

School of Engineering
B.TECH Mechanical Engineering
Mid Term Examination - Nov 2023

Duration : 90 Minutes
 Max Marks : 50

Sem III - C1UC321T - Mathematics-III Functions of Complex Variables and Transforms

General Instructions
 Answer to the specific question asked
 Draw neat, labelled diagrams wherever necessary
 Approved data hand books are allowed subject to verification by the Invigilator

- 1) Explain the types of singularities. K2 (2)
- 2) Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{2n}{n^2} (z - 3i)^n$. K1 (3)
- 3) Show that sum of the residues of $\frac{z^2}{(z+2)(z-1)^2}$ over all poles is 1. K2 (4)
- 4) By using Cauchy's Integral Formula, Show that :- K2 (6)

1. $\oint_C \frac{\sin^6 z}{(z-\pi/6)^3} dz = \frac{21\pi i}{16}$, where C is the circle $|z| = 1$

2. $\int_C \bar{z} dz = 2\pi i$, where C is the circle $|z| = 1$.

- 5) Solve the integral: $\oint_C \frac{dz}{(z^2+9)}$ where C is (a) $|z - 3i| = 4$ (b) $|z + 3i| = 2$. K3 (6)
- 6) Develop a Taylor series expansion of $f(z) = \frac{1}{z^2 - z - 6}$ about the points: (i) $z = -1$ (ii) $z = 1$. K3 (9)
- 7) Classify the nature of singularities of the following functions: K4 (8)

1. $\tan z$

2. $\frac{1}{z(1-z^2)}$

3. $\frac{e^{-z}}{(z-3)^4}$

- 8) Compute the integral $\int_0^{2\pi} \frac{d\theta}{3 + 2 \sin \theta}$ K4 (12)

OR

Examine the function $f(z) = \cosh z$ for analyticity and find $f'(z)$. K4 (12)