ADMISSION NUMBER

School of Basic Sciences

Bachelor of Science Honours in Mathematics
Semester End Examination - Jun 2024

Duration : 180 Minutes
Max Marks : 100

Sem IV - C1UC404T - Algebra

General Instructions
Answer to the specific question asked
Draw neat, labelled diagrams wherever necessary
Approved data hand books are allowed subject to verification by the Invigilator

1) Find the rank of $\begin{bmatrix} -1 & 0 & 3 & 0 \\ 1 & 3 & 2 & 9 \\ -9 & 7 & -5 & 7 \end{bmatrix}$. K1(3)

2) Show that for any two $n \times n$ invertible matrices A and B, the matrix AB is also invertible. Also find the inverse. K2(4)

3) Show that $\mathrm{R} = \{((\mathrm{a}, \mathrm{b}), (\mathrm{c}, \mathrm{d})) \mid \mathrm{ad}(\mathrm{b} + \mathrm{c}) = \mathrm{bc}(\mathrm{a} + \mathrm{d})\}$ is an equivalence relation on $\mathbb{R} \times \mathbb{R}$ K2(6)

4) Solve the following system of equations: K3(6)
$x - 3y + z = 1,$
$2x + y - 4z = -1,$
$6x - 7y + 8z = 7.$

5) Solve the following system of equations: K3(6)
$x - 4y + 7z = 8,$
$3x + 8y - 2z = 6,$
$7x - 8y + 26z = 31.$

6) Solve the linear congruence $37x \equiv 21 \mod 72,$ if possible K3(9)

7) Find the characteristic polynomial, eigenvalues and eigenvectors of the matrix A = $\begin{bmatrix} 3 & -1 \\ 5 & 7 \end{bmatrix}$. K3(9)

8) Solve the equation $x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 = 0.$ K4(8)

9) Examine whether the matrix $\begin{bmatrix} 3 & 1 & -1 \\ -1 & 2 & 1 \\ 1 & 1 & 5 \end{bmatrix}$ is diagonalizable or not. Also find an invertible matrix P such that $P^{-1}AP = D.$ K4(12)

10) Solve the given system of equations by LU decomposition method K5(10)
$x + y + z = 1,$
$3x + y + 3z = 5,$
$x - 2y - 5z = 10.$

11) 1. State and prove Remainder theorem. K5(15)

2. Prove that every subset of a countable set is countable.

OR

1. Using Mathematical induction on n, prove that if a is an odd integer, then $a^{2^n} \equiv 1 (mod\ 2^{n+2})$ for any n ≥ 1. K5(15)
2. Prove that if $(a, b) = 1$ and $(ac, b) = d$, then d divides c.

12) Show that the only real value of λ, for which the system K6(12)

$$x + 2y + 3z = \lambda x,$$
$$3x + y + 2z = \lambda y,$$
$$2x + 3y + z = \lambda z$$

has non-zero solution is 6 and solve it for λ = 6.

OR

Find the value of 'k' for which the given set of equations has infinite solutions K6(12)

$$(3k - 8)x + 3y + 3z = 0,$$
$$3x + (3k - 8)y + 3z = 0,$$
$$3x + 3y + (3k - 8)z = 0.$$