ADMISSION NUMBER

School of Basic Sciences

Master of Science in Mathematics
Semester End Examination - May 2024

Duration : 180 Minutes
Max Marks : 100

Sem IV - MSCM327 - Measure Theory

General Instructions
Answer to the specific question asked
Draw neat, labelled diagrams wherever necessary
Approved data hand books are allowed subject to verification by the Invigilator

1) Shows that every interval is measurable. K1 (3)

2) Show that union of finite collection of measurable sets is measurable. K2 (4)

3) Show that the product of complete measure spaces is not necessarily complete. K2 (6)

4) Show that every Lebesgue measurable is not necessarily borel set. K3 (6)

5) Define a complex measures. K3 (6)

6) How would you demonstrate that Lebesgue measure is invariant under all isometries of $\mathbb{R}^n$? K3 (9)

7) Show that every Borel set is Lebesgue measurable. K3 (9)

8) How can you describe $\lambda^*([a,b]) = b - a$. K4 (8)

9) Show that every signed measure is the difference of two positive measures, at least one of which is finite. K4 (12)

10) If $\{A_k\}_{k=1}^{\infty}$ is an infinite increasing sequence of measurable sets, then show that K5 (10)

$$m\left(\bigcup_{k=1}^{\infty} A_k\right) = \lim_{k\to\infty} m(A_k).$$

11) Show that every open cover of [0,1] has a finite subcover. K5 (15)

OR

Established the statement and proof of the bounded convergence theorem. K5 (15)

12) **Let $A \subseteq \mathbb{R}^n$ such that $\lambda^*(A) < \infty$. Then $A \in \mathrm{L}_0$ iff for every $\epsilon < 0$ there is a compact set K and open set G such that $K \subseteq A \subseteq G$ and $\lambda^*(G/A) < \epsilon$.** K6 (12)

OR

State and prove the Riesz-Fischer Theorem. K6 (12)