

School of Basic Sciences

**Bachelor of Science Honours in Mathematics
Semester End Examination - Jun 2024**

**Duration : 180 Minutes
Max Marks : 100**

Sem II - C1UC202B - Ordinary Differential equations and Mechanics

General Instructions

Answer to the specific question asked

Draw neat, labelled diagrams wherever necessary

Approved data hand books are allowed subject to verification by the Invigilator

- 1) Show that the solutions $y_1(x) = e^{2x}$, $y_2(x) = e^{3x}$ are linearly independent solutions of $D^2 - 5D + 6)y = 0$. K1(3)
- 2) A hemisphere rests in equilibrium on a sphere of equal radius. Show that the equilibrium is unstable when the curved surface of the hemisphere rests on sphere. K2(4)
- 3) Solve $x^2p^2 + xyp - 6y^2 = 0$. K2(6)
- 4) Solve $(dy/dx) + x\sin 2y = x^3\cos^2 y$ K3(6)
- 5) Apply the concept of central orbits to find the law of force to the pole if the path of particle is cardioid $r = a(1 + \cos\theta)$. K3(6)
- 6) Using method of undetermined coefficient to solve $\left(\frac{d^2y}{dx^2} - 2\frac{dy}{dx}\right)y = e^x \sin x$. K3(9)
- 7) Using method of undetermined coefficient to solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x - e^x$ K3(9)
- 8) Test whether or not the equation is exact $x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$ and hence solve the equation. K4(8)
- 9) Solve the system $2\frac{dx}{dt} - 2\frac{dy}{dt} - 3x = t, 2\frac{dx}{dt} + 2\frac{dy}{dt} + 3x + 8y = 2$ K4(12)
- 10) Solve $p^2x(x - 2) + p(2y - 2xy - x + 2) + y^2 + y = 0$ K5(10)
- 11) Solve the simultaneous differential equation $t\frac{dx}{dt} + y = 0, t\frac{dy}{dt} + x = 0$ given $x(1) = 1, y(-1) = 0$. K5(15)

OR

A system of forces given by (X,Y,Z;L,M,N) is replaced by two forces, one acting along the axis x and another force. Deduce that K5(15)

the magnitudes of the forces are $\frac{LX+MY+NZ}{L}$ and $\frac{[(MY+NZ)^2+L^2(Y^2+Z^2)]^{1/2}}{L}$ and also find the equation of the line of action of the other force.

12) Solve by the method of variation of parameters $\frac{d^2y}{dx^2} + 4y = 4\tan 2x$. K6(12)

OR

Solve by the method of variation of parameters K6(12)

$$(1-x)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = (1-x)^2$$