GALGOTIAS UNIVERSITY

ADMISSION NUMBER												

School of Basic Sciences

Master of Science in Mathematics
Semester End Examination - May 2024

Duration : 180 Minutes
Max Marks : 100

Sem IV - MSCM425 - Finite Element Method

General Instructions
Answer to the specific question asked
Draw neat, labelled diagrams wherever necessary
Approved data hand books are allowed subject to verification by the Invigilator

1) Explain the isoperimetric element terms. K1 (3)

2) Evaluate the integral $I = \int_{-1}^{1}(1 + x + x^2)dx$ by the help of Gauss quadrature formula. K2 (4)

3) Write down the jacobian matrix for four noded quadrilateral elements. K2 (6)

4) Consider the differential equation $\frac{d^2y}{dx^2} = -\cos \pi x, \quad 0 \le x \le 1$ K3 (6)
Subject to the following two sets of boundary conditions:
$u(0) = 0, \quad \left.\frac{du}{dx}\right|_{x=1} = 0.$
$\left.\frac{du}{dx}\right|_{x=0} = 0, \quad \left.\frac{du}{dx}\right|_{x=1} = 0.$
Determine a three- parameter solution, with trigonometric functions, using the least square method

5) Explain LST element. K3 (6)

6) Find a one-parameter approximate solution of the nonlinear equation K3 (9)

Subject to the boundary conditions and compare it with the exact solution Use (a) the Galerkin method

7) Discuss Lagrange's element for Linear and Quadratic case K3 (9)

8) Construct the matrix taking four quadratic element for the boundary value problem K4 (8)
$\frac{d^2y}{dt^2} + y = t^2 \; for \; 0 < t < 1$
satisfying
$y(0) = 1, \; y(1) = 0.$

9) For any quadrilateral element, calculate $\frac{\partial \psi_i}{\partial x}$ and $\frac{\partial \psi_i}{\partial y}$ at the point $\left(\frac{1}{3}, \frac{1}{3}\right)$ and $\left(\frac{1}{4}, \frac{1}{4}\right)$. K4 (12)

10) Discuss Nine-Node Quadrilateral element. K5 (10)

11) For the differential equation K5 (15)

$-\frac{1}{x}\left(\frac{d}{dx}\left[\frac{du}{dx}\right]\right) = 0 \quad \text{for } R_i < x < R_o$

(a) Construct weak form

(b) Find linear element and quadratic element

OR

Find linear and quadratic elements in one dimensional and solution of assembled system for K5 (15)

y"+P(x)y=Q(x) for 0<x<1

Satisfying

y(0)=1,y(1)=0

12) Derive the Lagrange cubic interpolation functions for a two node (one-dimensional) element using the alternative procedure based on interpolation properties. K6 (12)

OR

Solve the differential equation K6 (12)

$-\frac{d^2u}{dx^2} = \cos \pi x, \quad 0 < x < 1; \quad \left.\frac{du}{dx}\right|_{x=0} = 0, \quad \left.\frac{du}{dx}\right|_{x=1} = 0$

Use the uniform mesh of three linear elements to solve the problem and compare against the exact solution

$u(x) = \frac{\cos \pi x}{\pi^2}$