Name. _____ **Student Admn. No.:** _____	**Printed Pages:01**

School of Basic Sciences
Summer Term Examination – July - August 2024
[Programme: M. Sc] [Semester: III] [Batch: 2022-24]

Course Title: Functional Analysis **Course Code: MSCM301**	**Max Marks: 100** **Time: 3 Hrs.**

Instructions:
1. *All questions are compulsory.*
2. *Assume missing data suitably, if any.*

		K Level	COs	Marks
	SECTION-A (15 Marks) 5 Marks each			
1.	Show that a linear transformation $T: N \to N'$ is bounded iff T is continuous, where N and N' are Normed linear spaces.	K2	CO2	5
2.	If X is a complex inner product space and $\alpha, \beta, \gamma \in C$, then find $(x, \beta y + \gamma z)$, where (.) means inner product.	K3	CO3	5
3.	If X is a complex inner product space and $\alpha, \beta, \gamma \in C$, then show that $(x, \beta y - \gamma z) = \overline{\beta}(x, y) - \overline{\gamma}(x, z)$.	K2	CO2	5
	SECTION-B (40 Marks) 10 Marks each			
4.	Discuss that if N is a normed space and M is any finite dimensional subspace of N, then M is closed.	K4	CO4	10
5.	If N and N' are normed linear space and $T: N \to N'$, then prove that the following are equivalent. (a) $\|T\| = \sup sup \left\{ \frac{\|T(x)\|}{\|x\|} : x \in N,\ x \neq 0 \right\}$ (b) $\|T\| = \sup sup \{ \|T(x)\| : x \in N, \|x\| \leq 1 \}$ (c) $\|T\| = \sup sup \{ \|T(x)\| : x \in N, \|x\| = 1 \}$	K3	CO3	10
6.	Show that, If x *and* y are two vectors in a Hilbert space, then $\|(x + y)\|^2 + \|(x - y)\|^2 = 2\left(\|x\|^2 + \|y\|^2\right)$.	K3	CO3	10
7.	Show that $H\left(e^t - \pi\right) = \{0\ t < \log log\ \pi\ \ 1\ t > \log log\ \pi$	K4	CO4	10
	SECTION-C (45 Marks) 15 Marks each			
8.	Prove that an orthonormal set in a Hilbert space is linearly independent.	K3	CO3	15
9.	State and prove open mapping theorem.	K2	CO2	15
10	Apply the theory of norms to show that the linear spaces R^n is a normed space under the norm $\|x\| = \left(\sum_{i=1}^{n} \left| x_i \right|^2 \right)^{\frac{1}{2}}$, where $x = \left(x_1, x_2, \ldots x_n \right)$	K3	CO3	15