

School of Basic Sciences

Master of Science in Mathematics
Mid Term Examination - Mar 2024

Duration : 90 Minutes
Max Marks : 50

Sem IV - MSCM425 - Finite Element Method

General Instructions

Answer to the specific question asked

Draw neat, labelled diagrams wherever necessary

Approved data hand books are allowed subject to verification by the Invigilator

- 1) Distinguish between essential boundary condition and natural boundary condition. K2 (2)

- 2) Give the comparison between finite element methods and finite difference methods. K1 (3)

- 3) Solve the following equations using Galerkin's method (Use at least a two-parameter solution) K2 (4)

$$\frac{dy}{dx} = 5(1 + \cos x) - 0.05y, \quad 0 \leq x \leq 2\pi, \quad y(0) = 20$$

- 4) Using the Rayleigh Ritz method, find the solution of K2 (6)

$$\frac{d}{dx} \left[\frac{du}{dx} \right] - u + x^2 = 0 \quad \text{for } 0 < x < 1$$
 subject to the boundary conditions

$$u(0) = 1, \quad \frac{du}{dx} = 0 \quad \text{at } x = 1.$$

- 5) Consider the differential equation $\frac{d^2y}{dx^2} = -\cos \pi x, \quad 0 \leq x \leq 1$ K3 (6)
 Subject to the following two sets of boundary conditions:

$$u(0) = 0, \quad \left. \frac{du}{dx} \right|_{x=1} = 0.$$

$$\left. \frac{du}{dx} \right|_{x=0} = 0, \quad \left. \frac{du}{dx} \right|_{x=1} = 0.$$
 Determine a solution, with trigonometric functions, using the Ritz method

- 6) Consider the differential equation $\frac{d^2y}{dx^2} = -\cos \pi x, \quad 0 \leq x \leq 1$ K3 (9)
 Subject to the following two sets of boundary conditions:

$$u(0) = 0, \quad \left. \frac{du}{dx} \right|_{x=1} = 0.$$

$$\left. \frac{du}{dx} \right|_{x=0} = 0, \quad \left. \frac{du}{dx} \right|_{x=1} = 0.$$
 Determine a three- parameter solution, with trigonometric functions, using the collocation at $x = \frac{1}{4}, \frac{1}{2}$ and $\frac{3}{4}$.

- 7) Discuss the assembly for linear, quadratic and cubic element in 1D. K4 (8)

- 8) Construct the matrix taking two quadratic element for the boundary value problem K4 (12)

$$\frac{d^2y}{dt^2} + y = t^2 \text{ for } 0 < t < 1 \text{ satisfying } y(0)=1, y(1)=0.$$

OR

1. Discuss basic steps of finite element analysis. K4 (12)
2. Discuss connectivity of elements with examples.
3. Develop the weak form and find the solution

$$\frac{d^2v}{dx^2} - q = 0$$

Subject to boundary condition

$$v(0) = 0, \frac{dv}{dx}(1) = 0.$$