GALGOTIAS UNIVERSITY

ADMISSION NUMBER

School of Basic Sciences

Master of Science in Mathematics
Semester End Examination - Nov 2023

Duration : 180 Minutes
Max Marks : 100

Sem III - MSCM303 - Integral Equations and Calculus of Variation

General Instructions
Answer to the specific question asked
Draw neat, labelled diagrams wherever necessary
Approved data hand books are allowed subject to verification by the Invigilator

1) Define the Abel's integral equation. K1 (2)

2) Estimate an extremal and extremum value of the functional $[y(x)] = \int_0^2 (x - y')^2 dx$, K2 (4)

3) **Show that** $y = sinx - \frac{x}{2}cosx$ **is the required extremals of the functional** $I[y(x)]y = \int_0^{\pi/2} (y^2 - y'^2 - 2ysinx)dx, y(0) = 0, y(\pi/2) = 1$ K2 (6)

4) **Solve the differential equation:** K3 (9)
y''+y=x; y'(0)=1, y(2)=2

5) Solve the extremal of the functional $\int_0^1 (1 + y'^2 + z'^2)^{1/2} dx$ that satisfy the boundary conditions: $y(0) = 0, y(1) = 2, z(0) = 0, z(1) = 4.$ K3 (9)

6) Evaluate the extremal of the functional $I = \int_0^1 y'^2\, dx$,$y(0) = 0,\ y(1) = 1$ is extremum, subject to the condition $J = \int_0^1 y dx = 2$. K5 (10)

7) Analyze the extremal of the functional $\int_0^\pi (y'^2 - y^2) dx$ under the conditions $y(0) = 0, y(\pi) = 1$ and subject to the constraint $\int_0^\pi y dx = 1.$ K4 (12)

8) Obtain the solution of the boundary value problems by using Green's functions: $\frac{d^4y}{dx^4} = 1;$ $y(0) = y'(0) = y''(0) = y'''(0) = 0$ K5 (15)

9) Prove that $y(x) = e^{2x}$ is the solution of the integral equations: $y(x) = e^x + \int_0^x e^{x-t} y(t)dt$ K5 (15)

10) Discussed the Neumann series for the solution of the integral equation: K6 (18)
y(x)= $1 + x + \lambda \int_0^x (x - t)y(t)dt$