GALGOTIAS UNIVERSITY

ADMISSION NUMBER												

School of Basic Sciences

Bachelor of Science Honours in Mathematics
Mid Term Examination - Nov 2023

Duration : 90 Minutes
Max Marks : 50

Sem III - C1UC304T - Real Analysis

General Instructions
Answer to the specific question asked
Draw neat, labelled diagrams wherever necessary
Approved data hand books are allowed subject to verification by the Invigilator

1) Show that $\lim_{n\to\infty} 2^{-n}n^2 = 0$ K2 (2)

2) Solve: $\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{n^2}$. K1 (3)

3) Show that: (i) supremum of a non-empty set S of real numbers, whenever it exists, is unique. (ii) infimum of a non-empty set S of real numbers, whenever it exists, is unique. K2 (4)

4) Solve: $\lim_{n\to\infty} f_n, where\ f_n = \frac{1}{n}\{(n+1)(n+2)(n+3)\ldots(n+n)\}^{\frac{1}{n}}$ K2 (6)

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5) Differentiate limit point of a set and limit point of a sequence. Find limit points of the following sets: (i) closed interval [a, b] (ii) open interval (a, b). K3 (6)

6) If a sequence <f_n> converges to l, then every subsequences of < f_n >converges to l. K3 (9)

7) Using Cauchy's criterion of convergence, test for convergence of the sequence K4 (8)
$S_n = 1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \ldots + \frac{1}{2n-1}$.
Is it convergent?

8) Show that for any real numbers x and y $|x+y|^2 + |x-y|^2 = 2|x|^2 + 2|y|^2$ K4 (12)

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OR

Show that a sub-set of real numbers is closed iff it contains all its limit points. K4 (12)