

School of Basic Sciences
Bachelor of Science Honours in Mathematics
Semester End Examination - Nov 2023

Duration : 180 Minutes
Max Marks : 100

Sem V - C1UC502T - Transforms and Calculus of variation

General Instructions

Answer to the specific question asked

Draw neat, labelled diagrams wherever necessary

Approved data hand books are allowed subject to verification by the Invigilator

- 1) Identify the definite integral $\int_{x_1}^{x_2} f(x, y, \frac{dy}{dx}) dx$ K1 (2)
- 2) Demonstrate the iterative kernels K2 (4)
- 3) Analyse the Fourier cosine transform of $\frac{1}{1+x^2}$. Hence write Fourier sine transform of $\frac{x}{1+x^2}$ K2 (6)
- 4) Find the extremal of the functionals $\int_{x_0}^{x_1} (x + y')y' dx$ K3 (9)
- 5) Find the curves on which the functional $\int_1^2 \frac{x^3}{y'^2} dx$ with $y(1)=0$ and $y(2)=3$ can be extremized. K3 (9)
- 6) Determine the Jacobi condition is fulfilled for the functional $\int_0^1 (1 + y'^2) dx$ with fixed boundaries A(0,0) and B(1,0). K5 (10)
- 7) Explain the Fourier cosine transform and Fourier sine transform of $f(t)$, where $f(t) = \begin{cases} 2 + t, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$ K4 (12)
- 8) Compute the (complex) Fourier transform of the function $F(x) = \begin{cases} 1 - x^2; & |x| \leq 1 \\ 0; & |x| > 1 \end{cases}$ and hence evaluate $\int_0^{\infty} \left(\frac{s \cos s - \sin s}{s^3} \right) \cos \frac{s}{2} ds$. K5 (15)
- 9) Find a function $y(x)$ for which $\int_0^1 (x^2 - y'^2) dx$ is stationary, given that $\int_0^1 y^2 dx = 2$, $y(0)=0$, $y(1)=0$ K5 (15)
- 10) Find the minimum time taken by the particle while moving along a curve. K6 (18)