GALGOTIAS UNIVERSITY

ADMISSION NUMBER

School of Basic Sciences

Bachelor of Science Honours in Mathematics
Semester End Examination - Jun 2024

Duration : 180 Minutes
Max Marks : 100

Sem II - C1UC203T - Real Analysis I

General Instructions
Answer to the specific question asked
Draw neat, labelled diagrams wherever necessary
Approved data hand books are allowed subject to verification by the Invigilator

1)	State Rolle's theorem and Cauchy's mean value theorem. Give an example that satisfies all conditions of Rolle's theorem.	K1(3)
2)	Define discontinuity of second kind with an example. Show that the function $f(x) = 2^{\frac{1}{x}}$ has discontinuity of second kind at 0.	K2(4)
3)	Check the convergence of the following series (i) $\sum_{n=1}^{\infty}\frac{1}{n}$ (iii) $\sum_{n=1}^{\infty}\frac{1}{n^3}$ (ii) $\sum_{n=1}^{\infty}\frac{1}{n^2}$ (iv) $\sum_{n=1}^{\infty}\frac{1}{n^3}$ (b) Check the function $f(x) = \frac{x^2-a^2}{x-a}$ $at\ x = a\ is\ continuous\ or\ not?$	K2(6)
4)	For what values of k the following function $f(x) = \begin{cases} kx^2, & x \le 2 \\ 2x+k, & x > 2 \end{cases}$ is continuous everywhere.	K3(6)
5)	Show that $\sin x$ is uniformly continuous on [0, ∞).	K3(6)
6)	Find the value of p for which the series $\sum_{n=1}^{\infty}\frac{1}{n\,(\log n)^p}$ is convergent	K3(9)
7)	Find the condition on r such that the positive term infinite series $1 + r^2 + r^3 + r^4 + \cdots + r^n + \cdots$ is convergent.	K3(9)
8)	Using Cauchy's criterion of convergence, test for convergence of the sequence: (i)$<1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ ...+\frac{1}{n}>$ (ii) $<(\frac{n^n}{n})>$.	K4(8)
9)	Show that the series $\sum_{n=0}^{\infty}\frac{2^n+5}{3^n}$ is convergent and its sum is 10.5 .	K4(12)
10)	If x and y are two given real numbers with x >0, then prove that there exists a natural number n such that $nx > y$.	K5(10)
11)	Prove that every uniformly continuous function on an interval is continuous on that interval, but the converse is not true.	K5(15)

OR

Prove that the series $\sum_{n=1}^{\infty}\frac{(-1)^n \log n}{n^2}$ is convergent. Is it absolutely convergent? K5(15)

12) For what value of a and b the function K6(12)

$$f(x)=\begin{cases} x^2+3x+a, if\ x\le 1 \\ bx+2, \quad if\ x>1 \end{cases}$$

is differentiable.

OR

Apply Cauchy's integral test to examine the convergence of the series $\sum_{n=1}^{\infty}\frac{1}{(n^2+1)}$. K6(12)