GALGOTIAS UNIVERSITY

ADMISSION NUMBER

School of Engineering

B.TECH Electronics and Communication Engineering
Semester End Examination - Jun 2024

Duration : 180 Minutes
Max Marks : 100

Sem IV - C1UC420T - Probability and Stochastic Process

General Instructions
Answer to the specific question asked
Draw neat, labelled diagrams wherever necessary
Approved data hand books are allowed subject to verification by the Invigilator

1) The joint probability density function of X&Y is $\begin{cases} c(2x+y); & 0 \leq x \leq 2, 0 \leq y \leq 3 \\ 0; & else \end{cases}$ K1(3)

Then find the value of constant c and also find joint distribution function

2) Derive the system of steady state equations for a continuous time birth and death process and hence find the value of p0, where p0 is the steady state probability in initial state. K2(4)

3) A bank counter is currently served by two tellers. Customers entering the bank join a single queue and go to the next available teller when they reach the head of the line. On the average, the service time for a customer is 3 minutes, and 15 customers enter the bank per hour. Assuming that the arrivals process is Poisson and the service time is an exponential r.v., find the probability that a customer entering the bank will have to wait for service. K2(6)

4) In a university computer center, 80 jobs an hour are submitted on the average. Assuming that the computer service is modeled as an M/M/1 queueing system, what should the service rate be if the average turn around time(time at submission to time of getting job back) is to be less than 10 minutes? K3(6)

5) Consider a continuous Markov chain with two states S={0,1}. Assume the holding time parameters are given by λ0=λ1=λ. That is, the time that the chain spends in each state before going to the other state has an Exponential(λ) distribution. (a) Draw the state diagram of the embedded (jump) chain. (b) Find the transition matrix P(t). K3(6)

6) Let (Xn, n≥0) be a three state Markov Chain with transition K3(9) probability matrix

$$P = \begin{pmatrix} .75 & .25 & 0 \\ .25 & .50 & .25 \\ 0 & .75 & .25 \end{pmatrix}$$

With initial distribution $P[X_0 = i] = 1/3, i = 1, 2, 3$. Find $P(X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2)$.

7) A drive-in banking service is modeled as an M/M/1 queueing system with customer arrival rate of 2 per minute. It is desired to have fewer than 5 customers line up 99 percent of the time. How fast should the service rate be? K3(9)

8) Compute E[T], the expected time until the pattern h, h, h, t, h, h, h appears, when a coin that comes up heads with probability p and tails with probability q = 1 − p is continually flipped. K4(8)

9) The joint probability function of two discrete random variables X and Y is given by f(x, y) = c (2x+ y), where x and y can assume all integers such that 0 ≤ x ≤ 2, 0 ≤y ≤ 3, and f(x, y) = 0 otherwise. (a) Find the value of the constant c. (b) Find P (X≥ 1, Y ≤ 2). (c) Find P(X= 2, Y= 1). K4(12)

10) What is traffic intensity for M/M/1 queuing system? Consider a single server queuing system with Poisson input and exponential service times. Suppose the mean arrival rate is 3 calling units per hour, the expected service time is .25 hour. Calculate the traffic intensity of this system. K5(10)

11) Consider the density function K5(15)

$$f(x) = \begin{cases} k\sqrt{x}, & 0 < x < 1 \\ 0, & elsewhere \end{cases}$$

(a) Evaluate *k*. (b) Find CDF *F*(*x*) and use it to evaluate *P*(0.3 < *X* < 0.6).

OR

The daily amount of coffee, in liters, dispensed by a machine located in an airport lobby is a random variable X having a continuous uniform distribution with A = 7 and B = 10. Find the probability that on a given day the amount of coffee dispensed by this machine will be (a) at most 8.8 liters; (b) more than 7.4 liters but less than 9.5 liters; (c) at least 8.5 literss; (c) at least 8.5 liters K5(15)

12) Suppose that the probability of a dry day following a rainy day is 1/3 and probability of a rainy day following a dry day is1/2. Given that July 1 is a dry day find the probability that July 3 is a dry day. K6(12)

OR

Determine the closed set, probability of ultimate return to the states, periodicity of states, and mean recurrence time of the states of the Markov chain having the following transition matrix. Is the chain irreducible? K6(12)

$$\begin{bmatrix} 0 & 1 & 0 \\ \frac{3}{4} & 0 & \frac{1}{4} \\ 0 & 1 & 0 \end{bmatrix}$$