' 48''$; $c = .5886$.
4. $72° 12' 59''$.
5. 14.

XXXVII.

2. $A = 51° 18' 21''$; $C = 88° 41' 39''$; or $A = 128° 41' 39''$; $C = 11° 18' 21''$.
3. $B = 38° 38' 24''$; $C = 91° 91' 36''$; $c = 155.3$. 4. No.

XXXVIII.

1. $28° 35' 39''$.
2. $104° 44' 39''$.
3. $32° 20' 48''$.
4. $43° 40'$.
5. $128° 23' 13''$.
6. 106531 ft.
7. 3437.6 yds.
8. 1728.2 chains.
9. 25376 yds.
10. $A = 66° 27' 48''$; $B = 12° 55' 12''$.
11. $A = 92° 12' 53''$; $B = 35° 37' 7''$.
12. $B = 29° 1' 40''$; $C = 74° 55' 50''$.
13. $B = 70° 35' 24''$, or $109° 24' 36''$.
14. $B = 51° 56' 17''$, or $128° 3' 43''$.
15. $B = 62° 6' 10''$, or $117° 53' 50''$.
16. Very nearly $90°$.
17. 1319.6 yds.

XLII.

1. $\sin A = \frac{3}{5}$; $\cos A = \frac{4}{5}$. 2. $\frac{80}{3}\sqrt{3}$ ft. $= 46.19 \cdots$ ft. 6. 4227.47 ft.

9. $30°, 60°, 90°, 120°$, etc., have for sine $\frac{1}{2}, \sqrt{\frac{3}{2}}, 1, \sqrt{\frac{3}{2}}, \frac{1}{2}, 0, -\frac{1}{2}, -\sqrt{\frac{3}{2}} - 1, -\sqrt{\frac{3}{2}}, -\frac{1}{2}$, respectively.

12. The other sides are 765.4321 ft.; 1006.6 ft.

15. $30°, 60°, 90°$, etc., have for tan $\frac{1}{3}\sqrt{3}, \sqrt{3}, \infty, -\sqrt{3}, -\frac{1}{3}\sqrt{3}, 0, \frac{1}{3}\sqrt{3}, \infty, -\sqrt{3}, -\frac{1}{3}\sqrt{3}$, respectively.

17. $\frac{168}{193}, \frac{168}{195}, \frac{32592}{193 \times 195}$. 19. $\sec A = \frac{3}{4}\sqrt{2}$. 20. (i.) $\frac{15}{2}$; (ii.) -15.

21. $+\sin 18°$, $-\cos 18°$, $-\tan 18°$; $-\sin 30°$, $-\cos 36°$, $+\tan 36°$; $-\sin 36°$, $+\cos 36°$, $-\tan 36°$. 22. $\cos 5a = 16\cos^5 a - 20\cos^3 a + 5\cos a$.

23. (i.) $0, n\pi, \frac{1}{3}\pi$; (ii.) $\cos\theta = \frac{1}{2}$, or $\sin(\theta - 45°) = \frac{1}{\sqrt{2}}$.

24. $\frac{1}{26}\sqrt{651} = .981 \cdots$. 25. $\frac{11}{630}A$ radians; $5.72965°$.

26. sine, $\frac{4}{5}$; tan, $\frac{4}{3}$; cot, $\frac{3}{4}$; cosec, $\frac{5}{4}$; sec, $\frac{5}{3}$.

29. (i.) $6\log_{10}3+3\log_{10}2-3$; (ii.) $\frac{1}{3}\{\log_{10}7+1-\log_{10}2\}$; (iii.) $3\log_{10}7+3\log_{10}2-2\log_{10}3-2$.

31. $\frac{630}{11}\theta$ deg.; 19.09854°. **38.** $-\frac{2}{3}\sqrt{2}$.

34. $\frac{9}{2}$; -9. **39.** π; $\frac{1}{3}\pi$; $\frac{5}{3}\pi$.

36. $C=18^\circ$, $a=c=2\div\sqrt{(10-2\sqrt{5})}$. **41.** 1 ft., 120°, 30°; or 2 ft., 60°, 90°.

37. -1320°. **42.** $104^\circ\,28'\,39''$.

43. -630°. **44.** $-\frac{1}{5}\sqrt{5}$. **46.** 0; π; $\frac{2}{3}\pi$; $\frac{4}{3}\pi$.

48. $\frac{1}{2}\{\sqrt{6}\pm\sqrt{2}\}$ and 15°, 135°; or, 105°, 45°.

49. 9°; $286^\circ\,28'\,41.16''$; $\frac{7}{8}$.

52. (i.) $n\pi\pm\frac{1}{3}\pi$; (ii.) $\frac{1}{2}n\pi\pm\frac{1}{8}\pi$, or $n\pi+(-1)^n\frac{1}{4}\pi$.

53. $37\frac{1}{2}$ sq. ft. **54.** $40^\circ\,29'\,19.85''$. **55.** $\frac{1}{1080}\pi$; $\frac{1}{10}\pi$; $\frac{5}{4}$.

56. $\tan\alpha=4\sqrt{3}$, $\operatorname{cosec}\alpha=\frac{7}{12}\sqrt{3}$.

58. (i.) $n\pi\pm\frac{1}{4}\pi$; (ii.) $\frac{1}{2}n\pi\pm\frac{1}{8}\pi$; or, $2n\pi\pm\frac{1}{4}\pi$.

59. $2\sqrt{14}$ sq. ft. **60.** $38^\circ\,25'\,32.7''$.

64. (i.) $1-\log_{10}5+\frac{1}{2}\log_{10}7$; $1-4\log_{10}5+5\log_{10}3$; $2-5\log_{10}5-2\log_{10}3+2\log_{10}7$.

66. 192 ft., 185 ft., and 9234 sq. ft. **67.** 2.3 radians $=131.779926^\circ$.

69. .69897; .77815; $\bar{2}.33445$; $\bar{1}.91480$.

72. $78^\circ\,10'$; $70^\circ\,30'$; 9234 sq. ft. **73.** $\frac{4}{35}\pi$; $\frac{9}{35}\pi$; $\frac{14}{35}\pi$; $\frac{19}{35}\pi$; $\frac{24}{35}\pi$.

76. 116.6. **78.** $135^\circ, 15^\circ$; or $45^\circ, 105^\circ$. **79.** $\frac{2}{21}\pi$; $\frac{4}{21}\pi$; $\frac{6}{21}\pi$; $\frac{8}{21}\pi$; $\frac{10}{21}\pi$; $\frac{12}{21}\pi$.

81. 32.92…. **84.** 7 ft.; $\sqrt{19}$ ft.; $\frac{15}{4}\sqrt{3}$ sq. ft. **88.** 1035.43 ft.; 765.4321 ft.; 66°.

89. 6.981 ft. **90.** $\frac{1}{2}$; $-\frac{1}{2}$. **91.** 4.49999. **94.** 3210.793.

XLIII.

1. $A=95^\circ\,56'\,18''$; $B=46^\circ\,7'\,15''$; $c=75^\circ\,56'\,27''$.

2. $A=86^\circ\,54'\,53''$; $B=108^\circ\,18'\,23''$; $c=91^\circ\,1'\,12''$.

3. $c=82^\circ\,37'\,24''$; $A=64^\circ\,9'\,34''$; $b=73^\circ\,5'\,32''$.

4. $A=127^\circ\,22'$; $B=51^\circ\,18'$; $C=72^\circ\,27'$.

5. $a=146^\circ\,35'\,20''$; $b=27^\circ\,19'$; $c=137^\circ\,59'\,33''$.

6. $A=129^\circ\,49'\,44''$; $C=46^\circ\,32'\,48''$; $B=78^\circ\,47'\,48''$.

7. $B=68^\circ\,18'$; $A=132^\circ\,33'\,48''$; $a=131^\circ\,15'\,48''$; or, $B=111^\circ\,42'$; $A=77^\circ\,4'\,36''$; $a=95^\circ\,50'$.

9. $a=69^\circ\,56'\,30''$; $b=82^\circ\,16'\,28''$; $c=105^\circ\,57'$.

10. $a=72^\circ\,44'$; $b=36^\circ\,22'$; $C=56^\circ\,12'$.

11. Impossible.

12. $B=60^\circ\,45'$; $b=43^\circ\,34'$; $c=52^\circ\,11'$; or, $B=119^\circ\,15'$; $b=136^\circ\,25'$; $c=127^\circ\,49'$.

13. $a=60^\circ\,31'\,24''$; $B=143^\circ\,50'$; $b=147^\circ\,32'\,6''$.

14. $a=55^\circ$; $c=138^\circ\,10'$; $A=42^\circ\,30'$.

15. $a=85^\circ\,15'$; $b=105^\circ\,15'\,15''$; $c=91^\circ\,15'$.

16. $a=121^\circ\,27'\,30''$; $b=37^\circ\,15'\,30''$; $C=161^\circ\,22'$.

17. $c=23^\circ\,57'$; $B=59^\circ\,11'$; $C=29^\circ\,9'$; or, $c=55^\circ\,42'$; $B=120^\circ\,49'$; $C=97^\circ\,44'$.

18. $c = 82° 16'$;
$A = 29° 2'$;
$B = 45° 44'$.

19. $a = 58° 33' 37''$;
$b = 41° 47' 40''$;
$C = 104° 48' 4''$.

20. $A = 74° 2' 53''$;
$B = 56° 11' 55''$;
$c = 74° 41' 4''$.

21. $a = 29° 26' 30''$;
$b = 84° 37' 15''$;
$c = 85° 19' 2''$.

TABLES

LOGARITHMIC AND TRIGONOMETRIC

CALCULATED TO FIVE PLACES OF DECIMALS

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TABLES

LOGARITHMIC AND TRIGONOMETRIC

CALCULATED TO

FIVE PLACES OF DECIMALS

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BRIEF EXPLANATIONS AND RULES

FOR THE

USE OF THESE TABLES

THE logarithm of a number consists in general of two parts, — an integral part, and a decimal. The integral part is called the **characteristic,** and the decimal part, when it is so written that it is positive, is called the **mantissa.**

RULE I. *The characteristic of the logarithm of a number greater than unity is less by one than the number of digits in its integral part, and is positive.*

Thus, the characteristic of the logarithm of 48226 is 4.

RULE II. *The characteristic of the logarithm of a decimal fraction is greater by unity than the number of ciphers immediately after the decimal point, and is negative.*

Thus, the characteristic of the logarithm of .048226 is $\bar{2}$. We indicate that the characteristic is negative by writing the minus sign above it.

TABLE I

To find the logarithm of a number.

(*a*) *When the number is between 1 and 100.*

The logarithm is on page 1.

(*b*) *When the number consists of one or two significant figures.*

The mantissa is on page 1.

The characteristic is found by Rule I. or II.

Thus,

$$\log 8.5 = 0.92942\,;$$
$$\log .85 = \bar{1}.92942\,;$$
$$\log .085 = \bar{2}.9242.$$

(*c*) *When the number contains three significant figures.*

In the column headed *No.*, find the number. On a line with it, and in the column having 0 at the top, is the mantissa. Prefix the characteristic (Rules I. and II.).

Thus,
$$\log 574 = 2.75891;$$
$$\log 57.40 = 1.75891;$$
$$\log .0574 = \bar{2}.75891.$$

(*d*) *When the number contains four significant figures.*

In the column headed *No.*, find the first three significant figures. On a line with these, and in the column having at the top the fourth significant figure, is the mantissa. Prefix the proper characteristic (Rules I. and II.).

Thus,
$$\log 9275 = 3.96731;$$
$$\log 9.275 = 0.96731;$$
$$\log .09275 = \bar{2}.96731.$$

(*e*) *When the number contains more than four significant figures.*

Suppose the logarithm of 62543 is required. Since the number lies between 62540 and 62550, its logarithm lies between their logarithms. In the column headed *No.*, find the first three figures. On a line with these, and in the columns having 4 and 5 at the top, are the mantissæ .79616 and .79623. Prefixing the proper characteristic, we have

$$\log 62550 = 4.79623$$
$$\log 62540 = 4.79616$$

Differences, 10 .00007

Here we see that while the number increases from 62540 to 62550, the logarithm increases .00007. Now our number, 62543, is $\frac{3}{10}$ of the way from 62540 to 62550; hence, if to the logarithm of 62540 we add $\frac{3}{10}$ of .00007, a nearly correct logarithm of 62543 is obtained.

Thus,
$$\log 62540 = 4.79616$$
$$\text{correction} = .00002$$
$$\therefore \log 62543 = 4.79618$$

We have here assumed that the differences of logarithms are proportional to the differences of their corresponding numbers, which gives us results that are approximately correct. For greater accuracy we must use tables of more places.

To find the number corresponding to a logarithm.

(*a*) *When the given mantissa can be found in the table.*

The first three figures of the number are in the column headed *No.*, and on a line with the mantissa; the fourth figure is at the top of the column containing the mantissa.

Find the number whose logarithm is 2.93202.

The mantissa, found on page 17, corresponds to the number 8551. As the characteristic is 2, the required number is 855.1.

(*b*) *When the given mantissa cannot be found in the table.*

Find the number whose logarithm is 8.82252 – 10 or $\bar{2}.82252$.

As the exact mantissa is not in the table, take out the next larger, .82256, and the next smaller, .82249, and retain the characteristic in arranging the work.

Thus, the number corresponding to	$\bar{2}.82256$	is	.06646
and the number corresponding to	$\bar{2}.82249$	is	.06645
Differences,	.00007		.00001

Now the given logarithm, $\bar{2}.82252$, is .00003 greater than the smaller of the two logarithms, and the difference in logarithms of .00007 corresponds to a difference in numbers of .00001; therefore we should increase the number corresponding to the logarithm $\bar{2}.82249$ by $\frac{.00003}{.00007}$, or $\frac{3}{7}$ of .00001.

Thus, the number corresponding to $\bar{2}.82249$	= .06645.
and the correction ($\frac{3}{7}$ of .00001)	= .000004
∴ the number corresponding to $\bar{2}.82252$	= .066454

TABLE II

This table contains the logarithmic sine, tangent, cotangent, and cosine for every ten seconds from 0° to 2°, and for every minute from 1° to 89°.

To find the logarithmic sine, tangent, cotangent, or cosine of an angle less than 90°.

When the angle is less than 45°, use the column headings at the top of the page and the *left-hand* minute column. When the angle is greater than 45°, use the column headings at the bottom of the page and the *right-hand* minute column.

Find $\log \sin 20° \; 25' \; 12''$.

On page 43, we find

$$\log \sin 20° \; 25' = 9.54263.$$

This logarithm must be increased by $\frac{12}{60}$ of the difference between it and $\log \sin 20° \; 26'$.

$$\therefore \log \sin 20° \; 25' \; 12'' = 9.54263 + \tfrac{1}{5} \text{ of } .00034 = 9.54270.$$

Find $\log \tan 52° \; 17' \; 10''$.

On page 52, we find

$$\log \tan 52° \; 17' = 10.11162.$$

This value must be increased by $\frac{10}{60}$ of the difference between it and $\log \tan 52° \; 18'$.

$$\therefore \log \tan 52° \; 17' \; 10'' = 10.11162 + \tfrac{1}{6} \text{ of } .00026 = 10.11166.$$

NOTE. In finding log sin or log tan we **add** the correction, but **subtract** in finding log cos or log cot.

For closer work, larger tables, such as those of Vega, should be employed.

To find an angle less than 90°, having given its logarithmic sine, tangent, cotangent, or cosine.

Find the angle for which $\log \cos = 9.94065 - 10$.

On page 48, we find the next smaller logarithm, $9.94062 - 10$, which corresponds to the angle 29° 17′, and the next larger logarithm, which corresponds to the angle 29° 16′.

$$\begin{array}{lrr} \text{Thus,} & 9.94069 - 10 = \log\cos 29^\circ\ 16' \\ & 9.94062 - 10 = \log\cos 29^\circ\ 17' \\ \hline \text{Differences,} & .00007 \qquad\qquad\qquad 1' \end{array}$$

The given logarithm is .00003 larger than the smaller of these logarithms; therefore we have a correction of $\frac{.00003}{.00007}$ or $\frac{3}{7}$ of $60''$ to make.

Thus, log cos 9.94065 − 10 corresponds to angle $29^\circ\ 17' - \frac{3}{7}$ of $60'' = 29^\circ\ 16'\ 26''$.

Find the angle for which log tan = 0.15782.

On page 50, we find

$$\begin{array}{lr} & 0.15800 = 55^\circ\ 12' \\ & 0.15773 = 55^\circ\ 11' \\ \hline \text{Differences,} & .00027 \qquad\quad 1' \end{array}$$

The given logarithm is .00009 larger than the smaller of these logarithms; therefore we have a correction of $\frac{.00009}{.00027}$ or $\frac{1}{3}$ of $60''$ to make.

Thus, log tan 0.15782 corresponds to angle $55^\circ\ 11' + \frac{1}{3}$ of $60''$ $= 55^\circ\ 11'\ 20''$.

NOTE. In finding the angle corresponding to log sin or log tan we **add** the correction, but **subtract** for log cos or log cot.

TABLE III (*a*), (*b*), (*c*), (*d*)

These tables contain the natural trigonometric functions from 0° to 90° at intervals of 6′.

TABLE I

THE

COMMON LOGARITHMS

OF THE

NATURAL NUMBERS

FROM 1 TO 10,000

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
1	0.00000	21	1.32222	41	1.61278	61	1.78533	81	1.90849
2	0.30103	22	1.34242	42	1.62325	62	1.79239	82	1.91381
3	0.47712	23	1.36173	43	1.63347	63	1.79934	83	1.91908
4	0.60206	24	1.38021	44	1.64345	64	1.80618	84	1.92428
5	0.69897	25	1.39794	45	1.65321	65	1.81291	85	1.92942
6	0.77815	26	1.41497	46	1.66276	66	1.81954	86	1.93450
7	0.84510	27	1.43136	47	1.67210	67	1.82607	87	1.93952
8	0.90309	28	1.44716	48	1.68124	68	1.83251	88	1.94448
9	0.95424	29	1.46240	49	1.69020	69	1.83885	89	1.94939
10	1.00000	30	1.47712	50	1.69897	70	1.84510	90	1.95424
11	1.04139	31	1.49136	51	1.70757	71	1.85126	91	1.95904
12	1.07918	32	1.50515	52	1.71600	72	1.85733	92	1.96379
13	1.11394	33	1.51851	53	1.72428	73	1.86332	93	1.96848
14	1.14613	34	1.53148	54	1.73239	74	1.86923	94	1.97313
15	1.17609	35	1.54407	55	1.74036	75	1.87506	95	1.97772
16	1.20412	36	1.55630	56	1.74819	76	1.88081	96	1.98227
17	1.23045	37	1.56820	57	1.75587	77	1.88649	97	1.98677
18	1.25527	38	1.57978	58	1.76343	78	1.89209	98	1.99123
19	1.27875	39	1.59106	59	1.77085	79	1.89763	99	1.99564
20	1.30103	40	1.60206	60	1.77815	80	1.90309	100	2.00000

No.	0	1	2	3	4	5	6	7	8	9
100	00 000	00 043	00 087	00 130	00 173	00 217	00 260	00 303	00 346	00 389
101	00 432	00 475	00 518	00 561	00 604	00 647	00 689	00 732	00 775	00 817
102	00 860	00 903	00 945	00 988	01 030	01 072	01 115	01 157	01 199	01 242
103	01 284	01 326	01 368	01 410	01 452	01 494	01 536	01 578	01 620	01 662
104	01 703	01 745	01 787	01 828	01 870	01 912	01 953	01 995	02 036	02 078
105	02 119	02 160	02 202	02 243	02 284	02 325	02 366	02 407	02 449	02 490
106	02 531	02 572	02 612	02 653	02 694	02 735	02 776	02 816	02 857	02 898
107	02 938	02 979	03 019	03 060	03 100	03 141	03 181	03 222	03 262	03 302
108	03 342	03 383	03 423	03 463	03 503	03 543	03 583	03 623	03 663	03 703
109	03 743	03 782	03 822	03 862	03 902	03 941	03 981	04 021	04 060	04 100
110	04 139	04 179	04 218	04 258	04 297	04 336	04 376	04 415	04 454	04 493
111	04 532	04 571	04 610	04 650	04 689	04 727	04 766	04 805	04 844	04 883
112	04 922	04 961	04 999	05 038	05 077	05 115	05 154	05 192	05 231	05 269
113	05 308	05 346	05 385	05 423	05 461	05 500	05 538	05 576	05 614	05 652
114	05 690	05 729	05 767	05 805	05 843	05 881	05 918	05 956	05 994	06 032
115	06 070	06 108	06 145	06 183	06 221	06 258	06 296	06 333	06 371	06 408
116	06 446	06 483	06 521	06 558	06 595	06 633	06 670	06 707	06 744	06 781
117	06 819	06 856	06 893	06 930	06 967	07 004	07 041	07 078	07 115	07 151
118	07 188	07 225	07 262	07 298	07 335	07 372	07 408	07 445	07 482	07 518
119	07 555	07 591	07 628	07 664	07 700	07 737	07 773	07 809	07 846	07 882
120	07 918	07 954	07 990	08 027	08 063	08 099	08 135	08 171	08 207	08 243
121	08 279	08 314	08 350	08 386	08 422	08 458	08 493	08 529	08 565	08 600
122	08 636	08 672	08 707	08 743	08 778	08 814	08 849	08 884	08 920	08 955
123	08 991	09 026	09 061	09 096	09 132	09 167	09 202	09 237	09 272	09 307
124	09 342	09 377	09 412	09 447	09 482	09 517	09 552	09 587	09 621	09 656
125	09 691	09 726	09 760	09 795	09 830	09 864	09 899	09 934	09 968	10 003
126	10 037	10 072	10 106	10 140	10 175	10 209	10 243	10 278	10 312	10 346
127	10 380	10 415	10 449	10 483	10 517	10 551	10 585	10 619	10 653	10 687
128	10 721	10 755	10 789	10 823	10 857	10 890	10 924	10 958	10 992	11 025
129	11 059	11 093	11 126	11 160	11 193	11 227	11 261	11 294	11 327	11 361
130	11 394	11 428	11 461	11 494	11 528	11 561	11 594	11 628	11 661	11 694
131	11 727	11 760	11 793	11 826	11 860	11 893	11 926	11 959	11 992	12 024
132	12 057	12 090	12 123	12 156	12 189	12 222	12 254	12 287	12 320	12 352
133	12 385	12 418	12 450	12 483	12 516	12 548	12 581	12 613	12 646	12 678
134	12 710	12 743	12 775	12 808	12 840	12 872	12 905	12 937	12 969	13 001
135	13 033	13 066	13 098	13 130	13 162	13 194	13 226	13 258	13 290	13 322
136	13 354	13 386	13 418	13 450	13 481	13 513	13 545	13 577	13 609	13 640
137	13 672	13 704	13 735	13 767	13 799	13 830	13 862	13 893	13 925	13 956
138	13 988	14 019	14 051	14 082	14 114	14 145	14 176	14 208	14 239	14 270
139	14 301	14 333	14 364	14 395	14 426	14 457	14 489	14 520	14 551	14 582
140	14 613	14 644	14 675	14 706	14 737	14 768	14 799	14 829	14 860	14 891
141	14 922	14 953	14 983	15 014	15 045	15 076	15 106	15 137	15 168	15 198
142	15 229	15 259	15 290	15 320	15 351	15 381	15 412	15 442	15 473	15 503
143	15 534	15 564	15 594	15 625	15 655	15 685	15 715	15 746	15 776	15 806
144	15 836	15 866	15 897	15 927	15 957	15 987	16 017	16 047	16 077	16 107
145	16 137	16 167	16 197	16 227	16 256	16 286	16 316	16 346	16 376	16 406
146	16 435	16 465	16 495	16 524	16 554	16 584	16 613	16 643	16 673	16 702
147	16 732	16 761	16 791	16 820	16 850	16 879	16 909	16 938	16 967	16 997
148	17 026	17 056	17 085	17 114	17 143	17 173	17 202	17 231	17 260	17 289
149	17 319	17 348	17 377	17 406	17 435	17 464	17 493	17 522	17 551	17 580
150	17 609	17 638	17 667	17 696	17 725	17 754	17 782	17 811	17 840	17 869

No.	0	1	2	3	4	5	6	7	8	9
150	17 609	17 638	17 667	17 696	17 725	17 754	17 782	17 811	17 840	17 869
151	17 898	17 926	17 955	17 984	18 013	18 041	18 070	18 099	18 127	18 156
152	18 184	18 213	18 241	18 270	18 298	18 327	18 355	18 384	18 412	18 441
153	18 469	18 498	18 526	18 554	18 583	18 611	18 639	18 667	18 696	18 724
154	18 752	18 780	18 808	18 837	18 865	18 893	18 921	18 949	18 977	19 005
155	19 033	19 061	19 089	19 117	19 145	19 173	19 201	19 229	19 257	19 285
156	19 312	19 340	19 368	19 396	19 424	19 451	19 479	19 507	19 535	19 562
157	19 590	19 618	19 645	19 673	19 700	19 728	19 756	19 783	19 811	19 838
158	19 866	19 893	19 921	19 948	19 976	20 003	20 030	20 058	20 085	20 112
159	20 140	20 167	20 194	20 222	20 249	20 276	20 303	20 330	20 358	20 385
160	20 412	20 439	20 466	20 493	20 520	20 548	20 575	20 602	20 629	20 656
161	20 683	20 710	20 737	20 763	20 790	20 817	20 844	20 871	20 898	20 925
162	20 952	20 978	21 005	21 032	21 059	21 085	21 112	21 139	21 165	21 192
163	21 219	21 245	21 272	21 299	21 325	21 352	21 378	21 405	21 431	21 458
164	21 484	21 511	21 537	21 564	21 590	21 617	21 643	21 669	21 696	21 722
165	21 748	21 775	21 801	21 827	21 854	21 880	21 906	21 932	21 958	21 985
166	22 011	22 037	22 063	22 089	22 115	22 141	22 167	22 194	22 220	22 246
167	22 272	22 298	22 324	22 350	22 376	22 401	22 427	22 453	22 479	22 505
168	22 531	22 557	22 583	22 608	22 634	22 660	22 686	22 712	22 737	22 763
169	22 789	22 814	22 840	22 866	22 891	22 917	22 943	22 968	22 994	23 019
170	23 045	23 070	23 096	23 121	23 147	23 172	23 198	23 223	23 249	23 274
171	23 300	23 325	23 350	23 376	23 401	23 426	23 452	23 477	23 502	23 528
172	23 553	23 578	23 603	23 629	23 654	23 679	23 704	23 729	23 754	23 779
173	23 805	23 830	23 855	23 880	23 905	23 930	23 955	23 980	24 005	24 030
174	24 055	24 080	24 105	24 130	24 155	24 180	24 204	24 229	24 254	24 279
175	24 304	24 329	24 353	24 378	24 403	24 428	24 452	24 477	24 502	24 527
176	24 551	24 576	24 601	24 625	24 650	24 674	24 699	24 724	24 748	24 773
177	24 797	24 822	24 846	24 871	24 895	24 920	24 944	24 969	24 993	25 018
178	25 042	25 066	25 091	25 115	25 139	25 164	25 188	25 212	25 237	25 261
179	25 285	25 310	25 334	25 358	25 382	25 406	25 431	25 455	25 479	25 503
180	25 527	25 551	25 575	25 600	25 624	25 648	25 672	25 696	25 720	25 744
181	25 768	25 792	25 816	25 840	25 864	25 888	25 912	25 935	25 959	25 983
182	26 007	26 031	26 055	26 079	26 102	26 126	26 150	26 174	26 198	26 221
183	26 245	26 269	26 293	26 316	26 340	26 364	26 387	26 411	26 435	26 458
184	26 482	26 505	26 529	26 553	26 576	26 600	26 623	26 647	26 670	26 694
185	26 717	26 741	26 764	26 788	26 811	26 834	26 858	26 881	26 905	26 928
186	26 951	26 975	26 998	27 021	27 045	27 068	27 091	27 114	27 138	27 161
187	27 184	27 207	27 231	27 254	27 277	27 300	27 323	27 346	27 370	27 393
188	27 416	27 439	27 462	27 485	27 508	27 531	27 554	27 577	27 600	27 623
189	27 646	27 669	27 692	27 715	27 738	27 761	27 784	27 807	27 830	27 852
190	27 875	27 898	27 921	27 944	27 967	27 989	28 012	28 035	28 058	28 081
191	28 103	28 126	28 149	28 171	28 194	28 217	28 240	28 262	28 285	28 307
192	28 330	28 353	28 375	28 398	28 421	28 443	28 466	28 488	28 511	28 533
193	28 556	28 578	28 601	28 623	28 646	28 668	28 691	28 713	28 735	28 758
194	28 780	28 803	28 825	28 847	28 870	28 892	28 914	28 937	28 959	28 981
195	29 003	29 026	29 048	29 070	29 092	29 115	29 137	29 159	29 181	29 203
196	29 226	29 248	29 270	29 292	29 314	29 336	29 358	29 380	29 403	29 425
197	29 447	29 469	29 491	29 513	29 535	29 557	29 579	29 601	29 623	29 645
198	29 667	29 688	29 710	29 732	29 754	29 776	29 798	29 820	29 842	29 863
199	29 885	29 907	29 929	29 951	29 973	29 994	30 016	30 038	30 060	30 081
200	30 103	30 125	30 146	30 168	30 190	30 211	30 233	30 255	30 276	30 298

No.	0	1	2	3	4	5	6	7	8	9
200	30 103	30 125	30 146	30 168	30 190	30 211	30 233	30 255	30 276	30 298
201	30 320	30 341	30 363	30 384	30 406	30 428	30 449	30 471	30 492	30 514
202	30 535	30 557	30 578	30 600	30 621	30 643	30 664	30 685	30 707	30 728
203	30 750	30 771	30 792	30 814	30 835	30 856	30 878	30 899	30 920	30 942
204	30 963	30 984	31 006	31 027	31 048	31 069	31 091	31 112	31 133	31 154
205	31 175	31 197	31 218	31 239	31 260	31 281	31 302	31 323	31 345	31 366
206	31 387	31 408	31 429	31 450	31 471	31 492	31 513	31 534	31 555	31 576
207	31 597	31 618	31 639	31 660	31 681	31 702	31 723	31 744	31 765	31 785
208	31 806	31 827	31 848	31 869	31 890	31 911	31 931	31 952	31 973	31 994
209	32 015	32 035	32 056	32 077	32 098	32 118	32 139	32 160	32 181	32 201
210	32 222	32 243	32 263	32 284	32 305	32 325	32 346	32 366	32 387	32 408
211	32 428	32 449	32 469	32 490	32 510	32 531	32 552	32 572	32 593	32 613
212	32 634	32 654	32 675	32 695	32 715	32 736	32 756	32 777	32 797	32 818
213	32 838	32 858	32 879	32 899	32 919	32 940	32 960	32 980	33 001	33 021
214	33 041	33 062	33 082	33 102	33 122	33 143	33 163	33 183	33 203	33 224
215	33 244	33 264	33 284	33 304	33 325	33 345	33 365	33 385	33 405	33 425
216	33 445	33 465	33 486	33 506	33 526	33 546	33 566	33 586	33 606	33 626
217	33 646	33 666	33 686	33 706	33 726	33 746	33 766	33 786	33 806	33 826
218	33 846	33 866	33 885	33 905	33 925	33 945	33 965	33 985	34 005	34 025
219	34 044	34 064	34 084	34 104	34 124	34 143	34 163	34 183	34 203	34 223
220	34 242	34 262	34 282	34 301	34 321	34 341	34 361	34 380	34 400	34 420
221	34 439	34 459	34 479	34 498	34 518	34 537	34 557	34 577	34 596	34 616
222	34 635	34 655	34 674	34 694	34 713	34 733	34 753	34 772	34 792	34 811
223	34 830	34 850	34 869	34 889	34 908	34 928	34 947	34 967	34 986	35 005
224	35 025	35 044	35 064	35 083	35 102	35 122	35 141	35 160	35 180	35 199
225	35 218	35 238	35 257	35 276	35 295	35 315	35 334	35 353	35 372	35 392
226	35 411	35 430	35 449	35 468	35 488	35 507	35 526	35 545	35 564	35 583
227	35 603	35 622	35 641	35 660	35 679	35 698	35 717	35 736	35 755	35 774
228	35 793	35 813	35 832	35 851	35 870	35 889	35 908	35 927	35 946	35 965
229	35 984	36 003	36 021	36 040	36 059	36 078	36 097	36 116	36 135	36 154
230	36 173	36 192	36 211	36 229	36 248	36 267	36 286	36 305	36 324	36 342
231	36 361	36 380	36 399	36 418	36 436	36 455	36 474	36 493	36 511	36 530
232	36 549	36 568	36 586	36 605	36 624	36 642	36 661	36 680	36 698	36 717
233	36 736	36 754	36 773	36 791	36 810	36 829	36 847	36 866	36 884	36 903
234	36 922	36 940	36 959	36 977	36 996	37 014	37 033	37 051	37 070	37 088
235	37 107	37 125	37 144	37 162	37 181	37 199	37 218	37 236	37 254	37 273
236	37 291	37 310	37 328	37 346	37 365	37 383	37 401	37 420	37 438	37 457
237	37 475	37 493	37 511	37 530	37 548	37 566	37 585	37 603	37 621	37 639
238	37 658	37 676	37 694	37 712	37 731	37 749	37 767	37 785	37 803	37 822
239	37 840	37 858	37 876	37 894	37 912	37 931	37 949	37 967	37 985	38 003
240	38 021	38 039	38 057	38 075	38 093	38 112	38 130	38 148	38 166	38 184
241	38 202	38 220	38 238	38 256	38 274	38 292	38 310	38 328	38 346	38 364
242	38 382	38 399	38 417	38 435	38 453	38 471	38 489	38 507	38 525	38 543
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247	39 270	39 287	39 305	39 322	39 340	39 358	39 375	39 393	39 410	39 428
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249	39 620	39 637	39 655	39 672	39 690	39 707	39 724	39 742	39 759	39 777
250	39 794	39 811	39 829	39 846	39 863	39 881	39 898	39 915	39 933	39 950

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252	40 140	40 157	40 175	40 192	40 209	40 226	40 243	40 261	40 278	40 295
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258	41 162	41 179	41 196	41 212	41 229	41 246	41 263	41 280	41 296	41 313
259	41 330	41 347	41 363	41 380	41 397	41 414	41 430	41 447	41 464	41 481
260	41 497	41 514	41 531	41 547	41 564	41 581	41 597	41 614	41 631	41 647
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265	42 325	42 341	42 357	42 374	42 390	42 406	42 423	42 439	42 455	42 472
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271	43 297	43 313	43 329	43 345	43 361	43 377	43 393	43 409	43 425	43 441
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277	44 248	44 264	44 279	44 295	44 311	44 326	44 342	44 358	44 373	44 389
278	44 404	44 420	44 436	44 451	44 467	44 483	44 498	44 514	44 529	44 545
279	44 560	44 576	44 592	44 607	44 623	44 638	44 654	44 669	44 685	44 700
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283	45 179	45 194	45 209	45 225	45 240	45 255	45 271	45 286	45 301	45 317
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287	45 788	45 803	45 818	45 834	45 849	45 864	45 879	45 894	45 909	45 924
288	45 939	45 954	45 969	45 984	46 000	46 015	46 030	46 045	46 060	46 075
289	46 090	46 105	46 120	46 135	46 150	46 165	46 180	46 195	46 210	46 225
290	46 240	46 255	46 270	46 285	46 300	46 315	46 330	46 345	46 359	46 374
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292	46 538	46 553	46 568	46 583	46 598	46 613	46 627	46 642	46 657	46 672
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297	47 276	47 290	47 305	47 319	47 334	47 349	47 363	47 378	47 392	47 407
298	47 422	47 436	47 451	47 465	47 480	47 494	47 509	47 524	47 538	47 553
299	47 567	47 582	47 596	47 611	47 625	47 640	47 654	47 669	47 683	47 698
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303	48 144	48 159	48 173	48 187	48 202	48 216	48 230	48 244	48 259	48 273
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305	48 430	48 444	48 458	48 473	48 487	48 501	48 515	48 530	48 544	48 558
306	48 572	48 586	48 601	48 615	48 629	48 643	48 657	48 671	48 686	48 700
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311	49 276	49 290	49 304	49 318	49 332	49 346	49 360	49 374	49 388	49 402
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350	54 407	54 419	54 432	54 444	54 456	54 469	54 481	54 494	54 506	54 518

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367	56 467	56 478	56 490	56 502	56 514	56 526	56 538	56 549	56 561	56 573
368	56 585	56 597	56 608	56 620	56 632	56 644	56 656	56 667	56 679	56 691
369	56 703	56 714	56 726	56 738	56 750	56 761	56 773	56 785	56 797	56 808
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371	56 937	56 949	56 961	56 972	56 984	56 996	57 008	57 019	57 031	57 043
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373	57 171	57 183	57 194	57 206	57 217	57 229	57 241	57 252	57 264	57 276
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387	58 771	58 782	58 794	58 805	58 816	58 827	58 838	58 850	58 861	58 872
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400	60 206	60 217	60 228	60 239	60 249	60 260	60 271	60 282	60 293	60 304

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405	60 746	60 756	60 767	60 778	60 788	60 799	60 810	60 821	60 831	60 842
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416	61 909	61 920	61 930	61 941	61 951	61 962	61 972	61 982	61 993	62 003
417	62 014	62 024	62 034	62 045	62 055	62 066	62 076	62 086	62 097	62 107
418	62 118	62 128	62 138	62 149	62 159	62 170	62 180	62 190	62 201	62 211
419	62 221	62 232	62 242	62 252	62 263	62 273	62 284	62 294	62 304	62 315
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422	62 531	62 542	62 552	62 562	62 572	62 583	62 593	62 603	62 613	62 624
423	62 634	62 644	62 655	62 665	62 675	62 685	62 696	62 706	62 716	62 726
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425	62 839	62 849	62 859	62 870	62 880	62 890	62 900	62 910	62 921	62 931
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428	63 144	63 155	63 165	63 175	63 185	63 195	63 205	63 215	63 225	63 236
429	63 246	63 256	63 266	63 276	63 286	63 296	63 306	63 317	63 327	63 337
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438	64 147	64 157	64 167	64 177	64 187	64 197	64 207	64 217	64 227	64 237
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447	65 031	65 040	65 050	65 060	65 070	65 079	65 089	65 099	65 108	65 118
448	65 128	65 137	65 147	65 157	65 167	65 176	65 186	65 196	65 205	65 215
449	65 225	65 234	65 244	65 254	65 263	65 273	65 283	65 292	65 302	65 312
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452	65 514	65 523	65 533	65 543	65 552	65 562	65 571	65 581	65 591	65 600
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455	65 801	65 811	65 820	65 830	65 839	65 849	65 858	65 868	65 877	65 887
456	65 896	65 906	65 916	65 925	65 935	65 944	65 954	65 963	65 973	65 982
457	65 992	66 001	66 011	66 020	66 030	66 039	66 049	66 058	66 068	66 077
458	66 087	66 096	66 106	66 115	66 124	66 134	66 143	66 153	66 162	66 172
459	66 181	66 191	66 200	66 210	66 219	66 229	66 238	66 247	66 257	66 266
460	66 276	66 285	66 295	66 304	66 314	66 323	66 332	66 342	66 351	66 361
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489	68 931	68 940	68 949	68 958	68 966	68 975	68 984	68 993	69 002	69 011
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511	70 842	70 851	70 859	70 868	70 876	70 885	70 893	70 902	70 910	70 919
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575	75 967	75 974	75 982	75 989	75 997	76 005	76 012	76 020	76 027	76 035
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579	76 268	76 275	76 283	76 290	76 298	76 305	76 313	76 320	76 328	76 335
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620	79 239	79 246	79 253	79 260	79 267	79 274	79 281	79 288	79 295	79 302
621	79 309	79 316	79 323	79 330	79 337	79 344	79 351	79 358	79 365	79 372
622	79 379	79 386	79 393	79 400	79 407	79 414	79 421	79 428	79 435	79 442
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624	79 518	79 525	79 532	79 539	79 546	79 553	79 560	79 567	79 574	79 581
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626	79 657	79 664	79 671	79 678	79 685	79 692	79 699	79 706	79 713	79 720
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635	80 277	80 284	80 291	80 298	80 305	80 312	80 318	80 325	80 332	80 339
636	80 346	80 353	80 359	80 366	80 373	80 380	80 387	80 393	80 400	80 407
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638	80 482	80 489	80 496	80 502	80 509	80 516	80 523	80 530	80 536	80 543
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641	80 686	80 693	80 699	80 706	80 713	80 720	80 726	80 733	80 740	80 747
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654	81 558	81 564	81 571	81 578	81 584	81 591	81 598	81 604	81 611	81 617
655	81 624	81 631	81 637	81 644	81 651	81 657	81 664	81 671	81 677	81 684
656	81 690	81 697	81 704	81 710	81 717	81 723	81 730	81 737	81 743	81 750
657	81 757	81 763	81 770	81 776	81 783	81 790	81 796	81 803	81 809	81 816
658	81 823	81 829	81 836	81 842	81 849	81 856	81 862	81 869	81 875	81 882
659	81 889	81 895	81 902	81 908	81 915	81 921	81 928	81 935	81 941	81 948
660	81 954	81 961	81 968	81 974	81 981	81 987	81 994	82 000	82 007	82 014
661	82 020	82 027	82 033	82 040	82 046	82 053	82 060	82 066	82 073	82 079
662	82 086	82 092	82 099	82 105	82 112	82 119	82 125	82 132	82 138	82 145
663	82 151	82 158	82 164	82 171	82 178	82 184	82 191	82 197	82 204	82 210
664	82 217	82 223	82 230	82 236	82 243	82 249	82 256	82 263	82 269	82 276
665	82 282	82 289	82 295	82 302	82 308	82 315	82 321	82 328	82 334	82 341
666	82 347	82 354	82 360	82 367	82 373	82 380	82 387	82 393	82 400	82 406
667	82 413	82 419	82 426	82 432	82 439	82 445	82 452	82 458	82 465	82 471
668	82 478	82 484	82 491	82 497	82 504	82 510	82 517	82 523	82 530	82 536
669	82 543	82 549	82 556	82 562	82 569	82 575	82 582	82 588	82 595	82 601
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671	82 672	82 679	82 685	82 692	82 698	82 705	82 711	82 718	82 724	82 730
672	82 737	82 743	82 750	82 756	82 763	82 769	82 776	82 782	82 789	82 795
673	82 802	82 808	82 814	82 821	82 827	82 834	82 840	82 847	82 853	82 860
674	82 866	82 872	82 879	82 885	82 892	82 898	82 905	82 911	82 918	82 924
675	82 930	82 937	82 943	82 950	82 956	82 963	82 969	82 975	82 982	82 988
676	82 995	83 001	83 008	83 014	83 020	83 027	83 033	83 040	83 046	83 052
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678	83 123	83 129	83 136	83 142	83 149	83 155	83 161	83 168	83 174	83 181
679	83 187	83 193	83 200	83 206	83 213	83 219	83 225	83 232	83 238	83 245
680	83 251	83 257	83 264	83 270	83 276	83 283	83 289	83 296	83 302	83 308
681	83 315	83 321	83 327	83 334	83 340	83 347	83 353	83 359	83 366	83 372
682	83 378	83 385	83 391	83 398	83 404	83 410	83 417	83 423	83 429	83 436
683	83 442	83 448	83 455	83 461	83 467	83 474	83 480	83 487	83 493	83 499
684	83 506	83 512	83 518	83 525	83 531	83 537	83 544	83 550	83 556	83 563
685	83 569	83 575	83 582	83 588	83 594	83 601	83 607	83 613	83 620	83 626
686	83 632	83 639	83 645	83 651	83 658	83 664	83 670	83 677	83 683	83 689
687	83 696	83 702	83 708	83 715	83 721	83 727	83 734	83 740	83 746	83 753
688	83 759	83 765	83 771	83 778	83 784	83 790	83 797	83 803	83 809	83 816
689	83 822	83 828	83 835	83 841	83 847	83 853	83 860	83 866	83 872	83 879
690	83 885	83 891	83 897	83 904	83 910	83 916	83 923	83 929	83 935	83 942
691	83 948	83 954	83 960	83 967	83 973	83 979	83 985	83 992	83 998	84 004
692	84 011	84 017	84 023	84 029	84 036	84 042	84 048	84 055	84 061	84 067
693	84 073	84 080	84 086	84 092	84 098	84 105	84 111	84 117	84 123	84 130
694	84 136	84 142	84 148	84 155	84 161	84 167	84 173	84 180	84 186	84 192
695	84 198	84 205	84 211	84 217	84 223	84 230	84 236	84 242	84 248	84 255
696	84 261	84 267	84 273	84 280	84 286	84 292	84 298	84 305	84 311	84 317
697	84 323	84 330	84 336	84 342	84 348	84 354	84 361	84 367	84 373	84 379
698	84 386	84 392	84 398	84 404	84 410	84 417	84 423	84 429	84 435	84 442
699	84 448	84 454	84 460	84 466	84 473	84 479	84 485	84 491	84 497	84 504
700	84 510	84 516	84 522	84 528	84 535	84 541	84 547	84 553	84 559	84 566

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700	84 510	84 516	84 522	84 528	84 535	84 541	84 547	84 553	84 559	84 566
701	84 572	84 578	84 584	84 590	84 597	84 603	84 609	84 615	84 621	84 628
702	84 634	84 640	84 646	84 652	84 658	84 665	84 671	84 677	84 683	84 689
703	84 696	84 702	84 708	84 714	84 720	84 726	84 733	84 739	84 745	84 751
704	84 757	84 763	84 770	84 776	84 782	84 788	84 794	84 800	84 807	84 813
705	84 819	84 825	84 831	84 837	84 844	84 850	84 856	84 862	84 868	84 874
706	84 880	84 887	84 893	84 899	84 905	84 911	84 917	84 924	84 930	84 936
707	84 942	84 948	84 954	84 960	84 967	84 973	84 979	84 985	84 991	84 997
708	85 003	85 009	85 016	85 022	85 028	85 034	85 040	85 046	85 052	85 058
709	85 065	85 071	85 077	85 083	85 089	85 095	85 101	85 107	85 114	85 120
710	85 126	85 132	85 138	85 144	85 150	85 156	85 163	85 169	85 175	85 181
711	85 187	85 193	85 199	85 205	85 211	85 217	85 224	85 230	85 236	85 242
712	85 248	85 254	85 260	85 266	85 272	85 278	85 285	85 291	85 297	85 303
713	85 309	85 315	85 321	85 327	85 333	85 339	85 345	85 352	85 358	85 364
714	85 370	85 376	85 382	85 388	85 394	85 400	85 406	85 412	85 418	85 425
715	85 431	85 437	85 443	85 449	85 455	85 461	85 467	85 473	85 479	85 485
716	85 491	85 497	85 503	85 509	85 516	85 522	85 528	85 534	85 540	85 546
717	85 552	85 558	85 564	85 570	85 576	85 582	85 588	85 594	85 600	85 606
718	85 612	85 618	85 625	85 631	85 637	85 643	85 649	85 655	85 661	85 667
719	85 673	85 679	85 685	85 691	85 697	85 703	85 709	85 715	85 721	85 727
720	85 733	85 739	85 745	85 751	85 757	85 763	85 769	85 775	85 781	85 788
721	85 794	85 800	85 806	85 812	85 818	85 824	85 830	85 836	85 842	85 848
722	85 854	85 860	85 866	85 872	85 878	85 884	85 890	85 896	85 902	85 908
723	85 914	85 920	85 926	85 932	85 938	85 944	85 950	85 956	85 962	85 968
724	85 974	85 980	85 986	85 992	85 998	86 004	86 010	86 016	86 022	86 028
725	86 034	86 040	86 046	86 052	86 058	86 064	86 070	86 076	86 082	86 088
726	86 094	86 100	86 106	86 112	86 118	86 124	86 130	86 136	86 141	86 147
727	86 153	86 159	86 165	86 171	86 177	86 183	86 189	86 195	86 201	86 207
728	86 213	86 219	86 225	86 231	86 237	86 243	86 249	86 255	86 261	86 267
729	86 273	86 279	86 285	86 291	86 297	86 303	86 308	86 314	86 320	86 326
730	86 332	86 338	86 344	86 350	86 356	86 362	86 368	86 374	86 380	86 386
731	86 392	86 398	86 404	86 410	86 415	86 421	86 427	86 433	86 439	86 445
732	86 451	86 457	86 463	86 469	86 475	86 481	86 487	86 493	86 499	86 504
733	86 510	86 516	86 522	86 528	86 534	86 540	86 546	86 552	86 558	86 564
734	86 570	86 576	86 581	86 587	86 593	86 599	86 605	86 611	86 617	86 623
735	86 629	86 635	86 641	86 646	86 652	86 658	86 664	86 670	86 676	86 682
736	86 688	86 694	86 700	86 705	86 711	86 717	86 723	86 729	86 735	86 741
737	86 747	86 753	86 759	86 764	86 770	86 776	86 782	86 788	86 794	86 800
738	86 806	86 812	86 817	86 823	86 829	86 835	86 841	86 847	86 853	86 859
739	86 864	86 870	86 876	86 882	86 888	86 894	86 900	86 906	86 911	86 917
740	86 923	86 929	86 935	86 941	86 947	86 953	86 958	86 964	86 970	86 976
741	86 982	86 988	86 994	86 999	87 005	87 011	87 017	87 023	87 029	87 035
742	87 040	87 046	87 052	87 058	87 064	87 070	87 075	87 081	87 087	87 093
743	87 099	87 105	87 111	87 116	87 122	87 128	87 134	87 140	87 146	87 151
744	87 157	87 163	87 169	87 175	87 181	87 186	87 192	87 198	87 204	87 210
745	87 216	87 221	87 227	87 233	87 239	87 245	87 251	87 256	87 262	87 268
746	87 274	87 280	87 286	87 291	87 297	87 303	87 309	87 315	87 320	87 326
747	87 332	87 338	87 344	87 349	87 355	87 361	87 367	87 373	87 379	87 384
748	87 390	87 396	87 402	87 408	87 413	87 419	87 425	87 431	87 437	87 442
749	87 448	87 454	87 460	87 466	87 471	87 477	87 483	87 489	87 495	87 500
750	87 506	87 512	87 518	87 523	87 529	87 535	87 541	87 547	87 552	87 558

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750	87 506	87 512	87 518	87 523	87 529	87 535	87 541	87 547	87 552	87 558
751	87 564	87 570	87 576	87 581	87 587	87 593	87 599	87 604	87 610	87 616
752	87 622	87 628	87 633	87 639	87 645	87 651	87 656	87 662	87 668	87 674
753	87 679	87 685	87 691	87 697	87 703	87 708	87 714	87 720	87 726	87 731
754	87 737	87 743	87 749	87 754	87 760	87 766	87 772	87 777	87 783	87 789
755	87 795	87 800	87 806	87 812	87 818	87 823	87 829	87 835	87 841	87 846
756	87 852	87 858	87 864	87 869	87 875	87 881	87 887	87 892	87 898	87 904
757	87 910	87 915	87 921	87 927	87 933	87 938	87 944	87 950	87 955	87 961
758	87 967	87 973	87 978	87 984	87 990	87 996	88 001	88 007	88 013	88 018
759	88 024	88 030	88 036	88 041	88 047	88 053	88 058	88 064	88 070	88 076
760	88 081	88 087	88 093	88 098	88 104	88 110	88 116	88 121	88 127	88 133
761	88 138	88 144	88 150	88 156	88 161	88 167	88 173	88 178	88 184	88 190
762	88 195	88 201	88 207	88 213	88 218	88 224	88 230	88 235	88 241	88 247
763	88 252	88 258	88 264	88 270	88 275	88 281	88 287	88 292	88 298	88 304
764	88 309	88 315	88 321	88 326	88 332	88 338	88 343	88 349	88 355	88 360
765	88 366	88 372	88 377	88 383	88 389	88 395	88 400	88 406	88 412	88 417
766	88 423	88 429	88 434	88 440	88 446	88 451	88 457	88 463	88 468	88 474
767	88 480	88 485	88 491	88 497	88 502	88 508	88 513	88 519	88 525	88 530
768	88 536	88 542	88 547	88 553	88 559	88 564	88 570	88 576	88 581	88 587
769	88 593	88 598	88 604	88 610	88 615	88 621	88 627	88 632	88 638	88 643
770	88 649	88 655	88 660	88 666	88 672	88 677	88 683	88 689	88 694	88 700
771	88 705	88 711	88 717	88 722	88 728	88 734	88 739	88 745	88 750	88 756
772	88 762	88 767	88 773	88 779	88 784	88 790	88 795	88 801	88 807	88 812
773	88 818	88 824	88 829	88 835	88 840	88 846	88 852	88 857	88 863	88 868
774	88 874	88 880	88 885	88 891	88 897	88 902	88 908	88 913	88 919	88 925
775	88 930	88 936	88 941	88 947	88 953	88 958	88 964	88 969	88 975	88 981
776	88 986	88 992	88 997	89 003	89 009	89 014	89 020	89 025	89 031	89 037
777	89 042	89 048	89 053	89 059	89 064	89 070	89 076	89 081	89 087	89 092
778	89 098	89 104	89 109	89 115	89 120	89 126	89 131	89 137	89 143	89 148
779	89 154	89 159	89 165	89 170	89 176	89 182	89 187	89 193	89 198	89 204
780	89 209	89 215	89 221	89 226	89 232	89 237	89 243	89 248	89 254	89 260
781	89 265	89 271	89 276	89 282	89 287	89 293	89 298	89 304	89 310	89 315
782	89 321	89 326	89 332	89 337	89 343	89 348	89 354	89 360	89 365	89 371
783	89 376	89 382	89 387	89 393	89 398	89 404	89 409	89 415	89 421	89 426
784	89 432	89 437	89 443	89 448	89 454	89 459	89 465	89 470	89 476	89 481
785	89 487	89 492	89 498	89 504	89 509	89 515	89 520	89 526	89 531	89 537
786	89 542	89 548	89 553	89 559	89 564	89 570	89 575	89 581	89 586	89 592
787	89 597	89 603	89 609	89 614	89 620	89 625	89 631	89 636	89 642	89 647
788	89 653	89 658	89 664	89 669	89 675	89 680	89 686	89 691	89 697	89 702
789	89 708	89 713	89 719	89 724	89 730	89 735	89 741	89 746	89 752	89 757
790	89 763	89 768	89 774	89 779	89 785	89 790	89 796	89 801	89 807	89 812
791	89 818	89 823	89 829	89 834	89 840	89 845	89 851	89 856	89 862	89 867
792	89 873	89 878	89 883	89 889	89 894	89 900	89 905	89 911	89 916	89 922
793	89 927	89 933	89 938	89 944	89 949	89 955	89 960	89 966	89 971	89 977
794	89 982	89 988	89 993	89 998	90 004	90 009	90 015	90 020	90 026	90 031
795	90 037	90 042	90 048	90 053	90 059	90 064	90 069	90 075	90 080	90 086
796	90 091	90 097	90 102	90 108	90 113	90 119	90 124	90 129	90 135	90 140
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798	90 200	90 206	90 211	90 217	90 222	90 227	90 233	90 238	90 244	90 249
799	90 255	90 260	90 266	90 271	90 276	90 282	90 287	90 293	90 298	90 304
800	90 309	90 314	90 320	90 325	90 331	90 336	90 342	90 347	90 352	90 358

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800	90 309	90 314	90 320	90 325	90 331	90 336	90 342	90 347	90 352	90 358
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802	90 417	90 423	90 428	90 434	90 439	90 445	90 450	90 455	90 461	90 466
803	90 472	90 477	90 482	90 488	90 493	90 499	90 504	90 509	90 515	90 520
804	90 526	90 531	90 536	90 542	90 547	90 553	90 558	90 563	90 569	90 574
805	90 580	90 585	90 590	90 596	90 601	90 607	90 612	90 617	90 623	90 628
806	90 634	90 639	90 644	90 650	90 655	90 660	90 666	90 671	90 677	90 682
807	90 687	90 693	90 698	90 703	90 709	90 714	90 720	90 725	90 730	90 736
808	90 741	90 747	90 752	90 757	90 763	90 768	90 773	90 779	90 784	90 789
809	90 795	90 800	90 806	90 811	90 816	90 822	90 827	90 832	90 838	90 843
810	90 849	90 854	90 859	90 865	90 870	90 875	90 881	90 886	90 891	90 897
811	90 902	90 907	90 913	90 918	90 924	90 929	90 934	90 940	90 945	90 950
812	90 956	90 961	90 966	90 972	90 977	90 982	90 988	90 993	90 998	91 004
813	91 009	91 014	91 020	91 025	91 030	91 036	91 041	91 046	91 052	91 057
814	91 062	91 068	91 073	91 078	91 084	91 089	91 094	91 100	91 105	91 110
815	91 116	91 121	91 126	91 132	91 137	91 142	91 148	91 153	91 158	91 164
816	91 169	91 174	91 180	91 185	91 190	91 196	91 201	91 206	91 212	91 217
817	91 222	91 228	91 233	91 238	91 243	91 249	91 254	91 259	91 265	91 270
818	91 275	91 281	91 286	91 291	91 297	91 302	91 307	91 312	91 318	91 323
819	91 328	91 334	91 339	91 344	91 350	91 355	91 360	91 365	91 371	91 376
820	91 381	91 387	91 392	91 397	91 403	91 408	91 413	91 418	91 424	91 429
821	91 434	91 440	91 445	91 450	91 455	91 461	91 466	91 471	91 477	91 482
822	91 487	91 492	91 498	91 503	91 508	91 514	91 519	91 524	91 529	91 535
823	91 540	91 545	91 551	91 556	91 561	91 566	91 572	91 577	91 582	91 587
824	91 593	91 598	91 603	91 609	91 614	91 619	91 624	91 630	91 635	91 640
825	91 645	91 651	91 656	91 661	91 666	91 672	91 677	91 682	91 687	91 693
826	91 698	91 703	91 709	91 714	91 719	91 724	91 730	91 735	91 740	91 745
827	91 751	91 756	91 761	91 766	91 772	91 777	91 782	91 787	91 793	91 798
828	91 803	91 808	91 814	91 819	91 824	91 829	91 834	91 840	91 845	91 850
829	91 855	91 861	91 866	91 871	91 876	91 882	91 887	91 892	91 897	91 903
830	91 908	91 913	91 918	91 924	91 929	91 934	91 939	91 944	91 950	91 955
831	91 960	91 965	91 971	91 976	91 981	91 986	91 991	91 997	92 002	92 007
832	92 012	92 018	92 023	92 028	92 033	92 038	92 044	92 049	92 054	92 059
833	92 065	92 070	92 075	92 080	92 085	92 091	92 096	92 101	92 106	92 111
834	92 117	92 122	92 127	92 132	92 137	92 143	92 148	92 153	92 158	92 163
835	92 169	92 174	92 179	92 184	92 189	92 195	92 200	92 205	92 210	92 215
836	92 221	92 226	92 231	92 236	92 241	92 247	92 252	92 257	92 262	92 267
837	92 273	92 278	92 283	92 288	92 293	92 298	92 304	92 309	92 314	92 319
838	92 324	92 330	92 335	92 340	92 345	92 350	92 355	92 361	92 366	92 371
839	92 376	92 381	92 387	92 392	92 397	92 402	92 407	92 412	92 418	92 423
840	92 428	92 433	92 438	92 443	92 449	92 454	92 459	92 464	92 469	92 474
841	92 480	92 485	92 490	92 495	92 500	92 505	92 511	92 516	92 521	92 526
842	92 531	92 536	92 542	92 547	92 552	92 557	92 562	92 567	92 572	92 578
843	92 583	92 588	92 593	92 598	92 603	92 609	92 614	92 619	92 624	92 629
844	92 634	92 639	92 645	92 650	92 655	92 660	92 665	92 670	92 675	92 681
845	92 686	92 691	92 696	92 701	92 706	92 711	92 716	92 722	92 727	92 732
846	92 737	92 742	92 747	92 752	92 758	92 763	92 768	92 773	92 778	92 783
847	92 788	92 793	92 799	92 804	92 809	92 814	92 819	92 824	92 829	92 834
848	92 840	92 845	92 850	92 855	92 860	92 865	92 870	92 875	92 881	92 886
849	92 891	92 896	92 901	92 906	92 911	92 916	92 921	92 927	92 932	92 937
850	92 942	92 947	92 952	92 957	92 962	92 967	92 973	92 978	92 983	92 988

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850	92 942	92 947	92 952	92 957	92 962	92 967	92 973	92 978	92 983	92 988
851	92 993	92 988	93 003	93 008	93 013	93 018	93 024	93 029	93 034	93 039
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853	93 095	93 100	93 105	93 110	93 115	93 120	93 125	93 131	93 136	93 141
854	93 146	93 151	93 156	93 161	93 166	93 171	93 176	93 181	93 186	93 192
855	93 197	93 202	93 207	93 212	93 217	93 222	93 227	93 232	93 237	93 242
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857	93 298	93 303	93 308	93 313	93 318	93 323	93 328	93 334	93 339	93 344
858	93 349	93 354	93 359	93 364	93 369	93 374	93 379	93 384	93 389	93 394
859	93 399	93 404	93 409	93 414	93 420	93 425	93 430	93 435	93 440	93 445

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861	93 500	93 505	93 510	93 515	93 520	93 526	93 531	93 536	93 541	93 546
862	93 551	93 556	93 561	93 566	93 571	93 576	93 581	93 586	93 591	93 596
863	93 601	93 606	93 611	93 616	93 621	93 626	93 631	93 636	93 641	93 646
864	93 651	93 656	93 661	93 666	93 671	93 676	93 682	93 687	93 692	93 697
865	93 702	93 707	93 712	93 717	93 722	93 727	93 732	93 737	93 742	93 747
866	93 752	93 757	93 762	93 767	93 772	93 777	93 782	93 787	93 792	93 797
867	93 802	93 807	93 812	93 817	93 822	93 827	93 832	93 837	93 842	93 847
868	93 852	93 857	93 862	93 867	93 872	93 877	93 882	93 887	93 892	93 897
869	93 902	93 907	93 912	93 917	93 922	93 927	93 932	93 937	93 942	93 947

No.	0	1	2	3	4	5	6	7	8	9
870	93 952	93 957	93 962	93 967	93 972	93 977	93 982	93 987	93 992	93 997
871	94 002	94 007	94 012	94 017	94 022	94 027	94 032	94 037	94 042	94 047
872	94 052	94 057	94 062	94 067	94 072	94 077	94 082	94 086	94 091	94 096
873	94 101	94 106	94 111	94 116	94 121	94 126	94 131	94 136	94 141	94 146
874	94 151	94 156	94 161	94 166	94 171	94 176	94 181	94 186	94 191	94 196
875	94 201	94 206	94 211	94 216	94 221	94 226	94 231	94 236	94 240	94 245
876	94 250	94 255	94 260	94 265	94 270	94 275	94 280	94 285	94 290	94 295
877	94 300	94 305	94 310	94 315	94 320	94 325	94 330	94 335	94 340	94 345
878	94 349	94 354	94 359	94 364	94 369	94 374	94 379	94 384	94 389	94 394
879	94 399	94 404	94 409	94 414	94 419	94 424	94 429	94 433	94 438	94 443

No.	0	1	2	3	4	5	6	7	8	9
880	94 448	94 453	94 458	94 463	94 468	94 473	94 478	94 483	94 488	94 493
881	94 498	94 503	94 507	94 512	94 517	94 522	94 527	94 532	94 537	94 542
882	94 547	94 552	94 557	94 562	94 567	94 571	94 576	94 581	94 586	94 591
883	94 596	94 601	94 606	94 611	94 616	94 621	94 626	94 630	94 635	94 640
884	94 645	94 650	94 655	94 660	94 665	94 670	94 675	94 680	94 685	94 689
885	94 694	94 699	94 704	94 709	94 714	94 719	94 724	94 729	94 734	94 738
886	94 743	94 748	94 753	94 758	94 763	94 768	94 773	94 778	94 783	94 787
887	94 792	94 797	94 802	94 807	94 812	94 817	94 822	94 827	94 832	94 836
888	94 841	94 846	94 851	94 856	94 861	94 866	94 871	94 876	94 880	94 885
889	94 890	94 895	94 900	94 905	94 910	94 915	94 919	94 924	94 929	94 934

No.	0	1	2	3	4	5	6	7	8	9
890	94 939	94 944	94 949	94 954	94 959	94 963	94 968	94 973	94 978	94 983
891	94 988	94 993	94 998	95 002	95 007	95 012	95 017	95 022	95 027	95 032
892	95 036	95 041	95 046	95 051	95 056	95 061	95 066	95 071	95 075	95 080
893	95 085	95 090	95 095	95 100	95 105	95 109	95 114	95 119	95 124	95 129
894	95 134	95 139	95 143	95 148	95 153	95 158	95 163	95 168	95 173	95 177
895	95 182	95 187	95 192	95 197	95 202	95 207	95 211	95 216	95 221	95 226
896	95 231	95 236	95 240	95 245	95 250	95 255	95 260	95 265	95 270	95 274
897	95 279	95 284	95 289	95 294	95 299	95 303	95 308	95 313	95 318	95 323
898	95 328	95 332	95 337	95 342	95 347	95 352	95 357	95 361	95 366	95 371
899	95 376	95 381	95 386	95 390	95 395	95 400	95 405	95 410	95 415	95 419
900	95 424	95 429	95 434	95 439	95 444	95 448	95 453	95 458	95 463	95 468

No.	0	1	2	3	4	5	6	7	8	9
900	95 424	95 429	95 434	95 439	95 444	95 448	95 453	95 458	95 463	95 468
901	95 472	95 477	95 482	95 487	95 492	95 497	95 501	95 506	95 511	95 516
902	95 521	95 525	95 530	95 535	95 540	95 545	95 550	95 554	95 559	95 564
903	95 569	95 574	95 578	95 583	95 588	95 593	95 598	95 602	95 607	95 612
904	95 617	95 622	95 626	95 631	95 636	95 641	95 646	95 650	95 655	95 660
905	95 665	95 670	95 674	95 679	95 684	95 689	95 694	95 698	95 703	95 708
906	95 713	95 718	95 722	95 727	95 732	95 737	95 742	95 746	95 751	95 756
907	95 761	95 766	95 770	95 775	95 780	95 785	95 789	95 794	95 799	95 804
908	95 809	95 813	95 818	95 823	95 828	95 832	95 837	95 842	95 847	95 852
909	95 856	95 861	95 866	95 871	95 875	95 880	95 885	95 890	95 895	95 899
910	95 904	95 909	95 914	95 918	95 923	95 928	95 933	95 938	95 942	95 947
911	95 952	95 957	95 961	95 966	95 971	95 976	95 980	95 985	95 990	95 995
912	95 999	96 004	96 009	96 014	96 019	96 023	96 028	96 033	96 038	96 042
913	96 047	96 052	96 057	96 061	96 066	96 071	96 076	96 080	96 085	96 090
914	96 095	96 099	96 104	96 109	96 114	96 118	96 123	96 128	96 133	96 137
915	96 142	96 147	96 152	96 156	96 161	96 166	96 171	96 175	96 180	96 185
916	96 190	96 194	96 199	96 204	96 209	96 213	96 218	96 223	96 227	96 232
917	96 237	96 242	96 246	96 251	96 256	96 261	96 265	96 270	96 275	96 280
918	96 284	96 289	96 294	96 298	96 303	96 308	96 313	96 317	96 322	96 327
919	96 332	96 336	96 341	96 346	96 350	96 355	96 360	96 365	96 369	96 374
920	96 379	96 384	96 388	96 393	96 398	96 402	96 407	96 412	96 417	96 421
921	96 426	96 431	96 435	96 440	96 445	96 450	96 454	96 459	96 464	96 468
922	96 473	96 478	96 483	96 487	96 492	96 497	96 501	96 506	96 511	96 515
923	96 520	96 525	96 530	96 534	96 539	96 544	96 548	96 553	96 558	96 562
924	96 567	96 572	96 577	96 581	96 586	96 591	96 595	96 600	96 605	96 609
925	96 614	96 619	96 624	96 628	96 633	96 638	96 642	96 647	96 652	96 656
926	96 661	96 666	96 670	96 675	96 680	96 685	96 689	96 694	96 699	96 703
927	96 708	96 713	96 717	96 722	96 727	96 731	96 736	96 741	96 745	96 750
928	96 755	96 759	96 764	96 769	96 774	96 778	96 783	96 788	96 792	96 797
929	96 802	96 806	96 811	96 816	96 820	96 825	96 830	96 834	96 839	96 844
930	96 848	96 853	96 858	96 862	96 867	96 872	96 876	96 881	96 886	96 890
931	96 895	96 900	96 904	96 909	96 914	96 918	96 923	96 928	96 932	96 937
932	96 942	96 946	96 951	96 956	96 960	96 965	96 970	96 974	96 979	96 984
933	96 988	96 993	96 997	97 002	97 007	97 011	97 016	97 021	97 025	97 030
934	97 035	97 039	97 044	97 049	97 053	97 058	97 063	97 067	97 072	97 077
935	97 081	97 086	97 090	97 095	97 100	97 104	97 109	97 114	97 118	97 123
936	97 128	97 132	97 137	97 142	97 146	97 151	97 155	97 160	97 165	97 169
937	97 174	97 179	97 183	97 188	97 192	97 197	97 202	97 206	97 211	97 216
938	97 220	97 225	97 230	97 234	97 239	97 243	97 248	97 253	97 257	97 262
939	97 267	97 271	97 276	97 280	97 285	97 290	97 294	97 299	97 304	97 308
940	97 313	97 317	97 322	97 327	97 331	97 336	97 340	97 345	97 350	97 354
941	97 359	97 364	97 368	97 373	97 377	97 382	97 387	97 391	97 396	97 400
942	97 405	97 410	97 414	97 419	97 424	97 428	97 433	97 437	97 442	97 447
943	97 451	97 456	97 460	97 465	97 470	97 474	97 479	97 483	97 488	97 493
944	97 497	97 502	97 506	97 511	97 516	97 520	97 525	97 529	97 534	97 539
945	97 543	97 548	97 552	97 557	97 562	97 566	97 571	97 575	97 580	97 585
946	97 589	97 594	97 598	97 603	97 607	97 612	97 617	97 621	97 626	97 630
947	97 635	97 640	97 644	97 649	97 653	97 658	97 663	97 667	97 672	97 676
948	97 681	97 685	97 690	97 695	97 699	97 704	97 708	97 713	97 717	97 722
949	97 727	97 731	97 736	97 740	97 745	97 749	97 754	97 759	97 763	97 768
950	97 772	97 777	97 782	97 786	97 791	97 795	97 800	97 804	97 809	97 813

No.	0	1	2	3	4	5	6	7	8	9
950	97 772	97 777	97 782	97 786	97 791	97 795	97 800	97 804	97 809	97 813
951	97 818	97 823	97 827	97 832	97 836	97 841	97 845	97 850	97 855	97 859
952	97 864	97 868	97 873	97 877	97 882	97 886	97 891	97 896	97 900	97 905
953	97 909	97 914	97 918	97 923	97 928	97 932	97 937	97 941	97 946	97 950
954	97 955	97 959	97 964	97 968	97 973	97 978	97 982	97 987	97 991	97 996
955	98 000	98 005	98 009	98 014	98 019	98 023	98 028	98 032	98 037	98 041
956	98 046	98 050	98 055	98 059	98 064	98 068	98 073	98 078	98 082	98 087
957	98 091	98 096	98 100	98 105	98 109	98 114	98 118	98 123	98 127	98 132
958	98 137	98 141	98 146	98 150	98 155	98 159	98 164	98 168	98 173	98 177
959	98 182	98 186	98 191	98 195	98 200	98 204	98 209	98 214	98 218	98 223
960	98 227	98 232	98 236	98 241	98 245	98 250	98 254	98 259	98 263	98 268
961	98 272	98 277	98 281	98 286	98 290	98 295	98 299	98 304	98 308	98 313
962	98 318	98 322	98 327	98 331	98 336	98 340	98 345	98 349	98 354	98 358
963	98 363	98 367	98 372	98 376	98 381	98 385	98 390	98 394	98 399	98 403
964	98 408	98 412	98 417	98 421	98 426	98 430	98 435	98 439	98 444	98 448
965	98 453	98 457	98 462	98 466	98 471	98 475	98 480	98 484	98 489	98 493
966	98 498	98 502	98 507	98 511	98 516	98 520	98 525	98 529	98 534	98 538
967	98 543	98 547	98 552	98 556	98 561	98 565	98 570	98 574	98 579	98 583
968	98 588	98 592	98 597	98 601	98 605	98 610	98 614	98 619	98 623	98 628
969	98 632	98 637	98 641	98 646	98 650	98 655	98 659	98 664	98 668	98 673
970	98 677	98 682	98 686	98 691	98 695	98 700	98 704	98 709	98 713	98 717
971	98 722	98 726	98 731	98 735	98 740	98 744	98 749	98 753	98 758	98 762
972	98 767	98 771	98 776	98 780	98 784	98 789	98 793	98 798	98 802	98 807
973	98 811	98 816	98 820	98 825	98 829	98 834	98 838	98 843	98 847	98 851
974	98 856	98 860	98 865	98 869	98 874	98 878	98 883	98 887	98 892	98 896
975	98 900	98 905	98 909	98 914	98 918	98 923	98 927	98 932	98 936	98 941
976	98 945	98 949	98 954	98 958	98 963	98 967	98 972	98 976	98 981	98 985
977	98 989	98 994	98 998	99 003	99 007	99 012	99 016	99 021	99 025	99 029
978	99 034	99 038	99 043	99 047	99 052	99 056	99 061	99 065	99 069	99 074
979	99 078	99 083	99 087	99 092	99 096	99 100	99 105	99 109	99 114	99 118
980	99 123	99 127	99 131	99 136	99 140	99 145	99 149	99 154	99 158	99 162
981	99 167	99 171	99 176	99 180	99 185	99 189	99 193	99 198	99 202	99 207
982	99 211	99 216	99 220	99 224	99 229	99 233	99 238	99 242	99 247	99 251
983	99 255	99 260	99 264	99 269	99 273	99 277	99 282	99 286	99 291	99 295
984	99 300	99 304	99 308	99 313	99 317	99 322	99 326	99 330	99 335	99 339
985	99 344	99 348	99 352	99 357	99 361	99 366	99 370	99 374	99 379	99 383
986	99 388	99 392	99 396	99 401	99 405	99 410	99 414	99 419	99 423	99 427
987	99 432	99 436	99 441	99 445	99 449	99 454	99 458	99 463	99 467	99 471
988	99 476	99 480	99 484	99 489	99 493	99 498	99 502	99 506	99 511	99 515
989	99 520	99 524	99 528	99 533	99 537	99 542	99 546	99 550	99 555	99 559
990	99 564	99 568	99 572	99 577	99 581	99 585	99 590	99 594	99 599	99 603
991	99 607	99 612	99 616	99 621	99 625	99 629	99 634	99 638	99 642	99 647
992	99 651	99 656	99 660	99 664	99 669	99 673	99 677	99 682	99 686	99 691
993	99 695	99 699	99 704	99 708	99 712	99 717	99 721	99 726	99 730	99 734
994	99 739	99 743	99 747	99 752	99 756	99 760	99 765	99 769	99 774	99 778
995	99 782	99 787	99 791	99 795	99 800	99 804	99 808	99 813	99 817	99 822
996	99 826	99 830	99 835	99 839	99 843	99 848	99 852	99 856	99 861	99 865
997	99 870	99 874	99 878	99 883	99 887	99 891	99 896	99 900	99 904	99 909
998	99 913	99 917	99 922	99 926	99 930	99 935	99 939	99 944	99 948	99 952
999	99 957	99 961	99 965	99 970	99 974	99 978	99 983	99 987	99 991	99 996
1000	00 000	00 004	00 009	00 013	00 017	00 022	00 026	00 030	00 035	00 039

TABLE II

LOGARITHMS

OF THE

TRIGONOMETRIC FUNCTIONS

0°

′	″	log sin	d.	log tan	d.	log cot	log cos	″	′
0	0						10.000 00	0	60
	10	5.685 57		5.685 57		4.314 43	10.000 00	50	
	20	5.986 60	30103	5.986 60	30103	4.013 40	10.000 00	40	
	30	6.162 70	17610	6.162 70	17610	3.837 30	10.000 00	30	
	40	6.287 63	12493	6.287 63	12493	3.712 37	10.000 00	20	
	50	6.384 54	9691	6.384 54	9691	3.615 46	10.000 00	10	
1	0	6.463 73	7919	6.463 73	7919	3.536 27	10.000 00	0	59
	10	6.530 67	6694	6.530 67	6694	3.469 33	10.000 00	50	
	20	6.588 66	5799	6.588 66	5799	3.411 34	10.000 00	40	
	30	6.639 82	5116	6.639 82	5116	3.360 18	10.000 00	30	
	40	6.685 57	4575	6.685 57	4575	3.314 43	10.000 00	20	
	50	6.726 97	4140	6.726 97	4140	3.273 03	10.000 00	10	
2	0	6.764 76	3779	6.764 76	3779	3.235 24	10.000 00	0	58
	10	6.799 52	3476	6.799 52	3476	3.200 48	10.000 00	50	
	20	6.831 70	3218	6.831 70	3218	3.168 30	10.000 00	40	
	30	6.861 67	2997	6.861 67	2997	3.138 33	10.000 00	30	
	40	6.889 69	2802	6.889 69	2802	3.110 31	10.000 00	20	
	50	6.916 02	2633	6.916 02	2633	3.083 98	10.000 00	10	
3	0	6.940 85	2483	6.940 85	2483	3.059 15	10.000 00	0	57
	10	6.964 33	2348	6.964 33	2348	3.035 67	10.000 00	50	
	20	6.986 60	2227	6.986 61	2228	3.013 40	10.000 00	40	
	30	7.007 79	2119	7.007 79	2118	2.992 21	10.000 00	30	
	40	7.027 00	2021	7.028 00	2021	2.972 00	10.000 00	20	
	50	7.047 30	1930	7.047 30	1930	2.952 70	10.000 00	10	
4	0	7.065 79	1849	7.065 79	1849	2.934 21	10.000 00	0	56
	10	7.083 51	1772	7.083 52	1773	2.916 48	10.000 00	50	
	20	7.100 55	1704	7.100 55	1703	2.899 45	10.000 00	40	
	30	7.116 94	1639	7.116 94	1639	2.883 06	10.000 00	30	
	40	7.132 73	1579	7.132 73	1579	2.867 27	10.000 00	20	
	50	7.147 97	1524	7.147 97	1524	2.852 03	10.000 00	10	
5	0	7.162 70	1473	7.162 70	1473	2.837 30	10.000 00	0	55
	10	7.176 94	1424	7.176 94	1424	2.823 06	10.000 00	50	
	20	7.190 72	1378	7.190 73	1379	2.809 27	10.000 00	40	
	30	7.204 09	1337	7.204 09	1337	2.795 91	10.000 00	30	
	40	7.217 05	1296	7.217 05	1296	2.782 95	10.000 00	20	
	50	7.229 64	1259	7.229 64	1259	2.770 36	10.000 00	10	
6	0	7.241 88	1224	7.241 88	1224	2.758 12	10.000 00	0	54
	10	7.253 78	1190	7.253 78	1190	2.746 22	10.000 00	50	
	20	7.265 36	1158	7.265 36	1158	2.734 64	10.000 00	40	
	30	7.276 64	1128	7.276 64	1128	2.723 36	10.000 00	30	
	40	7.287 63	1099	7.287 64	1100	2.712 36	10.000 00	20	
	50	7.298 36	1073	7.298 36	1072	2.701 64	10.000 00	10	
7	0	7.308 82	1046	7.308 82	1046	2.691 18	10.000.00	0	53
	10	7.319 04	1022	7.319 04	1022	2.680 96	10.000 00	50	
	20	7.329 03	999	7.329 03	999	2.670 97	10.000 00	40	
	30	7.338 79	976	7.338 79	976	2.661 21	10.000 00	30	
	40	7.348 33	954	7.348 33	954	2.651 67	10.000 00	20	
	50	7.357 67	934	7.357 67	934	2.642 33	10.000 00	10	
8	0	7.366 82	915	7.366 82	915	2.633 18	10.000 00	0	52
	10	7.375 77	895	7.375 77	895	2.624 23	10.000 00	50	
	20	7.384 54	877	7.384 55	878	2.615 45	10.000 00	40	
	30	7.393 14	860	7.393 15	860	2.606 85	10.000 00	30	
	40	7.401 58	844	7.401 58	843	2.598 42	10.000 00	20	
	50	7.409 85	827	7.409 85	827	2.590 15	10.000 00	10	
9	0	7.417 97	812	7.417 97	812	2.582 03	10.000 00	0	51
	10	7.425 94	797	7.425 94	797	2.574 06	10.000 00	50	
	20	7.433 76	782	7.433 76	782	2.566 24	10.000 00	40	
	30	7.441 45	769	7.441 45	769	2.558 55	10.000 00	30	
	40	7.449 00	755	7.449 00	755	2.551 00	10.000 00	20	
	50	7.456 43	743	7.456 43	743	2.543 57	10.000 00	10	
10	0	7.463 73	730	7.463 73	730	2.536 27	10.000 00	0	50
′	″	log cos	d.	log cot	d.	log tan	log sin	″	′

89°

′	″	log sin	d.	log tan	d.	log cot	log cos	″	′
10	0	7.463 73	717	7.463 73	718	2.536 27	10.000 00	0	**50**
	10	7.470 90	707	7.470 91	706	2.529 09	10.000 00	50	
	20	7.477 97	694	7.477 97	695	2.522 03	10.000 00	40	
	30	7.484 91	684	7.484 92	684	2.515 08	10.000 00	30	
	40	7.491 75	674	7.491 76	673	2.508 24	10.000 00	20	
	50	7.498 49	663	7.498 49	663	2.501 51	10.000 00	10	
11	0	7.505 12	653	7.505 12	653	2.494 88	10.000 00	0	**49**
	10	7.511 65	643	7.511 65	644	2.488 35	10.000 00	50	
	20	7.518 08	634	7.518 09	634	2.481 91	10.000 00	40	
	30	7.524 42	625	7.524 43	624	2.475 57	10.000 00	30	
	40	7.530 67	616	7.530 67	616	2.469 33	10.000 00	20	
	50	7.536 83	608	7.536 83	608	2.463 17	10.000 00	10	
12	0	7.542 91	599	7.542 91	599	2.457 09	10.000 00	0	**48**
	10	7.548 90	591	7.548 90	591	2.451 10	10.000 00	50	
	20	7.554 81	583	7.554 81	583	2.445 19	10.000 00	40	
	30	7.560 64	575	7.560 64	575	2.439 36	10.000 00	30	
	40	7.566 39	567	7.566 39	568	2.433 61	10.000 00	20	
	50	7.572 06	561	7.572 07	560	2.427 93	10.000 00	10	
13	0	7.577 67	553	7.577 67	553	2.422 33	10.000 00	0	**47**
	10	7.583 20	546	7.583 20	547	2.416 80	10.000 00	50	
	20	7.588 66	540	7.588 67	539	2.411 33	10.000 00	40	
	30	7.594 06	533	7.594 06	533	2.405 94	10.000 00	30	
	40	7.599 39	526	7.599 39	527	2.400 61	10.000 00	20	
	50	7.604 65	520	7.604 66	520	2.395 34	10.000 00	10	
14	0	7.609 85	514	7.609 86	514	2.390 14	10.000 00	0	**46**
	10	7.614 99	508	7.615 00	508	2.385 00	10.000 00	50	
	20	7.620 07	502	7.620 08	502	2.379 92	10.000 00	40	
	30	7.625 09	497	7.625 10	496	2.374 90	10.000 00	30	
	40	7.630 06	490	7.630 06	491	2.369 94	10.000 00	20	
	50	7.634 96	486	7.634 97	485	2.365 03	10.000 00	10	
15	0	7.639 82	479	7.639 82	480	2.360 18	10.000 00	0	**45**
	10	7.644 61	475	7.644 62	475	2.355 38	10.000 00	50	
	20	7.649 36	470	7.649 37	469	2.350 63	10.000 00	40	
	30	7.654 06	464	7.654 06	465	2.345 94	10.000 00	30	
	40	7.658 70	460	7.658 71	459	2.341 29	10.000 00	20	
	50	7.663 30	454	7.663 30	455	2.336 70	10.000 00	10	
16	0	7.667 84	451	7.667 85	450	2.332 15	10.000 00	0	**44**
	10	7.672 35	445	7.672 35	445	2.327 65	10.000 00	50	
	20	7.676 80	441	7.676 80	441	2.323 20	10.000 00	40	
	30	7.681 21	436	7.681 21	437	2.318 79	10.000 00	30	
	40	7.685 57	432	7.685 58	432	2.314 42	9.999 99	20	
	50	7.689 89	428	7.689 90	428	2.310 10	9.999 99	10	
17	0	7.694 17	424	7.694 18	424	2.305 82	9.999 99	0	**43**
	10	7.698 41	420	7.698 42	419	2.301 58	9.999 99	50	
	20	7.702 61	416	7.702 61	416	2.297 39	9.999 99	40	
	30	7.706 76	412	7.706 77	411	2.293 23	9.999 99	30	
	40	7.710 88	408	7.710 88	408	2.289 12	9.999 99	20	
	50	7.714 96	404	7.714 96	404	2.285 04	9.999 99	10	
18	0	7.719 00	400	7.719 00	401	2.281 00	9.999 99	0	**42**
	10	7.723 00	397	7.723 01	396	2.276 99	9.999 99	50	
	20	7.726 97	393	7.726 97	393	2.273 03	9.999 99	40	
	30	7.730 90	389	7.730 90	390	2.269 10	9.999 99	30	
	40	7.734 79	386	7.734 80	386	2.265 20	9.999 99	20	
	50	7.738 65	383	7.738 66	382	2.261 34	9.999 99	10	
19	0	7.742 48	379	7.742 48	380	2.257 52	9.999 99	0	**41**
	10	7.746 27	376	7.746 28	376	2.253 72	9.999 99	50	
	20	7.750 03	373	7.750 04	373	2.249 96	9.999 99	40	
	30	7.753 76	369	7.753 77	369	2.246 23	9.999 99	30	
	40	7.757 45	367	7.757 46	367	2.242 54	9.999 99	20	
	50	7.761 12	363	7.761 13	363	2.238 87	9.999 99	10	
20	0	7.764 75		7.764 76		2.235 24	9.999 99	0	**40**
′	″	log cos	d.	log cot	d.	log tan	log sin	″	′

0°

′	″	log sin	d.	log tan	d.	log cot	log cos	″	′
20	0	7.764 75	361	7.764 76	361	2.235 24	9.999 99	0	40
	10	7.768 36	357	7.768 37	357	2.231 63	9.999 99	50	
	20	7.771 93	355	7.771 94	355	2.228 06	9.999 99	40	
	30	7.775 48	351	7.775 49	351	2.224 51	9.999 99	30	
	40	7.778 99	349	7.779 00	349	2.221 00	9.999 99	20	
	50	7.782 48	346	7.782 49	346	2.217 51	9.999 99	10	
21	0	7.785 94	344	7.785 95	343	2.214 05	9.999 99	0	39
	10	7.789 38	340	7.789 38	341	2.210 62	9.999 99	50	
	20	7.792 78	338	7.792 79	338	2.207 21	9.999 99	40	
	30	7.796 16	336	7.796 17	335	2.203 83	9.999 99	30	
	40	7.799 52	332	7.799 52	333	2.200 48	9.999 99	20	
	50	7.802 84	331	7.802 85	330	2.197 15	9.999 99	10	
22	0	7.806 15	327	7.806 15	328	2.193 85	9.999 99	0	38
	10	7.809 42	326	7.809 43	326	2.190 57	9.999 99	50	
	20	7.812 68	323	7.812 69	322	2.187 31	9.999 99	40	
	30	7.815 91	320	7.815 91	321	2.184 09	9.999 99	30	
	40	7.819 11	318	7.819 12	318	2.180 88	9.999 99	20	
	50	7.822 29	316	7.822 30	316	2.177 70	9.999 99	10	
23	0	7.825 45	314	7.825 46	314	2.174 54	9.999 99	0	37
	10	7.828 59	311	7.828 60	311	2.171 40	9.999 99	50	
	20	7.831 70	309	7.831 71	309	2.168 29	9.999 99	40	
	30	7.834 79	307	7.834 80	307	2.165 20	9.999 99	30	
	40	7.837 86	305	7.837 87	305	2.162 13	9.999 99	20	
	50	7.840 91	302	7.840 92	302	2.159 08	9.999 99	10	
24	0	7.843 93	301	7.843 94	301	2.156 06	9.999 99	0	36
	10	7.846 94	298	7.846 95	299	2.153 05	9.999 99	50	
	20	7.849 92	297	7.849 94	296	2.150 07	9.999 99	40	
	30	7.852 89	294	7.852 90	294	2.147 10	9.999 99	30	
	40	7.855 83	293	7.855 84	293	2.144 16	9.999 99	20	
	50	7.858 76	290	7.858 77	290	2.141 23	9.999 99	10	
25	0	7.861 66	289	7.861 67	289	2.138 33	9.999 99	0	35
	10	7.864 55	286	7.864 56	287	2.135 44	9.999 99	50	
	20	7.867 41	285	7.867 43	284	2.132 57	9.999 99	40	
	30	7.870 26	283	7.870 27	283	2.129 73	9.999 99	30	
	40	7.873 09	281	7.873 10	281	2.126 90	9.999 99	20	
	50	7.875 90	280	7.875 91	280	2.124 09	9.999 99	10	
26	0	7.878 70	277	7.878 71	277	2.121 29	9.999 99	0	34
	10	7.881 47	276	7.881 48	276	2.118 52	9.999 99	50	
	20	7.884 23	274	7.884 24	274	2.115 76	9.999 99	40	
	30	7.886 97	272	7.886 98	272	2.113 02	9.999 99	30	
	40	7.889 69	271	7.889 70	271	2.110 30	9.999 99	20	
	50	7.892 40	269	7.892 41	269	2.107 59	9.999 99	10	
27	0	7.895 09	267	7.895 10	267	2.104 90	9.999 99	0	33
	10	7.897 76	265	7.897 77	266	2.102 23	9.999 99	50	
	20	7.900 41	264	7.900 43	264	2.099 57	9.999 99	40	
	30	7.903 05	263	7.903 07	262	2.096 93	9.999 99	30	
	40	7.905 68	261	7.905 69	261	2.094 31	9.999 99	20	
	50	7.908 29	259	7.908 30	259	2.091 70	9.999 99	10	
28	0	7.910 88	258	7.910 89	258	2.089 11	9.999 99	0	32
	10	7.913 46	256	7.913 47	256	2.086 53	9.999 99	50	
	20	7.916 02	255	7.916 03	255	2.083 97	9.999 99	40	
	30	7.918 57	253	7.918 58	253	2.081 42	9.999 99	30	
	40	7.921 10	252	7.921 11	252	2.078 89	9.999 98	20	
	50	7.923 62	250	7.923 63	250	2.076 37	9.999 98	10	
29	0	7.926 12	249	7.926 13	249	2.073 87	9.999 98	0	31
	10	7.928 61	247	7.928 62	248	2.071 38	9.999 98	50	
	20	7.931 08	246	7.931 10	246	2.068 90	9.999 98	40	
	30	7.933 54	245	7.933 56	245	2.066 44	9.999 98	30	
	40	7.935 99	243	7.936 01	243	2.063 99	9.999 98	20	
	50	7.938 42	242	7.938 44	242	2.061 56	9.999 98	10	
30	0	7.940 84		7.940 86		2.059 14	9.999 98	0	30
′	″	log cos	d.	log cot	d.	log tan	log sin	″	′

89°

′	″	log sin	d.	log tan	d.	log cot	log cos	″	′
30	0	7.940 84	241	7.940 86	240	2.059 14	[illegible]	0	30
	10	7.934 25	239	7.943 26	240	2.056 74	[illegible]	50	
	20	7.945 64	238	7.945 66	238	2.054 34	9.9[illegible] 98	40	
	30	7.948 02	237	7.948 04	236	2.051 96	9.999 98	30	
	40	7.950 39	235	7.950 40	236	2.049 60	9.999 98	20	
	50	7.952 74	234	7.952 76	234	2.047 24	9.999 98	10	
31	0	7.955 08	233	7.955 10	233	2.044 90	9.999 98	0	29
	10	7.957 41	232	7.957 43	231	2.042 57	9.999 98	50	
	20	7.959 73	230	7.959 74	231	2.040 26	9.999 98	40	
	30	7.962 03	229	7.962 05	229	2.037 95	9.999 98	30	
	40	7.964 32	228	7.964 34	228	2.035 66	9.999 98	20	
	50	7.966 60	227	7.966 62	227	2.033 38	9.999 98	10	
32	0	7.968 87	226	7.968 89	225	2.031 11	9.999 98	0	28
	10	7.971 13	224	7.971 14	225	2.028 86	9.999 98	50	
	20	7.973 37	223	7.973 39	223	2.026 61	9.999 98	40	
	30	7.975 60	222	7.975 62	222	2.024 38	9.999 98	30	
	40	7.977 82	221	7.977 84	221	2.022 16	9.999 98	20	
	50	7.980 03	220	7.980 05	220	2.019 95	9.999 98	10	
33	0	7.982 23	219	7.982 25	219	2.017 75	9.999 98	0	27
	10	7.984 42	218	7.984 44	218	2.015 56	9.999 98	50	
	20	7.986 60	216	7.986 62	216	2.013 38	9.999 98	40	
	30	7.988 76	216	7.988 78	216	2.011 22	9.999 98	30	
	40	7.990 92	214	7.990 94	214	2.009 06	9.999 98	20	
	50	7.993 06	214	7.993 08	214	2.006 92	9.999 98	10	
34	0	7.995 20	212	7.995 22	212	2.004 78	9.999 98	0	26
	10	7.997 32	211	7.997 34	212	2.002 66	9.999 98	50	
	20	7.999 43	211	7.999 46	210	2.000 54	9.999 98	40	
	30	8.001 54	209	8.001 56	209	1.998 44	9.999 98	30	
	40	8.003 63	208	8.003 65	209	1.996 35	9.999 98	20	
	50	8.005 71	208	8.005 74	207	1.994 26	9.999 98	10	
35	0	8.007 79	206	8.007 81	206	1.992 19	9.999 98	0	25
	10	8.009 85	205	8.009 87	206	1.990 13	9.999 98	50	
	20	8.011 90	205	8.011 93	204	1.988 07	9.999 98	40	
	30	8.013 95	203	8.013 97	203	1.986 03	9.999 98	30	
	40	8.015 98	203	8.016 00	203	1.984 00	9.999 98	20	
	50	8.018 01	201	8.018 03	201	1.981 97	9.999 98	10	
36	0	8.020 02	201	8.020 04	201	1.979 96	9.999 98	0	24
	10	8.022 03	199	8.022 05	200	1.977 95	9.999 98	50	
	20	8.024 02	199	8.024 05	198	1.975 95	9.999 98	40	
	30	8.026 01	198	8.026 03	198	1.973 96	9.999 98	30	
	40	8.027 99	197	8.028 01	197	1.971 99	9.999 98	20	
	50	8.029 96	196	8.029 98	196	1.970 02	9.999 98	10	
37	0	8.031 92	195	8.031 94	196	1.968 06	9.999 97	0	23
	10	8.033 87	194	8.033 90	194	1.966 10	9.999 97	50	
	20	8.035 81	194	8.035 84	193	1.964 16	9.999 97	40	
	30	8.037 75	192	8.037 77	193	1.962 23	9.999 97	30	
	40	8.039 67	192	8.039 70	192	1.960 30	9.999 97	20	
	50	8.041 59	191	8.041 62	191	1.958 38	9.999 97	10	
38	0	8.043 50	190	8.043 53	190	1.956 47	9.999 97	0	22
	10	8.045 40	189	8.045 43	189	1.954 57	9.999 97	50	
	20	8.047 29	189	8.047 32	189	1.952 68	9.999 97	40	
	30	8.049 18	187	8.049 21	187	1.950 80	9.999 97	30	
	40	8.051 05	187	8.051 08	187	1.948 92	9.999 97	20	
	50	8.052 92	186	8.052 95	186	1.947 05	9.999 97	10	
39	0	8.054 78	185	8.054 81	185	1.945 19	9.999 97	0	21
	10	8.056 63	185	8.056 66	185	1.943 34	9.999 97	50	
	20	8.058 48	183	8.058 51	183	1.941 49	9.999 97	40	
	30	8 060 31	183	8.060 34	183	1.939 66	9.999 97	30	
	40	8.062 14	182	8.062 17	182	1.937 83	9.999 97	20	
	50	8.063 96	182	8.063 99	182	1.936 01	9.999 97	10	
40	0	8.065 78		8.065 81		1.934 19	9.999 97	0	20
′	″	log cos	d.	log cot	d.	log tan	log sin	″	′

′	″	log sin	d.	log tan	d.	log cot	log cos	″	′
40	0	8.065 78	180	8.065 81	180	1.934 19	9.999 97	0	**20**
	10	8.067 58	180	8.067 61	180	1.932 39	9.999 97	50	
	20	8.069 38	179	8.069 41	179	1.930 59	9.999 97	40	
	30	8.071 17	178	8.071 20	179	1.928 80	9.999 97	30	
	40	8.072 95	178	8.072 99	177	1.927 02	9.999 97	20	
	50	8.074 73	177	8.074 76	177	1.925 24	9.999 97	10	
41	0	8.076 50	176	8.076 53	176	1.923 47	9.999 97	0	**19**
	10	8.078 26	176	8.078 29	176	1.921 71	9.999 97	50	
	20	8.080 02	174	8.080 05	175	1.919 95	9.999 97	40	
	30	8.081 76	174	8.081 80	174	1.918 20	9.999 97	30	
	40	8.083 50	174	8.083 54	173	1.916 46	9.999 97	20	
	50	8.085 24	172	8.085 27	173	1.914 73	9.999 97	10	
42	0	8.086 96	172	8.087 00	172	1.913 00	9.999 97	0	**18**
	10	8.088 68	172	8.088 72	171	1.911 28	9.999 97	50	
	20	8.090 40	170	8.090 43	171	1.909 57	9.999 97	40	
	30	8.092 10	170	8.092 14	170	1.907 86	9.999 97	30	
	40	8.093 80	170	8.093 84	169	1.906 16	9.999 97	20	
	50	8.095 50	168	8.095 53	169	1.904 47	9.999 97	10	
43	0	8.097 18	168	8.097 22	168	1.902 78	9.999 97	0	**17**
	10	8.098 86	168	8.098 90	167	1.901 10	9.999 97	50	
	20	8.100 54	166	8.100 57	167	1.899 43	9.999 97	40	
	30	8.102 20	166	8.102 24	166	1.897 76	9.999 97	30	
	40	8.103 86	166	8.103 90	165	1.896 10	9.999 97	20	
	50	8.105 52	165	8.105 55	165	1.894 45	9.999 96	10	
44	0	8.107 17	164	8.107 20	165	1.892 80	9.999 96	0	**16**
	10	8.108 81	163	8.108 84	164	1.891 16	9.999 96	50	
	20	8.110 44	163	8.110 48	163	1.889 52	9.999 96	40	
	30	8.112 07	163	8.112 11	162	1.887 89	9.999 96	30	
	40	8.113 70	161	8.113 73	162	1.886 27	9.999 96	20	
	50	8.115 31	162	8.115 35	161	1.884 65	9.999 96	10	
45	0	8.116 93	160	8.116 96	161	1.883 04	9.999 96	0	**15**
	10	8.118 53	160	8.118 57	160	1.881 43	9.999 96	50	
	20	8.120 13	159	8.120 17	159	1.879 83	9.999 96	40	
	30	8.121 72	159	8.121 76	159	1.878 24	9.999 96	30	
	40	8.123 31	158	8.123 35	158	1.876 65	9.999 96	20	
	50	8.124 89	158	8.124 93	158	1.875 07	9.999 96	10	
46	0	8.126 47	157	8.126 51	157	1.873 50	9.999 96	0	**14**
	10	8.128 04	157	8.128 08	157	1.871 92	9.999 96	50	
	20	8.129 61	156	8.129 65	156	1.870 35	9.999 96	40	
	30	8.131 17	155	8.131 21	155	1.868 79	9.999 96	30	
	40	8.132 72	155	8.132 76	155	1.867 24	9.999 96	20	
	50	8.134 27	154	8.134 31	154	1.865 69	9.999 96	10	
47	0	8.135 81	154	8.135 85	154	1.864 15	9.999 96	0	**13**
	10	8.137 35	153	8.137 39	153	1.862 61	9.999 96	50	
	20	8.138 88	153	8.138 92	153	1.861 08	9.999 96	40	
	30	8.140 41	152	8.140 45	152	1.859 55	9.999 96	30	
	40	8.141 93	151	8.141 97	152	1.858 03	9.999 96	20	
	50	8.143 44	151	8.143 48	152	1.856 52	9.999 96	10	
48	0	8.144 95	151	8.145 00	150	1.855 00	9.999 96	0	**12**
	10	8.146 46	150	8.146 50	150	1.853 50	9.999 96	50	
	20	8.147 96	149	8.148 00	150	1.852 00	9.999 96	40	
	30	8.149 45	149	8.149 50	149	1.850 50	9.999 96	30	
	40	8.150 94	149	8.150 99	148	1.849 01	9.999 96	20	
	50	8.152 43	148	8.152 47	148	1.847 53	9.999 96	10	
49	0	8.153 91	147	8.153 95	148	1.846 05	9.999 96	0	**11**
	10	8.155 38	147	8.155 43	147	1.844 57	9.999 96	50	
	20	8.156 85	147	8.156 90	146	1.843 10	9.999 96	40	
	30	8.158 32	146	8.158 36	146	1.841 64	9.999 96	30	
	40	8.159 78	145	8.159 82	146	1.840 18	9.999 95	20	
	50	8.161 23	145	8.161 28	145	1.838 72	9.999 95	10	
50	0	8.162 68		8.162 73		1.837 27	9.999 95	0	**10**
′	″	log cos	d.	log cot	d.	log tan	log sin	″	′

′	″	log sin	d.	log tan	d.	log cot	log cos	″	′
50	0	8.162 68		8.162 73		1.837 27	9.999 95	0	10
	10	8.164 13	145	8.164 17	144	1.835 83	9.999 95	50	
	20	8.165 57	144	8.165 61	144	1.834 39	9.999 95	40	
	30	8.167 00	143	8.167 05	144	1.832 95	9.999 95	30	
	40	8.168 43	143	8.168 48	143	1.831 52	9.999 95	20	
	50	8.169 86	143	8.169 91	143	1.830 09	9.999 95	10	
51	0	8.171 28	142	8.171 33	142	1.828 67	9.999 95	0	9
	10	8.172 70	142	8.172 75	142	1.827 25	9.999 95	50	
	20	8.174 11	141	8.174 16	141	1.825 84	9.999 95	40	
	30	8.175 52	141	8.175 57	141	1.824 43	9.999 95	30	
	40	8.176 92	140	8.176 97	140	1.823 03	9.999 95	20	
	50	8.178 32	140	8.178 37	140	1.821 63	9.999 95	10	
52	0	8.179 71	139	8.179 76	139	1.820 24	9.999 95	0	8
	10	8.181 10	139	8.181 15	139	1.818 85	9.999 95	50	
	20	8.182 49	139	8.182 54	139	1.817 46	9.999 95	40	
	30	8.183 87	138	8.183 92	138	1.816 08	9.999 95	30	
	40	8.185 25	138	8.185 30	138	1.814 70	9.999 95	20	
	50	8.186 62	137	8.186 67	137	1.813 33	9.999 95	10	
53	0	8.187 98	136	8.188 04	137	1.811 96	9.999 95	0	7
	10	8.189 35	137	8.189 40	136	1.810 60	9.999 95	50	
	20	8.190 71	136	8.190 76	136	1.809 24	9.999 95	40	
	30	8.192 06	135	8.192 12	136	1.807 89	9.999 95	30	
	40	8.193 41	135	8.193 47	135	1.806 53	9.999 95	20	
	50	8.194 76	135	8.194 81	134	1.805 19	9.999 95	10	
54	0	8.196 10	134	8.196 16	135	1.803 84	9.999 95	0	6
	10	8.197 44	134	8.197 49	133	1.802 51	9.999 95	50	
	20	8.198 77	133	8.198 83	134	1.801 17	9.999 95	40	
	30	8.200 10	133	8.200 16	133	1.799 84	9.999 95	30	
	40	8.201 43	133	8.201 49	133	1.798 51	9.999 95	20	
	50	8.202 75	132	8.202 81	132	1.797 19	9.999 94	10	
55	0	8.204 07	132	8.204 13	132	1.795 87	9.999 94	0	5
	10	8.205 38	131	8.205 44	131	1.794 56	9.999 94	50	
	20	8.206 69	131	8.206 75	131	1.793 25	9.999 94	40	
	30	8.208 00	131	8.208 06	131	1.791 94	9.999 94	30	
	40	8.209 30	130	8.209 36	130	1.790 64	9.999 94	20	
	50	8.210 60	130	8.210 66	130	1.789 34	9.999 94	10	
56	0	8.211 89	129	8.211 95	129	1.788 05	9.999 94	0	4
	10	8.213 19	130	8.213 24	129	1.786 76	9.999 94	50	
	20	8.214 47	128	8.214 53	129	1.785 47	9.999 94	40	
	30	8.215 76	129	8.215 81	128	1.784 19	9.999 94	30	
	40	8.217 03	127	8.217 09	128	1.782 91	9.999 94	20	
	50	8.218 31	128	8.218 37	128	1.781 63	9.999 94	10	
57	0	8.219 58	127	8.219 64	127	1.780 36	9.999 94	0	3
	10	8.220 85	127	8.220 91	127	1.779 10	9.999 94	50	
	20	8.222 11	126	8.222 17	126	1.777 83	9.999 94	40	
	30	8.223 37	126	8.223 43	126	1.776 57	9.999 94	30	
	40	8.224 63	126	8.224 69	126	1.775 31	9.999 94	20	
	50	8.225 88	125	8.225 95	126	1.774 05	9.999 94	10	
58	0	8.227 13	125	8.227 20	125	1.772 80	9.999 94	0	2
	10	8.228 38	125	8.228 44	124	1.771 56	9.999 94	50	
	20	8.229 62	124	8.229 68	124	1.770 32	9.999 94	40	
	30	8.230 86	124	8.230 92	124	1.769 08	9.999 94	30	
	40	8.232 10	124	8.232 16	124	1.767 84	9.999 94	20	
	50	8.233 33	123	8.233 39	124	1.766 61	9.999 94	10	
59	0	8.234 56	123	8.234 62	123	1.765 38	9.999 94	0	1
	10	8.235 78	122	8.235 85	123	1.764 15	9.999 94	50	
	20	8.237 00	122	8.237 07	122	1.762 93	9.999 94	40	
	30	8.238 22	122	8.238 29	122	1.761 71	9.999 94	30	
	40	8.239 44	122	8.239 50	121	1.760 50	9.999 93	20	
	50	8.240 65	121	8.240 71	121	1.759 29	9.999 93	10	
60	0	8.241 86	121	8.241 92	121	1.758 08	9.999 93	0	0
′	″	log cos	d.	log cot	d.	log tan	log sin	″	′

′	″	log sin	d.	log tan	d.	log cot	log cos	″	′
0	0	8.241 86	120	8.241 92	121	1.758 08	9.999 93	0	60
	10	8.243 06	120	8.243 13	120	1.756 87	9.999 93	50	
	20	8.244 26	120	8.244 33	120	1.755 67	9.999 93	40	
	30	8.245 46	119	8.245 53	119	1.754 47	9.999 93	30	
	40	8.246 65	120	8.246 72	119	1.753 28	9.999 93	20	
	50	8.247 85	118	8.247 91	119	1.752 09	9.999 93	10	
1	0	8.249 03	119	8.249 10	119	1.750 90	9.999 93	0	59
	10	8.250 22	118	8.250 29	118	1.749 71	9.999 93	50	
	20	8.251 40	118	8.251 47	118	1.748 53	9.999 93	40	
	30	8.252 58	117	8.252 65	117	1.747 35	9.999 93	30	
	40	8.253 75	118	8.253 82	118	1.746 18	9.999 93	20	
	50	8.254 93	116	8.255 00	116	1.745 00	9.999 93	10	
2	0	8.256 09	117	8.256 16	117	1.743 84	9.999 93	0	58
	10	8.257 26	116	8.257 33	116	1.742 67	9.999 93	50	
	20	8.258 42	116	8.258 49	116	1.741 51	9.999 93	40	
	30	8.259 58	116	8.259 65	116	1.740 35	9.999 93	30	
	40	8.260 74	115	8.260 81	115	1.739 19	9.999 93	20	
	50	8.261 89	115	8.261 96	116	1.738 04	9.999 93	10	
3	0	8.263 04	115	8.263 12	114	1.736 88	9.999 93	0	57
	10	8.264 19	114	8.264 26	115	1.735 74	9.999 93	50	
	20	8.265 33	115	8.265 41	114	1.734 59	9.999 93	40	
	30	8.266 48	113	8.266 55	114	1.733 45	9.999 93	30	
	40	8.267 61	114	8.267 69	113	1.732 31	9.999 93	20	
	50	8.268 75	113	8.268 82	114	1.731 18	9.999 93	10	
4	0	8.269 88	113	8.269 96	113	1.730 04	9.999 92	0	56
	10	8.271 01	113	8.271 09	112	1.728 91	9.999 92	50	
	20	8.272 14	112	8.272 21	113	1.727 79	9.999 92	40	
	30	8.273 26	112	8.273 34	112	1.726 66	9.999 92	30	
	40	8.274 38	112	8.274 46	112	1.725 54	9.999 92	20	
	50	8.275 50	111	8.275 58	111	1.724 42	9.999 92	10	
5	0	8.276 61	112	8.276 69	111	1.723 31	9.999 92	0	55
	10	8.277 73	110	8.277 80	111	1.722 20	9.999 92	50	
	20	8.278 83	111	8.278 91	111	1.721 09	9.999 92	40	
	30	8.279 94	110	8.280 02	110	1.719 98	9.999 92	30	
	40	8.281 04	111	8.281 12	111	1.718 88	9.999 92	20	
	50	8.282 15	109	8.282 23	109	1.717 77	9.999 92	10	
6	0	8.283 24	110	8.283 32	110	1.716 68	9.999 92	0	54
	10	8.284 34	109	8.284 42	109	1.715 58	9.999 92	50	
	20	8.285 43	109	8.285 51	109	1.714 49	9.999 92	40	
	30	8.286 52	109	8.286 60	109	1.713 40	9.999 92	30	
	40	8.287 61	108	8.287 69	108	1.712 31	9.999 92	20	
	50	8.288 69	108	8.288 77	109	1.711 23	9.999 92	10	
7	0	8.289 77	108	8.289 86	108	1.710 14	9.999 92	0	53
	10	8.290 85	108	8.290 94	107	1.709 06	9.999 92	50	
	20	8.291 93	107	8.292 01	108	1.707 99	9.999 92	40	
	30	8.293 00	107	8.293 09	107	1.706 91	9.999 92	30	
	40	8.294 07	106	8.294 16	107	1.705 84	9.999 92	20	
	50	8.295 14	107	8.295 23	106	1.704 77	9.999 92	10	
8	0	8.296 21	106	8.296 29	107	1.703 71	9.999 92	0	52
	10	8.297 27	106	8.297 36	106	1.702 64	9.999 91	50	
	20	8.298 33	106	8.298 42	105	1.701 58	9.999 91	40	
	30	8.299 39	105	8.299 47	106	1.700 53	9.999 91	30	
	40	8.300 44	106	8.300 53	105	1.699 47	9.999 91	20	
	50	8.301 50	105	8.301 58	105	1.698 42	9.999 91	10	
9	0	8.302 55	104	8.302 63	105	1.697 37	9.999 91	0	51
	10	8.303 59	105	8.303 68	105	1.696 32	9.999 91	50	
	20	8.304 64	104	8.304 73	104	1.695 27	9.999 91	40	
	30	8.305 68	104	8.305 77	104	1.694 23	9.999 91	30	
	40	8.306 72	104	8.306 81	104	1.693 19	9.999 91	20	
	50	8.307 76	103	8.307 85	103	1.692 15	9.999 91	10	
10	0	8.308 79		8.308 88		1.691 12	9.999 91	0	50
′	″	log cos	d.	log cot	d.	log tan	log sin	″	′

′	″	log sin	d.	log tan	d.	log cot	log cos	″	′
10	0	8.308 79	104	8.308 88	104	1.691 12	9.999 91	0	50
	10	8.309 83	103	8.309 92	103	1.690 08	9.999 91	50	
	20	8.310 86	102	8.310 95	103	1.689 05	9.999 91	40	
	30	8.311 88	103	8.311 98	102	1.688 02	9.999 91	30	
	40	8.312 91	102	8.313 00	103	1.687 00	9.999 91	20	
	50	8.313 93	102	8.314 03	102	1.685 97	9.999 91	10	
11	0	8.314 95	102	8.315 05	101	1.684 95	9.999 91	0	49
	10	8.315 97	102	8.316 06	102	1.683 94	9.999 91	50	
	20	8.316 99	101	8.317 08	101	1.682 92	9.999 91	40	
	30	8.318 00	101	8.318 09	102	1.681 91	9.999 91	30	
	40	8.319 01	101	8.319 11	101	1.680 89	9.999 91	20	
	50	8.320 02	101	8.320 12	100	1.679 88	9.999 91	10	
12	0	8.321 03	100	8.321 12	101	1.678 88	9.999 90	0	48
	10	8.322 03	100	8.322 13	100	1.677 87	9.999 90	50	
	20	8.323 03	100	8.323 13	100	1.676 87	9.999 90	40	
	30	8.324 03	100	8.324 13	100	1.675 87	9.999 90	30	
	40	8.325 03	99	8.325 13	99	1.674 87	9.999 90	20	
	50	8.326 02	100	8.326 12	99	1.673 88	9.999 90	10	
13	0	8.327 02	99	8.327 11	100	1.672 89	9.999 90	0	47
	10	8.328 01	98	8.328 11	98	1.671 90	9.999 90	50	
	20	8.328 99	99	8.329 09	99	1.670 91	9.999 90	40	
	30	8.329 98	98	8.330 08	98	1.669 92	9.999 90	30	
	40	8.330 96	99	8.331 06	99	1.668 94	9.999 90	20	
	50	8.331 95	97	8.332 05	97	1.667 95	9.999 90	10	
14	0	8.332 92	98	8.333 02	98	1.666 98	9.999 90	0	46
	10	8.333 90	98	8.334 00	98	1.666 00	9.999 90	50	
	20	8.334 88	97	8.334 98	97	1.665 02	9.999 90	40	
	30	8.335 85	97	8.335 95	97	1.664 05	9.999 90	30	
	40	8.336 82	97	8.336 92	97	1.663 08	9.999 90	20	
	50	8.337 79	96	8.337 89	97	1.662 11	9.999 90	10	
15	0	8.338 75	97	8.338 86	96	1.661 14	9.999 90	0	45
	10	8.339 72	96	8.339 82	96	1.660 18	9.999 90	50	
	20	8.340 68	96	8.340 78	96	1.659 22	9.999 90	40	
	30	8.341 64	96	8.341 74	96	1.658 26	9.999 90	30	
	40	8.342 60	95	8.342 70	96	1.657 30	9.999 89	20	
	50	8.343 55	95	8.343 66	95	1.656 34	9.999 89	10	
16	0	8.344 50	96	8.344 61	95	1.655 39	9.999 89	0	44
	10	8.345 46	94	8.345 56	95	1.654 44	9.999 89	50	
	20	8.346 40	95	8.346 51	95	1.653 49	9.999 89	40	
	30	8.347 35	95	8.347 46	94	1.652 54	9.999 89	30	
	40	8.348 30	94	8.348 40	95	1.651 60	9.999 89	20	
	50	8.349 24	94	8.349 35	94	1.650 65	9.999 89	10	
17	0	8.350 18	94	8.350 29	94	1.649 71	9.999 89	0	43
	10	8.351 12	94	8.351 23	94	1.648 77	9.999 89	50	
	20	8.352 06	93	8.352 17	93	1.647 83	9.999 89	40	
	30	8.352 99	93	8.353 10	93	1.646 90	9.999 89	30	
	40	8.353 92	93	8.354 03	94	1.645 97	9.999 89	20	
	50	8.354 85	93	8.354 97	93	1.645 03	9.999 89	10	
18	0	8.355 78	93	8.355 90	92	1.644 10	9.999 89	0	42
	10	8.356 71	93	8.356 82	93	1.643 18	9.999 89	50	
	20	8.357 64	92	8.357 75	92	1.642 25	9.999 89	40	
	30	8.358 56	92	8.358 67	92	1.641 33	9.999 89	30	
	40	8.359 48	92	8.359 59	92	1.640 40	9.999 89	20	
	50	8.360 40	92	8.360 51	92	1.639 49	9.999 89	10	
19	0	8.361 32	91	8.361 43	92	1.638 57	9.999 89	0	41
	10	8.362 23	91	8.362 35	91	1.637 65	9.999 88	50	
	20	8.363 14	91	8.363 26	91	1.636 74	9.999 88	40	
	30	8.364 05	91	8.364 17	91	1.635 83	9.999 88	30	
	40	8.364 96	91	8.365 08	91	1.634 92	9.999 88	20	
	50	8.365 87	91	8.365 99	90	1.634 01	9.999 88	10	
20	0	8.366 78		8.366 89		1.633 11	9.999 88	0	40
′	″	log cos	d.	log cot	d.	log tan	log sin	″	′

1°

′	″	log sin	d.	log tan	d.	log cot	log cos	″	′
20	0	8.366 78	90	8.366 89	91	1.633 11	9.999 88	0	40
	10	8.367 68	90	8.367 80	90	1.632 20	9.999 88	50	
	20	8.368 58	90	8.368 70	90	1.631 30	9.999 88	40	
	30	8.369 48	90	8.369 60	90	1.630 40	9.999 88	30	
	40	8.370 38	90	8.370 50	90	1.629 50	9.999 88	20	
	50	8.371 28	89	8.371 40	89	1.628 60	9.999 88	10	
21	0	8.372 17	89	8.372 29	89	1.627 71	9.999 88	0	39
	10	8.373 06	89	8.373 18	90	1.626 82	9.999 88	50	
	20	8.373 95	89	8.374 08	89	1.625 92	9.999 88	40	
	30	8.374 84	89	8.374 97	88	1.625 03	9.999 88	30	
	40	8.375 73	89	8.375 85	89	1.624 15	9.999 88	20	
	50	8.376 62	88	8.376 74	88	1.623 26	9.999 88	10	
22	0	8.377 50	88	8.377 62	88	1.622 38	9.999 88	0	38
	10	8.378 38	88	8.378 50	88	1.621 50	9.999 88	50	
	20	8.379 26	88	8.379 38	88	1.620 62	9.999 88	40	
	30	8.380 14	87	8.380 26	88	1.619 74	9.999 87	30	
	40	8.381 01	88	8.381 14	88	1.618 86	9.999 87	20	
	50	8.381 89	87	8.382 02	87	1.617 98	9.999 87	10	
23	0	8.382 76	87	8.382 89	87	1.617 11	9.999 87	0	37
	10	8.383 63	87	8.383 76	87	1.616 24	9.999 87	50	
	20	8.384 50	87	8.384 63	87	1.615 37	9.999 87	40	
	30	8.385 37	87	8.385 50	86	1.614 50	9.999 87	30	
	40	8.386 24	86	8.386 36	87	1.613 64	9.990 87	20	
	50	8.387 10	86	8.387 23	86	1.612 77	9.999 87	10	
24	0	8.387 96	86	8.388 09	86	1.611 91	9.999 87	0	36
	10	8.388 82	86	8.388 95	86	1.611 05	9.999 87	50	
	20	8.389 68	86	8.389 81	86	1.610 19	9.999 87	40	
	30	8.390 54	85	8.390 67	86	1.609 33	9.999 87	30	
	40	8.391 39	86	8.391 53	85	1.608 47	9.999 87	20	
	50	8.392 25	85	8.392 38	85	1.607 62	9.999 87	10	
25	0	8.393 10	85	8.393 23	85	1.606 77	9.999 87	0	35
	10	8.393 95	85	8.394 08	85	1.605 92	9.999 87	50	
	20	8.394 80	85	8.394 93	85	1.605 07	9.999 87	40	
	30	8.395 65	84	8.395 78	85	1.604 22	9.999 87	30	
	40	8.396 49	85	8.396 63	84	1.603 37	9.999 87	20	
	50	8.397 34	84	8.397 47	85	1.602 53	9.999 86	10	
26	0	8.398 18	84	8.398 32	84	1.601 68	9.999 86	0	34
	10	8.399 02	84	8.399 16	84	1.600 84	9.999 86	50	
	20	8.399 86	84	8.400 00	83	1.600 00	9.999 86	40	
	30	8.400 70	83	8.400 83	84	1.599 17	9.999 86	30	
	40	8.401 53	84	8.401 67	84	1.598 33	9.999 86	20	
	50	8.402 37	83	8.402 51	83	1.597 50	9.999 86	10	
27	0	8.403 20	83	8.403 34	83	1.596 66	9.999 86	0	33
	10	8.404 03	83	8.404 17	83	1.595 83	9.999 86	50	
	20	8.404 86	83	8.405 00	83	1.595 00	9.999 86	40	
	30	8.405 69	82	8.405 83	82	1.594 17	9.999 86	30	
	40	8.406 51	83	8.406 65	83	1.593 35	9.999 86	20	
	50	8.407 34	82	8.407 48	82	1.592 52	9.999 86	10	
28	0	8.408 16	82	8.408 30	83	1.591 70	9.999 86	0	32
	10	8.408 98	82	8.409 13	82	1.590 87	9.999 86	50	
	20	8.409 80	82	8.409 95	82	1.590 05	9.999 86	40	
	30	8.410 62	82	8.410 77	81	1.589 23	9.999 86	30	
	40	8.411 44	81	8.411 58	82	1.588 42	9.999 86	20	
	50	8.412 25	81	8.412 40	81	1.587 60	9.999 86	10	
29	0	8.413 07	81	8.413 21	82	1.586 79	9.999 85	0	31
	10	8.413 88	81	8.414 03	81	1.585 97	9.999 85	50	
	20	8.414 69	81	8.414 84	81	1.585 16	9.999 85	40	
	30	8.415 50	81	8.415 65	81	1.584 35	9.999 85	30	
	40	8.416 31	80	8.416 46	80	1.583 54	9.999 85	20	
	50	8.417 11	81	8.417 26	81	1.582 74	9.999 85	10	
30	0	8.417 92		8.418 07		1.581 93	9.999 85	0	30
′	″	log cos	d.	log cot	d.	log tan	log sin	″	′

88°

′	″	log sin	d.	log tan	d.	log cot	log cos	″	′
30	0	8.417 92	80	8.418 07	80	1.581 93	[illegible]	0	30
	10	8.418 72	80	8.418 87	80	1.581 13	[illegible]	50	
	20	8.419 52	80	8.419 67	81	1.580 33	[illegible]	40	
	30	8.420 32	80	8.420 48	79	1.579 53	[illegible]	30	
	40	8.421 12	80	8.421 27	80	1.578 73	[illegible]	20	
	50	8.421 92	80	8.422 07	80	1.577 93	9.999 85	10	
31	0	8.422 72	79	8.422 87	79	1.577 13	9.999 85	0	29
	10	8.423 51	79	8.423 66	80	1.576 34	9.999 85	50	
	20	8.424 30	80	8.424 46	79	1.575 54	9.999 85	40	
	30	8.425 10	79	8.425 25	79	1.574 75	9.999 85	30	
	40	8.425 89	78	8.426 04	79	1.573 96	9.999 85	20	
	50	8.426 67	79	8.426 83	79	1.573 17	9.999 85	10	
32	0	8.427 46	79	8.427 62	78	1.572 38	9.999 84	0	28
	10	8.428 25	78	8.428 40	79	1.571 60	9.999 84	50	
	20	8.429 03	79	8.429 19	78	1.570 81	9.999 84	40	
	30	8.429 82	78	8.429 97	78	1.570 03	9.999 84	30	
	40	8.430 60	78	8.430 75	79	1.569 25	9.999 84	20	
	50	8.431 38	78	8.431 54	78	1.568 46	9.999 84	10	
33	0	8.432 16	77	8.432 32	77	1.567 69	9.999 84	0	27
	10	8.432 93	78	8.433 09	78	1.566 91	9.999 84	50	
	20	8.433 71	77	8.433 87	77	1.566 13	9.999 84	40	
	30	8.434 48	78	8.434 64	78	1.565 36	9.999 84	30	
	40	8.435 26	77	8.435 42	77	1.564 58	9.999 84	20	
	50	8.436 03	77	8.436 19	77	1.563 81	9.999 84	10	
34	0	8.436 80	77	8.436 96	77	1.563 04	9.999 84	0	26
	10	8.437 57	77	8.437 73	77	1.562 27	9.999 84	50	
	20	8.438 34	76	8.438 50	77	1.561 50	9.999 84	40	
	30	8.439 10	77	8.439 27	76	1.560 73	9.999 84	30	
	40	8.439 87	76	8.440 03	77	1.559 97	9.999 84	20	
	50	8.440 63	76	8.440 80	76	1.559 20	9.999 83	10	
35	0	8.441 39	77	8.441 56	76	1.558 44	9.999 83	0	25
	10	8.442 16	76	8.442 32	76	1.557 68	9.999 83	50	
	20	8.442 92	76	8.443 08	76	1.556 92	9.999 83	40	
	30	8.443 67	75	8.443 84	76	1.556 16	9.999 83	30	
	40	8.444 43	76	8.444 60	76	1.555 40	9.999 83	20	
	50	8.445 19	75	8.445 36	75	1.554 64	9.999 83	10	
36	0	8.445 94	75	8.446 11	75	1.553 89	9.999 83	0	24
	10	8.446 69	76	8.446 86	76	1.553 14	9.999 83	50	
	20	8.447 45	75	8.447 62	75	1.552 38	9.999 83	40	
	30	8.448 20	75	8.448 37	75	1.551 63	9.999 83	30	
	40	8.448 95	74	8.449 12	75	1.550 88	9.999 83	20	
	50	8.449 69	75	8.449 87	74	1.550 13	9.999 83	10	
37	0	8.450 44	75	8.450 61	75	1.549 39	9.999 83	0	23
	10	8.451 19	74	8.451 36	74	1.548 64	9.999 83	50	
	20	8.451 93	74	8.452 10	75	1.547 90	9.999 83	40	
	30	8.452 67	74	8.452 85	74	1.547 15	9.999 83	30	
	40	8.453 41	74	8.453 59	74	1.546 41	9.999 82	20	
	50	8.454 15	74	8.454 33	74	1.545 67	9.999 82	10	
38	0	8.454 89	74	8.455 07	74	1.544 93	9.999 82	0	22
	10	8.455 63	74	8.455 81	74	1.544 19	9.999 82	50	
	20	8.456 37	73	8.456 55	73	1.543 45	9.999 82	40	
	30	8.457 10	74	8.457 28	74	1.542 72	9.999 82	30	
	40	8.457 84	73	8.458 02	73	1.541 98	9.999 82	20	
	50	8.458 57	73	8.458 75	73	1.541 25	9.999 82	10	
39	0	8.459 30	73	8.459 48	73	1.540 52	9.999 82	0	21
	10	8.460 03	73	8.460 21	73	1.539 79	9.999 82	50	
	20	8.460 76	73	8.460 94	73	1.539 06	9.999 82	40	
	30	8.461 49	73	8.461 67	73	1.538 33	9.999 82	30	
	40	8.462 22	72	8.462 40	72	1.537 60	9.999 82	20	
	50	8.462 94	72	8.463 12	73	1.536 88	9.999 82	10	
40	0	8.463 66		8.463 85		1.536 15	9.999 82	0	20
′	″	log cos	d.	log cot	d.	log tan	log sin	″	′

′	″	log sin	d.	log tan	d.	log cot	log cos	″	′
40	0	8.463 66	73	8.463 85	72	1.536 15	9.999 82	0	**20**
	10	8.464 39	72	8.464 57	72	1.535 43	9.999 82	50	
	20	8.465 11	72	8.465 29	73	1.534 71	9.999 82	40	
	30	8.465 83	72	8.466 02	72	1.533 98	9.999 81	30	
	40	8.466 55	72	8.466 74	71	1.533 26	9.999 81	20	
	50	8.467 27	72	8.467 45	72	1.532 55	9.999 81	10	
41	0	8.467 99	71	8.468 17	72	1.531 83	9.999 81	0	**19**
	10	8.468 70	72	8.468 89	71	1.531 11	9.999 81	50	
	20	8.469 42	71	8.469 60	72	1.530 40	9.999 81	40	
	30	8.470 13	71	8.470 32	71	1.529 68	9.999 81	30	
	40	8.470 84	71	8.471 03	71	1.528 97	9.999 81	20	
	50	8.471 55	71	8.471 74	71	1.528 26	9.999 81	10	
42	0	8.472 26	71	8.472 45	71	1.527 55	9.999 81	0	**18**
	10	8.472 97	71	8.473 16	71	1.526 84	9.999 81	50	
	20	8.473 68	71	8.473 87	71	1.526 13	9.999 81	40	
	30	8.474 39	70	8.474 58	70	1.525 42	9.999 81	30	
	40	8.475 09	71	8.475 28	71	1.524 72	9.999 81	20	
	50	8.475 80	70	8.475 99	70	1.524 01	9.999 81	10	
43	0	8.476 50	70	8.476 69	71	1.523 31	9.999 81	0	**17**
	10	8.477 20	70	8.477 40	70	1.522 60	9.999 80	50	
	20	8.477 90	70	8.478 10	70	1.521 90	9.999 80	40	
	30	8.478 60	70	8.478 80	70	1.521 20	9.999 80	30	
	40	8.479 30	70	8.479 50	70	1.520 50	9.999 80	20	
	50	8.480 00	69	8.480 20	69	1.519 81	9.999 80	10	
44	0	8.480 69	70	8.480 89	70	1.519 11	9.999 80	0	**16**
	10	8.481 39	69	8.481 59	69	1.518 41	9.999 80	50	
	20	8.482 08	70	8.482 28	70	1.517 72	9.999 80	40	
	30	8.482 78	69	8.482 98	69	1.517 02	9.999 80	30	
	40	8.483 47	69	8.483 67	69	1.516 33	9.999 80	20	
	50	8.484 16	69	8.484 36	69	1.515 64	9.999 80	10	
45	0	8.484 85	69	8.485 05	69	1.514 95	9.999 80	0	**15**
	10	8.485 54	68	8.485 74	69	1.514 26	9.999 80	50	
	20	8.486 22	69	8.486 43	68	1.513 57	9.999 80	40	
	30	8.486 91	69	8.487 11	69	1.512 89	9.999 80	30	
	40	8.487 60	68	8.487 80	69	1.512 20	9.999 79	20	
	50	8.488 28	68	8.488 49	68	1.511 51	9.999 79	10	
46	0	8.488 96	69	8.489 17	68	1.510 83	9.999 79	0	**14**
	10	8.489 65	68	8.489 85	68	1.510 15	9.999 79	50	
	20	8.490 33	68	8.490 53	68	1.509 47	9.999 79	40	
	30	8.491 01	68	8.491 21	68	1.508 79	9.999 79	30	
	40	8.491 69	67	8.491 89	68	1.508 11	9.999 79	20	
	50	8.492 36	68	8.492 57	68	1.507 43	9.999 79	10	
47	0	8.493 04	68	8.493 25	68	1.506 75	9.999 79	0	**13**
	10	8.493 72	67	8.493 93	67	1.506 07	9.999 79	50	
	20	8.494 39	67	8.494 60	68	1.505 40	9.999 79	40	
	30	8.495 06	68	8.495 28	67	1.504 72	9.999 79	30	
	40	8.495 74	67	8.495 95	67	1.504 05	9.999 79	20	
	50	8.496 41	67	8.496 62	67	1.503 38	9.999 79	10	
48	0	8.497 08	67	8.497 29	67	1.502 71	9.999 79	0	**12**
	10	8.497 75	67	8.497 96	67	1.502 04	9.999 79	50	
	20	8.498 42	66	8.498 63	67	1.501 37	9.999 78	40	
	30	8.499 08	67	8.499 30	67	1.500 70	9.999 78	30	
	40	8.499 75	67	8.499 97	66	1.500 03	9.999 78	20	
	50	8.500 42	66	8.500 63	67	1.499 37	9.999 78	10	
49	0	8.501 08	66	8.501 30	66	1.498 70	9.999 78	0	**11**
	10	8.501 74	67	8.501 96	67	1.498 04	9.999 78	50	
	20	8.502 41	66	8.502 63	66	1.497 37	9.999 78	40	
	30	8.503 07	66	8.503 29	66	1.496 71	9.999 78	30	
	40	8.503 73	66	8.503 95	66	1.496 05	9.999 78	20	
	50	8.504 39	65	8.504 61	66	1.495 39	9.999 78	10	
50	0	8.505 04		8.505 27		1.494 73	9.999 78	0	**10**
′	″	log cos	d.	log cot	d.	log tan	log sin	″	′

′	″	log sin	d.	log tan	d.	log cot	log cos	″	′
50	0	8.505 04	66	8.505 27	66	1.494 73	9.999 78	0	10
	10	8.505 70	66	8.505 93	65	1.494 08	9.999 78	50	
	20	8.506 36	65	8.506 58	66	1.493 42	9.999 78	40	
	30	8.507 01	66	8.507 24	65	1.492 76	9.999 78	30	
	40	8.507 67	65	8.507 89	66	1.492 11	9.999 77	20	
	50	8.508 32	65	8.508 55	65	1.491 45	9.999 77	10	
51	0	8.508 97	66	8.509 20	65	1.490 80	9.999 77	0	9
	10	8.509 63	65	8.509 85	65	1.490 15	9.999 77	50	
	20	8.510 28	64	8.510 50	65	1.489 50	9.999 77	40	
	30	8.510 92	65	8.511 15	65	1.488 85	9.999 77	30	
	40	8.511 57	65	8.511 80	65	1.488 20	9.999 77	20	
	50	8.512 22	65	8.512 45	65	1.487 55	9.999 77	10	
52	0	8.512 87	64	8.513 10	64	1.486 90	9.999 77	0	8
	10	8.513 51	65	8.513 74	65	1.486 26	9.999 77	50	
	20	8.514 16	64	8.514 39	64	1.485 61	9.999 77	40	
	30	8.514 80	64	8.515 03	65	1.484 97	9.999 77	30	
	40	8.515 44	65	8.515 68	64	1.484 32	9.999 77	20	
	50	8.516 09	64	8.516 32	64	1.483 68	9.999 77	10	
53	0	8.516 73	64	8.516 96	64	1.483 04	9.999 77	0	7
	10	8.517 37	64	8.517 60	64	1.482 40	9.999 76	50	
	20	8.518 01	63	8.518 24	64	1.481 76	9.999 76	40	
	30	8.518 64	64	8.518 88	64	1.481 12	9.999 76	30	
	40	8.519 28	64	8.519 52	63	1.480 48	9.999 76	20	
	50	8.519 92	63	8.520 15	64	1.479 85	9.999 76	10	
54	0	8.520 55	64	8.520 79	64	1.479 21	9.999 76	0	6
	10	8.521 19	63	8.521 43	63	1.478 57	9.999 76	50	
	20	8.521 82	63	8.522 06	63	1.477 94	9.999 76	40	
	30	8.522 45	63	8.522 69	63	1.477 31	9.999 76	30	
	40	8.523 08	63	8.523 32	64	1.476 68	9.999 76	20	
	50	8.523 71	63	8.523 96	63	1.476 04	9.999 76	10	
55	0	8.524 34	63	8.524 59	63	1.475 41	9.999 76	0	5
	10	8.524 97	63	8.525 22	62	1.474 78	9.999 76	50	
	20	8.525 60	63	8.525 84	63	1.474 16	9.999 76	40	
	30	8.526 23	62	8.526 47	63	1.473 53	9.999 75	30	
	40	8.526 85	63	8.527 10	62	1.472 90	9.999 75	20	
	50	8.527 48	62	8.527 72	63	1.472 28	9.999 75	10	
56	0	8.528 10	63	8.528 35	62	1.471 65	9.999 75	0	4
	10	8.528 73	62	8.528 97	63	1.471 03	9.999 75	50	
	20	8.529 35	62	8.529 60	62	1.470 40	9.999 75	40	
	30	8.529 97	62	8.530 22	62	1.469 78	9.999 75	30	
	40	8.530 59	62	8.530 84	62	1.469 16	9.999 75	20	
	50	8.531 21	62	8.531 46	62	1.468 54	9.999 75	10	
57	0	8.531 83	62	8.532 08	62	1.467 92	9.999 75	0	3
	10	8.532 45	61	8.532 70	62	1.467 30	9.999 75	50	
	20	8.533 06	62	8.533 32	61	1.466 68	9.999 75	40	
	30	8.533 68	61	8.533 93	62	1.466 07	9.999 75	30	
	40	8.534 29	62	8.534 55	61	1.465 45	9.999 75	20	
	50	8.534 91	61	8.535 16	62	1.464 84	9.999 74	10	
58	0	8.535 52	62	8.535 78	61	1.464 22	9.999 74	0	2
	10	8.536 14	61	8.536 39	61	1.463 61	9.999 74	50	
	20	8.536 75	61	8.537 00	62	1.463 00	9.999 74	40	
	30	8.537 36	61	8.537 62	61	1.462 38	9.999 74	30	
	40	8.537 97	61	8.538 23	61	1.461 77	9.999 74	20	
	50	8.538 58	61	8.538 84	61	1.461 16	9.999 74	10	
59	0	8.539 19	60	8.539 45	60	1.460 55	9.999 74	0	1
	10	8.539 79	61	8.540 05	61	1.459 95	9.999 74	50	
	20	8.540 40	61	8.540 66	61	1.459 34	9.999 74	40	
	30	8.541 01	60	8.541 27	60	1.458 73	9.999 74	30	
	40	8.541 61	61	8.541 87	61	1.458 13	9.999 74	20	
	50	8.542 22	60	8.542 48	60	1.457 52	9.999 74	10	
60	0	8.542 82		8.543 08		1.456 92	9.999 74	0	0
′	″	log cos	d.	log cot	d.	log tan	log sin	″	′

′	1° log sin	1° log tan	1° log cot	1° log cos	2° log sin	2° log tan	2° log cot	2° log cos	
0	8.24 186	8.24 192	1.75 808	9.99 993	8.54 282	8.54 308	1.45 692	9.99 974	60
1	8.24 903	8.24 910	1.75 090	9.99 993	8.54 642	8.54 669	1.45 331	9.99 973	59
2	8.25 609	8.25 616	1.74 384	9.99 993	8.54 999	8.55 027	1.44 973	9.99 973	58
3	8.26 304	8.26 312	1.73 688	9.99 993	8.55 354	8.55 382	1.44 618	9.99 972	57
4	8.26 988	8.26 996	1.73 004	9.99 992	8.55 705	8.55 734	1.44 266	9.99 972	56
5	8.27 661	8.27 669	1.72 331	9.99 992	8.56 054	8.56 083	1.43 917	9.99 971	55
6	8.28 324	8.28 332	1.71 668	9.99 992	8.56 400	8.56 429	1.43 571	9.99 971	54
7	8.28 977	8.28 986	1.71 014	9.99 992	8.56 743	8.56 773	1.43 227	9.99 970	53
8	8.29 621	8.29 629	1.70 371	9.99 992	8.57 084	8.57 114	1.42 886	9.99 970	52
9	8.30 255	8.30 263	1.69 737	9.99 991	8.57 421	8.57 452	1.42 548	9.99 969	51
10	8.30 879	8.30 888	1.69 112	9.99 991	8.57 757	8.57 788	1.42 212	9.99 969	50
11	8.31 495	8.31 505	1.68 495	9.99 991	8.58 089	8.58 121	1.41 879	9.99 968	49
12	8.32 103	8.32 112	1.67 888	9.99 990	8.58 419	8.58 451	1.41 549	9.99 968	48
13	8.32 702	8.32 711	1.67 289	9.99 990	8.58 747	8.58 779	1.41 221	9.99 967	47
14	8.33 292	8.33 302	1.66 698	9.99 990	8.59 072	8.59 105	1.40 895	9.99 967	46
15	8.33 875	8.33 886	1.66 114	9.99 990	8.59 395	8.59 428	1.40 572	9.99 967	45
16	8.34 450	8.34 461	1.65 539	9.99 989	8.59 715	8.59 749	1.40 251	9.99 966	44
17	8.35 018	8.35 029	1.64 971	9.99 989	8.60 033	8.60 068	1.39 932	9.99 966	43
18	8.35 578	8.35 590	1.64 410	9.99 989	8.60 349	8.60 384	1.39 616	9.99 965	42
19	8.36 131	8.36 143	1.63 857	9.99 989	8.60 662	8.60 698	1.39 302	9.99 964	41
20	8.36 678	8.36 689	1.63 311	9.99 988	8.60 973	8.61 009	1.38 991	9.99 964	40
21	8.37 217	8.37 229	1.62 771	9.99 988	8.61 282	8.61 319	1.38 681	9.99 963	39
22	8.37 750	8.37 762	1.62 238	9.99 988	8.61 589	8.61 626	1.38 374	9.99 963	38
23	8.38 276	8.38 289	1.61 711	9.99 987	8.61 894	8.61 931	1.38 069	9.99 962	37
24	8.38 796	8.38 809	1.61 191	9.99 987	8.62 196	8.62 234	1 37 766	9.99 962	36
25	8.39 310	8.39 323	1.60 677	9.99 987	8.62 497	8.62 535	1.37 465	9.99 961	35
26	8.39 818	8.39 832	1.60 168	9.99 986	8.62 795	8.62 834	1.37 166	9.99 961	34
27	8.40 320	8.40 334	1.59 666	9.99 986	8.63 091	8.63 131	1.36 869	9.99 960	33
28	8.40 816	8.40 830	1.59 170	9.99 986	8.63 385	8.63 426	1.36 574	9.99 960	32
29	8.41 307	8.41 321	1.58 679	9.99 985	8.63 678	8.63 718	1.36 282	9.99 959	31
30	8.41 792	8.41 807	1.58 193	9.99 985	8.63 968	8.64 009	1.35 991	9.99 959	30
31	8.42 272	8.42 287	1.57 713	9.99 985	8.64 256	8.64 298	1.35 702	9.99 958	29
32	8.42 746	8.42 762	1.57 238	9.99 984	8.64 543	8.64 585	1.35 415	9.99 958	28
33	8.43 216	8.43 232	1.56 768	9.99 984	8.64 827	8.64 870	1.35 130	9.90 957	27
34	8.43 680	8.43 696	1.56 304	9.99 984	8.65 110	8.65 154	1.34 846	9.99 956	26
35	8.44 139	8.44 156	1.55 844	9.99 983	8.65 391	8.65 435	1.34 565	9.99 956	25
36	8.44 594	8.44 611	1.55 389	9.99 983	8.65 670	8.65 715	1.34 285	9.99 955	24
37	8.45 044	8.45 061	1.54 939	9.99 983	8.65 947	8.65 993	1.34 007	9.99 955	23
38	8.45 489	8.45 507	1.54 493	9.99 982	8.66 223	8.66 269	1.33 731	9.99 954	22
39	8.45 930	8.45 948	1.54 052	9.99 982	8.66 497	8.66 543	1.33 457	9.99 954	21
40	8.46 366	8.46 385	1.53 615	9.99 982	8.66 769	8.66 816	1.33 184	9.99 953	20
41	8.46 799	8.46 817	1.53 183	9.99 981	8.67 039	8.67 087	1.32 913	9.99 952	19
42	8.47 226	8.47 245	1.52 755	9.99 981	8.67 308	8.67 356	1.32 644	9.99 952	18
43	8.47 650	8.47 669	1.52 331	9.99 981	8.67 575	8.67 624	1.32 376	9.99 951	17
44	8.48 069	8.48 089	1.51 911	9.99 980	8.67 841	8.67 890	1.32 110	9.99 951	16
45	8.48 485	8.48 505	1.51 495	9.99 980	8.68 104	8.68 154	1.31 846	9.99 950	15
46	8.48 896	8.48 917	1.51 083	9.99 979	8.68 367	8.68 417	1.31 583	9.99 949	14
47	8.49 304	8.49 325	1.50 675	9.99 979	8.68 627	8.68 678	1.31 322	9.99 949	13
48	8.49 708	8.49 729	1.50 271	9.99 979	8.68 886	8.68 938	1.31 062	9.99 948	12
49	8.50 108	8.50 130	1.49 870	9.99 978	8.69 144	8.69 196	1.30 804	9.99 948	11
50	8.50 504	8.50 527	1.49 473	9.99 978	8.69 400	8.69 453	1.30 547	9.99 947	10
51	8.50 897	8.50 920	1.49 080	9.99 977	8.69 654	8.69 708	1.30 292	9.99 946	9
52	8.51 287	8.51 310	1.48 690	9.99 977	8.69 907	8.69 962	1.30 038	9.99 946	8
53	8.51 673	8.51 696	1.48 304	9.99 977	8.70 159	8.70 214	1.29 786	9.99 945	7
54	8.52 055	8.52 079	1.47 921	9.99 976	8.70 409	8.70 465	1.29 535	9.99 944	6
55	8.52 434	8.52 459	1.47 541	9.99 976	8.70 658	8.70 714	1.29 286	9.99 944	5
56	8.52 810	8.52 835	1.47 165	9.99 975	8.70 905	8.70 962	1.29 038	9.99 943	4
57	8.53 183	8.53 208	1.46 792	9.99 975	8.71 151	8.71 208	1.28 792	9.99 942	3
58	8.53 552	8.53 578	1.46 422	9.99 974	8.71 395	8.71 453	1.28 547	9.99 942	2
59	8.53 919	8.53 945	1.46 055	9.99 974	8.71 638	8.71 697	1.28 303	9.99 941	1
60	8.54 282	8.54 308	1.45 692	9.99 974	8.71 880	8.71 940	1.28 060	9.99 940	0
	88° log cos	88° log cot	88° log tan	88° log sin	87° log cos	87° log cot	87° log tan	87° log sin	′

′	log sin	log tan	log cot	log cos	log sin	log tan	log cot	log cos	
0	8.71 880	8.71 940	1.28 060	9.99 940	8.84 358	8.84 464	1.15 536	9.99 894	60
1	8.72 120	8.72 181	1.27 819	9.99 940	8.84 539	8.84 646	1.15 354	9.99 893	59
2	8.72 359	8.72 420	1.27 580	9.99 939	8.84 718	8.84 826	1.15 174	[illegible]	58
3	8.72 597	8.72 659	1.27 341	9.99 938	8.84 897	8.85 006	[illegible]	[illegible]	57
4	8.72 834	8.72 896	1.27 104	9.99 938	8.85 075	8.85 185	1.14 815	[illegible]	56
5	8.73 069	8.73 132	1.26 868	9.99 937	8.85 252	8.85 363	[illegible]	[illegible]	55
6	8.73 303	8.73 366	1.26 634	9.99 936	8.85 429	8.85 540	1.14 460	[illegible]	54
7	8.73 535	8.73 600	1.26 400	9.99 936	8.85 605	8.85 717	1.14 283	[illegible]	53
8	8.73 767	8.73 832	1.26 168	9.99 935	8.85 780	8.85 893	1.14 107	[illegible]	52
9	8.73 997	8.74 063	1.25 937	9.99 934	8.85 955	8.86 069	1.13 931	[illegible]	51
10	8.74 226	8.74 292	1.25 708	9.99 934	8.86 128	8.86 243	1.13 757	9.99 885	50
11	8.74 454	8.74 521	1.25 479	9.99 933	8.86 301	8.86 417	1.13 583	9.99 884	49
12	8.74 680	8.74 748	1.25 252	9.99 932	8.86 474	8.86 591	1.13 409	9.99 883	48
13	8.74 906	8.74 974	1.25 026	9.99 932	8.86 645	8.86 763	1.13 237	9.99 882	47
14	8.75 130	8.75 199	1.24 801	9.99 931	8.86 816	8.86 935	1.13 065	9.99 881	46
15	8.75 353	8.75 423	1.24 577	9.99 930	8.86 987	8.87 106	1.12 894	9.99 880	45
16	8.75 575	8.75 645	1.24 355	9.99 929	8.87 156	8.87 277	1.12 723	9.99 879	44
17	8.75 795	8.75 867	1.24 133	9.99 929	8.87 325	8.87 447	1.12 553	9.99 879	43
18	8.76 015	8.76 087	1.23 913	9.99 928	8.87 494	8.87 616	1.12 384	9.99 878	42
19	8.76 234	8.76 306	1.23 694	9.99 927	8.87 661	8.87 785	1.12 215	9.99 877	41
20	8.76 451	8.76 525	1.23 475	9.99 926	8.87 829	8.87 953	1.12 047	9.99 876	40
21	8.76 667	8.76 742	1.23 258	9.99 926	8.87 995	8.88 120	1.11 880	9.99 875	39
22	8.76 883	8.76 958	1.23 042	9.99 925	8.88 161	8.88 287	1.11 713	9.99 874	38
23	8.77 097	8.77 173	1.22 827	9.99 924	8.88 326	8.88 453	1.11 547	9.99 873	37
24	8.77 310	8.77 387	1.22 613	9.99 923	8.88 490	8.88 618	1.11 382	9.99 872	36
25	8.77 522	8.77 600	1.22 400	9.99 923	8.88 654	8.88 783	1.11 217	9.99 871	35
26	8.77 733	8.77 811	1.22 189	9.99 922	8.88 817	8.88 948	1.11 052	9.99 870	34
27	8.77 943	8.78 022	1.21 978	9.99 921	8.88 980	8.89 111	1.10 889	9.99 869	33
28	8.78 152	8.78 232	1.21 768	9.99 920	8.89 142	8.89 274	1.10 726	9.99 868	32
29	8.78 360	8.78 441	1.21 559	9.99 920	8.89 304	8.89 437	1.10 563	9.99 867	31
30	8.78 568	8.78 649	1.21 351	9.99 919	8.89 464	8.89 598	1.10 402	9.99 866	30
31	8.78 774	8.78 855	1.21 145	9.99 918	8.89 625	8.89 760	1.10 240	9.99 865	29
32	8.78 979	8.79 061	1.20 939	9.99 917	8.89 784	8.89 920	1.10 080	9.99 864	28
33	8.79 183	8.79 266	1.20 734	9.99 917	8.89 943	8.90 080	1.09 920	9.99 863	27
34	8.79 386	8.79 470	1.20 530	9.99 916	8.90 102	8.90 240	1.09 760	9.99 862	26
35	8.79 588	8.79 673	1.20 327	9.99 915	8.90 260	8.90 399	1.09 601	9.99 861	25
36	8.79 789	8.79 875	1.20 125	9.99 914	8.90 417	8.90 557	1.09 443	9.99 860	24
37	8.79 990	8.80 076	1.19 924	9.99 913	8.90 574	8.90 715	1.09 285	9.99 859	23
38	8.80 189	8.80 277	1.19 723	9.99 913	8.90 730	8.90 872	1.09 128	9.99 858	22
39	8.80 388	8.80 476	1.19 524	9.99 912	8.90 885	8.91 029	1.08 971	9.99 857	21
40	8.80 585	8.80 674	1.19 326	9.99 911	8.91 040	8.91 185	1.08 815	9.99 856	20
41	8.80 782	8.80 872	1.19 128	9.99 910	8.91 195	8.91 340	1.08 660	9.99 855	19
42	8.80 978	8.81 068	1.18 932	9.99 909	8.91 349	8.91 495	1.08 505	9.99 854	18
43	8.81 173	8.81 264	1.18 736	9.99 909	8.91 502	8.91 650	1.08 350	9.99 853	17
44	8.81 367	8.81 459	1.18 541	9.99 908	8.91 655	8.91 803	1.08 197	9.99 852	16
45	8.81 560	8.81 653	1.18 347	9.99 907	8.91 807	8.91 957	1.08 043	9.99 851	15
46	8.81 752	8.81 846	1.18 154	9.99 906	8.91 959	8.92 110	1.07 890	9.99 850	14
47	8.81 944	8.82 038	1.17 962	9.99 905	8.92 110	8.92 262	1.07 738	9.99 848	13
48	8.82 134	8.82 230	1.17 770	9.99 904	8.92 261	8.92 414	1.07 586	9.99 847	12
49	8.82 324	8.82 420	1.17 580	9.99 904	8.92 411	8.92 565	1.07 435	9.99 846	11
50	8.82 513	8.82 610	1.17 390	9.99 903	8.92 561	8.92 710	[illegible]	9.99 845	10
51	8.82 701	8.82 799	1.17 201	9.99 902	8.92 710	8.92 860	1.07 140	9.99 844	9
52	8.82 888	8.82 987	1.17 013	9.99 901	8.92 859	8.93 016	1.06 984	9.99 843	8
53	8.83 075	8.83 175	1.16 825	9.99 900	8.93 007	8.93 165	1.06 835	9.99 842	7
54	8.83 261	8.83 361	1.16 639	9.99 899	8.93 154	8.93 313	[illegible]	9.99 841	6
55	8.83 446	8.83 547	1.16 453	9.99 898	8.93 301	8.93 462	[illegible]	9.99 840	5
56	8.83 630	8.83 732	1.16 268	9.99 898	8.93 448	8.93 609	1.06 391	[illegible]	4
57	8.83 813	8.83 916	1.16 084	9.99 897	8.93 594	8.93 756	1.06 244	[illegible]	3
58	8.83 996	8.84 100	1.15 900	9.99 896	8.93 740	[illegible]	[illegible]	[illegible]	2
59	8.84 177	8.84 282	1.15 718	9.99 895	8.93 885	[illegible]	[illegible]	[illegible]	1
60	8.84 358	8.84 464	1.15 536	9.99 894	8.94 030	[illegible]	[illegible]	[illegible]	0
	log cos	log cot	log tan	log sin	log cos	log cot	log tan	log sin	′

′	5° log sin	5° log tan	5° log cot	5° log cos	6° log sin	6° log tan	6° log cot	6° log cos	
0	8.94 030	8.94 195	1.05 805	9.99 834	9.01 923	9.02 162	0.97 838	9.99 761	60
1	8.94 174	8.94 340	1.05 660	9.99 833	9.02 043	9.02 283	0.97 717	9.99 760	59
2	8.94 317	8.94 485	1.05 515	9.99 832	9.02 163	9.02 404	0.97 596	9.99 759	58
3	8.94 461	8.94 630	1.05 370	9.99 831	9.02 283	9.02 525	0.97 475	9.99 757	57
4	8.94 603	8.94 773	1.05 227	9.99 830	9.02 402	9.02 645	0.97 355	9.99 756	56
5	8.94 746	8.94 917	1.05 083	9.99 829	9.02 520	9.02 766	0.97 234	9.99 755	55
6	8.94 887	8.95 060	1.04 940	9.99 828	9.02 639	9.02 885	0.97 115	9.99 753	54
7	8.95 029	8.95 202	1.04 798	9.99 827	9.02 757	9.03 005	0.96 995	9.99 752	53
8	8.95 170	8.95 344	1.04 656	9.99 825	9.02 874	9.03 124	0.96 876	9.99 751	52
9	8.95 310	8.95 486	1.04 514	9.99 824	9.02 992	9.03 242	0.96 758	9.99 749	51
10	8.95 450	8.95 627	1.04 373	9.99 823	9.03 109	9.03 361	0.96 639	9.99 748	50
11	8.95 589	8.95 767	1.04 233	9.99 822	9.03 226	9.03 479	0.96 521	9.99 747	49
12	8.95 728	8.95 908	1.04 092	9.99 821	9.03 342	9.03 597	0.96 403	9.99 745	48
13	8.95 867	8.96 047	1.03 953	9.99 820	9.03 458	9.03 714	0.96 286	9.99 744	47
14	8.96 005	8.96 187	1.03 813	9.99 819	9.03 574	9.03 832	0.96 168	9.99 742	46
15	8.96 143	8.96 325	1.03 675	9.99 817	9.03 690	9.03 948	0.96 052	9.99 741	45
16	8.96 280	8.96 464	1.03 536	9.99 816	9.03 805	9.04 065	0.95 935	9.99 740	44
17	8.96 417	8.96 602	1.03 398	9.99 815	9.03 920	9.04 181	0.95 819	9.99 738	43
18	8.96 553	8.96 739	1.03 261	9.99 814	9.04 034	9.04 297	0.95 703	9.99 737	42
19	8.96 689	8.96 877	1.03 123	9.99 813	9.04 149	9.04 413	0.95 587	9.99 736	41
20	8.96 825	8.97 013	1.02 987	9.99 812	9.04 262	9.04 528	0.95 472	9.99 734	40
21	8.96 960	8.97 150	1.02 850	9.99 810	9.04 376	9.04 643	0.95 357	9.99 733	39
22	8.97 095	8.97 285	1.02 715	9.99 809	9.04 490	9.04 758	0.95 242	9.99 731	38
23	8.97 229	8.97 421	1.02 579	9.99 808	9.04 603	9.04 873	0.95 127	9.99 730	37
24	8.97 363	8.97 556	1.02 444	9.99 807	9.04 715	9.04 987	0.95 013	9.99 728	36
25	8.97 496	8.97 691	1.02 309	9.99 806	9.04 828	9.05 101	0.94 899	9.99 727	35
26	8.97 629	8.97 825	1.02 175	9.99 804	9.04 940	9.05 214	0.94 786	9.99 726	34
27	8.97 762	8.97 959	1.02 041	9.99 803	9.05 052	9.05 328	0.94 672	9.99 724	33
28	8.97 894	8.98 092	1.01 908	9.99 802	9.05 164	9.05 441	0.94 559	9.99 723	32
29	8.98 026	8.98 225	1.01 775	9.99 801	9.05 275	9.05 553	0.94 447	9.99 721	31
30	8.98 157	8.98 358	1.01 642	9.99 800	9.05 386	9.05 666	0.94 334	9.99 720	30
31	8.98 288	8.98 490	1.01 510	9.99 798	9.05 497	9.05 778	0.94 222	9.99 718	29
32	8.98 419	8.98 622	1.01 378	9.99 797	9.05 607	9.05 890	0.94 110	9.99 717	28
33	8.98 549	8.98 753	1.01 247	9.99 796	9.05 717	9.06 002	0.93 998	9.99 716	27
34	8.98 679	8.98 884	1.01 116	9.99 795	9.05 827	9.06 113	0.93 887	9.99 714	26
35	8.98 808	8.99 015	1.00 985	9.99 793	9.05 937	9.06 224	0.93 776	9.99 713	25
36	8.98 937	8.99 145	1.00 855	9.99 792	9.06 046	9.06 335	0.93 665	9.99 711	24
37	8.99 066	8.99 275	1.00 725	9.99 791	9.06 155	9.06 445	0.93 555	9.99 710	23
38	8.99 194	8.99 405	1.00 595	9.99 790	9.06 264	9.06 556	0.93 444	9.99 708	22
39	8.99 322	8.99 534	1.00 466	9.99 788	9.06 372	9.06 666	0.93 334	9.99 707	21
40	8.99 450	8.99 662	1.00 338	9.99 787	9.06 481	9.06 775	0.93 225	9.99 705	20
41	8.99 577	8.99 791	1.00 209	9.99 786	9.06 589	9.06 885	0.93 115	9.99 704	19
42	8.99 704	8.99 919	1.00 081	9.99 785	9.06 696	9.06 994	0.93 006	9.99 702	18
43	8.99 830	9.00 046	0.99 954	9.99 783	9.06 804	9.07 103	0.92 897	9.99 701	17
44	8.99 956	9.00 174	0.99 826	9.99 782	9.06 911	9.07 211	0.92 789	9.99 699	16
45	9.00 082	9.00 301	0.99 699	9.99 781	9.07 018	9.07 320	0.92 680	9.99 698	15
46	9.00 207	9.00 427	0.99 573	9.99 780	9.07 124	9.07 428	0.92 572	9.99 696	14
47	9.00 332	9.00 553	0.99 447	9.99 778	9.07 231	9.07 536	0.92 464	9.99 695	13
48	9.00 456	9.00 679	0.99 321	9.99 777	9.07 337	9.07 643	0.92 357	9.99 693	12
49	9.00 581	9.00 805	0.99 195	9.99 776	9.07 442	9.07 751	0.92 249	9.99 692	11
50	9.00 704	9.00 930	0.99 070	9.99 775	9.07 548	9.07 858	0.92 142	9.99 690	10
51	9.00 828	9.01 055	0.98 945	9.99 773	9.07 653	9.07 964	0.92 036	9.99 689	9
52	9.00 951	9.01 179	0.98 821	9.99 772	9.07 758	9.08 071	0.91 929	9.99 687	8
53	9.01 074	9.01 303	0.98 697	9.99 771	9.07 863	9.08 177	0.91 823	9.99 686	7
54	9.01 196	9.01 427	0.98 573	9.99 769	9.07 968	9.08 283	0.91 717	9.99 684	6
55	9.01 318	9.01 550	0.98 450	9.99 768	9.08 072	9.08 389	0.91 611	9.99 683	5
56	9.01 440	9.01 673	0.98 327	9.99 767	9.08 176	9.08 495	0.91 505	9.99 681	4
57	9.01 561	9.01 796	0.98 204	9.99 765	9.08 280	9.08 600	0.91 400	9.99 680	3
58	9.01 682	9.01 918	0.98 082	9.99 764	9.08 383	9.08 705	0.91 295	9.99 678	2
59	9.01 803	9.02 040	0.97 960	9.99 763	9.08 486	9.08 810	0.91 190	9.99 677	1
60	9.01 923	9.02 162	0.97 838	9.99 761	9.08 589	9.08 914	0.91 086	9.99 675	0
	log cos	log cot	log tan	log sin	log cos	log cot	log tan	log sin	′
	84°				83°				

	7°				8°				
′	log sin	log tan	log cot	log cos	log sin	log tan	log cot	log cos	
0	9.08 589	9.08 914	0.91 086	9.99 675	9.14 356	9.14 780	0.85 220	9.99 575	60
1	9.08 692	9.09 019	0.90 981	9.99 674	9.14 445	9.14 872	0.85 128	9.99 574	59
2	9.08 795	9.09 123	0.90 877	9.99 672	9.14 535	9.14 963	0.85 037	9.99 572	58
3	9.08 897	9.09 227	0.90 773	9.99 670	9.14 624	9.15 054	0.84 946	9.99 570	57
4	9.08 999	9.09 330	0.90 670	9.99 669	9.14 714	9.15 145	0.84 855	9.99 568	56
5	9.09 101	9.09 434	0.90 566	9.99 667	9.14 803	9.15 236	0.84 764	9.99 566	55
6	9.09 202	9.09 537	0.90 463	9.99 666	9.14 891	9.15 327	0.84 673	9.99 565	54
7	9.09 304	9.09 640	0.90 360	9.99 664	9.14 980	9.15 417	0.84 583	9.99 563	53
8	9.09 405	9.09 742	0.90 258	9.99 663	9.15 069	9.15 508	0.84 492	9.99 561	52
9	9.09 506	9.09 845	0.90 155	9.99 661	9.15 157	9.15 598	0.84 402	9.99 559	51
10	9.09 606	9.09 947	0.90 053	9.99 659	9.15 245	9.15 688	0.84 312	9.99 557	50
11	9.09 707	9.10 049	0.89 951	9.99 658	9.15 333	9.15 777	0.84 223	9.99 556	49
12	9.09 807	9.10 150	0.89 850	9.99 656	9.15 421	9.15 867	0.84 133	9.99 554	48
13	9.09 907	9.10 252	0.89 748	9.99 655	9.15 508	9.15 956	0.84 044	9.99 552	47
14	9.10 006	9.10 353	0.89 647	9.99 653	9.15 596	9.16 046	0.83 954	9.99 550	46
15	9.10 106	9.10 454	0.89 546	9.99 651	9.15 683	9.16 135	0.83 865	9.99 548	45
16	9.10 205	9.10 555	0.89 445	9.99 650	9.15 770	9.16 224	0.83 776	9.99 546	44
17	9.10 304	9.10 656	0.89 344	9.99 648	9.15 857	9.16 312	0.83 688	9.99 545	43
18	9.10 402	9.10 756	0.89 244	9.99 647	9.15 944	9.16 401	0.83 599	9.99 543	42
19	9.10 501	9.10 856	0.89 144	9.99 645	9.16 030	9.16 489	0.83 511	9.99 541	41
20	9.10 599	9.10 956	0.89 044	9.99 643	9.16 116	9.16 577	0.83 423	9.99 539	40
21	9.10 697	9.11 056	0.88 944	9.99 642	9.16 203	9.16 665	0.83 335	9.99 537	39
22	9.10 795	9.11 155	0.88 845	9.99 640	9.16 289	9.16 753	0.83 247	9.99 535	38
23	9.10 893	9.11 254	0.88 746	9.99 638	9.16 374	9.16 841	0.83 159	9.99 533	37
24	9.10 990	9.11 353	0.88 647	9.99 637	9.16 460	9.16 928	0.83 072	9.99 532	36
25	9.11 087	9.11 452	0.88 548	9.99 635	9.16 545	9.17 016	0.82 984	9.99 530	35
26	9.11 184	9.11 551	0.88 449	9.99 633	9.16 631	9.17 103	0.82 897	9.99 528	34
27	9.11 281	9.11 649	0.88 351	9.99 632	9.16 716	9.17 190	0.82 810	9.99 526	33
28	9.11 377	9.11 747	0.88 253	9.99 630	9.16 801	9.17 277	0.82 723	9.99 524	32
29	9.11 474	9.11 845	0.88 155	9.99 629	9.16 886	9.17 363	0.82 637	9.99 522	31
30	9.11 570	9.11 943	0.88 057	9.99 627	9.16 970	9.17 450	0.82 550	9.99 520	30
31	9.11 666	9.12 040	0.87 960	9.99 625	9.17 055	9.17 536	0.82 464	9.99 518	29
32	9.11 761	9.12 138	0.87 862	9.99 624	9.17 139	9.17 622	0.82 378	9.99 517	28
33	9.11 857	9.12 235	0.87 765	9.99 622	9.17 223	9.17 708	0.82 292	9.99 515	27
34	9.11 952	9.12 332	0.87 668	9.99 620	9.17 307	9.17 794	0.82 206	9.99 513	26
35	9.12 047	9.12 428	0.87 572	9.99 618	9.17 391	9.17 880	0.82 120	9.99 511	25
36	9.12 142	9.12 525	0.87 475	9.99 617	9.17 474	9.17 965	0.82 035	9.99 509	24
37	9.12 236	9.12 621	0.87 379	9.99 615	9.17 558	9.18 051	0.81 949	9.99 507	23
38	9.12 331	9.12 717	0.87 283	9.99 613	9.17 641	9.18 136	0.81 864	9.99 505	22
39	9.12 425	9.12 813	0.87 187	9.99 612	9.17 724	9.18 221	0.81 779	9.99 503	21
40	9.12 519	9.12 909	0.87 091	9.99 610	9.17 807	9.18 306	0.81 694	9.99 501	20
41	9.12 612	9.13 004	0.86 996	9.99 608	9.17 890	9.18 391	0.81 609	9.99 499	19
42	9.12 706	9.13 099	0.86 901	9.99 607	9.17 973	9.18 475	0.81 525	9.99 497	18
43	9.12 799	9.13 194	0.86 806	9.99 605	9.18 055	9.18 560	0.81 440	9.99 495	17
44	9.12 892	9.13 289	0.86 711	9.99 603	9.18 137	9.18 644	0.81 356	9.99 494	16
45	9.12 985	9.13 384	0.86 616	9.99 601	9.18 220	9.18 728	0.81 272	9.99 492	15
46	9.13 078	9.13 478	0.86 522	9.99 600	9.18 302	9.18 812	0.81 188	9.99 490	14
47	9.13 171	9.13 573	0.86 427	9.99 598	9.18 383	9.18 896	0.81 104	9.99 488	13
48	9.13 263	9.13 667	0.86 333	9.99 596	9.18 465	9.18 979	0.81 021	9.99 486	12
49	9.13 355	9.13 761	0.86 239	9.99 595	9.18 547	9.19 063	0.80 937	9.99 484	11
50	9.13 447	9.13 854	0.86 146	9.99 593	9.18 628	9.19 146	0.80 854	9.99 482	10
51	9.13 539	9.13 948	0.86 052	9.99 591	9.18 709	9.19 229	0.80 771	9.99 480	9
52	9.13 630	9.14 041	0.85 959	9.99 589	9.18 790	9.19 312	0.80 688	9.99 478	8
53	9.13 722	9.14 134	0.85 866	9.99 588	9.18 871	9.19 395	0.80 605	9.99 476	7
54	9.13 813	9.14 227	0.85 773	9.99 586	9.18 952	9.19 478	0.80 522	9.99 474	6
55	9.13 904	9.14 320	0.85 680	9.99 584	9.19 033	9.19 561	0.80 439	9.99 472	5
56	9.13 994	9.14 412	0.85 588	9.99 582	9.19 113	9.19 643	0.80 357	9.99 470	4
57	9.14 085	9.14 504	0.85 496	9.99 581	9.19 193	9.19 725	0.80 275	9.99 468	3
58	9.14 175	9.14 597	0.85 403	9.99 579	9.19 273	9.19 807	0.80 193	9.99 466	2
59	9.14 266	9.14 688	0.85 312	9.99 577	9.19 353	9.19 889	0.80 111	9.99 464	1
60	9.14 356	9.14 780	0.85 220	9.99 575	9.19 433	9.19 971	0.80 029	9.99 462	0
	log cos	log cot	log tan	log sin	log cos	log cot	log tan	log sin	′
	82°				81°				

′	9° log sin	9° log tan	9° log cot	9° log cos	10° log sin	10° log tan	10° log cot	10° log cos	
0	9.19 433	9.19 971	0.80 029	9.99 462	9.23 967	9.24 632	0.75 368	9.99 335	60
1	9.19 513	9.20 053	0.79 947	9.99 460	9.24 039	9.24 706	0.75 294	9.99 333	59
2	9.19 592	9.20 134	0.79 866	9.99 458	9.24 110	9.24 779	0.75 221	9.99 331	58
3	9.19 672	9.20 210	0.79 784	9.99 456	9.24 181	9.24 853	0.75 147	9.99 328	57
4	9.19 751	9.20 297	0.79 703	9.99 454	9.24 253	9.24 926	0.75 074	9.99 326	56
5	9.19 830	9.20 378	0.79 622	9.99 452	9.24 324	9.25 000	0.75 000	9.99 324	55
6	9.19 909	9.20 459	0.79 541	9.99 450	9.24 395	9.25 073	0.74 927	9.99 322	54
7	9.19 988	9.20 540	0.79 460	9.99 448	9.24 466	9.25 146	0.74 854	9.99 319	53
8	9.20 067	9.20 621	0.79 379	9.99 446	9.24 536	9.25 219	0.74 781	9.99 317	52
9	9.20 145	9.20 701	0.79 299	9.99 444	9.24 607	9.25 292	0.74 708	9.99 315	51
10	9.20 223	9.20 782	0.79 218	9.99 442	9.24 677	9.25 365	0.74 635	9.99 313	50
11	9.20 302	9.20 862	0.79 138	9.99 440	9.24 748	9.25 437	0.74 563	9.99 310	49
12	9.20 380	9.20 942	0.79 058	9.99 438	9.24 818	9.25 510	0.74 490	9.99 308	48
13	9.20 458	9.21 022	0.78 978	9.99 436	9.24 888	9.25 582	0.74 418	9.99 306	47
14	9.20 535	9.21 102	0.78 898	9.99 434	9.24 958	9.25 655	0.74 345	9.99 304	46
15	9.20 613	9.21 182	0.78 818	9.99 432	9.25 028	9.25 727	0.74 273	9.99 301	45
16	9.20 691	9.21 261	0.78 739	9.99 429	9.25 098	9.25 799	0.74 201	9.99 299	44
17	9.20 768	9.21 341	0.78 659	9.99 427	9.25 168	9.25 871	0.74 129	9.99 297	43
18	9.20 845	9.21 420	0.78 580	9.99 425	9.25 237	9.25 943	0.74 057	9.99 294	42
19	9.20 922	9.21 499	0.78 501	9.99 423	9.25 307	9.26 015	0.73 985	9.99 292	41
20	9.20 999	9.21 578	0.78 422	9.99 421	9.25 376	9.26 086	0.73 914	9.99 290	40
21	9.21 076	9.21 657	0.78 343	9.99 419	9.25 445	9.26 158	0.73 842	9.99 288	39
22	9.21 153	9.21 736	0.78 264	9.99 417	9.25 514	9.26 229	0.73 771	9.99 285	38
23	9.21 229	9.21 814	0.78 186	9.99 415	9.25 583	9.26 301	0.73 699	9.99 283	37
24	9.21 306	9.21 893	0.78 107	9.99 413	9.25 652	9.26 372	0.73 628	9.99 281	36
25	9.21 382	9.21 971	0.78 029	9.99 411	9.25 721	9.26 443	0.73 557	9.99 278	35
26	9.21 458	9.22 049	0.77 951	9.99 409	9.25 790	9.26 514	0.73 486	9.99 276	34
27	9.21 534	9.22 127	0.77 873	9.99 407	9.25 858	9.26 585	0.73 415	9.99 274	33
28	9.21 610	9.22 205	0.77 795	9.99 404	9.25 927	9.26 655	0.73 345	9.99 271	32
29	9.21 685	9.22 283	0.77 717	9.99 402	9.25 995	9.26 726	0.73 274	9.99 269	31
30	9.21 761	9.22 361	0.77 639	9.99 400	9.26 063	9.26 797	0.73 203	9.99 267	30
31	9.21 836	9.22 438	0.77 562	9.99 398	9.26 131	9.26 867	0.73 133	9.99 264	29
32	9.21 912	9.22 516	0.77 484	9.99 396	9.26 199	9.26 937	0.73 063	9.99 262	28
33	9.21 987	9.22 593	0.77 407	9.99 394	9.26 267	9.27 008	0.72 992	9.99 260	27
34	9.22 062	9.22 670	0.77 330	9.99 392	9.26 335	9.27 078	0.72 922	9.99 257	26
35	9.22 137	9.22 747	0.77 253	9.99 390	9.26 403	9.27 148	0.72 852	9.99 255	25
36	9.22 211	9.22 824	0.77 176	9.99 388	9.26 470	9.27 218	0.72 782	9.99 252	24
37	9.22 286	9.22 901	0.77 099	9.99 385	9.26 538	9.27 288	0.72 712	9.99 250	23
38	9.22 361	9.22 977	0.77 023	9.99 383	9.26 605	9.27 357	0.72 643	9.99 248	22
39	9.22 435	9.23 054	0.76 946	9.99 381	9.26 672	9.27 427	0.72 573	9.99 245	21
40	9.22 509	9.23 130	0.76 870	9.99 379	9.26 739	9.27 496	0.72 504	9.99 243	20
41	9.22 583	9.23 206	0.76 794	9.99 377	9.26 806	9.27 566	0.72 434	9.99 241	19
42	9.22 657	9.23 283	0.76 717	9.99 375	9.26 873	9.27 635	0.72 365	9.99 238	18
43	9.22 731	9.23 359	0.76 641	9.99 372	9.26 940	9.27 704	0.72 296	9.99 236	17
44	9.22 805	9.23 435	0.76 565	9.99 370	9.27 007	9.27 773	0.72 227	9.99 233	16
45	9.22 878	9.23 510	0.76 490	9.99 368	9.27 073	9.27 842	0.72 158	9.99 231	15
46	9.22 952	9.23 586	0.76 414	9.99 366	9.27 140	9.27 911	0.72 089	9.99 229	14
47	9.23 025	9.23 661	0.76 339	9.99 364	9.27 206	9.27 980	0.72 020	9.99 226	13
48	9.23 098	9.23 737	0.76 263	9.99 362	9.27 273	9.28 049	0.71 951	9.99 224	12
49	9.23 171	9.23 812	0.76 188	9.99 359	9.27 339	9.28 117	0.71 883	9.99 221	11
50	9.23 244	9.23 887	0.76 113	9.99 357	9.27 405	9.28 186	0.71 814	9.99 219	10
51	9.23 317	9.23 962	0.76 038	9.99 355	9.27 471	9.28 254	0.71 746	9.99 217	9
52	9.23 390	9.24 037	0.75 963	9.99 353	9.27 537	9.28 323	0.71 677	9.99 214	8
53	9.23 462	9.24 112	0.75 888	9.99 351	9.27 602	9.28 391	0.71 609	9.99 212	7
54	9.23 535	9.24 186	0.75 814	9.99 348	9.27 668	9.28 459	0.71 541	9.99 209	6
55	9.23 607	9.24 261	0.75 739	9.99 346	9.27 734	9.28 527	0.71 473	9.99 207	5
56	9.23 679	9.24 335	0.75 665	9.99 344	9.27 799	9.28 595	0.71 405	9.99 204	4
57	9.23 752	9.24 410	0.75 590	9.99 342	9.27 864	9.28 662	0.71 338	9.99 202	3
58	9.23 823	9.24 484	0.75 516	9.99 340	9.27 930	9.28 730	0.71 270	9.99 200	2
59	9.23 895	9.24 558	0.75 442	9.99 337	9.27 995	9.28 798	0.71 202	9.99 197	1
60	9.23 967	9.24 632	0.75 368	9.99 335	9.28 060	9.28 865	0.71 135	9.99 195	0
	log cos	log cot	log tan	log sin	log cos	log cot	log tan	log sin	′

′	11° log sin	log tan	log cot	log cos	12° log sin	log tan	log cot	log cos	′
0	9.28 060	9.28 865	0.71 135	9.99 195	9.31 788	9.32 747	0.67 253	9.99 040	60
1	9.28 125	9.28 933	0.71 067	9.99 192	9.31 847	9.32 810	0.67 190	9.99 038	59
2	9.28 190	9.29 000	0.71 000	9.99 190	9.31 907	9.32 872	0.67 128	9.99 035	58
3	9.28 254	9.29 067	0.70 933	9.99 187	9.31 966	9.32 933	0.67 067	9.99 032	57
4	9.28 319	9.29 134	0.70 866	9.99 185	9.32 025	9.32 995	0.67 005	9.99 030	56
5	9.28 384	9.29 201	0.70 799	9.99 182	9.32 084	9.33 057	0.66 943	9.99 027	55
6	9.28 448	9.29 268	0.70 732	9.99 180	9.32 143	9.33 119	0.66 881	9.99 024	54
7	9.28 512	9.29 335	0.70 665	9.99 177	9.32 202	9.33 180	0.66 820	9.99 022	53
8	9.28 577	9.29 402	0.70 598	9.99 175	9.32 261	9.33 242	0.66 758	9.99 019	52
9	9.28 641	9.29 468	0.70 532	9.99 172	9.32 319	9.33 303	0.66 697	9.99 016	51
10	9.28 705	9.29 535	0.70 465	9.99 170	9.32 378	9.33 365	0.66 635	9.99 013	50
11	9.28 769	9.29 601	0.70 399	9.99 167	9.32 437	9.33 426	0.66 574	9.99 011	49
12	9.28 833	9.29 668	0.70 332	9.99 165	9.32 495	9.33 487	0.66 513	9.99 008	48
13	9.28 896	9.29 734	0.70 266	9.99 162	9.32 553	9.33 548	0.66 452	9.99 005	47
14	9.28 960	9.29 800	0.70 200	9.99 160	9.32 612	9.33 609	0.66 391	9.99 002	46
15	9.29 024	9.29 866	0.70 134	9.99 157	9.32 670	9.33 670	0.66 330	9.99 000	45
16	9.29 087	9.29 932	0.70 068	9.99 155	9.32 728	9.33 731	0.66 269	9.98 997	44
17	9.29 150	9.29 998	0.70 002	9.99 152	9.32 786	9.33 792	0.66 208	9.98 994	43
18	9.29 214	9.30 064	0.69 936	9.99 150	9.32 844	9.33 853	0.66 147	9.98 991	42
19	9.29 277	9.30 130	0.69 870	9.99 147	9.32 902	9.33 913	0.66 087	9.98 989	41
20	9.29 340	9.30 195	0.69 805	9.99 145	9.32 960	9.33 974	0.66 026	9.98 986	40
21	9.29 403	9.30 261	0.69 739	9.99 142	9.33 018	9.34 034	0.65 966	9.98 983	39
22	9.29 466	9.30 326	0.69 674	9.99 140	9.33 075	9.34 095	0.65 905	9.98 980	38
23	9.29 529	9.30 391	0.69 609	9.99 137	9.33 133	9.34 155	0.65 845	9.98 978	37
24	9.29 591	9.30 457	0.69 543	9.99 135	9.33 190	9.34 215	0.65 785	9.98 975	36
25	9.29 654	9.30 522	0.69 478	9.99 132	9.33 248	9.34 276	0.65 724	9.98 972	35
26	9.29 716	9.30 587	0.69 413	9.99 130	9.33 305	9.34 336	0.65 664	9.98 969	34
27	9.29 779	9.30 652	0.69 348	9.99 127	9.33 362	9.34 396	0.65 604	9.98 967	33
28	9.29 841	9.30 717	0.69 283	9.99 124	9.33 420	9.34 456	0.65 544	9.98 964	32
29	9.29 903	9.30 782	0.69 218	9.99 122	9.33 477	9.34 516	0.65 484	9.98 961	31
30	9.29 966	9.30 846	0.69 154	9.99 119	9.33 534	9.34 576	0.65 424	9.98 958	30
31	9.30 028	9.30 911	0.69 089	9.99 117	9.33 591	9.34 635	0.65 365	9.98 955	29
32	9.30 090	9.30 975	0.69 025	9.99 114	9.33 647	9.34 695	0.65 305	9.98 953	28
33	9.30 151	9.31 040	0.68 960	9.99 112	9.33 704	9.34 755	0.65 245	9.98 950	27
34	9.30 213	9.31 104	0.68 896	9.99 109	9.33 761	9.34 814	0.65 186	9.98 947	26
35	9.30 275	9.31 168	0.68 832	9.99 106	9.33 818	9.34 874	0.65 126	9.98 944	25
36	9.30 336	9.31 233	0.68 767	9.99 104	9.33 874	9.34 933	0.65 067	9.98 941	24
37	9.30 398	9.31 297	0.68 703	9.99 101	9.33 931	9.34 992	0.65 008	9.98 938	23
38	9.30 459	9.31 361	0.68 639	9.99 099	9.33 987	9.35 051	0.64 949	9.98 936	22
39	9.30 521	9.31 425	0.68 575	9.99 096	9.34 043	9.35 111	0.64 889	9.98 933	21
40	9.30 582	9.31 489	0.68 511	9.99 093	9.34 100	9.35 170	0.64 830	9.98 930	20
41	9.30 643	9.31 552	0.68 448	9.99 091	9.34 156	9.35 229	0.64 771	9.98 927	19
42	9.30 704	9.31 616	0.68 384	9.99 088	9.34 212	9.35 288	0.64 712	9.98 924	18
43	9.30 765	9.31 679	0.68 321	9.99 086	9.34 268	9.35 347	0.64 653	9.98 921	17
44	9.30 826	9.31 743	0.68 257	9.99 083	9.34 324	9.35 405	0.64 595	9.98 919	16
45	9.30 887	9.31 806	0.68 194	9.99 080	9.34 380	9.35 464	0.64 536	9.98 916	15
46	9.30 947	9.31 870	0.68 130	9.99 078	9.34 436	9.35 523	0.64 477	9.98 913	14
47	9.31 008	9.31 933	0.68 067	9.99 075	9.34 491	9.35 581	0.64 419	9.98 910	13
48	9.31 068	9.31 996	0.68 004	9.99 072	9.34 547	9.35 640	0.64 360	9.98 907	12
49	9.31 129	9.32 059	0.67 941	9.99 070	9.34 602	9.35 698	0.64 302	9.98 904	11
50	9.31 189	9.32 122	0.67 878	9.99 067	9.34 658	9.35 757	0.64 243	9.98 901	10
51	9.31 250	9.32 185	0.67 815	9.99 064	9.34 713	9.35 815	0.64 185	9.98 898	9
52	9.31 310	9.32 248	0.67 752	9.99 062	9.34 769	9.35 873	0.64 127	9.98 896	8
53	9.31 370	9.32 311	0.67 689	9.99 059	9.34 824	9.35 931	0.64 069	9.98 893	7
54	9.31 430	9.32 373	0.67 627	9.99 056	9.34 879	9.35 989	0.64 011	9.98 890	6
55	9.31 490	9.32 436	0.67 564	9.99 054	9.34 934	9.36 047	0.63 953	9.98 887	5
56	9.31 549	9.32 498	0.67 502	9.99 051	9.34 989	9.36 105	0.63 895	9.98 884	4
57	9.31 609	9.32 561	0.67 439	9.99 048	9.35 044	9.36 163	0.63 837	9.98 881	3
58	9.31 669	9.32 623	0.67 377	9.99 046	9.35 099	9.36 221	0.63 779	9.98 878	2
59	9.31 728	9.32 685	0.67 315	9.99 043	9.35 154	9.36 279	0.63 721	9.98 875	1
60	9.31 788	9.32 747	0.67 253	9.99 040	9.35 209	9.36 336	0.63 664	9.98 872	0
	log cos	log cot	log tan	log sin 78°	log cos	log cot	log tan	log sin 77°	′

	13°				14°				
′	log sin	log tan	log cot	log cos	log sin	log tan	log cot	log cos	
0	9.35 209	9.36 336	0.63 664	9.98 872	9.38 368	9.39 677	0.60 323	9.98 690	**60**
1	9.35 263	9.36 394	0.63 606	9.98 869	9.38 418	9.39 731	0.60 269	9.98 687	59
2	9.35 318	9.36 452	0.63 548	9.98 867	9.38 469	9.39 785	0.60 215	9.98 684	58
3	9.35 373	9.36 509	0.63 491	9.98 864	9.38 519	9.39 838	0.60 162	9.98 681	57
4	9.35 427	9.36 566	0.63 434	9.98 861	9.38 570	9.39 892	0.60 108	9.98 678	56
5	9.35 481	9.36 624	0.63 376	9.98 858	9.38 620	9.39 945	0.60 055	9.98 675	55
6	9.35 536	9.36 681	0.63 319	9.98 855	9.38 670	9.39 999	0.60 001	9.98 671	54
7	9.35 590	9.36 738	0.63 262	9.98 852	9.38 721	9.40 052	0.59 948	9.98 668	53
8	9.35 644	9.36 795	0.63 205	9.98 849	9.38 771	9.40 106	0.59 894	9.98 665	52
9	9.35 698	9.36 852	0.63 148	9.98 846	9.38 821	9.40 159	0.59 841	9.98 662	51
10	9.35 752	9.36 909	0.63 091	9.98 843	9.38 871	9.40 212	0.59 788	9.98 659	**50**
11	9.35 806	9.36 966	0.63 034	9.98 840	9.38 921	9.40 266	0.59 734	9.98 656	49
12	9.35 860	9.37 023	0.62 977	9.98 837	9.38 971	9.40 319	0.59 681	9.98 652	48
13	9.35 914	9.37 080	0.62 920	9.98 834	9.39 021	9.40 372	0.59 628	9.98 649	47
14	9.35 968	9.37 137	0.62 863	9.98 831	9.39 071	9.40 425	0.59 575	9.98 646	46
15	9.36 022	9.37 193	0.62 807	9.98 828	9.39 121	9.40 478	0.59 522	9.98 643	45
16	9.36 075	9.37 250	0.62 750	9.98 825	9.39 170	9.40 531	0.59 469	9.98 640	44
17	9.36 129	9.37 306	0.62 694	9.98 822	9.39 220	9.40 584	0.59 416	9.98 636	43
18	9.36 182	9.37 363	0.62 637	9.98 819	9.39 270	9.40 636	0.59 364	9.98 633	42
19	9.36 236	9.37 419	0.62 581	9.98 816	9.39 319	9.40 689	0.59 311	9.98 630	41
20	9.36 289	9.37 476	0.62 524	9.98 813	9.39 369	9.40 742	0.59 258	9.98 627	**40**
21	9.36 342	9.37 532	0.62 468	9.98 810	9.39 418	9.40 795	0.59 205	9.98 623	39
22	9.36 395	9.37 588	0.62 412	9.98 807	9.39 467	9.40 847	0.59 153	9.98 620	38
23	9.36 449	9.37 644	0.62 356	9.98 804	9.39 517	9.40 900	0.59 100	9.98 617	37
24	9.36 502	9.37 700	0.62 300	9.98 801	9.39 566	9.40 952	0.59 048	9.98 614	36
25	9.36 555	9.37 756	0.62 244	9.98 798	9.39 615	9.41 005	0.58 995	9.98 610	35
26	9.36 608	9.37 812	0.62 188	9.98 795	9.39 664	9.41 057	0.58 943	9.98 607	34
27	9.36 660	9.37 868	0.62 132	9.98 792	9.39 713	9.41 109	0.58 891	9.98 604	33
28	9.36 713	9.37 924	0.62 076	9.98 789	9.39 762	9.41 161	0.58 839	9.98 601	32
29	9.36 766	9.37 980	0.62 020	9.98 786	9.39 811	9.41 214	0.58 786	9.98 597	31
30	9.36 819	9.38 035	0.61 965	9.98 783	9.39 860	9.41 266	0.58 734	9.98 594	**30**
31	9.36 871	9.38 091	0.61 909	9.98 780	9.39 909	9.41 318	0.58 682	9.98 591	29
32	9.36 924	9.38 147	0.61 853	9.98 777	9.39 958	9.41 370	0.58 630	9.98 588	28
33	9.36 976	9.38 202	0.61 798	9.98 774	9.40 006	9.41 422	0.58 578	9.98 584	27
34	9.37 028	9.38 257	0.61 743	9.98 771	9.40 055	9.41 474	0.58 526	9.98 581	26
35	9.37 081	9.38 313	0.61 687	9.98 768	9.40 103	9.41 526	0.58 474	9.98 578	25
36	9.37 133	9.38 368	0.61 632	9.98 765	9.40 152	9.41 578	0.58 422	9.98 574	24
37	9.37 185	9.38 423	0.61 577	9.98 762	9.40 200	9.41 629	0.58 371	9.98 571	23
38	9.37 237	9.38 479	0.61 521	9.98 759	9.40 249	9.41 681	0.58 319	9.98 568	22
39	9.37 289	9.38 534	0.61 466	9.98 756	9.40 297	9.41 733	0.58 267	9.98 565	21
40	9.37 341	9.38 589	0.61 411	9.98 753	9.40 346	9.41 784	0.58 216	9.98 561	**20**
41	9.37 393	9.38 644	0.61 356	9.98 750	9.40 394	9.41 836	0.58 164	9.98 558	19
42	9.37 445	9.38 699	0.61 301	9.98 746	9.40 442	9.41 887	0.58 113	9.98 555	18
43	9.37 497	9.38 754	0.61 246	9.98 743	9.40 490	9.41 939	0.58 061	9.98 551	17
44	9.37 549	9.38 808	0.61 192	9.98 740	9.40 538	9.41 990	0.58 010	9.98 548	16
45	9.37 600	9.38 863	0.61 137	9.98 737	9.40 586	9.42 041	0.57 959	9.98 545	15
46	9.37 652	9.38 918	0.61 082	9.98 734	9.40 634	9.42 093	0.57 907	9.98 541	14
47	9.37 703	9.38 972	0.61 028	9.98 731	9.40 682	9.42 144	0.57 856	9.98 538	13
48	9.37 755	9.39 027	0.60 973	9.98 728	9.40 730	9.42 195	0.57 805	9.98 535	12
49	9.37 806	9.39 082	0.60 918	9.98 725	9.40 778	9.42 246	0.57 754	9.98 531	11
50	9.37 858	9.39 136	0.60 864	9.98 722	9.40 825	9.42 297	0.57 703	9.98 528	**10**
51	9.37 909	9.39 190	0.60 810	9.98 719	9.40 873	9.42 348	0.57 652	9.98 525	9
52	9.37 960	9.39 245	0.60 755	9.98 715	9.40 921	9.42 399	0.57 601	9.98 521	8
53	9.38 011	9.39 299	0.60 701	9.98 712	9.40 968	9.42 450	0.57 550	9.98 518	7
54	9.38 062	9.39 353	0.60 647	9.98 709	9.41 016	9.42 501	0.57 499	9.98 515	6
55	9.38 113	9.39 407	0.60 593	9.98 706	9.41 063	9.42 552	0.57 448	9.98 511	5
56	9.38 164	9.39 461	0.60 539	9.98 703	9.41 111	9.42 603	0.57 397	9.98 508	4
57	9.38 215	9.39 515	0.60 485	9.98 700	9.41 158	9.42 653	0.57 347	9.98 505	3
58	9.38 266	9.39 569	0.60 431	9.98 697	9.41 205	9.42 704	0.57 296	9.98 501	2
59	9.38 317	9.39 623	0.60 377	9.98 694	9.41 252	9.42 755	0.57 245	9.98 498	1
60	9.38 368	9.39 677	0.60 323	9.98 690	9.41 300	9.42 805	0.57 195	9.98 494	**0**
	log cos	log cot	log tan	log sin	log cos	log cot	log tan	log sin	′
	76°				75°				

′	15° log sin	log tan	log cot	log cos	16° log sin	log tan	log cot	log cos	
0	9.41 300	9.42 805	0.57 195	9.98 494	9.44 034	9.45 750	0.54 250	9.98 284	60
1	9.41 347	9.42 856	0.57 144	9.98 491	9.44 078	9.45 797	0.54 203	9.98 281	59
2	9.41 394	9.42 906	0.57 094	9.98 488	9.44 122	9.45 845	0.54 155	9.98 277	58
3	9.41 441	9.42 957	0.57 043	9.98 484	9.44 166	9.45 892	0.54 108	9.98 273	57
4	9.41 488	9.43 007	0.56 993	9.98 481	9.44 210	9.45 940	0.54 060	9.98 270	56
5	9.41 535	9.43 057	0.56 943	9.98 477	9.44 253	9.45 987	0.54 013	9.98 266	55
6	9.41 582	9.43 108	0.56 892	9.98 474	9.44 297	9.46 035	0.53 965	9.98 262	54
7	9.41 628	9.43 158	0.56 842	9.98 471	9.44 341	9.46 082	0.53 918	9.98 259	53
8	9.41 675	9.43 208	0.56 792	9.98 467	9.44 385	9.46 130	0.53 870	9.98 255	52
9	9.41 722	9.43 258	0.56 742	9.98 464	9.44 428	9.46 177	0.53 823	9.98 251	51
10	9.41 768	9.43 308	0.56 692	9.98 460	9.44 472	9.46 224	0.53 776	9.98 248	50
11	9.41 815	9.43 358	0.56 642	9.98 457	9.44 516	9.46 271	0.53 729	9.98 244	49
12	9.41 861	9.43 408	0.56 592	9.98 453	9.44 559	9.46 319	0.53 681	9.98 240	48
13	9.41 908	9.43 458	0.56 542	9.98 450	9.44 602	9.46 366	0.53 634	9.98 237	47
14	9.41 954	9.43 508	0.56 492	9.98 447	9.44 646	9.46 413	0.53 587	9.98 233	46
15	9.42 001	9.43 558	0.56 442	9.98 443	9.44 689	9.46 460	0.53 540	9.98 229	45
16	9.42 047	9.43 607	0.56 393	9.98 440	9.44 733	9.46 507	0.53 493	9.98 226	44
17	9.42 093	9.43 657	0.56 343	9.98 436	9.44 776	9.46 554	0.53 446	9.98 222	43
18	9.42 140	9.43 707	0.56 293	9.98 433	9.44 819	9.46 601	0.53 399	9.98 218	42
19	9.42 186	9.43 756	0.56 244	9.98 429	9.44 862	9.46 648	0.53 352	9.98 215	41
20	9.42 232	9.43 806	0.56 194	9.98 426	9.44 905	9.46 694	0.53 306	9.98 211	40
21	9.42 278	9.43 855	0.56 145	9.98 422	9.44 948	9.46 741	0.53 259	9.98 207	39
22	9.42 324	9.43 905	0.56 095	9.98 419	9.44 992	9.46 788	0.53 212	9.98 204	38
23	9.42 370	9.43 954	0.56 046	9.98 415	9.45 035	9.46 835	0.53 165	9.98 200	37
24	9.42 416	9.44 004	0.55 996	9.98 412	9.45 077	9.46 881	0.53 119	9.98 196	36
25	9.42 461	9.44 053	0.55 947	9.98 409	9.45 120	9.46 928	0.53 072	9.98 192	35
26	9.42 507	9.44 102	0.55 898	9.98 405	9.45 163	9.46 975	0.53 025	9.98 189	34
27	9.42 553	9.44 151	0.55 849	9.98 402	9.45 206	9.47 021	0.52 979	9.98 185	33
28	9.42 599	9.44 201	0.55 799	9.98 398	9.45 249	9.47 068	0.52 932	9.98 181	32
29	9.42 644	9.44 250	0.55 750	9.98 395	9.45 292	9.47 114	0.52 886	9.98 177	31
30	9.42 690	9.44 299	0.55 701	9.98 391	9.45 334	9.47 160	0.52 840	9.98 174	30
31	9.42 735	9.44 348	0.55 652	9.98 388	9.45 377	9.47 207	0.52 793	9.98 170	29
32	9.42 781	9.44 397	0.55 603	9.98 384	9.45 419	9.47 253	0.52 747	9.98 166	28
33	9.42 826	9.44 446	0.55 554	9.98 381	9.45 462	9.47 299	0.52 701	9.98 162	27
34	9.42 872	9.44 495	0.55 505	9.98 377	9.45 504	9.47 346	0.52 654	9.98 159	26
35	9.42 917	9.44 544	0.55 456	9.98 373	9.45 547	9.47 392	0.52 608	9.98 155	25
36	9.42 962	9.44 592	0.55 408	9.98 370	9.45 589	9.47 438	0.52 562	9.98 151	24
37	9.43 008	9.44 641	0.55 359	9.98 366	9.45 632	9.47 484	0.52 516	9.98 147	23
38	9.43 053	9.44 690	0.55 310	9.98 363	9.45 674	9.47 530	0.52 470	9.98 144	22
39	9.43 098	9.44 738	0.55 262	9.98 359	9.45 716	9.47 576	0.52 424	9.98 140	21
40	9.43 143	9.44 787	0.55 213	9.98 356	9.45 758	9.47 622	0.52 378	9.98 136	20
41	9.43 188	9.44 836	0.55 164	9.98 352	9.45 801	9.47 668	0.52 332	9.98 132	19
42	9.43 233	9.44 884	0.55 116	9.98 349	9.45 843	9.47 714	0.52 286	9.98 129	18
43	9.43 278	9.44 933	0.55 067	9.98 345	9.45 885	9.47 760	0.52 240	9.98 125	17
44	9.43 323	9.44 981	0.55 019	9.98 342	9.45 927	9.47 806	0.52 194	9.98 121	16
45	9.43 367	9.45 029	0.54 971	9.98 338	9.45 969	9.47 852	0.52 148	9.98 117	15
46	9.43 412	9.45 078	0.54 922	9.98 334	9.46 011	9.47 897	0.52 103	9.98 113	14
47	9.43 457	9.45 126	0.54 874	9.98 331	9.46 053	9.47 943	0.52 057	9.98 110	13
48	9.43 502	9.45 174	0.54 826	9.98 327	9.46 095	9.47 989	0.52 011	9.98 106	12
49	9.43 546	9.45 222	0.54 778	9.98 324	9.46 136	9.48 035	0.51 965	9.98 102	11
50	9.43 591	9.45 271	0.54 729	9.98 320	9.46 178	9.48 080	0.51 920	9.98 098	10
51	9.43 635	9.45 319	0.54 681	9.98 317	9.46 220	9.48 126	0.51 874	9.98 094	9
52	9.43 680	9.45 367	0.54 633	9.98 313	9.46 262	9.48 171	0.51 829	9.98 090	8
53	9.43 724	9.45 415	0.54 585	9.98 309	9.46 303	9.48 217	0.51 783	9.98 087	7
54	9.43 769	9.45 463	0.54 537	9.98 306	9.46 345	9.48 262	0.51 738	9.98 083	6
55	9.43 813	9.45 511	0.54 489	9.98 302	9.46 386	9.48 307	0.51 693	9.98 079	5
56	9.43 857	9.45 559	0.54 441	9.98 299	9.46 428	9.48 353	0.51 647	9.98 075	4
57	9.43 901	9.45 606	0.54 394	9.98 295	9.46 469	9.48 398	0.51 602	9.98 071	3
58	9.43 946	9.45 654	0.54 346	9.98 291	9.46 511	9.48 443	0.51 557	9.98 067	2
59	9.43 990	9.45 702	0.54 298	9.98 288	9.46 552	9.48 489	0.51 511	9.98 063	1
60	9.44 034	9.45 750	0.54 250	9.98 284	9.46 594	9.48 534	0.51 466	9.98 060	0
	log cos	log cot	log tan	log sin	log cos	log cot	log tan	log sin	′
	74°				73°				

′	17° log sin	17° log tan	17° log cot	17° log cos	18° log sin	18° log tan	18° log cot	18° log cos	
0	9.46 594	9.48 534	0.51 466	9.98 060	9.48 998	9.51 178	0.48 822	9.97 821	60
1	9.46 635	9.48 579	0.51 421	9.98 056	9.49 037	9.51 221	0.48 779	9.97 817	59
2	9.46 676	9.48 624	0.51 376	9.98 052	9.49 076	9.51 264	0.48 736	9.97 812	58
3	9.46 717	9.48 669	0.51 331	9.98 048	9.49 115	9.51 306	0.48 694	9.97 808	57
4	9.46 758	9.48 714	0.51 286	9.98 044	9.49 153	9.51 349	0.48 651	9.97 804	56
5	9.46 800	9.48 759	0.51 241	9.98 040	9.49 192	9.51 392	0.48 608	9.97 800	55
6	9.46 841	9.48 804	0.51 196	9.98 036	9.49 231	9.51 435	0.48 565	9.97 796	54
7	9.46 882	9.48 849	0.51 151	9.98 032	9.49 269	9.51 478	0.48 522	9.97 792	53
8	9.46 923	9.48 894	0.51 106	9.98 029	9.49 308	9.51 520	0.48 480	9.97 788	52
9	9.46 964	9.48 939	0.51 061	9.98 025	9.49 347	9.51 563	0.48 437	9.97 784	51
10	9.47 005	9.48 984	0.51 016	9.98 021	9.49 385	9.51 606	0.48 394	9.97 779	50
11	9.47 045	9.49 029	0.50 971	9.98 017	9.49 424	9.51 648	0.48 352	9.97 775	49
12	9.47 086	9.49 073	0.50 927	9.98 013	9.49 462	9.51 691	0.48 309	9.97 771	48
13	9.47 127	9.49 118	0.50 882	9.98 009	9.49 500	9.51 734	0.48 266	9.97 767	47
14	9.47 168	9.49 163	0.50 837	9.98 005	9.49 539	9.51 776	0.48 224	9.97 763	46
15	9.47 209	9.49 207	0.50 793	9.98 001	9.49 577	9.51 819	0.48 181	9.97 759	45
16	9.47 249	9.49 252	0.50 748	9.97 997	9.49 615	9.51 861	0.48 139	9.97 754	44
17	9.47 290	9.49 296	0.50 704	9.97 993	9.49 654	9.51 903	0.48 097	9.97 750	43
18	9.47 330	9.49 341	0.50 659	9.97 989	9.49 692	9.51 946	0.48 054	9.97 746	42
19	9.47 371	9.49 385	0.50 615	9.97 986	9.49 730	9.51 988	0.48 012	9.97 742	41
20	9.47 411	9.49 430	0.50 570	9.97 982	9.49 768	9.52 031	0.47 969	9.97 738	40
21	9.47 452	9.49 474	0.50 526	9.97 978	9.49 806	9.52 073	0.47 927	9.97 734	39
22	9.47 492	9.49 519	0.50 481	9.97 974	9.49 844	9.52 115	0.47 885	9.97 729	38
23	9.47 533	9.49 563	0.50 437	9.97 970	9.49 882	9.52 157	0.47 843	9.97 725	37
24	9.47 573	9.49 607	0.50 393	9.97 966	9.49 920	9.52 200	0.47 800	9.97 721	36
25	9.47 613	9.49 652	0.50 348	9.97 962	9.49 958	9.52 242	0.47 758	9.97 717	35
26	9.47 654	9.49 696	0.50 304	9.97 958	9.49 996	9.52 284	0.47 716	9.97 713	34
27	9.47 694	9.49 740	0.50 260	9.97 954	9.50 034	9.52 326	0.47 674	9.97 708	33
28	9.47 734	9.49 784	0.50 216	9.97 950	9.50 072	9.52 368	0.47 632	9.97 704	32
29	9.47 774	9.49 828	0.50 172	9.97 946	9.50 110	9.52 410	0.47 590	9.97 700	31
30	9.47 814	9.49 872	0.50 128	9.97 942	9.50 148	9.52 452	0.47 548	9.97 696	30
31	9.47 854	9.49 916	0.50 084	9.97 938	9.50 185	9.52 494	0.47 506	9.97 691	29
32	9.47 894	9.49 960	0.50 040	9.97 934	9.50 223	9.52 536	0.47 464	9.97 687	28
33	9.47 934	9.50 004	0.49 996	9.97 930	9.50 261	9.52 578	0.47 422	9.97 683	27
34	9.47 974	9.50 048	0.49 952	9.97 926	9.50 298	9.52 620	0.47 380	9.97 679	26
35	9.48 014	9.50 092	0.49 908	9.97 922	9.50 336	9.52 661	0.47 339	9.97 674	25
36	9.48 054	9.50 136	0.49 864	9.97 918	9.50 374	9.52 703	0.47 297	9.97 670	24
37	9.48 094	9.50 180	0.49 820	9.97 914	9.50 411	9.52 745	0.47 255	9.97 666	23
38	9.48 133	9.50 223	0.49 777	9.97 910	9.50 449	9.52 787	0.47 213	9.97 662	22
39	9.48 173	9.50 267	0.49 733	9.97 906	9.50 486	9.52 829	0.47 171	9.97 657	21
40	9.48 213	9.50 311	0.49 689	9.97 902	9.50 523	9.52 870	0.47 130	9.97 653	20
41	9.48 252	9.50 355	0.49 645	9.97 898	9.50 561	9.52 912	0.47 088	9.97 649	19
42	9.48 292	9.50 398	0.49 602	9.97 894	9.50 598	9.52 953	0.47 047	9.97 645	18
43	9.48 332	9.50 442	0.49 558	9.97 890	9.50 635	9.52 995	0.47 005	9.97 640	17
44	9.48 371	9.50 485	0.49 515	9.97 886	9.50 673	9.53 037	0.46 963	9.97 636	16
45	9.48 411	9.50 529	0.49 471	9.97 882	9.50 710	9.53 078	0.46 922	9.97 632	15
46	9.48 450	9.50 572	0.49 428	9.97 878	9.50 747	9.53 120	0.46 880	9.97 628	14
47	9.48 490	9.50 616	0.49 384	9.97 874	9.50 784	9.53 161	0.46 839	9.97 623	13
48	9.48 529	9.50 659	0.49 341	9.97 870	9.50 821	9.53 202	0.46 798	9.97 619	12
49	9.48 568	9.50 703	0.49 297	9.97 866	9.50 858	9.53 244	0.46 756	9.97 615	11
50	9.48 607	9.50 746	0.49 254	9.97 861	9.50 896	9.53 285	0.46 715	9.97 610	10
51	9.48 647	9.50 789	0.49 211	9.97 857	9.50 933	9.53 327	0.46 673	9.97 606	9
52	9.48 686	9.50 833	0.49 167	9.97 853	9.50 970	9.53 368	0.46 632	9.97 602	8
53	9.48 725	9.50 876	0.49 124	9.97 849	9.51 007	9.53 409	0.46 591	9.97 597	7
54	9.48 764	9.50 919	0.49 081	9.97 845	9.51 043	9.53 450	0.46 550	9.97 593	6
55	9.48 803	9.50 962	0.49 038	9.97 841	9.51 080	9.53 492	0.46 508	9.97 589	5
56	9.48 842	9.51 005	0.48 995	9.97 837	9.51 117	9.53 533	0.46 467	9.97 584	4
57	9.48 881	9.51 048	0.48 952	9.97 833	9.51 154	9.53 574	0.46 426	9.97 580	3
58	9.48 920	9.51 092	0.48 908	9.97 829	9.51 191	9.53 615	0.46 385	9.97 576	2
59	9.48 959	9.51 135	0.48 865	9.97 825	9.51 227	9.53 656	0.46 344	9.97 571	1
60	9.48 998	9.51 178	0.48 822	9.97 821	9.51 264	9.53 697	0.46 303	9.97 567	0
	72° log cos	72° log cot	72° log tan	72° log sin	71° log cos	71° log cot	71° log tan	71° log sin	′

′	19° log sin	log tan	log cot	log cos	20° log sin	log tan	log cot	log cos	′
0	9.51 264	9.53 697	0.46 303	9.97 567	9.53 405	9.56 107	0.43 893	9.97 299	60
1	9.51 301	9.53 738	0.46 262	9.97 563	9.53 440	9.56 140	0.43 854	9.97 294	59
2	9.51 338	9.53 779	0.46 221	9.97 558	9.53 475	9.56 185	0.43 815	9.97 289	58
3	9.51 374	9.53 820	0.46 180	9.97 554	9.53 509	9.56 224	0.43 776	9.97 285	57
4	9.51 411	9.53 861	0.46 139	9.97 550	9.53 544	9.56 264	0.43 736	9.97 280	56
5	9.51 447	9.53 902	0.46 098	9.97 545	9.53 578	9.56 303	0.43 697	9.97 276	55
6	9.51 484	9.53 943	0.46 057	9.97 541	9.53 613	9.56 342	0.43 658	9.97 271	54
7	9.51 520	9.53 984	0.46 016	9.97 536	9.53 647	9.56 381	0.43 619	9.97 266	53
8	9.51 557	9.54 025	0.45 975	9.97 532	9.53 682	9.56 420	0.43 580	9.97 262	52
9	9.51 593	9.54 065	0.45 935	9.97 528	9.53 716	9.56 459	0.43 541	9.97 257	51
10	9.51 629	9.54 106	0.45 894	9.97 523	9.53 751	9.56 498	0.43 502	9.97 252	50
11	9.51 666	9.54 147	0.45 853	9.97 519	9.53 785	9.56 537	0.43 463	9.97 248	49
12	9.51 702	9.54 187	0.45 813	9.97 515	9.53 819	9.56 576	0.43 424	9.97 243	48
13	9.51 738	9.54 228	0.45 772	9.97 510	9.53 854	9.56 615	0.43 385	9.97 238	47
14	9.51 774	9.54 269	0.45 731	9.97 506	9.53 888	9.56 654	0.43 346	9.97 234	46
15	9.51 811	9.54 309	0.45 691	9.97 501	9.53 922	9.56 693	0.43 307	9.97 229	45
16	9.51 847	9.54 350	0.45 650	9.97 497	9.53 957	9.56 732	0.43 268	9.97 224	44
17	9.51 883	9.54 390	0.45 610	9.97 492	9.53 991	9.56 771	0.43 229	9.97 220	43
18	9.51 919	9.54 431	0.45 569	9.97 488	9.54 025	9.56 810	0.43 190	9.97 215	42
19	9.51 955	9.54 471	0.45 529	9.97 484	9.54 059	9.56 849	0.43 151	9.97 210	41
20	9.51 991	9.54 512	0.45 488	9.97 479	9.54 093	9.56 887	0.43 113	9.97 206	40
21	9.52 027	9.54 552	0.45 448	9.97 475	9.54 127	9.56 926	0.43 074	9.97 201	39
22	9.52 063	9.54 593	0.45 407	9.97 470	9.54 161	9.56 965	0.43 035	9.97 196	38
23	9.52 099	9.54 633	0.45 367	9.97 466	9.54 195	9.57 004	0.42 996	9.97 192	37
24	9.52 135	9.54 673	0.45 327	9.97 461	9.54 229	9.57 042	0.42 958	9.97 187	36
25	9.52 171	9.54 714	0.45 286	9.97 457	9.54 263	9.57 081	0.42 919	9.97 182	35
26	9.52 207	9.54 754	0.45 246	9.97 453	9.54 297	9.57 120	0.42 880	9.97 178	34
27	9.52 242	9.54 794	0.45 206	9.97 448	9.54 331	9.57 158	0.42 842	9.97 173	33
28	9.52 278	9.54 835	0.45 165	9.97 444	9.54 365	9.57 197	0.42 803	9.97 168	32
29	9.52 314	9.54 875	0.45 125	9.97 439	9.54 399	9.57 235	0.42 765	9.97 163	31
30	9.52 350	9.54 915	0.45 085	9.97 435	9.54 433	9.57 274	0.42 726	9.97 159	30
31	9.52 385	9.54 955	0.45 045	9.97 430	9.54 466	9.57 312	0.42 688	9.97 154	29
32	9.52 421	9.54 995	0.45 005	9.97 426	9.54 500	9.57 351	0.42 649	9.97 149	28
33	9.52 456	9.55 035	0.44 965	9.97 421	9.54 534	9.57 389	0.42 611	9.97 145	27
34	9.52 492	9.55 075	0.44 925	9.97 417	9.54 567	9.57 428	0.42 572	9.97 140	26
35	9.52 527	9.55 115	0.44 885	9.97 412	9.54 601	9.57 466	0.42 534	9.97 135	25
36	9.52 563	9.55 155	0.44 845	9.97 408	9.54 635	9.57 504	0.42 496	9.97 130	24
37	9.52 598	9.55 195	0.44 805	9.97 403	9.54 668	9.57 543	0.42 457	9.97 126	23
38	9.52 634	9.55 235	0.44 765	9.97 399	9.54 702	9.57 581	0.42 419	9.97 121	22
39	9.52 669	9.55 275	0.44 725	9.97 394	9.54 735	9.57 619	0.42 381	9.97 116	21
40	9.52 705	9.55 315	0.44 685	9.97 390	9.54 769	9.57 658	0.42 342	9.97 111	20
41	9.52 740	9.55 355	0.44 645	9.97 385	9.54 802	9.57 696	0.42 304	9.97 107	19
42	9.52 775	9.55 395	0.44 605	9.97 381	9.54 836	9.57 734	0.42 266	9.97 102	18
43	9.52 811	9.55 434	0.44 566	9.97 376	9.54 869	9.57 772	0.42 228	9.97 097	17
44	9.52 846	9.55 474	0.44 526	9.97 372	9.54 903	9.57 810	0.42 190	9.97 092	16
45	9.52 881	9.55 514	0.44 486	9.97 367	9.54 936	9.57 849	0.42 151	9.97 087	15
46	9.52 916	9.55 554	0.44 446	9.97 363	9.54 969	9.57 887	0.42 113	9.97 083	14
47	9.52 951	9.55 593	0.44 407	9.97 358	9.55 003	9.57 925	0.42 075	9.97 078	13
48	9.52 986	9.55 633	0.44 367	9.97 353	9.55 036	9.57 963	0.42 037	9.97 073	12
49	9.53 021	9.55 673	0.44 327	9.97 349	9.55 069	9.58 001	0.41 999	9.97 068	11
50	9.53 056	9.55 712	0.44 288	9.97 344	9.55 102	9.58 039	0.41 961	9.97 063	10
51	9.53 092	9.55 752	0.44 248	9.97 340	9.55 136	9.58 077	0.41 923	9.97 059	9
52	9.53 126	9.55 791	0.44 209	9.97 335	9.55 169	9.58 115	0.41 885	9.97 054	8
53	9.53 161	9.55 831	0.44 169	9.97 331	9.55 202	9.58 153	0.41 847	9.97 049	7
54	9.53 196	9.55 870	0.44 130	9.97 326	9.55 235	9.58 191	0.41 809	9.97 044	6
55	9.53 231	9.55 910	0.44 090	9.97 322	9.55 268	9.58 229	0.41 771	9.97 039	5
56	9.53 266	9.55 949	0.44 051	9.97 317	9.55 301	9.58 267	0.41 733	9.97 035	4
57	9.53 301	9.55 989	0.44 011	9.97 312	9.55 334	9.58 304	0.41 696	9.97 030	3
58	9.53 336	9.56 028	0.43 972	9.97 308	9.55 367	9.58 342	0.41 658	9.97 025	2
59	9.53 370	9.56 067	0.43 933	9.97 303	9.55 400	9.58 380	0.41 620	9.97 020	1
60	9.53 405	9.56 107	0.43 893	9.97 299	9.55 433	9.58 418	0.41 582	9.97 015	0
	log cos	log cot	log tan	log sin	log cos	log cot	log tan	log sin	′
	70°				69°				

′	21° log sin	log tan	log cot	log cos	22° log sin	log tan	log cot	log cos	
0	9.55 433	9.58 418	0.41 582	9.97 015	9.57 358	9.60 641	0.39 359	9.96 717	60
1	9.55 466	9.58 455	0.41 545	9.97 010	9.57 389	9.60 677	0.39 323	9.96 711	59
2	9.55 499	9.58 493	0.41 507	9.97 005	9.57 420	9.60 714	0.39 286	9.96 706	58
3	9.55 532	9.58 531	0.41 469	9.97 001	9.57 451	9.60 750	0.39 250	9.96 701	57
4	9.55 564	9.58 569	0.41 431	9.96 996	9.57 482	9.60 786	0.39 214	9.96 696	56
5	9.55 597	9.58 606	0.41 394	9.96 991	9.57 514	9.60 823	0.39 177	9.96 691	55
6	9.55 630	9.58 644	0.41 356	9.96 986	9.57 545	9.60 859	0.39 141	9.96 686	54
7	9.55 663	9.58 681	0.41 319	9.96 981	9.57 576	9.60 895	0.39 105	9.96 681	53
8	9.55 695	9.58 719	0.41 281	9.96 976	9.57 607	9.60 931	0.39 069	9.96 676	52
9	9.55 728	9.58 757	0.41 243	9.96 971	9.57 638	9.60 967	0.39 033	9.96 670	51
10	9.55 761	9.58 794	0.41 206	9.96 966	9.57 669	9.61 004	0.38 996	9.96 665	50
11	9.55 793	9.58 832	0.41 168	9.96 962	9.57 700	9.61 040	0.38 960	9.96 660	49
12	9.55 826	9.58 869	0.41 131	9.96 957	9.57 731	9.61 076	0.38 924	9.96 655	48
13	9.55 858	9.58 907	0.41 093	9.96 952	9.57 762	9.61 112	0.38 888	9.96 650	47
14	9.55 891	9.58 944	0.41 056	9.96 947	9.57 793	9.61 148	0.38 852	9.96 645	46
15	9.55 923	9.58 981	0.41 019	9.96 942	9.57 824	9.61 184	0.38 816	9.96 640	45
16	9.55 956	9.59 019	0.40 981	9.96 937	9.57 855	9.61 220	0.38 780	9.96 634	44
17	9.55 988	9.59 056	0.40 944	9.96 932	9.57 885	9.61 256	0.38 744	9.96 629	43
18	9.56 021	9.59 094	0.40 906	9.96 927	9.57 916	9.61 292	0.38 708	9.96 624	42
19	9.56 053	9.59 131	0.40 869	9.96 922	9.57 947	9.61 328	0.38 672	9.96 619	41
20	9.56 085	9.59 168	0.40 832	9.96 917	9.57 978	9.61 364	0.38 636	9.96 614	40
21	9.56 118	9.59 205	0.40 795	9.96 912	9.58 008	9.61 400	0.38 600	9.96 608	39
22	9.56 150	9.59 243	0.40 757	9.96 907	9.58 039	9.61 436	0.38 564	9.96 603	38
23	9.56 182	9.59 280	0.40 720	9.96 903	9.58 070	9.61 472	0.38 528	9.96 598	37
24	9.56 215	9.59 317	0.40 683	9.96 898	9.58 101	9.61 508	0.38 492	9.96 593	36
25	9.56 247	9.59 354	0.40 646	9.96 893	9.58 131	9.61 544	0.38 456	9.96 588	35
26	9.56 279	9.59 391	0.40 609	9.96 888	9.58 162	9.61 579	0.38 421	9.96 582	34
27	9.56 311	9.59 429	0.40 571	9.96 883	9.58 192	9.61 615	0.38 385	9.96 577	33
28	9.56 343	9.59 466	0.40 534	9.96 878	9.58 223	9.61 651	0.38 349	9.96 572	32
29	9.56 375	9.59 503	0.40 497	9.96 873	9.58 253	9.61 687	0.38 313	9.96 567	31
30	9.56 408	9.59 540	0.40 460	9.96 868	9.58 284	9.61 722	0.38 278	9.96 562	30
31	9.56 440	9.59 577	0.40 423	9.96 863	9.58 314	9.61 758	0.38 242	9.96 556	29
32	9.56 472	9.59 614	0.40 386	9.96 858	9.58 345	9.61 794	0.38 206	9.96 551	28
33	9.56 504	9.59 651	0.40 349	9.96 853	9.58 375	9.61 830	0.38 170	9.96 546	27
34	9.56 536	9.59 688	0.40 312	9.96 848	9.58 406	9.61 865	0.38 135	9.96 541	26
35	9.56 568	9.59 725	0.40 275	9.96 843	9.58 436	9.61 901	0.38 099	9.96 535	25
36	9.56 599	9.59 762	0.40 238	9.96 838	9.58 467	9.61 936	0.38 064	9.96 530	24
37	9.56 631	9.59 799	0.40 201	9.96 833	9.58 497	9.61 972	0.38 028	9.96 525	23
38	9.56 663	9.59 835	0.40 165	9.96 828	9.58 527	9.62 008	0.37 992	9.96 520	22
39	9.56 695	9.59 872	0.40 128	9.96 823	9.58 557	9.62 043	0.37 957	9.96 514	21
40	9.56 727	9.59 909	0.40 091	9.96 818	9.58 588	9.62 079	0.37 921	9.96 509	20
41	9.56 759	9.59 946	0.40 054	9.96 813	9.58 618	9.62 114	0.37 886	9.96 504	19
42	9.56 790	9.59 983	0.40 017	9.96 808	9.58 648	9.62 150	0.37 850	9.96 498	18
43	9.56 822	9.60 019	0.39 981	9.96 803	9.58 678	9.62 185	0.37 815	9.96 493	17
44	9.56 854	9.60 056	0.39 944	9.96 798	9.58 709	9.62 221	0.37 779	9.96 488	16
45	9.56 886	9.60 093	0.39 907	9.96 793	9.58 739	9.62 256	0.37 744	9.96 483	15
46	9.56 917	9.60 130	0.39 870	9.96 788	9.58 769	9.62 292	0.37 708	9.96 477	14
47	9.56 949	9.60 166	0.39 834	9.96 783	9.58 799	9.62 327	0.37 673	9.96 472	13
48	9.56 980	9.60 203	0.39 797	9.96 778	9.58 829	9.62 362	0.37 638	9.96 467	12
49	9.57 012	9.60 240	0.39 760	9.96 772	9.58 859	9.62 398	0.37 602	9.96 461	11
50	9.57 044	9.60 276	0.39 724	9.96 767	9.58 889	9.62 433	0.37 567	9.96 456	10
51	9.57 075	9.60 313	0.39 687	9.96 762	9.58 919	9.62 468	0.37 532	9.96 451	9
52	9.57 107	9.60 349	0.39 651	9.96 757	9.58 949	9.62 504	0.37 496	9.96 445	8
53	9.57 138	9.60 386	0.39 614	9.96 752	9.58 979	9.62 539	0.37 461	9.96 440	7
54	9.57 169	9.60 422	0.39 578	9.96 747	9.59 009	9.62 574	0.37 426	9.96 435	6
55	9.57 201	9.60 459	0.39 541	9.96 742	9.59 039	9.62 609	0.37 391	9.96 429	5
56	9.57 232	9.60 495	0.39 505	9.96 737	9.59 069	9.62 645	0.37 355	9.96 424	4
57	9.57 264	9.60 532	0.39 468	9.96 732	9.59 098	9.62 680	0.37 320	9.96 419	3
58	9.57 295	9.60 568	0.39 432	9.96 727	9.59 128	9.62 715	0.37 285	9.96 413	2
59	9.57 326	9.60 605	0.39 395	9.96 722	9.59 158	9.62 750	0.37 250	9.96 408	1
60	9.57 358	9.60 641	0.39 359	9.96 717	9.59 188	9.62 785	0.37 215	9.96 403	0
	log cos	log cot	log tan	log sin 68°	log cos	log cot	log tan	log sin 67°	′

′	23° log sin	log tan	log cot	log cos	24° log sin	log tan	log cot	log cos	
0	9.59 188	9.62 785	0.37 215	9.96 403	9.60 931	9.64 858	0.35 142	9.96 073	60
1	9.59 218	9.62 820	0.37 180	9.96 397	9.60 960	9.64 892	0.35 108	9.96 067	59
2	9.59 247	9.62 855	0.37 145	9.96 392	9.60 988	9.64 926	0.35 074	9.96 062	58
3	9.59 277	9.62 890	0.37 110	9.96 387	9.61 016	9.64 960	0.35 040	9.96 056	57
4	9.59 307	9.62 926	0.37 074	9.96 381	9.61 045	9.64 994	0.35 006	9.96 050	56
5	9.59 336	9.62 961	0.37 039	9.96 376	9.61 073	9.65 028	0.34 972	9.96 045	55
6	9.59 366	9.62 996	0.37 004	9.96 370	9.61 101	9.65 062	0.34 938	9.96 039	54
7	9.59 396	9.63 031	0.36 969	9.96 365	9.61 129	9.65 096	0.34 904	9.96 034	53
8	9.59 425	9.63 066	0.36 934	9.96 360	9.61 158	9.65 130	0.34 870	9.96 028	52
9	9.59 455	9.63 101	0.36 899	9.96 354	9.61 186	9.65 164	0.34 836	9.96 022	51
10	9.59 484	9.63 135	0.36 865	9.96 349	9.61 214	9.65 197	0.34 803	9.96 017	50
11	9.59 514	9.63 170	0.36 830	9.96 343	9.61 242	9.65 231	0.34 769	9.96 011	49
12	9.59 543	9.63 205	0.36 795	9.96 338	9.61 270	9.65 265	0.34 735	9.96 005	48
13	9.59 573	9.63 240	0.36 760	9.96 333	9.61 298	9.65 299	0.34 701	9.96 000	47
14	9.59 602	9.63 275	0.36 725	9.96 327	9.61 326	9.65 333	0.34 667	9.95 994	46
15	9.59 632	9.63 310	0.36 690	9.96 322	9.61 354	9.65 366	0.34 634	9.95 988	45
16	9.59 661	9.63 345	0.36 655	9.96 316	9.61 382	9.65 400	0.34 600	9.95 982	44
17	9.59 690	9.63 379	0.36 621	9.96 311	9.61 411	9.65 434	0.34 566	9.95 977	43
18	9.59 720	9.63 414	0.36 586	9.96 305	9.61 438	9.65 467	0.34 533	9.95 971	42
19	9.59 749	9.63 449	0.36 551	9.96 300	9.61 466	9.65 501	0.34 499	9.95 965	41
20	9.59 778	9.63 484	0.36 516	9.96 294	9.61 494	9.65 535	0.34 465	9.95 960	40
21	9.59 808	9.63 519	0.36 481	9.96 289	9.61 522	9.65 568	0.34 432	9.95 954	39
22	9.59 837	9.63 553	0.36 447	9.96 284	9.61 550	9.65 602	0.34 398	9.95 948	38
23	9.59 866	9.63 588	0.36 412	9.96 278	9.61 578	9.65 636	0.34 364	9.95 942	37
24	9.59 895	9.63 623	0.36 377	9.96 273	9.61 606	9.65 669	0.34 331	9.95 937	36
25	9.59 924	9.63 657	0.36 343	9.96 267	9.61 634	9.65 703	0.34 297	9.95 931	35
26	9.59 954	9.63 692	0.36 308	9.96 262	9.61 662	9.65 736	0.34 264	9.95 925	34
27	9.59 983	9.63 726	0.36 274	9.96 256	9.61 689	9.65 770	0.34 230	9.95 920	33
28	9.60 012	9.63 761	0.36 239	9.96 251	9.61 717	9.65 803	0.34 197	9.95 914	32
29	9.60 041	9.63 796	0.36 204	9.96 245	9.61 745	9.65 837	0.34 163	9.95 908	31
30	9.60 070	9.63 830	0.36 170	9.96 240	9.61 773	9.65 870	0.34 130	9.95 902	30
31	9.60 099	9.63 865	0.36 135	9.96 234	9.61 800	9.65 904	0.34 096	9.95 897	29
32	9.60 128	9.63 899	0.36 101	9.96 229	9.61 828	9.65 937	0.34 063	9.95 891	28
33	9.60 157	9.63 934	0.36 066	9.96 223	9.61 856	9.65 971	0.34 029	9.95 885	27
34	9.60 186	9.63 968	0.36 032	9.96 218	9.61 883	9.66 004	0.33 996	9.95 879	26
35	9.60 215	9.64 003	0.35 997	9.96 212	9.61 911	9.66 038	0.33 962	9.95 873	25
36	9.60 244	9.64 037	0.35 963	9.96 207	9.61 939	9.66 071	0.33 929	9.95 868	24
37	9.60 273	9.64 072	0.35 928	9.96 201	9.61 966	9.66 104	0.33 896	9.95 862	23
38	9.60 302	9.64 106	0.35 894	9.96 196	9.61 994	9.66 138	0.33 862	9.95 856	22
39	9.60 331	9.64 140	0.35 860	9.96 190	9.62 021	9.66 171	0.33 829	9.95 850	21
40	9.60 359	9.64 175	0.35 825	9.96 185	9.62 049	9.66 204	0.33 796	9.95 844	20
41	9.60 388	9.64 209	0.35 791	9.96 179	9.62 076	9.66 238	0.33 762	9.95 839	19
42	9.60 417	9.64 243	0.35 757	9.96 174	9.62 104	9.66 271	0.33 729	9.95 833	18
43	9.60 446	9.64 278	0.35 722	9.96 168	9.62 131	9.66 304	0.33 696	9.95 827	17
44	9.60 474	9.64 312	0.35 688	9.96 162	9.62 159	9.66 337	0.33 663	9.95 821	16
45	9.60 503	9.64 346	0.35 654	9.96 157	9.62 186	9.66 371	0.33 629	9.95 815	15
46	9.60 532	9.64 381	0.35 619	9.96 151	9.62 214	9.66 404	0.33 596	9.95 810	14
47	9.60 561	9.64 415	0.35 585	9.96 146	9.62 241	9.66 437	0.33 563	9.95 804	13
48	9.60 589	9.64 449	0.35 551	9.96 140	9.62 268	9.66 470	0.33 530	9.95 798	12
49	9.60 618	9.64 483	0.35 517	9.96 135	9.62 296	9.66 503	0.33 497	9.95 792	11
50	9.60 646	9.64 517	0.35 483	9.96 129	9.62 323	9.66 537	0.33 463	9.95 786	10
51	9.60 675	9.64 552	0.35 448	9.96 123	9.62 350	9.66 570	0.33 430	9.95 780	9
52	9.60 704	9.64 586	0.35 414	9.96 118	9.62 377	9.66 603	0.33 397	9.95 775	8
53	9.60 732	9.64 620	0.35 380	9.96 112	9.62 405	9.66 636	0.33 364	9.95 769	7
54	9.60 761	9.64 654	0.35 346	9.96 107	9.62 432	9.66 669	0.33 331	9.95 763	6
55	9.60 789	9.64 688	0.35 312	9.96 101	9.62 459	9.66 702	0.33 298	9.95 757	5
56	9.60 818	9.64 722	0.35 278	9.96 095	9.62 486	9.66 735	0.33 265	9.95 751	4
57	9.60 846	9.64 756	0.35 244	9.96 090	9.62 513	9.66 768	0.33 232	9.95 745	3
58	9.60 875	9.64 790	0.35 210	9.96 084	9.62 541	9.66 801	0.33 199	9.95 739	2
59	9.60 903	9.64 824	0.35 176	9.96 079	9.62 568	9.66 834	0.33 166	9.95 733	1
60	9.60 931	9.64 858	0.35 142	9.96 073	9.62 595	9.66 867	0.33 133	9.95 728	0
	log cos	log cot	log tan	log sin	log cos	log cot	log tan	log sin	′
	66°				65°				

	25°				26°				
′	log sin	log tan	log cot	log cos	log sin	log tan	log cot	log cos	
0	9.62 595	9.66 867	0.33 133	9.95 728	9.64 184	9.68 818	0.31 182	9.95 366	60
1	9.62 622	9.66 900	0.33 100	9.95 722	9.64 210	9.68 850	0.31 150	9.95 360	59
2	9.62 649	9.66 933	0.33 067	9.95 716	9.64 236	9.68 882	0.31 118	9.95 354	58
3	9.62 676	9.66 966	0.33 034	9.95 710	9.64 262	9.68 914	0.31 086	9.95 348	57
4	9.62 703	9.66 999	0.33 001	9.95 704	9.64 288	9.68 946	0.31 054	9.95 341	56
5	9.62 730	9.67 032	0.32 968	9.95 698	9.64 313	9.68 978	0.31 022	9.95 335	55
6	9.62 757	9.67 065	0.32 935	9.95 692	9.64 339	9.69 010	0.30 990	9.95 329	54
7	9.62 784	9.67 098	0.32 902	9.95 686	9.64 365	9.69 042	0.30 958	9.95 323	53
8	9.62 811	9.67 131	0.32 869	9.95 680	9.64 391	9.69 074	0.30 926	9.95 317	52
9	9.62 838	9.67 163	0.32 837	9.95 674	9.64 417	9.69 106	0.30 894	9.95 310	51
10	9.62 865	9.67 196	0.32 804	9.95 668	9.64 442	9.69 138	0.30 862	9.95 304	50
11	9.62 892	9.67 229	0.32 771	9.95 663	9.64 468	9.69 170	0.30 830	9.95 298	49
12	9.62 918	9.67 262	0.32 738	9.95 657	9.64 494	9.69 202	0.30 798	9.95 292	48
13	9.62 945	9.67 295	0.32 705	9.95 651	9.64 519	9.69 234	0.30 766	9.95 286	47
14	9.62 972	9.67 327	0.32 673	9.95 645	9.64 545	9.69 266	0.30 734	9.95 279	46
15	9.62 999	9.67 360	0.32 640	9.95 639	9.64 571	9.69 298	0.30 702	9.95 273	45
16	9.63 026	9.67 393	0.32 607	9.95 633	9.64 596	9.69 329	0.30 671	9.95 267	44
17	9.63 052	9.67 426	0.32 574	9.95 627	9.64 622	9.69 361	0.30 639	9.95 261	43
18	9.63 079	9.67 458	0.32 542	9.95 621	9.64 647	9.69 393	0.30 607	9.95 254	42
19	9.63 106	9.67 491	0.32 509	9.95 615	9.64 673	9.69 425	0.30 575	9.95 248	41
20	9.63 133	9.67 524	0.32 476	9.95 609	9.64 698	9.69 457	0.30 543	9.95 242	40
21	9.63 159	9.67 556	0.32 444	9.95 603	9.64 724	9.69 488	0.30 512	9.95 236	39
22	9.63 186	9.67 589	0.32 411	9.95 597	9.64 749	9.69 520	0.30 480	9.95 229	38
23	9.63 213	9.67 622	0.32 378	9.95 591	9.64 775	9.69 552	0.30 448	9.95 223	37
24	9.63 239	9.67 654	0.32 346	9.95 585	9.64 800	9.69 584	0.30 416	9.95 217	36
25	9.63 266	9.67 687	0.32 313	9.95 579	9.64 826	9.69 615	0.30 385	9.95 211	35
26	9.63 292	9.67 719	0.32 281	9.95 573	9.64 851	9.69 647	0.30 353	9.95 204	34
27	9.63 319	9.67 752	0.32 248	9.95 567	9.64 877	9.69 679	0.30 321	9.95 198	33
28	9.63 345	9.67 785	0.32 215	9.95 561	9.64 902	9.69 710	0.30 290	9.95 192	32
29	9.63 372	9.67 817	0.32 183	9.95 555	9.64 927	9.69 742	0.30 258	9.95 185	31
30	9.63 398	9.67 850	0.32 150	9.95 549	9.64 953	9.69 774	0.30 226	9.95 179	30
31	9.63 425	9.67 882	0.32 118	9.95 543	9.64 978	9.69 805	0.30 195	9.95 173	29
32	9.63 451	9.67 915	0.32 085	9.95 537	9.65 003	9.69 837	0.30 163	9.95 167	28
33	9.63 478	9.67 947	0.32 053	9.95 531	9.65 029	9.69 868	0.30 132	9.95 160	27
34	9.63 504	9.67 980	0.32 020	9.95 525	9.65 054	9.69 900	0.30 100	9.95 154	26
35	9.63 531	9.68 012	0.31 988	9.95 519	9.65 079	9.69 932	0.30 068	9.95 148	25
36	9.63 557	9.68 044	0.31 956	9.95 513	9.65 104	9.69 963	0.30 037	9.95 141	24
37	9.63 583	9.68 077	0.31 923	9.95 507	9.65 130	9.69 995	0.30 005	9.95 135	23
38	9.63 610	9.68 109	0.31 891	9.95 500	9.65 155	9.70 026	0.29 974	9.95 129	22
39	9.63 636	9.68 142	0.31 858	9.95 494	9.65 180	9.70 058	0.29 942	9.95 122	21
40	9.63 662	9.68 174	0.31 826	9.95 488	9.65 205	9.70 089	0.29 911	9.95 116	20
41	9.63 689	9.68 206	0.31 794	9.95 482	9.65 230	9.70 121	0.29 879	9.95 110	19
42	9.63 715	9.68 239	0.31 761	9.95 476	9.65 255	9.70 152	0.29 848	9.95 103	18
43	9.63 741	9.68 271	0.31 729	9.95 470	9.65 281	9.70 184	0.29 816	9.95 097	17
44	9.63 767	9.68 303	0.31 697	9.95 464	9.65 306	9.70 215	0.29 785	9.95 090	16
45	9.63 794	9.68 336	0.31 664	9.95 458	9.65 331	9.70 247	0.29 753	9.95 084	15
46	9.63 820	9.68 368	0.31 632	9.95 452	9.65 356	9.70 278	0.29 722	9.95 078	14
47	9.63 846	9.68 400	0.31 600	9.95 446	9.65 381	9.70 309	0.29 691	9.95 071	13
48	9.63 872	9.68 432	0.31 568	9.95 440	9.65 406	9.70 341	0.29 659	9.95 065	12
49	9.63 898	9.68 465	0.31 535	9.95 434	9.65 431	9.70 372	0.29 628	9.95 059	11
50	9.63 924	9.68 497	0.31 503	9.95 427	9.65 456	9.70 404	0.29 596	9.95 052	10
51	9.63 950	9.68 529	0.31 471	9.95 421	9.65 481	9.70 435	0.29 565	9.95 046	9
52	9.63 976	9.68 561	0.31 439	9.95 415	9.65 506	9.70 466	0.29 534	9.95 039	8
53	9.64 002	9.68 593	0.31 407	9.95 409	9.65 531	9.70 498	0.29 502	9.95 033	7
54	9.64 028	9.68 626	0.31 374	9.95 403	9.65 556	9.70 529	0.29 471	9.95 027	6
55	9.64 054	9.68 658	0.31 342	9.95 397	9.65 580	9.70 560	0.29 440	9.95 020	5
56	9.64 080	9.68 690	0.31 310	9.95 391	9.65 605	9.70 592	0.29 408	9.95 014	4
57	9.64 106	9.68 722	0.31 278	9.95 384	9.65 630	9.70 623	0.29 377	9.95 007	3
58	9.64 132	9.68 754	0.31 246	9.95 378	9.65 655	9.70 654	0.29 346	9.95 001	2
59	9.64 158	9.68 786	0.31 214	9.95 372	9.65 680	9.70 685	0.29 315	9.94 995	1
60	9.64 184	9.68 818	0.31 182	9.95 366	9.65 705	9.70 717	0.29 283	9.94 988	0
	log cos	log cot	log tan	log sin	log cos	log cot	log tan	log sin	′
	64°				63°				

′	27° log sin	log tan	log cot	log cos	28° log sin	log tan	log cot	log cos	
0	9.65 705	9.70 717	0.29 283	9.94 988	9.67 161	9.72 567	0.27 433	9.94 593	60
1	9.65 729	9.70 748	0.29 252	9.94 982	9.67 185	9.72 598	0.27 402	9.94 587	59
2	9.65 754	9.70 779	0.29 221	9.94 975	9.67 208	9.72 628	0.27 372	9.94 580	58
3	9.65 779	9.70 810	0.29 190	9.94 969	9.67 232	9.72 659	0.27 341	9.94 573	57
4	9.65 804	9.70 841	0.29 159	9.94 962	9.67 256	9.72 689	0.27 311	9.94 567	56
5	9.65 828	9.70 873	0.29 127	9.94 956	9.67 280	9.72 720	0.27 280	9.94 560	55
6	9.65 853	9.70 904	0.29 096	9.94 949	9.67 303	9.72 750	0.27 250	9.94 553	54
7	9.65 878	9.70 935	0.29 065	9.94 943	9.67 327	9.72 780	0.27 220	9.94 546	53
8	9.65 902	9.70 966	0.29 034	9.94 936	9.67 350	9.72 811	0.27 189	9.94 540	52
9	9.65 927	9.70 997	0.29 003	9.94 930	9.67 374	9.72 841	0.27 159	9.94 533	51
10	9.65 952	9.71 028	0.28 972	9.94 923	9.67 398	9.72 872	0.27 128	9.94 526	50
11	9.65 976	9.71 059	0.28 941	9.94 917	9.67 421	9.72 902	0.27 098	9.94 519	49
12	9.66 001	9.71 090	0.28 910	9.94 911	9.67 445	9.72 932	0.27 068	9.94 513	48
13	9.66 025	9.71 121	0.28 879	9.94 904	9.67 468	9.72 963	0.27 037	9.94 506	47
14	9.66 050	9.71 153	0.28 847	9.94 898	9.67 492	9.72 993	0.27 007	9.94 499	46
15	9.66 075	9.71 184	0.28 816	9.94 891	9.67 515	9.73 023	0.26 977	9.94 492	45
16	9.66 099	9.71 215	0.28 785	9.94 885	9.67 539	9.73 054	0.26 946	9.94 485	44
17	9.66 124	9.71 246	0.28 754	9.94 878	9.67 562	9.73 084	0.26 916	9.94 479	43
18	9.66 148	9.71 277	0.28 723	9.94 871	9.67 586	9.73 114	0.26 886	9.94 472	42
19	9.66 173	9.71 308	0.28 692	9.94 865	9.67 609	9.73 144	0.26 856	9.94 465	41
20	9.66 197	9.71 339	0.28 661	9.94 858	9.67 633	9.73 175	0.26 825	9.94 458	40
21	9.66 221	9.71 370	0.28 630	9.94 852	9.67 656	9.73 205	0.26 795	9.94 451	39
22	9.66 246	9.71 401	0.28 599	9.94 845	9.67 680	9.73 235	0.26 765	9.94 445	38
23	9.66 270	9.71 431	0.28 569	9.94 839	9.67 703	9.73 265	0.26 735	9.94 438	37
24	9.66 295	9.71 462	0.28 538	9.94 832	9.67 726	9.73 295	0.26 705	9.94 431	36
25	9.66 319	9.71 493	0.28 507	9.94 826	9.67 750	9.73 326	0.26 674	9.94 424	35
26	9.66 343	9.71 524	0.28 476	9.94 819	9.67 773	9.73 356	0.26 644	9.94 417	34
27	9.66 368	9.71 555	0.28 445	9.94 813	9.67 796	9.73 386	0.26 614	9.94 410	33
28	9.66 392	9.71 586	0.28 414	9.94 806	9.67 820	9.73 416	0.26 584	9.94 404	32
29	9.66 416	9.71 617	0.28 383	9.94 799	9.67 843	9.73 446	0.26 554	9.94 397	31
30	9.66 441	9.71 648	0.28 352	9.94 793	9.67 866	9.73 476	0.26 524	9.94 390	30
31	9.66 465	9.71 679	0.28 321	9.94 786	9.67 890	9.73 507	0.26 493	9.94 383	29
32	9.66 489	9.71 709	0.28 291	9.94 780	9.67 913	9.73 537	0.26 463	9.94 376	28
33	9.66 513	9.71 740	0.28 260	9.94 773	9.67 936	9.73 567	0.26 433	9.94 369	27
34	9.66 537	9.71 771	0.28 229	9.94 767	9.67 959	9.73 597	0.26 403	9.94 362	26
35	9.66 562	9.71 802	0.28 198	9.94 760	9.67 982	9.73 627	0.26 373	9.94 355	25
36	9.66 586	9.71 833	0.28 167	9.94 753	9.68 006	9.73 657	0.26 343	9.94 349	24
37	9.66 610	9.71 863	0.28 137	9.94 747	9.68 029	9.73 687	0.26 313	9.94 342	23
38	9.66 634	9.71 894	0.28 106	9.94 740	9.68 052	9.73 717	0.26 283	9.94 335	22
39	9.66 658	9.71 925	0.28 075	9.94 734	9.68 075	9.73 747	0.26 253	9.94 328	21
40	9.66 682	9.71 955	0.28 045	9.94 727	9.68 098	9.73 777	0.26 223	9.94 321	20
41	9.66 706	9.71 986	0.28 014	9.94 720	9.68 121	9.73 807	0.26 193	9.94 314	19
42	9.66 731	9.72 017	0.27 983	9.94 714	9.68 144	9.73 837	0.26 163	9.94 307	18
43	9.66 755	9.72 048	0.27 952	9.94 707	9.68 167	9.73 867	0.26 133	9.94 300	17
44	9.66 779	9.72 078	0.27 922	9.94 700	9.68 190	9.73 897	0.26 103	9.94 293	16
45	9.66 803	9.72 109	0.27 891	9.94 694	9.68 213	9.73 927	0.26 073	9.94 286	15
46	9.66 827	9.72 140	0.27 860	9.94 687	9.68 237	9.73 957	0.26 043	9.94 279	14
47	9.66 851	9.72 170	0.27 830	9.94 680	9.68 260	9.73 987	0.26 013	9.94 273	13
48	9.66 875	9.72 201	0.27 799	9.94 674	9.68 283	9.74 017	0.25 983	9.94 266	12
49	9.66 899	9.72 231	0.27 769	9.94 667	9.68 305	9.74 047	0.25 953	9.94 259	11
50	9.66 922	9.72 262	0.27 738	9.94 660	9.68 328	9.74 077	0.25 923	9.94 252	10
51	9.66 946	9.72 293	0.27 707	9.94 654	9.68 351	9.74 107	0.25 893	9.94 245	9
52	9.66 970	9.72 323	0.27 677	9.94 647	9.68 374	9.74 137	0.25 863	9.94 238	8
53	9.66 994	9.72 354	0.27 646	9.94 640	9.68 397	9.74 166	0.25 834	9.94 231	7
54	9.67 018	9.72 384	0.27 616	9.94 634	9.68 420	9.74 196	0.25 804	9.94 224	6
55	9.67 042	9.72 415	0.27 585	9.94 627	9.68 443	9.74 226	0.25 774	9.94 217	5
56	9.67 066	9.72 445	0.27 555	9.94 620	9.68 466	9.74 256	0.25 744	9.94 210	4
57	9.67 090	9.72 476	0.27 524	9.94 614	9.68 489	9.74 286	0.25 714	9.94 203	3
58	9.67 113	9.72 506	0.27 494	9.94 607	9.68 512	9.74 316	0.25 684	9.94 196	2
59	9.67 137	9.72 537	0.27 463	9.94 600	9.68 534	9.74 345	0.25 655	9.94 189	1
60	9.67 161	9.72 567	0.27 433	9.94 593	9.68 557	9.74 375	0.25 625	9.94 182	0
	62° log cos	log cot	log tan	log sin	61° log cos	log cot	log tan	log sin	′

′	29° log sin	29° log tan	29° log cot	29° log cos	30° log sin	30° log tan	30° log cot	30° log cos	
0	9.68 557	9.74 375	0.25 625	9.94 182	9.69 897	9.76 144	0.23 856	9.93 753	60
1	9.68 580	9.74 405	0.25 595	9.94 175	9.69 919	9.76 173	0.23 827	9.93 746	59
2	9.68 603	9.74 435	0.25 565	9.94 168	9.69 941	9.76 202	0.23 798	9.93 738	58
3	9.68 625	9.74 465	0.25 535	9.94 161	9.69 963	9.76 231	0.23 769	9.93 731	57
4	9.68 648	9.74 494	0.25 506	9.94 154	9.69 984	9.76 261	0.23 739	9.93 724	56
5	9.68 671	9.74 524	0.25 476	9.94 147	9.70 006	9.76 290	0.23 710	9.93 717	55
6	9.68 694	9.74 554	0.25 446	9.94 140	9.70 028	9.76 319	0.23 681	9.93 709	54
7	9.68 716	9.74 583	0.25 417	9.94 133	9.70 050	9.76 348	0.23 652	9.93 702	53
8	9.68 739	9.74 613	0.25 387	9.94 126	9.70 072	9.76 377	0.23 623	9.93 695	52
9	9.68 762	9.74 643	0.25 357	9.94 119	9.70 093	9.76 406	0.23 594	9.93 687	51
10	9.68 784	9.74 673	0.25 327	9.94 112	9.70 115	9.76 435	0.23 565	9.93 680	50
11	9.68 807	9.74 702	0.25 298	9.94 105	9.70 137	9.76 464	0.23 536	9.93 673	49
12	9.68 829	9.74 732	0.25 268	9.94 098	9.70 159	9.76 493	0.23 507	9.93 665	48
13	9.68 852	9.74 762	0.25 238	9.94 090	9.70 180	9.76 522	0.23 478	9.93 658	47
14	9.68 875	9.74 791	0.25 209	9.94 083	9.70 202	9.76 551	0.23 449	9.93 650	46
15	9.68 897	9.74 821	0.25 179	9.94 076	9.70 224	9.76 580	0.23 420	9.93 643	45
16	9.68 920	9.74 851	0.25 149	9.94 069	9.70 245	9.76 609	0.23 391	9.93 636	44
17	9.68 942	9.74 880	0.25 120	9.94 062	9.70 267	9.76 639	0.23 361	9.93 628	43
18	9.68 965	9.74 910	0.25 090	9.94 055	9.70 288	9.76 668	0.23 332	9.93 621	42
19	9.68 987	9.74 939	0.25 061	9.94 048	9.70 310	9.76 697	0.23 303	9.93 614	41
20	9.69 010	9.74 969	0.25 031	9.94 041	9.70 332	9.76 725	0.23 275	9.93 606	40
21	9.69 032	9.74 998	0.25 002	9.94 034	9.70 353	9.76 754	0.23 246	9.93 599	39
22	9.69 055	9.75 028	0.24 972	9.94 027	9.70 375	9.76 783	0.23 217	9.93 591	38
23	9.69 077	9.75 058	0.24 942	9.94 020	9.70 396	9.76 812	0.23 188	9.93 584	37
24	9.69 100	9.75 087	0.24 913	9.94 012	9.70 418	9.76 841	0.23 159	9.93 577	36
25	9.69 122	9.75 117	0.24 883	9.94 005	9.70 439	9.76 870	0.23 130	9.93 569	35
26	9.69 144	9.75 146	0.24 854	9.93 998	9.70 461	9.76 899	0.23 101	9.93 562	34
27	9.69 167	9.75 176	0.24 824	9.93 991	9.70 482	9.76 928	0.23 072	9.93 554	33
28	9.69 189	9.75 205	0.24 795	9.93 984	9.70 504	9.76 957	0.23 043	9.93 547	32
29	9.69 212	9.75 235	0.24 765	9.93 977	9.70 525	9.76 986	0.23 014	9.93 539	31
30	9.69 234	9.75 264	0.24 736	9.93 970	9.70 547	9.77 015	0.22 985	9.93 532	30
31	9.69 256	9.75 294	0.24 706	9.93 963	9.70 568	9.77 044	0.22 956	9.93 525	29
32	9.69 279	9.75 323	0.24 677	9.93 955	9.70 590	9.77 073	0.22 927	9.93 517	28
33	9.69 301	9.75 353	0.24 647	9.93 948	9.70 611	9.77 101	0.22 899	9.93 510	27
34	9.69 323	9.75 382	0.24 618	9.93 941	9.70 633	9.77 130	0.22 870	9.93 502	26
35	9.69 345	9.75 411	0.24 589	9.93 934	9.70 654	9.77 159	0.22 841	9.93 495	25
36	9.69 368	9.75 441	0.24 559	9.93 927	9.70 675	9.77 188	0.22 812	9.93 487	24
37	9.69 390	9.75 470	0.24 530	9.93 920	9.70 697	9.77 217	0.22 783	9.93 480	23
38	9.69 412	9.75 500	0.24 500	9.93 912	9.70 718	9.77 246	0.22 754	9.93 472	22
39	9.69 434	9.75 529	0.24 471	9.93 905	9.70 739	9.77 274	0.22 726	9.93 465	21
40	9.69 456	9.75 558	0.24 442	9.93 898	9.70 761	9.77 303	0.22 697	9.93 457	20
41	9.69 479	9.75 588	0.24 412	9.93 891	9.70 782	9.77 332	0.22 668	9.93 450	19
42	9.69 501	9.75 617	0.24 383	9.93 884	9.70 803	9.77 361	0.22 639	9.93 442	18
43	9.69 523	9.75 647	0.24 353	9.93 876	9.70 824	9.77 390	0.22 610	9.93 435	17
44	9.69 545	9.75 676	0.24 324	9.93 869	9.70 846	9.77 418	0.22 582	9.93 427	16
45	9.69 567	9.75 705	0.24 295	9.93 862	9.70 867	9.77 447	0.22 553	9.93 420	15
46	9.69 589	9.75 735	0.24 265	9.93 855	9.70 888	9.77 476	0.22 524	9.93 412	14
47	9.69 611	9.75 764	0.24 236	9.93 847	9.70 909	9.77 505	0.22 495	9.93 405	13
48	9.69 633	9.75 793	0.24 207	9.93 840	9.70 931	9.77 533	0.22 467	9.93 397	12
49	9.69 655	9.75 822	0.24 178	9.93 833	9.70 952	9.77 562	0.22 438	9.93 390	11
50	9.69 677	9.75 852	0.24 148	9.93 826	9.70 973	9.77 591	0.22 409	9.93 382	10
51	9.69 699	9.75 881	0.24 119	9.93 819	9.70 994	9.77 619	0.22 381	9.93 375	9
52	9.69 721	9.75 910	0.24 090	9.93 811	9.71 015	9.77 648	0.22 352	9.93 367	8
53	9.69 743	9.75 939	0.24 061	9.93 804	9.71 036	9.77 677	0.22 323	9.93 360	7
54	9.69 765	9.75 969	0.24 031	9.93 797	9.71 058	9.77 706	0.22 294	9.93 352	6
55	9.69 787	9.75 998	0.24 002	9.93 789	9.71 079	9.77 734	0.22 266	9.93 344	5
56	9.69 809	9.76 027	0.23 973	9.93 782	9.71 100	9.77 763	0.22 237	9.93 337	4
57	9.69 831	9.76 056	0.23 944	9.93 775	9.71 121	9.77 791	0.22 209	9.93 329	3
58	9.69 853	9.76 086	0.23 914	9.93 768	9.71 142	9.77 820	0.22 180	9.93 322	2
59	9.69 875	9.76 115	0.23 885	9.93 760	9.71 163	9.77 849	0.22 151	9.93 314	1
60	9.69 897	9.76 144	0.23 856	9.93 753	9.71 184	9.77 877	0.22 123	9.93 307	0
	60° log cos	60° log cot	60° log tan	60° log sin	59° log cos	59° log cot	59° log tan	59° log sin	′

	31°				32°				
′	log sin	log tan	log cot	log cos	log sin	log tan	log cot	log cos	
0	9.71 184	9.77 877	0.22 123	9.93 307	9.72 421	9.79 579	0.20 421	9.92 842	60
1	9.71 205	9.77 906	0.22 094	9.93 299	9.72 441	9.79 607	0.20 393	9.92 834	59
2	9.71 226	9.77 935	0.22 065	9.93 291	9.72 461	9.79 635	0.20 365	9.92 826	58
3	9.71 247	9.77 963	0.22 037	9.93 284	9.72 482	9.79 663	0.20 337	9.92 818	57
4	9.71 268	9.77 992	0.22 008	9.93 276	9.72 502	9.79 691	0.20 309	9.92 810	56
5	9.71 289	9.78 020	0.21 980	9.93 269	9.72 522	9.79 719	0.20 281	9.92 803	55
6	9.71 310	9.78 049	0.21 951	9.93 261	9.72 542	9.79 747	0.20 253	9.92 795	54
7	9.71 331	9.78 077	0.21 923	9.93 253	9.72 562	9.79 776	0.20 224	9.92 787	53
8	9.71 352	9.78 106	0.21 894	9.93 246	9.72 582	9.79 804	0.20 196	9.92 779	52
9	9.71 373	9.78 135	0.21 865	9.93 238	9.72 602	9.79 832	0.20 168	9.92 771	51
10	9.71 393	9.78 163	0.21 837	9.93 230	9.72 622	9.79 860	0.20 140	9.92 763	50
11	9.71 414	9.78 192	0.21 808	9.93 223	9.72 643	9.79 888	0.20 112	9.92 755	49
12	9.71 435	9.78 220	0.21 780	9.93 215	9.72 663	9.79 916	0.20 084	9.92 747	48
13	9.71 456	9.78 249	0.21 751	9.93 207	9.72 683	9.79 944	0.20 056	9.92 739	47
14	9.71 477	9.78 277	0.21 723	9.93 200	9.72 703	9.79 972	0.20 028	9.92 731	46
15	9.71 498	9.78 306	0.21 694	9.93 192	9.72 723	9.80 000	0.20 000	9.92 723	45
16	9.71 519	9.78 334	0.21 666	9.93 184	9.72 743	9.80 028	0.19 972	9.92 715	44
17	9.71 539	9.78 363	0.21 637	9.93 177	9.72 763	9.80 056	0.19 944	9.92 707	43
18	9.71 560	9.78 391	0.21 609	9.93 169	9.72 783	9.80 084	0.19 916	9.92 699	42
19	9.71 581	9.78 419	0.21 581	9.93 161	9.72 803	9.80 112	0.19 888	9.92 691	41
20	9.71 602	9.78 448	0.21 552	9.93 154	9.72 823	9.80 140	0.19 860	9.92 683	40
21	9.71 622	9.78 476	0.21 524	9.93 146	9.72 843	9.80 168	0.19 832	9.92 675	39
22	9.71 643	9.78 505	0.21 495	9.93 138	9.72 863	9.80 195	0.19 805	9.92 667	38
23	9.71 664	9.78 533	0.21 467	9.93 131	9.72 883	9.80 223	0.19 777	9.92 659	37
24	9.71 685	9.78 562	0.21 438	9.93 123	9.72 902	9.80 251	0.19 749	9.92 651	36
25	9.71 705	9.78 590	0.21 410	9.93 115	9.72 922	9.80 279	0.19 721	9.92 643	35
26	9.71 726	9.78 618	0.21 382	9.93 108	9.72 942	9.80 307	0.19 693	9.92 635	34
27	9.71 747	9.78 647	0.21 353	9.93 100	9.72 962	9.80 335	0.19 665	9.92 627	33
28	9.71 767	9.78 675	0.21 325	9.93 092	9.72 982	9.80 363	0.19 637	9.92 619	32
29	9.71 788	9.78 704	0.21 296	9.93 084	9.73 002	9.80 391	0.19 609	9.92 611	31
30	9.71 809	9.78 732	0.21 268	9.93 077	9.73 022	9.80 419	0.19 581	9.92 603	30
31	9.71 829	9.78 760	0.21 240	9.93 069	9.73 041	9.80 447	0.19 553	9.92 595	29
32	9.71 850	9.78 789	0.21 211	9.93 061	9.73 061	9.80 474	0.19 526	9.92 587	28
33	9.71 870	9.78 817	0.21 183	9.93 053	9.73 081	9.80 502	0.19 498	9.92 579	27
34	9.71 891	9.78 845	0.21 155	9.93 046	9.73 101	9.80 530	0.19 470	9.92 571	26
35	9.71 911	9.78 874	0.21 126	9.93 038	9.73 121	9.80 558	0.19 442	9.92 563	25
36	9.71 932	9.78 902	0.21 098	9.93 030	9.73 140	9.80 586	0.19 414	9.92 555	24
37	9.71 952	9.78 930	0.21 070	9.93 022	9.73 160	9.80 614	0.19 386	9.92 546	23
38	9.71 973	9.78 959	0.21 041	9.93 014	9.73 180	9.80 642	0.19 358	9.92 538	22
39	9.71 994	9.78 987	0.21 013	9.93 007	9.73 200	9.80 669	0.19 331	9.92 530	21
40	9.72 014	9.79 015	0.20 985	9.92 999	9.73 219	9.80 697	0.19 303	9.92 522	20
41	9.72 034	9.79 043	0.20 957	9.92 991	9.73 239	9.80 725	0.19 275	9.92 514	19
42	9.72 055	9.79 072	0.20 928	9.92 983	9.73 259	9.80 753	0.19 247	9.92 506	18
43	9.72 075	9.79 100	0.20 900	9.92 976	9.73 278	9.80 781	0.19 219	9.92 498	17
44	9.72 096	9.79 128	0.20 872	9.92 968	9.73 298	9.80 808	0.19 192	9.92 490	16
45	9.72 116	9.79 156	0.20 844	9.92 960	9.73 318	9.80 836	0.19 164	9.92 482	15
46	9.72 137	9.79 185	0.20 815	9.92 952	9.73 337	9.80 864	0.19 136	9.92 473	14
47	9.72 157	9.79 213	0.20 787	9.92 944	9.73 357	9.80 892	0.19 108	9.92 465	13
48	9.72 177	9.79 241	0.20 759	9.92 936	9.73 377	9.80 919	0.19 081	9.92 457	12
49	9.72 198	9.79 269	0.20 731	9.92 929	9.73 396	9.80 947	0.19 053	9.92 449	11
50	9.72 218	9.79 297	0.20 703	9.92 921	9.73 416	9.80 975	0.19 025	9.92 441	10
51	9.72 238	9.79 326	0.20 674	9.92 913	9.73 435	9.81 003	0.18 997	9.92 433	9
52	9.72 259	9.79 354	0.20 646	9.92 905	9.73 455	9.81 030	0.18 970	9.92 425	8
53	9.72 279	9.79 382	0.20 618	9.92 897	9.73 474	9.81 058	0.18 942	9.92 417	7
54	9.72 299	9.79 410	0.20 590	9.92 889	9.73 494	9.81 086	0.18 914	9.92 408	6
55	9.72 320	9.79 438	0.20 562	9.92 881	9.73 513	9.81 113	0.18 887	9.92 400	5
56	9.72 340	9.79 466	0.20 534	9.92 874	9.73 533	9.81 141	0.18 859	9.92 392	4
57	9.72 360	9.79 495	0.20 505	9.92 866	9.73 552	9.81 169	0.18 831	9.92 384	3
58	9.72 381	9.79 523	0.20 477	9.92 858	9.73 572	9.81 196	0.18 804	9.92 376	2
59	9.72 401	9.79 551	0.20 449	9.92 850	9.73 591	9.81 224	0.18 776	9.92 367	1
60	9.72 421	9.79 579	0.20 421	9.92 842	9.73 611	9.81 252	0.18 748	9.92 359	0
	log cos	log cot	log tan	log sin	log cos	log cot	log tan	log sin	′
	58°				57°				

	33°				34°				
′	log sin	log tan	log cot	log cos	log sin	log tan	log cot	log cos	
0	9.73 611	9.81 252	0.18 748	9.92 359	9.74 756	9.82 899	0.17 101	9.91 857	60
1	9.73 630	9.81 279	0.18 721	9.92 351	9.74 775	9.82 926	0.17 074	9.91 849	59
2	9.73 650	9.81 307	0.18 693	9.92 343	9.74 794	9.82 953	0.17 047	9.91 840	58
3	9.73 669	9.81 335	0.18 665	9.92 335	9.74 812	9.82 980	0.17 020	9.91 832	57
4	9.73 689	9.81 362	0.18 638	9.92 326	9.74 831	9.83 008	0.16 992	9.91 823	56
5	9.73 708	9.81 390	0.18 610	9.92 318	9.74 850	9.83 035	0.16 965	9.91 815	55
6	9.73 727	9.81 418	0.18 582	9.92 310	9.74 868	9.83 062	0.16 938	9.91 806	54
7	9.73 747	9.81 445	0.18 555	9.92 302	9.74 887	9.83 089	0.16 911	9.91 798	53
8	9.73 766	9.81 473	0.18 527	9.92 293	9.74 906	9.83 117	0.16 883	9.91 789	52
9	9.73 785	9.81 500	0.18 500	9.92 285	9.74 924	9.83 144	0.16 856	9.91 781	51
10	9.73 805	9.81 528	0.18 472	9.92 277	9.74 943	9.83 171	0.16 829	9.91 772	50
11	9.73 824	9.81 556	0.18 444	9.92 269	9.74 961	9.83 198	0.16 802	9.91 763	49
12	9.73 843	9.81 583	0.18 417	9.92 260	9.74 980	9.83 225	0.16 775	9.91 755	48
13	9.73 863	9.81 611	0.18 389	9.92 252	9.74 999	9.83 252	0.16 748	9.91 746	47
14	9.73 882	9.81 638	0.18 362	9.92 244	9.75 017	9.83 280	0.16 720	9.91 738	46
15	9.73 901	9.81 666	0.18 334	9.92 235	9.75 036	9.83 307	0.16 693	9.91 729	45
16	9.73 921	9.81 693	0.18 307	9.92 227	9.75 054	9.83 334	0.16 666	9.91 720	44
17	9.73 940	9.81 721	0.18 279	9.92 219	9.75 073	9.83 361	0.16 639	9.91 712	43
18	9.73 959	9.81 748	0.18 252	9.92 211	9.75 091	9.83 388	0.16 612	9.91 703	42
19	9.73 978	9.81 776	0.18 224	9.92 202	9.75 110	9.83 415	0.16 585	9.91 695	41
20	9.73 997	9.81 803	0.18 197	9.92 194	9.75 128	9.83 442	0.16 558	9.91 686	40
21	9.74 017	9.81 831	0.18 169	9.92 186	9.75 147	9.83 470	0.16 530	9.91 677	39
22	9.74 036	9.81 858	0.18 142	9.92 177	9.75 165	9.83 497	0.16 503	9.91 669	38
23	9.74 055	9.81 886	0.18 114	9.92 169	9.75 184	9.83 524	0.16 476	9.91 660	37
24	9.74 074	9.81 913	0.18 087	9.92 161	9.75 202	9.83 551	0.16 449	9.91 651	36
25	9.74 093	9.81 941	0.18 059	9.92 152	9.75 221	9.83 578	0.16 422	9.91 643	35
26	9.74 113	9.81 968	0.18 032	9.92 144	9.75 239	9.83 605	0.16 395	9.91 634	34
27	9.74 132	9.81 996	0.18 004	9.92 136	9.75 258	9.83 632	0.16 368	9.91 625	33
28	9.74 151	9.82 023	0.17 977	9.92 127	9.75 276	9.83 659	0.16 341	9.91 617	32
29	9.74 170	9.82 051	0.17 949	9.92 119	9.75 294	9.83 686	0.16 314	9.91 608	31
30	9.74 189	9.82 078	0.17 922	9.92 111	9.75 313	9.83 713	0.16 287	9.91 599	30
31	9.74 208	9.82 106	0.17 894	9.92 102	9.75 331	9.83 740	0.16 260	9.91 591	29
32	9.74 227	9.82 133	0.17 867	9.92 094	9.75 350	9.83 768	0.16 232	9.91 582	28
33	9.74 246	9.82 161	0.17 839	9.92 086	9.75 368	9.83 795	0.16 205	9.91 573	27
34	9.74 265	9.82 188	0.17 812	9.92 077	9.75 386	9.83 822	0.16 178	9.91 565	26
35	9.74 284	9.82 215	0.17 785	9.92 069	9.75 405	9.83 849	0.16 151	9.91 556	25
36	9.74 303	9.82 243	0.17 757	9.92 060	9.75 423	9.83 876	0.16 124	9.91 547	24
37	9.74 322	9.82 270	0.17 730	9.92 052	9.75 441	9.83 903	0.16 097	9.91 538	23
38	9.74 341	9.82 298	0.17 702	9.92 044	9.75 459	9.83 930	0.16 070	9.91 530	22
39	9.74 360	9.82 325	0.17 675	9.92 035	9.75 478	9.83 957	0.16 043	9.91 521	21
40	9.74 379	9.82 352	0.17 648	9.92 027	9.75 496	9.83 984	0.16 016	9.91 512	20
41	9.74 398	9.82 380	0.17 620	9.92 018	9.75 514	9.84 011	0.15 989	9.91 504	19
42	9.74 417	9.82 407	0.17 593	9.92 010	9.75 533	9.84 038	0.15 962	9.91 495	18
43	9.74 436	9.82 435	0.17 565	9.92 002	9.75 551	9.84 065	0.15 935	9.91 486	17
44	9.74 455	9.82 462	0.17 538	9.91 993	9.75 569	9.84 092	0.15 908	9.91 477	16
45	9.74 474	9.82 489	0.17 511	9.91 985	9.75 587	9.84 119	0.15 881	9.91 469	15
46	9.74 493	9.82 517	0.17 483	9.91 976	9.75 605	9.84 146	0.15 854	9.91 460	14
47	9.74 512	9.82 544	0.17 456	9.91 968	9.75 624	9.84 173	0.15 827	9.91 451	13
48	9.74 531	9.82 571	0.17 429	9.91 959	9.75 642	9.84 200	0.15 800	9.91 442	12
49	9.74 549	9.82 599	0.17 401	9.91 951	9.75 660	9.84 227	0.15 773	9.91 433	11
50	9.74 568	9.82 626	0.17 374	9.91 942	9.75 678	9.84 254	0.15 746	9.91 425	10
51	9.74 587	9.82 653	0.17 347	9.91 934	9.75 696	9.84 280	0.15 720	9.91 416	9
52	9.74 606	9.82 681	0.17 319	9.91 925	9.75 714	9.84 307	0.15 693	9.91 407	8
53	9.74 625	9.82 708	0.17 292	9.91 917	9.75 733	9.84 334	0.15 666	9.91 398	7
54	9.74 644	9.82 735	0.17 265	9.91 908	9.75 751	9.84 361	0.15 639	9.91 389	6
55	9.74 662	9.82 762	0.17 238	9.91 900	9.75 769	9.84 388	0.15 612	9.91 381	5
56	9.74 681	9.82 790	0.17 210	9.91 891	9.75 787	9.84 415	0.15 585	9.91 372	4
57	9.74 700	9.82 817	0.17 183	9.91 883	9.75 805	9.84 442	0.15 558	9.91 363	3
58	9.74 719	9.82 844	0.17 156	9.91 874	9.75 823	9.84 469	0.15 531	9.91 354	2
59	9.74 737	9.82 871	0.17 129	9.91 866	9.75 841	9.84 496	0.15 504	9.91 345	1
60	9.74 756	9.82 899	0.17 101	9.91 857	9.75 859	9.84 523	0.15 477	9.91 336	0
	log cos	log cot	log tan	log sin	log cos	log cot	log tan	log sin	′
	56°				55°				

′	35° log sin	35° log tan	35° log cot	35° log cos	36° log sin	36° log tan	36° log cot	36° log cos	
0	9.75 859	9.84 523	0.15 477	9.91 336	9.76 922	9.86 126	0.13 874	9.90 796	60
1	9.75 877	9.84 550	0.15 450	9.91 328	9.76 939	9.86 153	0.13 847	9.90 787	59
2	9.75 895	9.84 576	0.15 424	9.91 319	9.76 957	9.86 179	0.13 821	9.90 777	58
3	9.75 913	9.84 603	0.15 397	9.91 310	9.76 974	9.86 206	0.13 794	9.90 768	57
4	9.75 931	9.84 630	0.15 370	9.91 301	9.76 991	9.86 232	0.13 768	9.90 759	56
5	9.75 949	9.84 657	0.15 343	9.91 292	9.77 009	9.86 259	0.13 741	9.90 750	55
6	9.75 967	9.84 684	0.15 316	9.91 283	9.77 026	9.86 285	0.13 715	9.90 741	54
7	9.75 985	9.84 711	0.15 289	9.91 274	9.77 043	9.86 312	0.13 688	9.90 731	53
8	9.76 003	9.84 738	0.15 262	9.91 266	9.77 061	9.86 338	0.13 662	9.90 722	52
9	9.76 021	9.84 764	0.15 236	9.91 257	9.77 078	9.86 365	0.13 635	9.90 713	51
10	9.76 039	9.84 791	0.15 209	9.91 248	9.77 095	9.86 392	0.13 608	9.90 704	50
11	9.76 057	9.84 818	0.15 182	9.91 239	9.77 112	9.86 418	0.13 582	9.90 694	49
12	9.76 075	9.84 845	0.15 155	9.91 230	9.77 130	9.86 445	0.13 555	9.90 685	48
13	9.76 093	9.84 872	0.15 128	9.91 221	9.77 147	9.86 471	0.13 529	9.90 676	47
14	9.76 111	9.84 899	0.15 101	9.91 212	9.77 164	9.86 498	0.13 502	9.90 667	46
15	9.76 129	9.84 925	0.15 075	9.91 203	9.77 181	9.86 524	0.13 476	9.90 657	45
16	9.76 146	9.84 952	0.15 048	9.91 194	9.77 199	9.86 551	0.13 449	9.90 648	44
17	9.76 164	9.84 979	0.15 021	9.91 185	9.77 216	9.86 577	0.13 423	9.90 639	43
18	9.76 182	9.85 006	0.14 994	9.91 176	9.77 233	9.86 603	0.13 397	9.90 630	42
19	9.76 200	9.85 033	0.14 967	9.91 167	9.77 250	9.86 630	0.13 370	9.90 620	41
20	9.76 218	9.85 059	0.14 941	9.91 158	9.77 268	9.86 656	0.13 344	9.90 611	40
21	9.76 236	9.85 086	0.14 914	9.91 149	9.77 285	9.86 683	0.13 317	9.90 602	39
22	9.76 253	9.85 113	0.14 887	9.91 141	9.77 302	9.86 709	0.13 291	9.90 592	38
23	9.76 271	9.85 140	0.14 860	9.91 132	9.77 319	9.86 736	0.13 264	9.90 583	37
24	9.76 289	9.85 166	0.14 834	9.91 123	9.77 336	9.86 762	0.13 238	9.90 574	36
25	9.76 307	9.85 193	0.14 807	9.91 114	9.77 353	9.86 789	0.13 211	9.90 565	35
26	9.76 324	9.85 220	0.14 780	9.91 105	9.77 370	9.86 815	0.13 185	9.90 555	34
27	9.76 342	9.85 247	0.14 753	9.91 096	9.77 387	9.86 842	0.13 158	9.90 546	33
28	9.76 360	9.85 273	0.14 727	9.91 087	9.77 405	9.86 868	0.13 132	9.90 537	32
29	9.76 378	9.85 300	0.14 700	9.91 078	9.77 422	9.86 894	0.13 106	9.90 527	31
30	9.76 395	9.85 327	0.14 673	9.91 069	9.77 439	9.86 921	0.13 079	9.90 518	30
31	9.76 413	9.85 354	0.14 646	9.91 060	9.77 456	9.86 947	0.13 053	9.90 509	29
32	9.76 431	9.85 380	0.14 620	9.91 051	9.77 473	9.86 974	0.13 026	9.90 499	28
33	9.76 448	9.85 407	0.14 593	9.91 042	9.77 490	9.87 000	0.13 000	9.90 490	27
34	9.76 466	9.85 434	0.14 566	9.91 033	9.77 507	9.87 027	0.12 973	9.90 480	26
35	9.76 484	9.85 460	0.14 540	9.91 023	9.77 524	9.87 053	0.12 947	9.90 471	25
36	9.76 501	9.85 487	0.14 513	9.91 014	9.77 541	9.87 079	0.12 921	9.90 462	24
37	9.76 519	9.85 514	0.14 486	9.91 005	9.77 558	9.87 106	0.12 894	9.90 452	23
38	9.76 537	9.85 540	0.14 460	9.90 996	9.77 575	9.87 132	0.12 868	9.90 443	22
39	9.76 554	9.85 567	0.14 433	9.90 987	9.77 592	9.87 158	0.12 842	9.90 434	21
40	9.76 572	9.85 594	0.14 406	9.90 978	9.77 609	9.87 185	0.12 815	9.90 424	20
41	9.76 590	9.85 620	0.14 380	9.90 969	9.77 626	9.87 211	0.12 789	9.90 415	19
42	9.76 607	9.85 647	0.14 353	9.90 960	9.77 643	9.87 238	0.12 762	9.90 405	18
43	9.76 625	9.85 674	0.14 326	9.90 951	9.77 660	9.87 264	0.12 736	9.90 396	17
44	9.76 642	9.85 700	0.14 300	9.90 942	9.77 677	9.87 290	0.12 710	9.90 386	16
45	9.76 660	9.85 727	0.14 273	9.90 933	9.77 694	9.87 317	0.12 683	9.90 377	15
46	9.76 677	9.85 754	0.14 246	9.90 924	9.77 711	9.87 343	0.12 657	9.90 368	14
47	9.76 695	9.85 780	0.14 220	9.90 915	9.77 728	9.87 369	0.12 631	9.90 358	13
48	9.76 712	9.85 807	0.14 193	9.90 906	9.77 744	9.87 396	0.12 604	9.90 349	12
49	9.76 730	9.85 834	0.14 166	9.90 896	9.77 761	9.87 422	0.12 578	9.90 339	11
50	9.76 747	9.85 860	0.14 140	9.90 887	9.77 778	9.87 448	0.12 552	9.90 330	10
51	9.76 765	9.85 887	0.14 113	9.90 878	9.77 795	9.87 475	0.12 525	9.90 320	9
52	9.76 782	9.85 913	0.14 087	9.90 869	9.77 812	9.87 501	0.12 499	9.90 311	8
53	9.76 800	9.85 940	0.14 060	9.90 860	9.77 829	9.87 527	0.12 473	9.90 301	7
54	9.76 817	9.85 967	0.14 033	9.90 851	9.77 846	9.87 554	0.12 446	9.90 292	6
55	9.76 835	9.85 993	0.14 007	9.90 842	9.77 862	9.87 580	0.12 420	9.90 282	5
56	9.76 852	9.86 020	0.13 980	9.90 832	9.77 879	9.87 606	0.12 394	9.90 273	4
57	9.76 870	9.86 046	0.13 954	9.90 823	9.77 896	9.87 633	0.12 367	9.90 263	3
58	9.76 887	9.86 073	0.13 927	9.90 814	9.77 913	9.87 659	0.12 341	9.90 254	2
59	9.76 904	9.86 100	0.13 900	9.90 805	9.77 930	9.87 685	0.12 315	9.90 244	1
60	9.76 922	9.86 126	0.13 874	9.90 796	9.77 946	9.87 711	0.12 289	9.90 235	0
	54° log cos	54° log cot	54° log tan	54° log sin	53° log cos	53° log cot	53° log tan	53° log sin	′

′	37° log sin	log tan	log cot	log cos	38° log sin	log tan	log cot	log cos	
0	9.77 946	9.87 711	0.12 289	9.90 235	9.78 934	9.89 281	0.10 719	9.89 653	60
1	9.77 963	9.87 738	0.12 262	9.90 225	9.78 950	9.89 307	0.10 693	9.89 643	59
2	9.77 980	9.87 764	0.12 236	9.90 216	9.78 967	9.89 333	0.10 667	9.89 633	58
3	9.77 997	9.87 790	0.12 210	9.90 206	9.78 983	9.89 359	0.10 641	9.89 624	57
4	9.78 013	9.87 817	0.12 183	9.90 197	9.78 999	9.89 385	0.10 615	9.89 614	56
5	9.78 030	9.87 843	0.12 157	9.90 187	9.79 015	9.89 411	0.10 589	9.89 604	55
6	9.78 047	9.87 869	0.12 131	9.90 178	9.79 031	9.89 437	0.10 563	9.89 594	54
7	9.78 063	9.87 895	0.12 105	9.90 168	9.79 047	9.89 463	0.10 537	9.89 584	53
8	9.78 080	9.87 922	0.12 078	9.90 159	9.79 063	9.89 489	0.10 511	9.89 574	52
9	9.78 097	9.87 948	0.12 052	9.90 149	9.79 079	9.89 515	0.10 485	9.89 564	51
10	9.78 113	9.87 974	0.12 026	9.90 139	9.79 095	9.89 541	0.10 459	9.89 554	50
11	9.78 130	9.88 000	0.12 000	9.90 130	9.79 111	9.89 567	0.10 433	9.89 544	49
12	9.78 147	9.88 027	0.11 973	9.90 120	9.79 128	9.89 593	0.10 407	9.89 534	48
13	9.78 163	9.88 053	0.11 947	9.90 111	9.79 144	9.89 619	0.10 381	9.89 524	47
14	9.78 180	9.88 079	0.11 921	9.90 101	9.79 160	9.89 645	0.10 355	9.89 514	46
15	9.78 197	9.88 105	0.11 895	9.90 091	9.79 176	9.89 671	0.10 329	9.89 504	45
16	9.78 213	9.88 131	0.11 869	9.90 082	9.79 192	9.89 697	0.10 303	9.89 495	44
17	9.78 230	9.88 158	0.11 842	9.90 072	9.79 208	9.89 723	0.10 277	9.89 485	43
18	9.78 246	9.88 184	0.11 816	9.90 063	9.79 224	9.89 749	0.10 251	9.89 475	42
19	9.78 263	9.88 210	0.11 790	9.90 053	9.79 240	9.89 775	0.10 225	9.89 465	41
20	9.78 280	9.88 236	0.11 764	9.90 043	9.79 256	9.89 801	0.10 199	9.89 455	40
21	9.78 296	9.88 262	0.11 738	9.90 034	9.79 272	9.89 827	0.10 173	9.89 445	39
22	9.78 313	9.88 289	0.11 711	9.90 024	9.79 288	9.89 853	0.10 147	9.89 435	38
23	9.78 329	9.88 315	0.11 685	9.90 014	9.79 304	9.89 879	0.10 121	9.89 425	37
24	9.78 346	9.88 341	0.11 659	9.90 005	9.79 319	9.89 905	0.10 095	9.89 415	36
25	9.78 362	9.88 367	0.11 633	9.89 995	9.79 335	9.89 931	0.10 069	9.89 405	35
26	9.78 379	9.88 393	0.11 607	9.89 985	9.79 351	9.89 957	0.10 043	9.89 395	34
27	9.78 395	9.88 420	0.11 580	9.89 976	9.79 367	9.89 983	0.10 017	9.89 385	33
28	9.78 412	9.88 446	0.11 554	9.89 966	9.79 383	9.90 009	0.09 991	9.89 375	32
29	9.78 428	9.88 472	0.11 528	9.89 956	9.79 399	9.90 035	0.09 965	9.89 364	31
30	9.78 445	9.88 498	0.11 502	9.89 947	9.79 415	9.90 061	0.09 939	9.89 354	30
31	9.78 461	9.88 524	0.11 476	9.89 937	9.79 431	9.90 086	0.09 914	9.89 344	29
32	9.78 478	9.88 550	0.11 450	9.89 927	9.79 447	9.90 112	0.09 888	9.89 334	28
33	9.78 494	9.88 577	0.11 423	9.89 918	9.79 463	9.90 138	0.09 862	9.89 324	27
34	9.78 510	9.88 603	0.11 397	9.89 908	9.79 478	9.90 164	0.09 836	9.89 314	26
35	9.78 527	9.88 629	0.11 371	9.89 898	9.79 494	9.90 190	0.09 810	9.89 304	25
36	9.78 543	9.88 655	0.11 345	9.89 888	9.79 510	9.90 216	0.09 784	9.89 294	24
37	9.78 560	9.88 681	0.11 319	9.89 879	9.79 526	9.90 242	0.09 758	9.89 284	23
38	9.78 576	9.88 707	0.11 293	9.89 869	9.79 542	9.90 268	0.09 732	9.89 274	22
39	9.78 592	9.88 733	0.11 267	9.89 859	9.79 558	9.90 294	0.09 706	9.89 264	21
40	9.78 609	9.88 759	0.11 241	9.89 849	9.79 573	9.90 320	0.09 680	9.89 254	20
41	9.78 625	9.88 786	0.11 214	9.89 840	9.79 589	9.90 346	0.09 654	9.89 244	19
42	9.78 642	9.88 812	0.11 188	9.89 830	9.79 605	9.90 371	0.09 629	9.89 233	18
43	9.78 658	9.88 838	0.11 162	9.89 820	9.79 621	9.90 397	0.09 603	9.89 223	17
44	9.78 674	9.88 864	0.11 136	9.89 810	9.79 636	9.90 423	0.09 577	9.89 213	16
45	9.78 691	9.88 890	0.11 110	9.89 801	9.79 652	9.90 449	0.09 551	9.89 203	15
46	9.78 707	9.88 916	0.11 084	9.89 791	9.79 668	9.90 475	0.09 525	9.89 193	14
47	9.78 723	9.88 942	0.11 058	9.89 781	9.79 684	9.90 501	0.09 499	9.89 183	13
48	9.78 739	9.88 968	0.11 032	9.89 771	9.79 699	9.90 527	0.09 473	9.89 173	12
49	9.78 756	9.88 994	0.11 006	9.89 761	9.79 715	9.90 553	0.09 447	9.89 162	11
50	9.78 772	9.89 020	0.10 980	9.89 752	9.79 731	9.90 578	0.09 422	9.89 152	10
51	9.78 788	9.89 046	0.10 954	9.89 742	9.79 746	9.90 604	0.09 396	9.89 142	9
52	9.78 805	9.89 073	0.10 927	9.89 732	9.79 762	9.90 630	0.09 370	9.89 132	8
53	9.78 821	9.89 099	0.10 901	9.89 722	9.79 778	9.90 656	0.09 344	9.89 122	7
54	9.78 837	9.89 125	0.10 875	9.89 712	9.79 793	9.90 682	0.09 318	9.89 112	6
55	9.78 853	9.89 151	0.10 849	9.89 702	9.79 809	9.90 708	0.09 292	9.89 101	5
56	9.78 869	9.89 177	0.10 823	9.89 693	9.79 825	9.90 734	0.09 266	9.89 091	4
57	9.78 886	9.89 203	0.10 797	9.89 683	9.79 840	9.90 759	0.09 241	9.89 081	3
58	9.78 902	9.89 229	0.10 771	9.89 673	9.79 856	9.90 785	0.09 215	9.89 071	2
59	9.78 918	9.89 255	0.10 745	9.89 663	9.79 872	9.90 811	0.09 189	9.89 060	1
60	9.78 934	9.89 281	0.10 719	9.89 653	9.79 887	9.90 837	0.09 163	9.89 050	0
	log cos	log cot	log tan	log sin 52°	log cos	log cot	log tan	log sin 51°	′

′	39° log sin	log tan	log cot	log cos	40° log sin	log tan	log cot	log cos	
0	9.79 887	9.90 837	0.09 163	9.89 050	9.80 807	9.92 381	0.07 619	9.88 425	60
1	9.79 903	9.90 863	0.09 137	9.89 040	9.80 822	9.92 407	0.07 593	9.88 415	59
2	9.79 918	9.90 889	0.09 111	9.89 030	9.80 837	9.92 433	0.07 567	9.88 404	58
3	9.79 934	9.90 914	0.09 086	9.89 020	9.80 852	9.92 458	0.07 542	9.88 394	57
4	9.79 950	9.90 940	0.09 060	9.89 009	9.80 867	9.92 484	0.07 516	9.88 383	56
5	9.79 965	9.90 966	0.09 034	9.88 999	9.80 882	9.92 510	0.07 490	9.88 372	55
6	9.79 981	9.90 992	0.09 008	9.88 989	9.80 897	9.92 535	0.07 465	9.88 362	54
7	9.79 996	9.91 018	0.08 982	9.88 978	9.80 912	9.92 561	0.07 439	9.88 351	53
8	9.80 012	9.91 043	0.08 957	9.88 968	9.80 927	9.92 587	0.07 413	9.88 340	52
9	9.80 027	9.91 069	0.08 931	9.88 958	9.80 942	9.92 612	0.07 388	9.88 330	51
10	9.80 043	9.91 095	0.08 905	9.88 948	9.80 957	9.92 638	0.07 362	9.88 319	50
11	9.80 058	9.91 121	0.08 879	9.88 937	9.80 972	9.92 663	0.07 337	9.88 308	49
12	9.80 074	9.91 147	0.08 853	9.88 927	9.80 987	9.92 689	0.07 311	9.88 298	48
13	9.80 089	9.91 172	0.08 828	9.88 917	9.81 002	9.92 715	0.07 285	9.88 287	47
14	9.80 105	9.91 198	0.08 802	9.88 906	9.81 017	9.92 740	0.07 260	9.88 276	46
15	9.80 120	9.91 224	0.08 776	9.88 896	9.81 032	9.92 766	0.07 234	9.88 266	45
16	9.80 136	9.91 250	0.08 750	9.88 886	9.81 047	9.92 792	0.07 208	9.88 255	44
17	9.80 151	9.91 276	0.08 724	9.88 875	9.81 061	9.92 817	0.07 183	9.88 244	43
18	9.80 166	9.91 301	0.08 699	9.88 865	9.81 076	9.92 843	0.07 157	9.88 234	42
19	9.80 182	9.91 327	0.08 673	9.88 855	9.81 091	9.92 868	0.07 132	9.88 223	41
20	9.80 197	9.91 353	0.08 647	9.88 844	9.81 106	9.92 894	0.07 106	9.88 212	40
21	9.80 213	9.91 379	0.08 621	9.88 834	9.81 121	9.92 920	0.07 080	9.88 201	39
22	9.80 228	9.91 404	0.08 596	9.88 824	9.81 136	9.92 945	0.07 055	9.88 191	38
23	9.80 244	9.91 430	0.08 570	9.88 813	9.81 151	9.92 971	0.07 029	9.88 180	37
24	9.80 259	9.91 456	0.08 544	9.88 803	9.81 166	9.92 996	0.07 004	9.88 169	36
25	9.80 274	9.91 482	0.08 518	9.88 793	9.81 180	9.93 022	0.06 978	9.88 158	35
26	9.80 290	9.91 507	0.08 493	9.88 782	9.81 195	9.93 048	0.06 952	9.88 148	34
27	9.80 305	9.91 533	0.08 467	9.88 772	9.81 210	9.93 073	0.06 927	9.88 137	33
28	9.80 320	9.91 559	0.08 441	9.88 761	9.81 225	9.93 099	0.06 901	9.88 126	32
29	9.80 336	9.91 585	0.08 415	9.88 751	9.81 240	9.93 124	0.06 876	9.88 115	31
30	9.80 351	9.91 610	0.08 390	9.88 741	9.81 254	9.93 150	0.06 850	9.88 105	30
31	9.80 366	9.91 636	0.08 364	9.88 730	9.81 269	9.93 175	0.06 825	9.88 094	29
32	9.80 382	9.91 662	0.08 338	9.88 720	9.81 284	9.93 201	0.06 799	9.88 083	28
33	9.80 397	9.91 688	0.08 312	9.88 709	9.81 299	9.93 227	0.06 773	9.88 072	27
34	9.80 412	9.91 713	0.08 287	9.88 699	9.81 314	9.93 252	0.06 748	9.88 061	26
35	9.80 428	9.91 739	0.08 261	9.88 688	9.81 328	9.93 278	0.06 722	9.88 051	25
36	9.80 443	9.91 765	0.08 235	9.88 678	9.81 343	9.93 303	0.06 697	9.88 040	24
37	9.80 458	9.91 791	0.08 209	9.88 668	9.81 358	9.93 329	0.06 671	9.88 029	23
38	9.80 473	9.91 816	0.08 184	9.88 657	9.81 372	9.93 354	0.06 646	9.88 018	22
39	9.80 489	9.91 842	0.08 158	9.88 647	9.81 387	9.93 380	0.06 620	9.88 007	21
40	9.80 504	9.91 868	0.08 132	9.88 636	9.81 402	9.93 406	0.06 594	9.87 996	20
41	9.80 519	9.91 893	0.08 107	9.88 626	9.81 417	9.93 431	0.06 569	9.87 985	19
42	9.80 534	9.91 919	0.08 081	9.88 615	9.81 431	9.93 457	0.06 543	9.87 975	18
43	9.80 550	9.91 945	0.08 055	9.88 605	9.81 446	9.93 482	0.06 518	9.87 964	17
44	9.80 565	9.91 971	0.08 029	9.88 594	9.81 461	9.93 508	0.06 492	9.87 953	16
45	9.80 580	9.91 996	0.08 004	9.88 584	9.81 475	9.93 533	0.06 467	9.87 942	15
46	9.80 595	9.92 022	0.07 978	9.88 573	9.81 490	9.93 559	0.06 441	9.87 931	14
47	9.80 610	9.92 048	0.07 952	9.88 563	9.81 505	9.93 584	0.06 416	9.87 920	13
48	9.80 625	9.92 073	0.07 927	9.88 552	9.81 519	9.93 610	0.06 390	9.87 909	12
49	9.80 641	9.92 099	0.07 901	9.88 542	9.81 534	9.93 636	0.06 364	9.87 898	11
50	9.80 656	9.92 125	0.07 875	9.88 531	9.81 549	9.93 661	0.06 339	9.87 887	10
51	9.80 671	9.92 150	0.07 850	9.88 521	9.81 563	9.93 687	0.06 313	9.87 877	9
52	9.80 686	9.92 176	0.07 824	9.88 510	9.81 578	9.93 712	0.06 288	9.87 866	8
53	9.80 701	9.92 202	0.07 798	9.88 499	9.81 592	9.93 738	0.06 262	9.87 855	7
54	9.80 716	9.92 227	0.07 773	9.88 489	9.81 607	9.93 763	0.06 237	9.87 844	6
55	9.80 731	9.92 253	0.07 747	9.88 478	9.81 622	9.93 789	0.06 211	9.87 833	5
56	9.80 746	9.92 279	0.07 721	9.88 468	9.81 636	9.93 814	0.06 186	9.87 822	4
57	9.80 762	9.92 304	0.07 696	9.88 457	9.81 651	9.93 840	0.06 160	9.87 811	3
58	9.80 777	9.92 330	0.07 670	9.88 447	9.81 665	9.93 865	0.06 135	9.87 800	2
59	9.80 792	9.92 356	0.07 644	9.88 436	9.81 680	9.93 891	0.06 109	9.87 789	1
60	9.80 807	9.92 381	0.07 619	9.88 425	9.81 694	9.93 916	0.06 084	9.87 778	0
	log cos	log cot	log tan	log sin	log cos	log cot	log tan	log sin	′
	50°				49°				

	41°				42°				
′	log sin	log tan	log cot	log cos	log sin	log tan	log cot	log cos	
0	9.81 694	9.93 916	0.06 084	9.87 778	9.82 551	9.95 444	0.04 556	9.87 107	60
1	9.81 709	9.93 942	0.06 058	9.87 767	9.82 565	9.95 469	0.04 531	9.87 096	59
2	9.81 723	9.93 967	0.06 033	9.87 756	9.82 579	9.95 495	0.04 505	9.87 085	58
3	9.81 738	9.93 993	0.06 007	9.87 745	9.82 593	9.95 520	0.04 480	9.87 073	57
4	9.81 752	9.94 018	0.05 982	9.87 734	9.82 607	9.95 545	0.04 455	9.87 062	56
5	9.81 767	9.94 044	0.05 956	9.87 723	9.82 621	9.95 571	0.04 429	9.87 050	55
6	9.81 781	9.94 069	0.05 931	9.87 712	9.82 635	9.95 596	0.04 404	9.87 039	54
7	9.81 796	9.94 095	0.05 905	9.87 701	9.82 649	9.95 622	0.04 378	9.87 028	53
8	9.81 810	9.94 120	0.05 880	9.87 690	9.82 663	9.95 647	0.04 353	9.87 016	52
9	9.81 825	9.94 146	0.05 854	9.87 679	9.82 677	9.95 672	0.04 328	9.87 005	51
10	9.81 839	9.94 171	0.05 829	9.87 668	9.82 691	9.95 698	0.04 302	9.86 993	50
11	9.81 854	9.94 197	0.05 803	9.87 657	9.82 705	9.95 723	0.04 277	9.86 982	49
12	9.81 868	9.94 222	0.05 778	9.87 646	9.82 719	9.95 748	0.04 252	9.86 970	48
13	9.81 882	9.94 248	0.05 752	9.87 635	9.82 733	9.95 774	0.04 226	9.86 959	47
14	9.81 897	9.94 273	0.05 727	9.87 624	9.82 747	9.95 799	0.04 201	9.86 947	46
15	9.81 911	9.94 299	0.05 701	9.87 613	9.82 761	9.95 825	0.04 175	9.86 936	45
16	9.81 926	9.94 324	0.05 676	9.87 601	9.82 775	9.95 850	0.04 150	9.86 924	44
17	9.81 940	9.94 350	0.05 650	9.87 590	9.82 788	9.95 875	0.04 125	9.86 913	43
18	9.81 955	9.94 375	0.05 625	9.87 579	9.82 802	9.95 901	0.04 099	9.86 902	42
19	9.81 969	9.94 401	0.05 599	9.87 568	9.82 816	9.95 926	0.04 074	9.86 890	41
20	9.81 983	9.94 426	0.05 574	9.87 557	9.82 830	9.95 952	0.04 048	9.86 879	40
21	9.81 998	9.94 452	0.05 548	9.87 546	9.82 844	9.95 977	0.04 023	9.86 867	39
22	9.82 012	9.94 477	0.05 523	9.87 535	9.82 858	9.96 002	0.03 998	9.86 855	38
23	9.82 026	9.94 503	0.05 497	9.87 524	9.82 872	9.96 028	0.03 972	9.86 844	37
24	9.82 041	9.94 528	0.05 472	9.87 513	9.82 885	9.96 053	0.03 947	9.86 832	36
25	9.82 055	9.94 554	0.05 446	9.87 501	9.82 899	9.96 078	0.03 922	9.86 821	35
26	9.82 069	9.94 579	0.05 421	9.87 490	9.82 913	9.96 104	0.03 896	9.86 809	34
27	9.82 084	9.94 604	0.05 396	9.87 479	9.82 927	9.96 129	0.03 871	9.86 798	33
28	9.82 098	9.94 630	0.05 370	9.87 468	9.82 941	9.96 155	0.03 845	9.86 786	32
29	9.82 112	9.94 655	0.05 345	9.87 457	9.82 955	9.96 180	0.03 820	9.86 775	31
30	9.82 126	9.94 681	0.05 319	9.87 446	9.82 968	9.96 205	0.03 795	9.86 763	30
31	9.82 141	9.94 706	0.05 294	9.87 434	9.82 982	9.96 231	0.03 769	9.86 752	29
32	9.82 155	9.94 732	0.05 268	9.87 423	9.82 996	9.96 256	0.03 744	9.86 740	28
33	9.82 169	9.94 757	0.05 243	9.87 412	9.83 010	9.96 281	0.03 719	9.86 728	27
34	9.82 184	9.94 783	0.05 217	9.87 401	9.83 023	9.96 307	0.03 693	9.86 717	26
35	9.82 198	9.94 808	0.05 192	9.87 390	9.83 037	9.96 332	0.03 668	9.86 705	25
36	9.82 212	9.94 834	0.05 166	9.87 378	9.83 051	9.96 357	0.03 643	9.86 694	24
37	9.82 226	9.94 859	0.05 141	9.87 367	9.83 065	9.96 383	0.03 617	9.86 682	23
38	9.82 240	9.94 884	0.05 116	9.87 356	9.83 078	9.96 408	0.03 592	9.86 670	22
39	9.82 255	9.94 910	0.05 090	9.87 345	9.83 092	9.96 433	0.03 567	9.86 659	21
40	9.82 269	9.94 935	0.05 065	9.87 334	9.83 106	9.96 459	0.03 541	9.86 647	20
41	9.82 283	9.94 961	0.05 039	9.87 322	9.83 120	9.96 484	0.03 516	9.86 635	19
42	9.82 297	9.94 986	0.05 014	9.87 311	9.83 133	9.96 510	0.03 490	9.86 624	18
43	9.82 311	9.95 012	0.04 988	9.87 300	9.83 147	9.96 535	0.03 465	9.86 612	17
44	9.82 326	9.95 037	0.04 963	9.87 288	9.83 161	9.96 560	0.03 440	9.86 600	16
45	9.82 340	9.95 062	0.04 938	9.87 277	9.83 174	9.96 586	0.03 414	9.86 589	15
46	9.82 354	9.95 088	0.04 912	9.87 266	9.83 188	9.96 611	0.03 389	9.86 577	14
47	9.82 368	9.95 113	0.04 887	9.87 255	9.83 202	9.96 636	0.03 364	9.86 565	13
48	9.82 382	9.95 139	0.04 861	9.87 243	9.83 215	9.96 662	0.03 338	9.86 554	12
49	9.82 396	9.95 164	0.04 836	9.87 232	9.83 229	9.96 687	0.03 313	9.86 542	11
50	9.82 410	9.95 190	0.04 810	9.87 221	9.83 242	9.96 712	0.03 288	9.86 530	10
51	9.82 424	9.95 215	0.04 785	9.87 209	9.83 256	9.96 738	0.03 262	9.86 518	9
52	9.82 439	9.95 240	0.04 760	9.87 198	9.83 270	9.96 763	0.03 237	9.86 507	8
53	9.82 453	9.95 266	0.04 734	9.87 187	9.83 283	9.96 788	0.03 212	9.86 495	7
54	9.82 467	9.95 291	0.04 709	9.87 175	9.83 297	9.96 814	0.03 186	9.86 483	6
55	9.82 481	9.95 317	0.04 683	9.87 164	9.83 310	9.96 839	0.03 161	9.86 472	5
56	9.82 495	9.95 342	0.04 658	9.87 153	9.83 324	9.96 864	0.03 136	9.86 460	4
57	9.82 509	9.95 368	0.04 632	9.87 141	9.83 338	9.96 890	0.03 110	9.86 448	3
58	9.82 523	9.95 393	0.04 607	9.87 130	9.83 351	9.96 915	0.03 085	9.86 436	2
59	9.82 537	9.95 418	0.04 582	9.87 119	9.83 365	9.96 940	0.03 060	9.86 425	1
60	9.82 551	9.95 444	0.04 556	9.87 107	9.83 378	9.96 966	0.03 034	9.86 413	0
	log cos	log cot	log tan	log sin	log cos	log cot	log tan	log sin	′
	48°				47°				

′	43° log sin	log tan	log cot	log cos	44° log sin	log tan	log cot	log cos	
0	9.83 378	9.96 966	0.03 034	9.86 413	9.84 177	9.98 484	0.01 516	9.85 693	60
1	9.83 392	9.96 991	0.03 009	9.86 401	9.84 190	9.98 509	0.01 491	9.85 681	59
2	9.83 405	9.97 016	0.02 984	9.86 389	9.84 203	9.98 534	0.01 466	9.85 669	58
3	9.83 419	9.97 042	0.02 958	9.86 377	9.84 216	9.98 560	0.01 440	9.85 657	57
4	9.83 432	9.97 067	0.02 933	9.86 366	9.84 229	9.98 585	0.01 415	9.85 645	56
5	9.83 446	9.97 092	0.02 908	9.86 354	9.84 242	9.98 610	0.01 390	9.85 632	55
6	9.83 459	9.97 118	0.02 882	9.86 342	9.84 255	9.98 635	0.01 365	9.85 620	54
7	9.83 473	9.97 143	0.02 857	9.86 330	9.84 269	9.98 661	0.01 339	9.85 608	53
8	9.83 486	9.97 168	0.02 832	9.86 318	9.84 282	9.98 686	0.01 314	9.85 596	52
9	9.83 500	9.97 193	0.02 807	9.86 306	9.84 295	9.98 711	0.01 289	9.85 583	51
10	9.83 513	9.97 219	0.02 781	9.86 295	9.84 308	9.98 737	0.01 263	9.85 571	50
11	9.83 527	9.97 244	0.02 756	9.86 283	9.84 321	9.98 762	0.01 238	9.85 559	49
12	9.83 540	9.97 269	0.02 731	9.86 271	9.84 334	9.98 787	0.01 213	9.85 547	48
13	9.83 554	9.97 295	0.02 705	9.86 259	9.84 347	9.98 812	0.01 188	9.85 534	47
14	9.83 567	9.97 320	0.02 680	9.86 247	9.84 360	9.98 838	0.01 162	9.85 522	46
15	9.83 581	9.97 345	0.02 655	9.86 235	9.84 373	9.98 863	0.01 137	9.85 510	45
16	9.83 594	9.97 371	0.02 629	9.86 223	9.84 385	9.98 888	0.01 112	9.85 497	44
17	9.83 608	9.97 396	0.02 604	9.86 211	9.84 398	9.98 913	0.01 087	9.85 485	43
18	9.83 621	9.97 421	0.02 579	9.86 200	9.84 411	9.98 939	0.01 061	9.85 473	42
19	9.83 634	9.97 447	0.02 553	9.86 188	9.84 424	9.98 964	0.01 036	9.85 460	41
20	9.83 648	9.97 472	0.02 528	9.86 176	9.84 437	9.98 989	0.01 011	9.85 448	40
21	9.83 661	9.97 497	0.02 503	9.86 164	9.84 450	9.99 015	0.00 985	9.85 436	39
22	9.83 674	9.97 523	0.02 477	9.86 152	9.84 463	9.99 040	0.00 960	9.85 423	38
23	9.83 688	9.97 548	0.02 452	9.86 140	9.84 476	9.99 065	0.00 935	9.85 411	37
24	9.83 701	9.97 573	0.02 427	9.86 128	9.84 489	9.99 090	0.00 910	9.85 399	36
25	9.83 715	9.97 598	0.02 402	9.86 116	9.84 502	9.99 116	0.00 884	9.85 386	35
26	9.83 728	9.97 624	0.02 376	9.86 104	9.84 515	9.99 141	0.00 859	9.85 374	34
27	9.83 741	9.97 649	0.02 351	9.86 092	9.84 528	9.99 166	0.00 834	9.85 361	33
28	9.83 755	9.97 674	0.02 326	9.86 080	9.84 540	9.99 191	0.00 809	9.85 349	32
29	9.83 768	9.97 700	0.02 300	9.86 068	9.84 553	9.99 217	0.00 783	9.85 337	31
30	9.83 781	9.97 725	0.02 275	9.86 056	9.84 566	9.99 242	0.00 758	9.85 324	30
31	9.83 795	9.97 750	0.02 250	9.86 044	9.84 579	9.99 267	0.00 733	9.85 312	29
32	9.83 808	9.97 776	0.02 224	9.86 032	9.84 592	9.99 293	0.00 707	9.85 299	28
33	9.83 821	9.97 801	0.02 199	9.86 020	9.84 605	9.99 318	0.00 682	9.85 287	27
34	9.83 834	9.97 826	0.02 174	9.86 008	9.84 618	9.99 343	0.00 657	9.85 274	26
35	9.83 848	9.97 851	0.02 149	9.85 996	9.84 630	9.99 368	0.00 632	9.85 262	25
36	9.83 861	9.97 877	0.02 123	9.85 984	9.84 643	9.99 394	0.00 606	9.85 250	24
37	9.83 874	9.97 902	0.02 098	9.85 972	9.84 656	9.99 419	0.00 581	9.85 237	23
38	9.83 887	9.97 927	0.02 073	9.85 960	9.84 669	9.99 444	0.00 556	9.85 225	22
39	9.83 901	9.97 953	0.02 047	9.85 948	9.84 682	9.99 469	0.00 531	9.85 212	21
40	9.83 914	9.97 978	0.02 022	9.85 936	9.84 694	9.99 495	0.00 505	9.85 200	20
41	9.83 927	9.98 003	0.01 997	9.85 924	9.84 707	9.99 520	0.00 480	9.85 187	19
42	9.83 940	9.98 029	0.01 971	9.85 912	9.84 720	9.99 545	0.00 455	9.85 175	18
43	9.83 954	9.98 054	0.01 946	9.85 900	9.84 733	9.99 570	0.00 430	9.85 162	17
44	9.83 967	9.98 079	0.01 921	9.85 888	9.84 745	9.99 596	0.00 404	9.85 150	16
45	9.83 980	9.98 104	0.01 896	9.85 876	9.84 758	9.99 621	0.00 379	9.85 137	15
46	9.83 993	9.98 130	0.01 870	9.85 864	9.84 771	9.99 646	0.00 354	9.85 125	14
47	9.84 006	9.98 155	0.01 845	9.85 851	9.84 784	9.99 672	0.00 328	9.85 112	13
48	9.84 020	9.98 180	0.01 820	9.85 839	9.84 796	9.99 697	0.00 303	9.85 100	12
49	9.84 033	9.98 206	0.01 794	9.85 827	9.84 809	9.99 722	0.00 278	9.85 087	11
50	9.84 046	9.98 231	0.01 769	9.85 815	9.84 822	9.99 747	0.00 253	9.85 074	10
51	9.84 059	9.98 256	0.01 744	9.85 803	9.84 835	9.99 773	0.00 227	9.85 062	9
52	9.84 072	9.98 281	0.01 719	9.85 791	9.84 847	9.99 798	0.00 202	9.85 049	8
53	9.84 085	9.98 307	0.01 693	9.85 779	9.84 860	9.99 823	0.00 177	9.85 037	7
54	9.84 098	9.98 332	0.01 668	9.85 766	9.84 873	9.99 848	0.00 152	9.85 024	6
55	9.84 112	9.98 357	0.01 643	9.85 754	9.84 885	9.99 874	0.00 126	9.85 012	5
56	9.84 125	9.98 383	0.01 617	9.85 742	9.84 898	9.99 899	0.00 101	9.84 999	4
57	9.84 138	9.98 408	0.01 592	9.85 730	9.84 911	9.99 924	0.00 076	9.84 986	3
58	9.84 151	9.98 433	0.01 567	9.85 718	9.84 923	9.99 949	0.00 051	9.84 974	2
59	9.84 164	9.98 458	0.01 542	9.85 706	9.84 936	9.99 975	0.00 025	9.84 961	1
60	9.84 177	9.98 484	0.01 516	9.85 693	9.84 949	0.00 000	0.00 000	9.84 949	0
	log cos	log cot	log tan	log sin 46°	log cos	log cot	log tan	log sin 45°	′

TABLE III (*a*)

NATURAL SINES

Deg.	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′
0	0000	0017	0035	0052	0070	0087	0105	0122	0140	0157
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332
2	0349	0366	0384	0401	0419	0436	0454	0471	0488	0506
3	0523	0541	0558	0576	0593	0610	0628	0645	0663	0680
4	0698	0715	0732	0750	0767	0785	0802	0819	0837	0854
5	0872	0889	0906	0924	0941	0958	0976	0993	1011	1028
6	1045	1063	1080	1097	1115	1132	1149	1167	1184	1201
7	1219	1236	1253	1271	1288	1305	1323	1340	1357	1374
8	1392	1409	1426	1444	1461	1478	1495	1513	1530	1547
9	1564	1582	1599	1616	1633	1650	1668	1685	1702	1719
10	1736	1754	1771	1788	1805	1822	1840	1857	1874	1891
11	1908	1925	1942	1959	1977	1994	2011	2028	2045	2062
12	2079	2096	2113	2130	2147	2164	2181	2198	2215	2232
13	2250	2267	2284	2300	2317	2334	2351	2368	2385	2402
14	2419	2436	2453	2470	2487	2504	2521	2538	2554	2571
15	2588	2605	2622	2639	2656	2672	2689	2706	2723	2740
16	2756	2773	2790	2807	2823	2840	2857	2874	2890	2907
17	2924	2940	2957	2974	2990	3007	3024	3040	3057	3074
18	3090	3107	3123	3140	3156	3173	3190	3206	3223	3239
19	3256	3272	3289	3305	3322	3338	3355	3371	3387	3404
20	3420	3437	3453	3469	3486	3502	3518	3535	3551	3567
21	3584	3600	3616	3633	3649	3665	3681	3697	3714	3730
22	3746	3762	3778	3795	3811	3827	3843	3859	3875	3891
23	3907	3923	3939	3955	3971	3987	4003	4019	4035	4051
24	4067	4083	4099	4115	4131	4147	4163	4179	4195	4210
25	4226	4242	4258	4274	4289	4305	4321	4337	4352	4368
26	4384	4399	4415	4431	4446	4462	4478	4493	4509	4524
27	4540	4555	4571	4586	4602	4617	4633	4648	4664	4679
28	4695	4710	4726	4741	4756	4772	4787	4802	4818	4833
29	4848	4863	4879	4894	4909	4924	4939	4955	4970	4985
30	5000	5015	5030	5045	5060	5075	5090	5105	5120	5135
31	5150	5165	5180	5195	5210	5225	5240	5255	5270	5284
32	5299	5314	5329	5344	5358	5373	5388	5402	5417	5432
33	5446	5461	5476	5490	5505	5519	5534	5548	5563	5577
34	5592	5606	5621	5635	5650	5664	5678	5693	5707	5721
35	5736	5750	5764	5779	5793	5807	5821	5835	5850	5864
36	5878	5892	5906	5920	5934	5948	5962	5976	5990	6004
37	6018	6032	6046	6060	6074	6088	6101	6115	6129	6143
38	6157	6170	6184	6198	6211	6225	6239	6252	6266	6280
39	6293	6307	6320	6334	6347	6361	6374	6388	6401	6414

Deg.	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′
40	6428	6441	6455	6468	6481	6494	6508	6521	6534	6547
41	6561	6574	6587	6600	6613	6626	6639	6652	6665	6678
42	6691	6704	6717	6730	6743	6756	6769	6782	6794	6807
43	6820	6833	6845	6858	6871	6884	6896	6909	6921	6934
44	6947	6959	6972	6984	6997	7009	7022	7034	7046	7059
45	7071	7083	7096	7108	7120	7133	7145	7157	7169	7181
46	7193	7206	7218	7230	7242	7254	7266	7278	7290	7302
47	7314	7325	7337	7349	7361	7373	7385	7396	7408	7420
48	7431	7443	7455	7466	7478	7490	7501	7513	7524	7536
49	7547	7558	7570	7581	7593	7604	7615	7627	7638	7649
50	7660	7672	7683	7694	7705	7716	7727	7738	7749	7760
51	7771	7782	7793	7804	7815	7826	7837	7848	7859	7869
52	7880	7891	7902	7912	7923	7934	7944	7955	7965	7976
53	7986	7997	8007	8018	8028	8039	8049	8059	8070	8080
54	8090	8100	8111	8121	8131	8141	8151	8161	8171	8181
55	8192	8202	8211	8221	8231	8241	8351	8261	8271	8281
56	8290	8300	8310	8320	8329	8339	8348	8358	8368	8377
57	8387	8396	8406	8415	8425	8434	8443	8453	8462	8471
58	8480	8490	8499	8508	8517	8526	8536	8545	8554	8563
59	8572	8581	8590	8599	8607	8616	8625	8634	8643	8652
60	8660	8669	8678	8686	8695	8704	8712	8721	8729	8738
61	8746	8755	8763	8771	8780	8788	8796	8805	8813	8821
62	8829	8838	8846	8854	8862	8870	8878	8886	8894	8902
63	8910	8918	8926	8934	8942	8949	8957	8965	8973	8980
64	8988	8996	9003	9011	9018	9026	9033	9041	9048	9056
65	9063	9070	9078	9085	9092	9100	9107	9114	9121	9128
66	9135	9143	9150	9157	9164	9171	9178	9184	9191	9198
67	9205	9212	9219	9225	9232	9239	9245	9252	9259	9265
68	9272	9278	9285	9291	9298	9304	9311	9317	9323	9330
69	9336	9342	9348	9354	9361	9367	9373	9379	9385	9391
70	9397	9403	9409	9415	9421	9426	9432	9438	9444	9449
71	9455	9461	9466	9472	9478	9483	9489	9494	9500	9505
72	9511	9516	9521	9527	9532	9537	9542	9548	9553	9558
73	9563	9568	9573	9578	9583	9588	9593	9598	9603	9608
74	9613	9617	9622	9627	9632	9636	9641	9646	9650	9655
75	9659	9664	9668	9673	9677	9681	9686	9690	9694	9699
76	9703	9707	9711	9715	9720	9724	9728	9732	9736	9740
77	9744	9748	9751	9755	9759	9763	9767	9770	9774	9778
78	9781	9785	9789	9792	9796	9799	9803	9806	9810	9813
79	9816	9820	9823	9826	9829	9833	9836	9839	9842	9845
80	9848	9851	9854	9857	9860	9863	9866	9869	9871	9874
81	9877	9880	9882	9885	9888	9890	9893	9895	9898	9900
82	9903	9905	9907	9910	9912	9914	9917	9919	9921	9923
83	9925	9928	9930	9932	9934	9936	9938	9940	9942	9943
84	9945	9947	9949	9951	9952	9954	9956	9957	9959	9960
85	9962	9963	9965	9966	9968	9969	9971	9972	9973	9974
86	9976	9977	9978	9979	9980	9981	9982	9983	9984	9985
87	9986	9987	9988	9989	9990	9990	9991	9992	9993	9993
88	9994	9995	9995	9996	9996	9997	9997	9997	9998	9998
89	9998	9999	9999	9999	9999	1.000	1.000	1.000	1.000	1.000

TABLE III (*b*)

NATURAL COSINES

Deg.	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′
0	1.000	1.000	1.000	1.000	1.000	9999	9999	9999	9999	9999
1	9998	9998	9998	9997	9997	9997	9996	9996	9995	9995
2	9994	9993	9993	9992	9991	9990	9990	9989	9988	9987
3	9986	9985	9984	9983	9982	9981	9980	9979	9978	9977
4	9976	9974	9973	9972	9971	9969	9968	9966	9965	9963
5	9962	9960	9959	9957	9956	9954	9952	9951	9949	9947
6	9945	9943	9942	9940	9938	9936	9934	9932	9930	9928
7	9925	9923	9921	9919	9917	9914	9912	9910	9907	9905
8	9903	9900	9898	9895	9893	9890	9888	9885	9882	9880
9	9877	9874	9871	9869	9866	9863	9860	9857	9854	9851
10	9848	9845	9842	9839	9836	9833	9829	9826	9823	9820
11	9816	9813	9810	9806	9803	9799	9796	9792	9789	9785
12	9781	9778	9774	9770	9767	9763	9759	9755	9751	9748
13	9744	9740	9736	9732	9728	9724	9720	9715	9711	9707
14	9703	9699	9694	9690	9686	9681	9677	9673	9668	9664
15	9659	9655	9650	9646	9641	9636	9632	9627	9622	9617
16	9613	9608	9603	9598	9593	9588	9583	9578	9573	9568
17	9563	9558	9553	9548	9542	9537	9532	9527	9521	9516
18	9511	9505	9500	9494	9489	9483	9478	9472	9466	9461
19	9455	9449	9444	9438	9432	9426	9421	9415	9409	9403
20	9397	9391	9385	9379	9373	9367	9361	9354	9348	9342
21	9336	9330	9323	9317	9311	9304	9298	9291	9285	9278
22	9272	9265	9259	9252	9245	9239	9232	9225	9219	9212
23	9205	9198	9191	9184	9178	9171	9164	9157	9150	9143
24	9135	9128	9121	9114	9107	9100	9092	9085	9078	9070
25	9063	9056	9048	9041	9033	9026	9018	9011	9003	8996
26	8988	8980	8973	8965	8957	8949	8942	8934	8926	8918
27	8910	8902	8894	8886	8878	8870	8862	8854	8846	8838
28	8829	8821	8813	8805	8796	8788	8780	8771	8763	8755
29	8746	8738	8729	8721	8712	8704	8695	8686	8678	8669
30	8660	8652	8643	8634	8625	8616	8607	8599	8590	8581
31	8572	8563	8554	8545	8536	8526	8517	8508	8499	8490
32	8480	8471	8462	8453	8443	8434	8425	8415	8406	8396
33	8387	8377	8368	8358	8348	8339	8329	8320	8310	8300
34	8290	8281	8271	8261	8251	8241	8231	8221	8211	8202
35	8192	8181	8171	8161	8151	8141	8131	8121	8111	8100
36	8090	8080	8070	8059	8049	8039	8028	8018	8007	7997
37	7986	7976	7965	7955	7944	7934	7923	7912	7902	7891
38	7880	7869	7859	7848	7837	7826	7815	7804	7793	7782
39	7771	7760	7749	7738	7727	7716	7705	7694	7683	7672

Deg.	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′
40	7660	7649	7638	7627	7615	7604	7593	7581	7570	7559
41	7547	7536	7524	7513	7501	7490	7478	7466	7455	7443
42	7431	7420	7408	7396	7385	7373	7361	7349	7337	7325
43	7314	7302	7290	7278	7266	7254	7242	7230	7218	7206
44	7193	7181	7169	7157	7145	7133	7120	7108	7096	7083
45	7071	7059	7046	7034	7022	7009	6997	6984	6972	6959
46	6947	6934	6921	6909	6896	6884	6871	6858	6845	6833
47	6820	6807	6794	6782	6769	6756	6743	6730	6717	6704
48	6691	6678	6665	6652	6639	6626	6613	6600	6587	6574
49	6561	6547	6534	6521	6508	6494	6481	6468	6455	6441
50	6428	6414	6401	6388	6374	6361	6347	6334	6320	6307
51	6293	6280	6266	6252	6239	6225	6211	6198	6184	6170
52	6157	6143	6129	6115	6101	6088	6074	6060	6046	6032
53	6018	6004	5990	5976	5962	5948	5934	5920	5906	5892
54	5878	5864	5850	5835	5821	5807	5793	5779	5764	5750
55	5736	5721	5707	5693	5678	5664	5650	5635	5621	5606
56	5592	5577	5563	5548	5534	5519	5505	5490	5476	5461
57	5446	5432	5417	5402	5388	5373	5358	5344	5329	5314
58	5299	5284	5270	5255	5240	5225	5210	5195	5180	5165
59	5150	5135	5120	5105	5090	5075	5060	5045	5030	5015
60	5000	4985	4970	4955	4939	4924	4909	4894	4879	4863
61	4848	4833	4818	4802	4787	4772	4756	4741	4726	4710
62	4695	4679	4664	4648	4633	4617	4602	4586	4571	4555
63	4540	4524	4509	4493	4478	4462	4446	4431	4415	4399
64	4384	4368	4352	4337	4321	4305	4289	4274	4258	4242
65	4226	4210	4195	4179	4163	4147	4131	4115	4099	4083
66	4067	4051	4035	4019	4003	3987	3971	3955	3939	3923
67	3907	3891	3875	3859	3843	3827	3811	3795	3778	3762
68	3746	3730	3714	3697	3681	3665	3649	3633	3616	3600
69	3584	3567	3551	3535	3518	3502	3486	3469	3453	3437
70	3420	3404	3387	3371	3355	3338	3322	3305	3289	3272
71	3256	3239	3223	3206	3190	3173	3156	3140	3123	3107
72	3090	3074	3057	3040	3024	3007	2990	2974	2957	2940
73	2924	2907	2890	2874	2857	2840	2823	2807	2790	2773
74	2756	2740	2723	2706	2689	2672	2656	2639	2622	2605
75	2588	2571	2554	2538	2521	2504	2487	2470	2453	2436
76	2419	2402	2385	2368	2351	2334	2317	2300	2284	2267
77	2250	2233	2215	2198	2181	2164	2147	2130	2113	2096
78	2079	2062	2045	2028	2011	1994	1977	1959	1942	1925
79	1908	1891	1874	1857	1840	1822	1805	1788	1771	1754
80	1736	1719	1702	1685	1668	1650	1633	1616	1599	1582
81	1564	1547	1530	1513	1495	1478	1461	1444	1426	1409
82	1392	1374	1357	1340	1323	1305	1288	1271	1253	1236
83	1219	1201	1184	1167	1149	1132	1115	1097	1080	1063
84	1045	1028	1011	0993	0976	0958	0941	0924	0906	0889
85	0872	0854	0837	0819	0802	0785	0767	0750	0732	0715
86	0698	0680	0663	0645	0628	0610	0593	0576	0558	0541
87	0523	0506	0488	0471	0454	0436	0419	0401	0384	0366
88	0349	0332	0314	0297	0279	0262	0244	0227	0209	0192
89	0175	0157	0140	0122	0105	0087	0070	0052	0035	0017

TABLE III (c)

NATURAL TANGENTS

Deg.	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′
0	.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157
1	.0175	0192	0209	0227	0244	0262	0279	0297	0314	0332
2	.0349	0367	0384	0402	0419	0437	0454	0472	0489	0507
3	.0524	0542	0559	0577	0594	0612	0629	0647	0664	0682
4	.0699	0717	0734	0752	0769	0787	0805	0822	0840	0857
5	.0875	0892	0910	0928	0945	0963	0981	0998	1016	1033
6	.1051	1069	1086	1104	1122	1139	1157	1175	1192	1210
7	.1228	1246	1263	1281	1299	1317	1334	1352	1370	1388
8	.1405	1423	1441	1459	1477	1495	1512	1530	1548	1566
9	.1584	1602	1620	1638	1655	1673	1691	1709	1727	1745
10	.1763	1781	1799	1817	1835	1853	1871	1890	1908	1926
11	.1944	1962	1980	1998	2016	2035	2053	2071	2089	2107
12	.2126	2144	2162	2180	2199	2217	2235	2254	2272	2290
13	.2309	2327	2345	2364	2382	2401	2419	2438	2456	2475
14	.2493	2512	2530	2549	2568	2586	2605	2623	2642	2661
15	.2679	2698	2717	2736	2754	2773	2792	2811	2830	2849
16	.2867	2886	2905	2924	2943	2962	2981	3000	3019	3038
17	.3057	3076	3096	3115	3134	3153	3172	3191	3211	3230
18	.3249	3269	3288	3307	3327	3346	3365	3385	3404	3424
19	.3443	3463	3482	3502	3522	3541	3561	3581	3600	3620
20	.3640	3659	3679	3699	3719	3739	3759	3779	3799	3819
21	.3839	3859	3879	3899	3919	3939	3959	3979	4000	4020
22	.4040	4061	4081	4101	4122	4142	4163	4183	4204	4224
23	.4245	4265	4286	4307	4327	4348	4369	4390	4411	4431
24	.4452	4473	4494	4515	4536	4557	4578	4599	4621	4642
25	.4663	4684	4706	4727	4748	4770	4791	4813	4834	4856
26	.4877	4899	4921	4942	4964	4986	5008	5029	5051	5073
27	.5095	5117	5139	5161	5184	5206	5228	5250	5272	5295
28	.5317	5340	5362	5384	5407	5430	5452	5475	5498	5520
29	.5543	5566	5589	5612	5635	5658	5681	5704	5727	5750
30	.5774	5797	5820	5844	5867	5890	5914	5938	5961	5985
31	.6009	6032	6056	6080	6104	6128	6152	6176	6200	6224
32	.6249	6273	6297	6322	6346	6371	6395	6420	6445	6469
33	.6494	6519	6544	6569	6594	6619	6644	6669	6694	6720
34	.6745	6771	6796	6822	6847	6873	6899	6924	6950	6976
35	.7002	7028	7054	7080	7107	7133	7159	7186	7212	7239
36	.7265	7292	7319	7346	7373	7400	7427	7454	7481	7508
37	.7536	7563	7590	7618	7646	7673	7701	7729	7757	7785
38	.7813	7841	7869	7898	7926	7954	7983	8012	8040	8069
39	.8098	8127	8156	8185	8214	8243	8273	8302	8332	8361

Deg.	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′
40	.8391	8421	8451	8481	8511	8541	8571	8601	8632	8662
41	.8693	8724	8754	8785	8816	8847	8878	8910	8941	8972
42	.9004	9036	9067	9099	9131	9163	9195	9228	9260	9293
43	.9325	9358	9391	9424	9457	9490	9523	9556	9590	9623
44	.9657	9691	9725	9759	9793	9827	9861	9896	9930	9965
45	1.0000	0035	0070	0105	0141	0176	0212	0247	0283	0319
46	1.0355	0392	0428	0464	0501	0538	0575	0612	0649	0686
47	1.0724	0761	0799	0837	0875	0913	0951	0990	1028	1067
48	1.1106	1145	1184	1224	1263	1303	1343	1383	1423	1463
49	1.1504	1544	1585	1626	1667	1708	1750	1792	1833	1875
50	1.1918	1960	2002	2045	2088	2131	2174	2218	2261	2305
51	1.2349	2393	2437	2482	2527	2572	2617	2662	2708	2753
52	1.2799	2846	2892	2938	2985	3032	3079	3127	3175	3222
53	1.3270	3319	3367	3416	3465	3514	3564	3613	3663	3713
54	1.3764	3814	3865	3916	3968	4019	4071	4124	4176	4229
55	1.4281	4335	4388	4442	4496	4550	4605	4659	4715	4770
56	1.4826	4882	4938	4994	5051	5108	5166	5224	5282	5340
57	1.5399	5458	5517	5577	5637	5697	5757	5818	5880	5941
58	1.6003	6066	6128	6191	6255	6319	6383	6447	6512	6577
59	1.6643	6709	6775	6842	6909	6977	7045	7113	7182	7251
60	1.7321	7391	7461	7532	7603	7675	7747	7820	7893	7966
61	1.8040	8115	8190	8265	8341	8418	8495	8572	8650	8728
62	1.8807	8887	8967	9047	9128	9210	9292	9375	9458	9542
63	1.9626	9711	9797	9883	9970	0057	0145	0233	0323	0413
64	2.0503	0594	0686	0778	0872	0965	1060	1155	1251	1348
65	2.1445	1543	1642	1742	1842	1943	2045	2148	2251	2355
66	2.2460	2566	2673	2781	2889	2998	3109	3220	3332	3445
67	2.3559	3673	3789	3906	4023	4142	4262	4383	4504	4627
68	2.4751	4876	5002	5129	5257	5386	5517	5649	5782	5916
69	2.6051	6187	6325	6464	6605	6746	6889	7034	7179	7326
70	2.7475	7625	7776	7929	8083	8239	8397	8556	8716	8878
71	2.9042	9208	9375	9544	9714	9887	0061	0237	0415	0595
72	3.0777	0961	1146	1334	1524	1716	1910	2106	2305	2506
73	3.2709	2914	3122	3332	3544	3759	3977	4197	4420	4646
74	3.4874	5105	5339	5576	5816	6059	6305	6554	6806	7062
75	3.7321	7583	7848	8118	8391	8667	8947	9232	9520	9812
76	4.0108	0408	0713	1022	1335	1653	1976	2303	2635	2972
77	4.3315	3662	4015	4374	4737	5107	5483	5864	6252	6646
78	4.7046	7453	7867	8288	8716	9152	9594	0045	0504	0970
79	5.1446	1929	2422	2924	3435	3955	4486	5026	5578	6140
80	5.6713	7297	7894	8502	9124	9758	$\bar{0}405$	$\bar{1}066$	$\bar{1}742$	$\bar{2}432$
81	6.3138	3859	4596	5350	6122	6912	7920	8548	9395	0264
82	7.1154	2066	3002	3962	4947	5958	6996	8062	9158	0285
83	8.1443	2636	3863	5126	6427	7769	9152	$\bar{0}579$	$\bar{2}052$	$\bar{3}572$
84	9.5144	9.677	9.845	10.02	10.20	10.39	10.58	10.78	10.99	11.20
85	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95
86	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46
87	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27
88	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08
89	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0

TABLE III (*d*)

NATURAL COTANGENTS

Deg.	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′
0	Inf.	573.0	286.5	191.0	143.2	114.6	95.49	81.85	71.62	63.66
1	57.29	52.08	47.74	44.07	40.92	38.19	35.80	33.69	31.82	30.14
2	28.64	27.27	26.03	24.90	23.86	22.90	22.02	21.20	20.45	19.74
3	19.08	18.46	17.89	17.34	16.83	16.35	15.89	15.46	15.06	14.67
4	14.30	13.95	13.62	13.30	13.00	12.71	12.43	12.16	11.91	11.66
5	11.43	11.20	10.99	10.78	10.58	10.39	10.20	10.02	9.845	9.677
6	9.5144	3572	2052	0579	$\overline{9}152$	$\overline{7}769$	$\overline{6}427$	$\overline{5}126$	$\overline{3}863$	$\overline{2}636$
7	8.1443	0285	$\overline{9}158$	$\overline{8}062$	$\overline{6}996$	$\overline{5}958$	$\overline{4}947$	$\overline{3}962$	$\overline{3}002$	$\overline{2}066$
8	7.1154	0264	$\overline{9}395$	$\overline{8}548$	$\overline{7}920$	$\overline{6}912$	$\overline{6}122$	$\overline{5}350$	$\overline{4}596$	$\overline{3}859$
9	6.3138	2432	1742	1066	0405	$\overline{9}758$	$\overline{9}124$	8502	7894	7297
10	5.6713	6140	5578	5026	4486	3955	3435	2924	2422	1929
11	5.1446	0970	0504	0045	$\overline{9}594$	$\overline{9}152$	$\overline{8}716$	$\overline{8}288$	$\overline{7}867$	$\overline{7}453$
12	4.7046	6646	6252	5864	5483	5107	4737	4374	4015	3662
13	4.3315	2972	2635	2303	1976	1653	1335	1022	0713	0408
14	4.0108	$\overline{9}812$	$\overline{9}520$	$\overline{9}232$	$\overline{8}947$	$\overline{8}667$	$\overline{8}391$	$\overline{8}118$	$\overline{7}848$	$\overline{7}583$
15	3.7321	7062	6806	6554	6305	6059	5816	5576	5339	5105
16	3.4874	4646	4420	4197	3977	3759	3544	3332	3122	2914
17	3.2709	2506	2305	2106	1910	1716	1524	1334	1146	0961
18	3.0777	0595	0415	0237	0061	$\overline{9}887$	$\overline{9}714$	$\overline{9}544$	$\overline{9}375$	$\overline{9}208$
19	2.9042	8878	8716	8556	8397	8239	8083	7929	7776	7625
20	2.7475	7326	7179	7034	6889	6746	6605	6464	6325	6187
21	2.6051	5916	5782	5649	5517	5386	5257	5129	5002	4876
22	2.4751	4627	4504	4383	4262	4142	4023	3906	3789	3673
23	2.3559	3445	3332	3220	3109	2998	2889	2781	2673	2566
24	2.2460	2355	2251	2148	2045	1943	1842	1742	1642	1543
25	2.1445	1348	1251	1155	1060	0965	0872	0778	0686	0594
26	2.0503	0413	0323	0233	0145	0057	$\overline{9}970$	$\overline{9}883$	$\overline{9}797$	$\overline{9}711$
27	1.9626	9542	9458	9375	9292	9210	9128	9047	8967	8887
28	1.8807	8728	8650	8572	8495	8418	8341	8265	8190	8115
29	1.8040	7966	7893	7820	7747	7675	7603	7532	7461	7391
30	1.7321	7251	7182	7113	7045	6977	6909	6842	6775	6709
31	1.6643	6577	6512	6447	6383	6319	6255	6191	6128	6066
32	1.6003	5941	5880	5818	5757	5697	5637	5577	5517	5458
33	1.5399	5340	5282	5224	5166	5108	5051	4994	4938	4882
34	1.4826	4770	4715	4659	4605	4550	4496	4442	4388	4335
35	1.4281	4229	4176	4124	4071	4019	3968	3916	3865	3814
36	1.3764	3713	3663	3613	3564	3514	3465	3416	3367	3319
37	1.3270	3222	3175	3127	3079	3032	2985	2938	2892	2846
38	1.2799	2753	2708	2662	2617	2572	2527	2482	2437	2393
39	1.2349	2305	2261	2218	2174	2131	2088	2045	2002	1960

Deg.	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′
40	1.1918	1875	1833	1792	1750	1708	1667	1626	1585	1544
41	1.1504	1463	1423	1383	1343	1303	1263	1224	1184	1145
42	1.1106	1067	1028	0990	0951	0913	0875	0837	0799	0761
43	1.0724	0686	0649	0612	0575	0538	0501	0464	0428	0392
44	1.0355	0319	0283	0247	0212	0176	0141	0105	0070	0035
45	1.0000	0.9965	0.9930	0.9896	0.9861	0.9827	0.9793	0.9759	0.9725	0.9691
46	.9657	9623	9590	9556	9523	9490	9457	9424	9391	9358
47	.9325	9293	9260	9228	9195	9163	9131	9099	9067	9036
48	.9004	8972	8941	8910	8878	8847	8816	8785	8754	8724
49	.8693	8662	8632	8601	8571	8541	8511	8481	8451	8421
50	.8391	8361	8332	8302	8273	8243	8214	8185	8156	8127
51	.8098	8069	8040	8012	7983	7954	7926	7898	7869	7841
52	.7813	7785	7757	7729	7701	7673	7646	7618	7590	7563
53	.7536	7508	7481	7454	7427	7400	7373	7346	7319	7292
54	.7265	7239	7212	7186	7159	7133	7107	7080	7054	7028
55	.7002	6976	6950	6924	6899	6873	6847	6822	6796	6771
56	.6745	6720	6694	6669	6644	6619	6594	6569	6544	6519
57	.6494	6469	6445	6420	6395	6371	6346	6322	6297	6273
58	.6249	6224	6200	6176	6152	6128	6104	6080	6056	6032
59	.6009	5985	5961	5938	5914	5890	5867	5844	5820	5797
60	.5774	5750	5727	5704	5681	5658	5635	5612	5589	5566
61	.5543	5520	5498	5475	5452	5430	5407	5384	5362	5340
62	.5317	5295	5272	5250	5228	5206	5184	5161	5139	5117
63	.5095	5073	5051	5029	5008	4986	4964	4942	4921	4899
64	.4877	4856	4834	4813	4791	4770	4748	4727	4706	4684
65	.4663	4642	4621	4599	4578	4557	4536	4515	4494	4473
66	.4452	4431	4411	4390	4369	4348	4327	4307	4286	4265
67	.4245	4224	4204	4183	4163	4142	4122	4101	4081	4061
68	.4040	4020	4000	3979	3959	3939	3919	3899	3879	3859
69	.3839	3819	3799	3779	3759	3739	3719	3699	3679	3659
70	.3640	3620	3600	3581	3561	3541	3522	3502	3482	3463
71	.3443	3424	3404	3385	3365	3346	3327	3307	3288	3269
72	.3249	3230	3211	3191	3172	3153	3134	3115	3096	3076
73	.3057	3038	3019	3000	2981	2962	2943	2924	2905	2886
74	.2867	2849	2830	2811	2792	2773	2754	2736	2717	2698
75	.2679	2661	2642	2623	2605	2586	2568	2549	2530	2512
76	.2493	2475	2456	2438	2419	2401	2382	2364	2345	2327
77	.2309	2290	2272	2254	2235	2217	2199	2180	2162	2144
78	.2126	2107	2089	2071	2053	2035	2016	1998	1980	1962
79	.1944	1926	1908	1890	1871	1853	1835	1817	1799	1781
80	.1763	1745	1727	1709	1691	1673	1655	1638	1620	1602
81	.1584	1566	1548	1530	1512	1495	1477	1459	1441	1423
82	.1405	1388	1370	1352	1334	1317	1299	1281	1263	1246
83	.1228	1210	1192	1175	1157	1139	1122	1104	1086	1069
84	.1051	1033	1016	0998	0981	0963	0945	0928	0910	0892
85	.0875	0857	0840	0822	0805	0787	0769	0752	0734	0717
86	.0699	0682	0664	0647	0629	0612	0594	0577	0559	0542
87	.0524	0507	0489	0472	0454	0437	0419	0402	0384	0367
88	.0349	0332	0314	0297	0279	0262	0244	0227	0209	0192
89	.0175	0157	0140	0122	0105	0087	0070	0052	0035	0017