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**School of Basic Sciences**

Master of Science in Mathematics  
Mid Term Examination - Mar 2024

Duration : 90 Minutes  
Max Marks : 50

**Sem IV - MSCM327 - Measure Theory**General Instructions

Answer to the specific question asked

Draw neat, labelled diagrams wherever necessary

Approved data hand books are allowed subject to verification by the Invigilator

- 1) Explain that empty set and whole set are measurable. K2 (2)
- 2) How would you identify outer measure of a set. K1 (3)
- 3) Give a counter example of measurable function that is not continuous also discuss the measurability of the provided function. K2 (4)
- 4) Every set of positive measure contains a nonmeasurable set. K2 (8)
- 5) Prove that the measure of Cantor set is zero. K3 (6)
- 6) Show that a real valued function with measurable domain is measurable if and only if inverse image of open set is measurable K3 (9)
- 7) Prove that the if A is subset B and  $\mu(A) < \infty$  then  $\mu(B - A) = \mu(B) - \mu(A)$ . K4 (8)
- 8) If  $f$  is integrable on  $[a, b]$  and  $\int_a^x f(t)dt = 0, \forall x \in [a, b]$ , then  $f(t) = 0$  for almost every  $t \in [a, b]$ . K4 (12)

**OR**

What criteria would you use to assess that when any property holds almost everywhere. K4 (12)