GALGOTIAS UNIVERSITY

ADMISSION NUMBER

School of Computing Science and Engineering

Bachelor of Computer Applications
Semester End Examination - Jul 2024

Duration : 180 Minutes
Max Marks : 100

Sem III - C1UC323T - Linear Algebra

General Instructions
Answer to the specific question asked
Draw neat, labelled diagrams wherever necessary
Approved data hand books are allowed subject to verification by the Invigilator

1)	Find the value of the constant '*b*' so that the rank of the matrix $\begin{pmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ b & 13 & 10 \end{pmatrix}$ is 2.	K1(3)
2)	Let C be the set of all continuous real valued functions defined on $\mathbb{R}$ and let D be the set of all differentiable real valued functions defined on $\mathbb{R}$. Show that C and D are subspaces of F, the vector space of all real-valued functions defined on $\mathbb{R}$	K2(4)
3)	Let $A = CBC^{-1}$. Then show that $A^6 = CB^6C^{-1}$.	K2(6)
4)	Let T: $\mathbb{R}^2 \rightarrow \mathbb{R}^3$ be a linear transformation defined as T(x, y) = (x, x + y, y). Find the nullity of T.	K3(6)
5)	Consider the matrix $A = \begin{pmatrix} 2 & 3 \\ x & y \end{pmatrix}$. If the eigen values of A are 4 and 8, then find the values of x and y.	K3(6)
6)	Let $A = \begin{pmatrix} 2 & 8 \\ 1 & -1 \end{pmatrix}$. Compute (i) the characteristic polynomial of A, (ii) the eigenvalues of A, (iii) a basis for each eigen space of A and (iv) the algebraic and geometric multiplicity of each eigen value.	K3(9)
7)	Solve the system of equations : $x_1 + x_2 + x_3 = 1, 3x_1 + x_2 - 3x_3 = 5$, $x_1 - 2x_2 - 5x_3 = 10$ by Gauss elimination method.	K3(9)
8)	Let $A = \begin{pmatrix} 1 & 2 & 2 \\ -3 & -1 & 3 \\ 2 & 0 & 1 \end{pmatrix}$. Compute (i) the characteristic polynomial of A, (ii) the A^{-1} .	K4(8)
9)	Let $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & 3 \end{pmatrix}$. Compute (i) the characteristic polynomial of A, (ii) the eigenvalues of A, (iii) a basis for each eigen space of A and (iv) the algebraic and geometric multiplicity of each eigen value.	K4(12)
10)	Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 7 \\ 0 & 0 & -1 \end{pmatrix}$.	K5(10)

11) K5(15)

1. Let $A = \begin{pmatrix} 2 & -0.1 \\ 0 & 3 \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} \frac{1}{2} & a \\ 0 & b \end{pmatrix}$. Then prove that (a + b) $=\frac{7}{20}$.
2. Prove that the system of equations 4x + 2y = 7, 2x + y = 6 has no solution.
3. Prove that the polynomials 1, x, x^2 $span\ P_2(x)$.

OR

K5(15)

For the matrix A = $\begin{pmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{pmatrix}$, prove that $(A^{-1})^T = (A^T)^{-1} (ii) (A^{-1})^{-1} = A$.
(ii) Prove that the system $x + 2y + 3z = \lambda x, 3x + y + 2z = \lambda y, 2x + 3y + z = \lambda z$ has non-zero solution for only one real value and then find the solution.

12) K6(12)

Determine, whether $A = \begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix}$ is diagonalizable and if so, find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

OR

K6(12)

Let $W = \text{Linear span}(x_1, x_2)$, where $x_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ and $x_2 = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$. Construct an orthogonal basis for W.