GALGOTIAS UNIVERSITY

ADMISSION NUMBER

School of Computing Science and Engineering

Bachelor of Computer Applications
Semester End Examination - Jun 2024

Duration : 180 Minutes
Max Marks : 100

Sem II - C1UC323T - Linear Algebra

General Instructions
Answer to the specific question asked
Draw neat, labelled diagrams wherever necessary
Approved data hand books are allowed subject to verification by the Invigilator

1) *Explain whether* $\mathbb{R}^2$ *is a subspace of* $\mathbb{R}^3$? K1(3)

2) Let C be the set of all continuous real valued functions defined on $\mathbb{R}$ and let D be the set of all differentiable real valued functions defined on $\mathbb{R}$. Show that C and D are subspaces of F, the vector space of all real-valued functions defined on $\mathbb{R}$. K2(4)

3) Verify the rank-nullity theorem for the linear transformation T: $\mathbb{R}^2$ → **R3**$\mathbb{R}^3$ *defined by T(x,y) = (x+y, x-y, y).* K2(6)

4) Classify the following matrices into singular and non-singular matrices: K3(6)

$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 \\ -3 & 6 \end{bmatrix}$

$C = \begin{bmatrix} 7 & 4 \\ 0 & 5 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ 2 & -1 \end{bmatrix}$

5) Consider the matrix $A = \begin{pmatrix} 2 & 3 \\ x & y \end{pmatrix}.$ If the eigen values of A are 4 and 8, then find the values of x and y. K3(6)

6) Let $A = \begin{pmatrix} 4 & -2 \\ -9 & 3 \end{pmatrix}$. Compute K3(9)

(*a*) the characteristic polynomial of A,
(b) the eigenvalues of A,
(c) a basis for each eigenspace of A and
(d) the algebraic and geometric multiplicity of each eigen value.

7) Let $A = \begin{pmatrix} 2 & 8 \\ 1 & -1 \end{pmatrix}$. Compute K3(9)

(*a*) the characteristic polynomial of A,
(b) the eigenvalues of A,

(c) a basis for each eigenspace of A and
(d) the algebraic and geometric multiplicity of each eigen value.

8) Let $A = \begin{pmatrix} 1 & 0 & 2 \\ 3 & -1 & 3 \\ 2 & 0 & 1 \end{pmatrix}$. Compute K4(8)
(*a*) the characteristic polynomial of A,
(b) the eigenvalues of A,
(c) a basis for each eigenspace of A and
(d) the algebraic and geometric multiplicity of each eigen value.

9) Let $A = \begin{pmatrix} 1 & 2 & 0 \\ -1 & -1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$. Compute K4(12)
(*a*) the characteristic polynomial of A,
(b) the eigenvalues of A,
(c) a basis for each eigenspace of A and
(d) the algebraic and geometric multiplicity of each eigen value.

10) Let the linear transformation T: $\mathbb{R}^3 \rightarrow \mathbb{R}^3$ be defined by T(x, y, z) = (x + y - z, x + y + z, y - z). Then find matrix of linear transformation T with respect to the ordered basis K5(10)
(a) B = {(0, 1, 0), (0, 0, 1),(1, 0, 0)} and
(b) B = {(1,0,0), (1,1,0), (1,1,1)}.

11) Find the values of λ and μ for which the system of linear equations: $x + 2y + z = 6,\ x + 4y + 3z = 10,\ x + 4y + \lambda z = \mu,$ has K5(15)
(a) a unique solution,
(b) infinite number of solutions,
(c) no solution.

OR

Solve the system of equations $x_1 + 3x_2 + 4x_3 = 4,\ -x_1 + 3x_2 + 2x_3 = 2,$ and $3x_1 + 9x_2 + 6x_3 = -6$ by Gauss elimination. Further, also verify that the obtained solution is correct. K5(15)

12) Determine whether $A = \begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix}$ is diagonalizable and if so, find an invertible matrix P and a diagonal matrix D such that . $P^{-1}AP = D$. K6(12)

OR

Determine whether $A = \begin{pmatrix} 1 & -1 \\ 3 & 2 \end{pmatrix}$ is diagonalizable and if so, find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$. K6(12)