

School of Computing Science and Engineering**Bachelor of Technology in Computer Science and Engineering
Semester End Examination - Jul 2024****Duration : 180 Minutes
Max Marks : 100****Sem IV - C1UC422T - Mathematical Puzzles and Games**General Instructions*Answer to the specific question asked**Draw neat, labelled diagrams wherever necessary**Approved data hand books are allowed subject to verification by the Invigilator*

- 1) State Wilson's Theorem. K1(3)
- 2) Use Fermat's Theorem to evaluate $(13)^{247} \pmod{17}$. K2(4)
- 3) Identify the twin primes among the given pair of numbers: (1, 3), (3, 5), (13, 19), (1, 2) K2(6)
- 4) In a small village, there are 87 families, of which 52 families have at most 2 children. In a rural development programme, 20 families are to be chosen for assistance, of which at least 18 families must have at most 2 children. In how many ways can the choice be made? K3(6)
- 5) Find the general solution of the following system. (i) $x \equiv 5 \pmod{25}$
(ii) $x \equiv 32 \pmod{23}$ K3(6)
- 6) Use mathematical induction to prove the inequality $n! \geq 2^n$ for all positive integers $n \geq 4$. K3(9)
- 7) Find all x such that $x \equiv 3 \pmod{4}$ and $x \equiv 2 \pmod{7}$. K3(9)
- 8) The H.C.F. of two numbers is 23 and the other two factors of their L.C.M. are 13 and 14. Find the numbers. K4(8)
- 9) Apply the Chinese remainder theorem to solve $x \equiv 3 \pmod{9}$ and $x \equiv 7 \pmod{13}$. K4(12)
- 10) Using Wilson's Theorem to find the remainder when $225!$ is divided by 227. K5(10)
- 11) Consider a group of 73 people. Then which of the following is necessarily true: (i) At least 10 persons were born in the same month. (ii) At least 2 months should have same person born in. (iii) At least one month should have same person born in. (iv) At most 6 persons should born in every month. K5(15)

OR

By using Mathematical induction, Prove that $n(n+1)(n+2)(n+3)$ is divisible by 24. K5(15)

- 12) Find $\phi(n)$ when n is a prime number and ϕ denotes the Euler's Phi function. K6(12)

OR

Find the number of arrangements of the letters of the word INDEPENDENCE. In how many of these arrangements, (i) Do the words start with P. (ii) do all the vowels always occur together? (iii) do the vowels never occur together? (iv) do the words begin with I and end in P? K6(12)