GALGOTIAS UNIVERSITY

ADMISSION NUMBER

School of Basic Sciences

Master of Science in Mathematics

Mid Term Examination - May 2024

Duration : 90 Minutes
Max Marks : 50

Sem II - C1PM201T - Functional Analysis

General Instructions
Answer to the specific question asked
Draw neat, labelled diagrams wherever necessary
Approved data hand books are allowed subject to verification by the Invigilator

1) State Zorn's lemma and give one example. K2 (2)

2) State complete metric space and give one example. K1 (3)

3) Define bounded linear transformation with example. K2 (4)

4) State and prove Holder's inequality for finite sequences. K2 (6)

5) Let T be a linear transformation of a Normed linear space N into another Normed linear space N', then prove that if T is continuous then it is continuous at the origin in the sense that $x_n \to 0 \Rightarrow T(x_n) \to 0$. K3 (6)

6) Norm is continuous in a normed linear space. K3 (9)

7) Show that the linear spaces $\mathbb{C}^n$ is a Banach space under the norm $\|x\| = (\sum_{i=1}^{n} |x_i|^2)^{\frac{1}{2}}$, where $x = (x_1, x_2, \ldots x_n)$. K4 (8)

8) The linear space $l_p, p > 1$ of all sequences $x = \{x_n\}$ for which $\sum_{n=1}^{\infty} |x_n|^p < \infty$, with the norm K4 (12)

$$\|x\| = \left(\sum_{i=1}^{n} |x_i|^p\right)^{\frac{1}{p}}$$

is a Banach Space.

OR

Prove that the linear space $C[a,b]$ of all continuous functions defined on $[a,b]$ is a Banach space with the norm $\|f\| = \sup\{|f(x)| : x \in [a,b]\}$. K4 (12)