GALGOTIAS UNIVERSITY

ADMISSION NUMBER

School of Basic Sciences

Bachelor of Science Honours in Physics
Semester End Examination - Jun 2024

Duration : 180 Minutes
Max Marks : 100

Sem II - C1UD201T - Mathematical Physics-II

General Instructions
Answer to the specific question asked
Draw neat, labelled diagrams wherever necessary
Approved data hand books are allowed subject to verification by the Invigilator

1) Explain what a Fourier series is and how it relates to periodic functions. K1 (3)

2) Form the PDE from $z = ax + by + a^2 + b^2$ K2 (4)

3) Find the ordinary points, Singular points, regular singular points, and irregular singular points of the differential equation: K2 (6)

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$$

4) Solve in series the differential equation K3 (6)

$$\frac{d^2y}{dx^2} + 4xy = 0$$

5) Evaluate $\int_0^2 x^2 (2-x)^3 dx$ K3 (6)

6) Evaluate $\int_0^\infty x^9 e^{-2x^2} dx$ K3 (9)

7) Prove the following relation $\left[J_{\frac{1}{2}}(x)\right]^2 + \left[J_{\frac{-1}{2}}(x)\right]^2 = \frac{2}{\pi x}$ K3 (9)

8) Form the PDE by eliminating arbitrary constants a and b from the relation $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ K4 (8)

9) Evaluate the integral $\int_0^{\frac{\pi}{2}} sin^2\theta cos^4\theta d\theta$ K4 (12)

10) Express $\int_0^1 x^m (1-x^p)^n dx$ in terms of gamma function and hence evaluate $\int_0^1 x^5 (1-x^3)^{10} dx$ K5 (10)

11) Find the Fourier series expansion of the periodic function f(x). K5 (15)

$f(x) = x + x^2, -\pi < x < \pi, f(x + 2\pi) = f(x)$

and hence deduce

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} + ... + \infty = \frac{\pi^2}{12}$$

OR

Given that the Fourier series expansion of K5 (15)

$f(x) = \left(\frac{\pi - x}{2}\right)^2, 0 \leq x \leq 2\pi, f(x + 2\pi) = f(x)$

Is defined as:

$$f(x) = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} (-1)^n \frac{cosnx}{n^2}$$

Hence deduce:

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + ... + \infty = \frac{\pi^2}{6}$$

12) Form the partial differential equation by eliminating arbitrary function $z = y^2 + f(\frac{1}{x} + logy)$ K6 (12)

OR

Evaluate $\int_0^{\infty} \frac{1}{1 + x^4} dx$ K6 (12)